

Fractal dynamics of human gait: a reassessment of the 1996 data of Hausdorff et al.

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Delignières D, Torre K. Fractal dynamics of human gait: a reassessment of the 1996 data of Hausdorff et al. *J Appl Physiol* 106: 1272–1279, 2009. First published February 19, 2009; doi:10.1152/jappphysiol.90757.2008.—We propose in this paper a reassessment of the original data of Hausdorff et al. (Hausdorff JM, Purdon PL, Peng C-K, Ladin Z, Wei JY, Goldberger AR. *J Appl Physiol* 80: 1448–1457, 1996). We confirm, using autoregressive fractionally integrated moving average modeling, the presence of genuine fractal correlations in stride interval series in self-paced conditions. In contrast with the conclusions of the authors, we show that correlations did not disappear in metronomic conditions. The series of stride intervals presented antipersistent correlations, and $1/f$ fluctuations were evidenced in the asynchronies to the metronome. We show that the super central pattern generator model (West B, Scafetta N. *Phys Rev E Stat Nonlin Soft Matter Phys* 67: 051917, 2003) allows accounting for the experimentally observed correlations in both self-paced and metronomic conditions, by the simple setting of the coupling strength parameter. We conclude that $1/f$ fluctuations in gait are not overridden by supraspinal influences when walking is paced by a metronome. The source of $1/f$ noise is still at work in this condition, but expressed differently under the influence of a continuous coupling process.

gait dynamics; $1/f$ fluctuations; metronomic driving; modeling

IN A SERIES OF PAPERS PUBLISHED some years ago in the *Journal of Applied Physiology*, Hausdorff and collaborators suggested that successive step durations during walking presented a typical structure over time, characterized by the presence of long-range dependence, or fractal correlations (14–17). This fractal organization was conceived by the authors as an essential characteristic of normal walking, determining the adaptability and the flexibility of locomotion. They showed that these fractal properties were significantly altered in aged participants and in patients suffering from Huntington's disease (14). In those cases, the fractal organization tended to disappear, and step dynamics became more random.

The presence of long-range fractal dependence in physiological or psychological time series is a common result. Fractals have been discovered, for example, in heartbeat series (20), in the time intervals produced in finger tapping (6, 11, 28), in serial reaction time (12, 29), or in relative phase in a bimanual coordination task (26). Evidencing fractal properties in the series produced by biological systems has important theoretical consequences. Fractal signals are considered the natural outcome of complex self-organized systems. Such systems possess a kind of intrinsic stability, emerging from a subtle cooperation between their many components. One of the most

classically considered signature of fractal processes is the presence of a scaling relationship, in the spectral domain, between power and frequency (f). In the special case of $1/f$ noise, power is roughly proportional to the inverse of frequency. This spectral signature suggests that the signal cannot be considered as generated by a dominant or a limited set of dominant subsystems, but is the result of the cooperation of a number of components acting at different time scales (23). The maintenance of a steady state in the system cannot then be considered as the result of a single feedback loop, controlled by a specific instance, but as an emergent property of the whole system. A homeostatic control, for example, is supposed to be revealed in the structure of the produced signals by the presence of short-term dependence, the actual observation being only influenced by the preceding one, or by a limited set of preceding values. These short-term autocorrelations can be easily accounted for by autoregressive or moving average processes, but the resulting signals do not possess fractal properties.

Nevertheless, the presence of long-range dependence in step duration remains a questionable statement. This conclusion was more suggested by Hausdorff and collaborators than effectively (and statistically) proven. The identification of fractal properties with classical methods is only based on the visual inspection of the log-log power spectrum or of the diffusion plot obtained with temporal methods, such as detrended fluctuation analysis (DFA). Generally, authors are satisfied with obtaining an approximate linear fit to their data, which is supposed to reveal the typical scale invariance of fractal processes. Nevertheless, this visual evaluation remains qualitative, and there is no statistical test for judging if a regression line is appropriate or not (30). Moreover, short-term dependence processes can sometimes mimic the spectrum or the diffusion plot of a fractal series (21, 27, 30). Wagenmakers et al. (30) proposed a number of examples of such ambiguous results obtained with short-range dependence processes.

