On Fair Allocation of Mixed Goods

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Preface

Abstract

In the world of fair allocation of indivisible items and fair division of divisible items it might be wise to look at the combination of these two field. This will be the fair allocation and division of mixed goods.

Contribution

Contents

List of Figures

List of Tables

List of Listings

Chapter 1

Introduction

1.1 Preliminaries

Collection of terminology, notation and definitions that are presumed to be known for the rest of the report.

Mixed Goods

When discussion mixed goods we are talking about a mix of both indivisible and divisible goods (often referred to as *cake*). A mixed goods instance can contain any combination of number of divisible and indivisible goods. A good means that all items will have a positive utility or value for all agents.

Problem Instance

An instance $\langle N, M, V, F \rangle$ of the fair division problem is defined by a:

- set N of $n \in N$ agents
- set M of $m \in M$ indivisible items
- set V of $v \in V$ valuation functions where v_i is valuation function for agent i
- density function F that divides divisible items

a set M of $m \in N$ indivisible items, and a valuation profile $V = v_1, v_2, ..., v_n$ that specifies the preferences of every agent $i \in N$ over each subset of the items in M via a valuation function $v_i : 2M \to R$.

Additive valuations

A well-studied subclass of monotone valuations is that of additive valuations, wherein an agent's value of any subset of items is equal to the sum of the values of individual items in the set, i.e., for any agent $i \in N$ and any set of items $S \subseteq M$, $v_i(S) := j \in Svi(j)$, where we assume that $v_i(\emptyset) = 0$. For simplicity, we will write $v_i(j)$ or $v_{i,j}$ to denote $v_i(j)$.

Bundle

The collection of items that an agent receives is called a bundle. A bundle is a subset of the items in M.

Allocation

An allocation $A := (A1, ..., An)$ is an n-partition of a subset of the set of items M, where $Ai \subseteq M$ is the *bundle* allocated to the agent i (note that A_i can be empty $\emptyset$). An allocation is said to be complete if it assigns all items in M, and is called partial otherwise.

Top-trading envy graph

The top-trading envy graph T_A of an allocation A is a subgraph of its envy graph G_A with a directed edge from agent i to agent k if $v_i(A_k) = max_{j \in N} v_i(A_j)$ and $v_i(A_k) > v_i(A_i)$, i.e., if agent i envies agent k and A_k is the most preferred bundle for agent i.

Envy Graph

The envy graph GA of an allocation A is a directed graph on the vertex set N with a directed edge from agent i to agent k if $vi(Ak) > vi(Ai)$, i.e., if agent i prefers the bundle Akover the bundle Ai.

Envy-freeness and its relaxations

EF

An allocation A is said to be envy-free (EF) if for every pair of agents $i, k \in N$, we have $v_i(A_i) \geq v_i(A_k)$,

EF1

An allocation A is said to be envy-free up to one item (EF1) if for every pair of agents $i, k \in N$ such that $A_i \cap A_k \backslash = \emptyset$, there exists an item $j \in A_i \cup A_k$ such that $v_i(A_i \backslash j) \geq v_i(A_k \backslash j)$.

Envy-freeness for mixed goods (EFM)
An allocation A is said to satisfy envy-freeness for mixed goods (EFM) if for any agents $i, j \in N$,

- if agent j's bundle consists of only indivisible goods, there exists $g \in A_j$ such that $u_i(A_i) \geq u_i(A_j \backslash g)$;
- otherwise, $u_i(A_i) \geq u_i(A_j)$.

It is easy to see that when the goods are all divisible, EFM reduces to EF; when goods are all indivisible, EFM reduces to EF1. Therefore EFM is a natural generalization of both EF and EF1 to the mixed goods setting.

1.2 Previous Work

Lots of articles that has discussed mixed goods/resources and their solutions etc.

1.2.1 Mixed Goods

[2] and [1] [4]

1.2.2 Mixed Resources

[3]

Chapter 2

Method

See my nice algorithm at Algorithm 1

Algorithm 1 Baseline

Require: $n \geq 0$
Ensure:
  $X \leftarrow x$
  $N \leftarrow n$
  while $N \neq 0$ **do**
    if N is even **then**
      $X \leftarrow X \times X$
      $N \leftarrow \frac{N}{2}$ ▷ This is a comment
    else if N is odd **then**
      $y \leftarrow y \times X$
      $N \leftarrow N - 1$
    end if
  end while

Chapter 3

Results

Chapter 4

Conclusions

Bibliography

[1] Xiaohui Bei, Shengxin Liu, Xinhang Lu, Jinyan Liu, and Zihao Li. *Fair Division of Mixed Divisible and Indivisible Goods.* Unknown, January 2021.

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[3] Umang Bhaskar, A. R. Sricharan, and Rohit Vaish. *On Approximate Envy-Freeness for Indivisible Chores and Mixed Resources.* Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2021.

[4] Rucha Kulkarni, Ruta Mehta, and Setareh Taki. *Indivisible Mixed Manna: On the Computability of MMS+PO Allocations.* Unknown, April 2021.