Reach System 1 Kinematic and Dynamic Properties

Blueprint Lab

Last Update: September 2019

1 Kinematic Properties

Link	d (mm)	θ	a (mm)	α
0	46.2	$\theta_0 + \pi$	20	$\pi/2$
1	0	$\theta_1 - \theta_a$	150.71	π
2	0	$\theta_2 - \theta_a$	20	$-\pi/2$
3	-180	$\theta_3 + \pi/2$	0	$\pi/2$
4	0	$-\pi/2$	0	0

Table 1: Standard DH Parameters for R5M with $\theta_a = \tan^{-1}\left(\frac{145.3}{40}\right)$
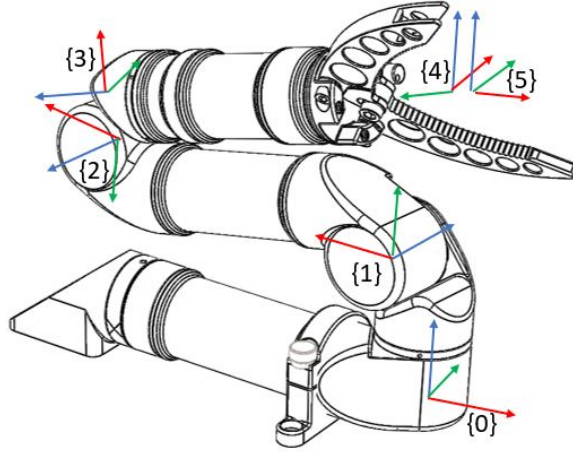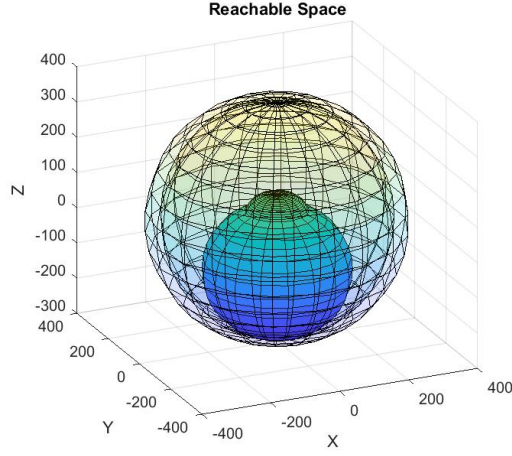

Figure 1: R5M joint frames (x,y,z)

Figure 2: Reachable workspace without self collision showing inner and outer limits

1.1 Workspace

The outer reachable limits form a torus

$$(\sqrt{x^2 + y^2} - a_0)^2 + (z - d_0)^2 \leq (a_1 + \sqrt{d_3^2 + a_2^2})^2 \tag{1}$$

The inner reachable limit is the torus

$$(\sqrt{x^2 + y^2} - a_0)^2 + (z - d_0)^2 \geq ((39.94 + a_2)^2 + (145.3 + d_3)^2) \tag{2}$$

The inner reachable limit with the arm in the downward position is the torus

$$(\sqrt{x^2 + y^2} - a_0)^2 + (z - d_0 + 145.3)^2 \geq (-d_3)^2 \tag{3}$$

These are shown in Figure 2.

1.2 Inverse Kinematics

From Figure 3 we can solve for the underarm solution

$$\theta_0 = tan2^1(\frac{y}{x}) + \pi \tag{4}$$

$$R = \sqrt{x^2 + y^2} \tag{5}$$

From Figure 4 we can solve for
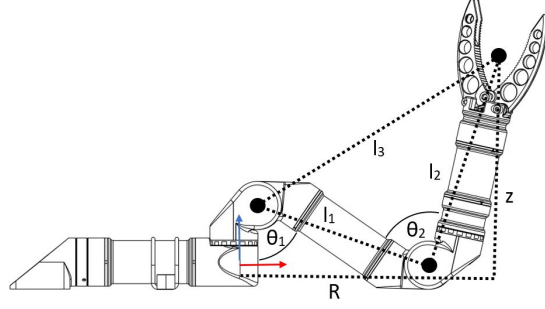
$$l_1 = a_1 \tag{6}$$

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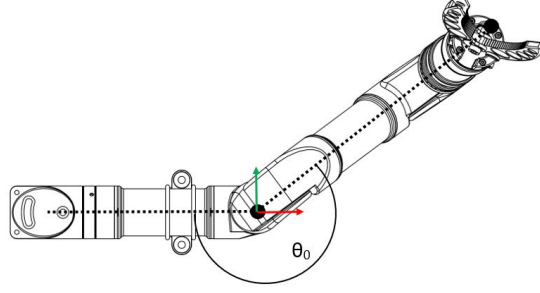
Figure 3: Calculation of θ_0