Hausdorff and coworkers (16) applied a surrogate data test, which was supposed to provide statistical evidence for the presence of fractal correlations in the series. This method consists of randomly shuffling data sets and estimating the fractal exponents of the obtained series. This procedure is supposed to remove the influence of sequential ordering, and the surrogate data sets are expected to present uncorrelated structures. An inferential test is then applied to compare the exponents obtained from the original series and from the surrogate data sets. As can be seen, the null hypothesis that is

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tested in this procedure is the absence of correlation in the series. This null hypothesis is surely not the most relevant, as the absence of correlation (purely white noise) in biological or psychological time series should be considered more as an exception than as the rule (24). The fundamental question is about the nature (short term vs. long term) of dependencies in the series (30). Clearly, classical methods are unable to adequately answer this question. Spectral analysis or DFA could be relevant for quantifying long-range dependence, when long-range dependence is supposed to be present a priori, but they are unable per se to validate such a presence. Theiler et al. (25) proposed a second family of surrogate methods testing the null hypothesis of short-range dependence. In a first proposition, surrogate data are composed of autoregressive moving average (ARMA) processes. An alternative proposition involves randomizing the phases of a Fourier transform. The efficiency of these tests, however, depends on the capability of the discriminating statistics to distinguish between surrogate and experimental series. As stated previously, classical fractal analysis methods, such as DFA or spectral analysis, seem unable to unambiguously discriminate these series (27, 30).

Wagenmakers et al. (30) and Torre et al. (27) proposed an inferential test for the presence of long-range dependence in time series, based on ARFIMA (autoregressive fractionally integrated moving average) modeling. The first aim of the present paper was to apply this new method for testing for the effective presence of long-range dependence in stride interval series.

Another interesting (and intriguing) result, presented by Hausdorff et al. (16), was the extinction of correlations in stride interval series when walking was driven by an auditory metronome. In this condition, fluctuations in the series could not be distinguished from white noise, suggesting that successive stride intervals became uncorrelated. This result could have important theoretical consequences. As argued by Hausdorff et al., an extinguishing of correlations in metronomic conditions could be interpreted as a complete overriding of the locomotor system's dynamics by supraspinal influences. In other words, the original complexity of the system, revealed by the typical fractal fluctuations of stride intervals in self-paced walking, is supposed to be entirely controlled by high-level, cognitive processes. However, Hausdorff et al. remained rather allusive concerning this result, just arguing that "there was little visually appreciable difference between the randomly shuffled time series and the original time series during metronomic walking." Recent theoretical advances on timing processes suggest the necessity of a deeper analysis of these data before any definitive conclusion.

Self-paced and metronomically driven cyclical movements have been studied in a number of experiments, especially in finger tapping (see, for example, Refs. 2, 28). These two conditions are referred to as "continuation" (a metronome gives the initial tempo for a short period, and then the metronome is removed, and participants have to continue to tap as regularly as possible, following the prescribe tempo) and "synchronization" (participants have to synchronize their taps with the beeps given by the metronome) paradigms. In continuation, a number of studies have shown that the series of intertap intervals present fractal properties close to $1/f$ noise (2, 11, 19, 28). In synchronization, intertap interval

series present a completely different temporal structure, characterized by the presence of antipersistent (negative) correlation. In contrast, the series of asynchronies (i.e., errors to the metronome) present a fractal structure close to $1/f$ noise (2, 28). The most important to note at this level is that using the same paradigm (self-paced vs. metronomic conditions) and obtaining the same results in self-paced condition ($1/f$ fluctuations) tapping experiments did not evidence the extinguishing of correlations reported by Hausdorff et al. (16) in the metronomic condition.