Figure 4: Calculation of θ_1 and θ_2

$$l_2 = \sqrt{a_2^2 + d_3^2} \tag{7}$$

$$l_3 = \sqrt{(R - a_0)^2 + (z - d_0)^2} \tag{8}$$

$$\theta_2 = cos^{-1}(\frac{l_1^2 + l_2^2 - l_3^2}{2l_1 l_2}) - sin^{-1}(\frac{2a_2}{l_1}) - sin^{-1}(\frac{a_2}{l_2}); \tag{9}$$

$$\theta_1 = \frac{\pi}{2} + tan2^{-1}(\frac{z - d_0}{R - a_0}) - cos^{-1}(\frac{l_1^2 + l_3^2 - l_2^2}{2l_1 l_3}) - sin^{-1}(\frac{2a_2}{l_1}) \tag{10}$$

Now we can also solve for the overarm solution from Figure 5

$$\theta_0 = tan2^1(\frac{y}{x}) \tag{11}$$

Finally from Figure 6

$$l_3 = \sqrt{(R + a_0)^2 + (z - d_0)^2} \tag{12}$$

$$\theta_2 = cos^{-1}(\frac{l_1^2 + l_2^2 - l_3^2}{2l_1 l_2}) - sin^{-1}(\frac{2a_2}{l_1}) - sin^{-1}(\frac{a_2}{l_2}); \tag{13}$$

$$\theta_1 = \frac{3\pi}{2} - tan2^{-1}(\frac{z - d_0}{R + a_0}) - cos^{-1}(\frac{l_1^2 + l_3^2 - l_2^2}{2l_1 l_3}) - sin^{-1}(\frac{2a_2}{l_1}) \tag{14}$$

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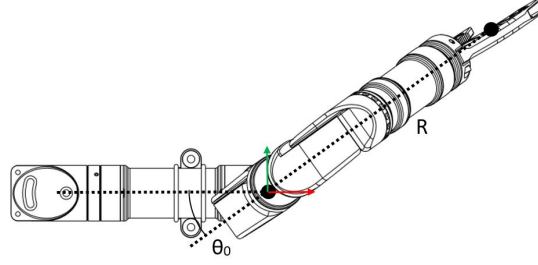
Figure 5: Calculation of θ_1 and θ_2
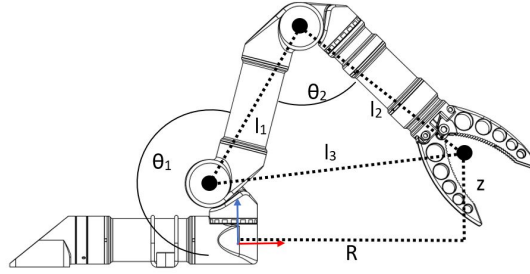
Figure 6: Calculation of θ_0

2 Inertial Properties

Link	Mass (kg)	COM (mm)	I $(kg.mm^2)$		
0	0.341	$(\ -75 \quad -6 \quad -3\ )$	$\begin{pmatrix} 99 & 139 & 115 \\ 139 & 2920 & 3 \\ 115 & 3 & 2934 \end{pmatrix}$		
1	0.194	$(\ 5 \quad -1 \quad 16\ )$	$\begin{pmatrix} 189 & 5 & 54 \\ 5 & 213 & 3 \\ 54 & 3 & 67 \end{pmatrix}$		
2	0.429	$(\ 73 \quad 0 \quad 0\ )$	$\begin{pmatrix} 87 & -76 & -10 \\ -76 & 3190 & 0 \\ -10 & 0 & 3213 \end{pmatrix}$		
3	0.115	$(\ 17 \quad -26 \quad -2\ )$	$\begin{pmatrix} 120 & -61 & -1 \\ -61 & 62 & 0 \\ -1 & 0 & 156 \end{pmatrix}$		
4	0.333	$(\ 0 \quad 3 \quad -98\ )$	$\begin{pmatrix} 3709 & 2 & -4 \\ 2 & 3734 & 0 \\ -4 & 0 & 79 \end{pmatrix}$		