Torre and Delignières (28) suggested that the long-range dependencies observed in intertap intervals in self-paced tapping and in asynchronies in synchronized tapping could share the same origin. They argued that the presence of long-range dependence in intertap intervals in self-paced tapping is determined by a central timekeeper possessing fractal properties (6). In the synchronization condition, this fractal timekeeper is still at work, but an autoregressive correction process controls the discrepancy between the periods produced by the timekeeper and those imposed by the metronome. The authors showed that these simple assumptions allowed accounting for the serial dependence patterns observed in both self-paced and synchronization conditions (28). In that line of thought, the system does not lose complexity when driven by an external metronomic signal. The original source of long-range dependence is still present, but expresses differently due to the influence of a corrective process that tends to reduce asynchronies.

A second important aim of the present paper will then be to check whether series collected in metronomic conditions actually exhibit random fluctuations, or possess correlation patterns similar to those observed in synchronized tapping. We propose to complete the original analyses of Hausdorff et al. (16) by the analysis of asynchrony series, which were not considered by the authors.

Finally, we checked whether the Super Central Pattern Generator (SCPG) model proposed by West and Scafetta (32) is able to account for the serial correlation patterns evidenced in all conditions. We choose this model because it contains the essential ingredients that were included in tapping models: 1) a source of fractal fluctuation that expresses in all conditions, and 2) a forcing process representing metronome pacing.

METHOD

We analyzed in the present paper the original series of the study by Hausdorff et al. (16) (available at <http://www.physionet.org/physiobank/database/umwdb/>). Note that these series were already used in a number of subsequent papers (4, 13, 22, 32). These data were collected from 10 healthy young men (mean age 21.7 yr, range 18–29 yr). In a first stage, participants were tested in three free-walking conditions: 1) 1 h at a self-selected or usual pace, 2) 1 h at a rate slower than normal, and 3) 1 h at a rate faster than normal. Then participants performed three additional 0.5-h trials, during which they had to walk in time with a metronome that was set to each participant's mean stride interval computed from each of the three previous 1-h walks. Stride interval was measured by using force-sensitive switches taped inside one shoe. Note that, in the metronomic walking trials, the metronome was set to run twice as fast as the participant's mean stride interval from the free walk, and the participant was instructed to make each (left and right) heel strike coincident with the sounding of the metronome. Further details concerning the experimental protocol can be seen in Hausdorff et al. (16).

The data provided by the authors are the individual series of stride intervals collected from each participant in the six experimental conditions. To test our second hypothesis, we tried to derive from the metronomic walking stride interval series the corresponding series of asynchronies. Asynchronies (A_i) can be considered the mathematical integration of stride intervals (I_i), as:

$$A_i = A_0 + \sum_{k=1}^i (I_{k-1} - T) \quad (1)$$

where T represents the period of the metronome. We set T to the mean stride interval in the corresponding free-walking condition. A_0 was arbitrarily set to zero, inducing a constant error in the obtained asynchrony series. This constant error is supposed to have no influence on the temporal structure of the obtained series. Note that, in some occasions, the obtained series presented some abrupt shifts, indicating a local disruption between walking and metronome (see Fig. 1). Generally, these shifts had amplitude corresponding to $T/2$, suggesting that walking was rephased very quickly with the metronome (within one stride). These shifts appeared in three series at low speed, four at normal speed, and four at fast speed. Within these series, shifts occurred occasionally (from 1 to 8 shifts, with a mean of 2.8, for a mean series length of $\sim 1,610$ points). We corrected these shifts by subtracting or adding $T/2$ to the following values. This procedure restored stationarity, without significantly altering the correlation properties of the series. We present in Fig. 2 three examples of series of free-walking stride durations, metronomic-walking stride duration, and asynchronies, for a representative participant.

Data processing. To statistically attest for the presence of long-range dependence in series, we first applied the ARMA/ARFIMA modeling procedure (27, 30). This method consists in fitting 18 models to the studied series. Nine of these models are ARMA (p, q) models, with p and q varying systematically from 0 to 2. In this notation, p and q represent the orders of the autoregressive and the moving average processes, respectively. These ARMA models contain only short-term dependence and represent the null hypothesis. The other nine models are the corresponding ARFIMA (p, d, q) models, differing from the previous ARMA models by the inclusion of the fractional parameter d . This fractional parameter is supposed to be necessary for accounting for long-term dependence, if present in the series (27, 30). When positive, d reveals the presence of persistent correlation typical of $1/f$ noise. When negative, it characterizes the series as antipersistent fractional Gaussian noise. The method tests each model using the Bayes Information Criterion, a goodness-of-fit statistic that is based on a trade-off between accuracy and parsimony: the best model is the one that gives a good account of the data with a minimum number of free parameters. The raw Bayes Information Criterion values are transformed into

weights, which can be conceived as the probability for the i th model to be the best model, given the data and the set of candidate models (for details, see Ref. 27). Note that the weights computed among a given set of models sum to one, allowing a comparison of the relative likelihood of each model.

On the basis of these weights, two criteria have been proposed for detecting the presence of long-range dependence in a set of series (27): 1) the best model (i.e., the model with the highest weight) should be an ARFIMA (p, d, q) for at least 90% of series, with d being significantly different from 0; and 2) the sum of the weights of the nine ARFIMA models should be at least 0.90 (i.e., ARFIMA models capture 90% of the overall likelihood). These criteria and their associated thresholds have been established on the basis of the analysis of series of simulated fractal and nonfractal series (27). Note that this procedure was conceived for working on a set of series representing a given condition, rather than on a single series.

The models' fitting was conducted using the ARFIMA package (8) for the matrix computing language Ox (7). We used, with some minor adaptations, the Ox code provided by Simon Farrell (available at the following web address: <http://eis.bristol.ac.uk/~pssaf/>).

In a second step, we applied the DFA (20) to estimate the fractal exponent of the series. This method is based on the analysis of the relationship between the mean magnitude of fluctuations in the series and the length of the intervals over which these fluctuations are observed. The algorithm of DFA consists first in integrating the series $x(t)$, calculating for every t the cumulated sum of the deviations of the mean:

$$X(i) = \sum_{t=1}^i [x(t) - \bar{x}] \quad \text{for } i = 1, 2, 3, \dots, N \quad (2)$$

where N corresponds to series length. This integrated series is then divided in nonoverlapping intervals of length n . In each interval, a least squares line is fit to the data (representing the trend in the interval). The $X(t)$ series is then locally detrended by subtracting to all values the theoretical value $X_{th}(t)$ given by the regression. For each interval length n , the characteristic magnitude of fluctuation $F(n)$ is calculated by:

$$F(n) = \sqrt{\frac{1}{N} \sum_{k=1}^N [X(k) - X_{th}(k)]^2} \quad (3)$$

As suggested by Hausdorff et al. (16), we considered interval lengths ranging from $n = 6$ to $n = 600$. For fractal series, a power law is expected, as

$$F(n) \propto n^\alpha \quad (4)$$

where α is the scaling exponent; α is estimated by the slope of the graph representing $F(n)$ as a function of n , in log-log coordinates. To avoid possible biases induced by the logarithmic distribution of points, we divided the $\log(n)$ series into 10 intervals of equal length and computed an averaged point within each interval. We then estimated the regression slope over these 10 points. The α is supposed to equal 0 for white noise, 1.5 for ordinary Brownian motion, and 1 for $1/f$ noise. To detect the effect of walking rate on scaling exponents, we applied one-way ANOVAs with repeated measures.

In a third step, we performed a spectral analysis of series, to obtain a complete characterization of the signal in the frequency domain. We applied the $^{low}PSD_{we}$ algorithm, which includes some preprocessing operations before the application of the fast Fourier transform (for details, see Refs. 5, 9). We considered the bilogarithmic representation of the power spectrum. Note that, in this graphical representation, fractal processes are revealed by a linear, negative slope in the low-frequency region of the spectrum. Uncorrelated (white) noise is revealed by a flat spectrum, and negative correlations by a positive