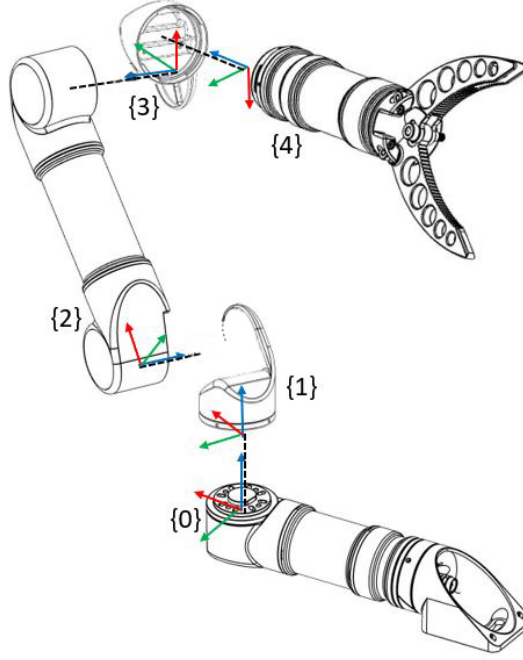Table 2: Inertial properties for R5M

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Figure 7: Inertial frames for each link (x,y,z)

3 Hydrodynamic Properties

3.1 Added Mass

We use standard equations for added mass assuming cylindrical sections with spherical ends.

Link	$(X_{\dot{u}}, Y_{\dot{v}}, Z_{\dot{w}})(kg)$			$(K_{\dot{p}}, M_{\dot{q}}, N_{\dot{r}})(kg.mm^2)$		
0	0.017ρ	0.189ρ	0.189ρ	0	1414ρ	1414ρ
1	0.032ρ	0.032ρ	0.017ρ	7ρ	7ρ	0
2	0.017ρ	0.201ρ	0.201ρ	0	1716ρ	1716ρ
3	0.032ρ	0.017ρ	0.032ρ	7ρ	0	7ρ
4	0.226ρ	0.226ρ	0.017ρ	2443ρ	2443ρ	0

Table 3: Added mass terms where $\rho \sim 1$ is the density in kg/L

3.2 Viscous Damping

We consider only quadratic drag assuming Reynolds numbers above the Stokes flow approximation at any significant velocity. We also consider only transla-

tional drag. We approximate the Reynolds number as

$$Re = \frac{\rho u L}{\mu} \sim \frac{10^3 * 0.1 * 0.1}{10^{-3}} \sim 10^{-4} \tag{15}$$

where ρ is the fluid density, u is the relative velocity, L is the characteristic length and μ is the dynamic viscosity, which gives a drag coefficient of $c_d \sim 0.5$. We now calculate the quadratic drag using

$$F_d = \frac{1}{2} \rho u^2 c_d A \tag{16}$$

where A is the cross sectional area.

The centre of drag is assumed to coincide with the centre of buoyancy

Link	$(X_{u|u|}, Y_{v|v|}, Z_{w|w|})$ $(N/\sqrt{ms^{-1}})$
0	$(\ 0.3\rho \quad 1.5\rho \quad 1.5\rho\ )$
1	$(\ 0.26\rho \quad 0.26\rho \quad 0.3\rho\ )$
2	$(\ 0.3\rho \quad 1.6\rho \quad 1.6\rho\ )$
3	$(\ 0.26\rho \quad 0.3\rho \quad 0.26\rho\ )$
4	$(\ 1.8\rho \quad 1.8\rho \quad 0.3\rho\ )$

Table 4: Drag terms where $\rho \sim 1$ is the density in kg/L

3.3   Buoyancy

Link	Volume (L)	COB (mm)
0	0.202	$(\ -75 \quad -6 \quad -3\ )$
1	0.018	$(\ -1 \quad -3 \quad 32\ )$
2	0.203	$(\ 73 \quad 0 \quad -2\ )$
3	0.025	$(\ 3 \quad 1 \quad -17\ )$
4	0.155	$(\ 0 \quad 3 \quad -98\ )$

Table 5: Buoyancy terms with Centre of Buoyancy (COB)
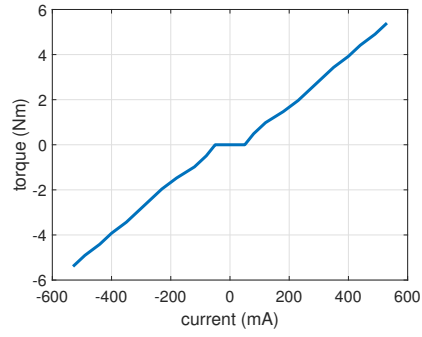
4   Actuator Properties

$$torque = 90.6 * (current \pm 43.0) \tag{17}$$

Figure 8: Plot of torque vs current for high torque joint