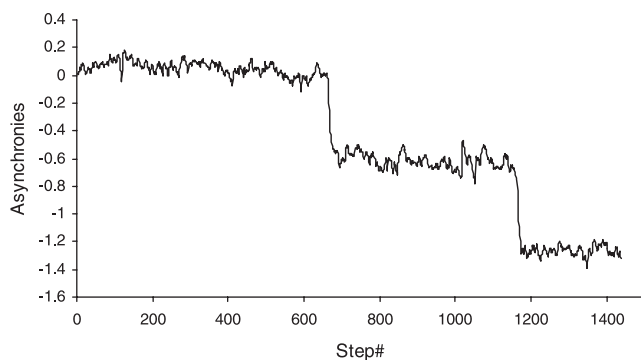


Fig. 1. An example of reconstructed asynchronies series. Two shifts can be discerned: the first one at step 654, and the second at step 1,153. Data are from participant 8 at normal speed.

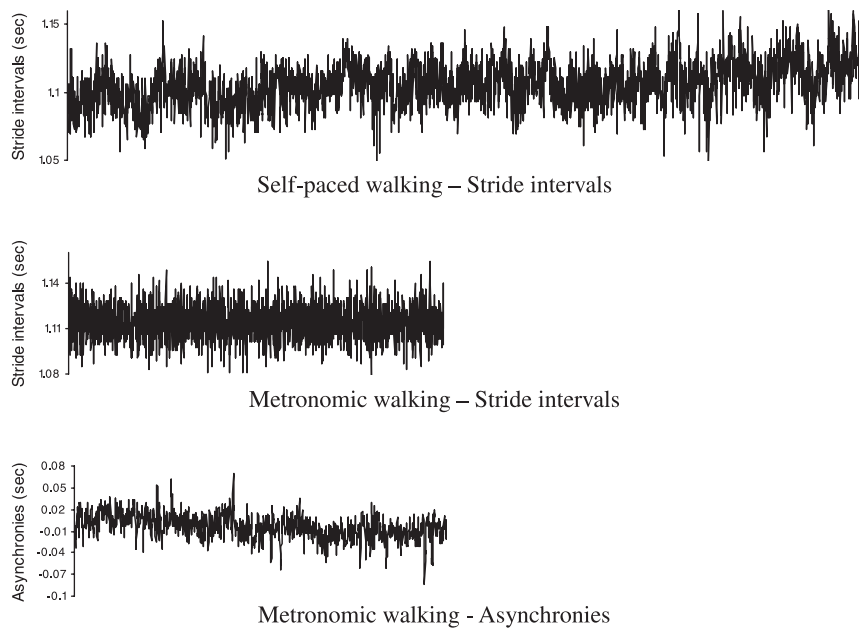


Fig. 2. Three representative time series. *Top*: stride durations, free-walking, normal speed. *Middle*: stride durations, metronomic walking, normal speed. *Bottom*: asynchronies, normal speed. Data are from participant 5.

slope. The bilogarithmic plot also allows characterizing the short-term behavior of series, by examining the slope at high frequencies. Note that spectral analysis was often used for measuring the strength of correlation within the series, through the estimation of the slope of the log-log spectrum, in the low-frequency region (9). The obtained spectral exponent is theoretically linearly related to the DFA exponent. Nevertheless, this method presents a rather low accuracy, compared with DFA (5), and we only consider in the present paper the qualitative graphical signatures provided by the spectra.

RESULTS

For ensuring a better readability, we reported all numerical results in Table 1.

Table 1. Mean results of time-series analyses

	Experimental Series			Model
	Slow speed	Normal speed	Fast speed	
N	10	10	10	100
SD, ms				
Self-paced-periods	48 (38)	18 (3)	19 (5)	19 (3)
Paced-periods	26 (11)	15 (2)	17 (7)	12 (1)
Paced- asynchronies	65 (24)	50 (18)	48 (20)	48 (12)
ARFIMA as best model				
Self-paced-periods	6/10	10/10	10/10	97/100
Paced-periods	10/10	10/10	10/10	99/100
Paced-asynchronies	10/10	10/10	10/10	97/100
Mean ARFIMA weight				
Self-paced-periods	0.77	1.00	0.99	0.94
Paced-periods	0.96	0.90	0.96	0.99
Paced-asynchronies	0.97	0.96	0.97	0.88
α DFA				
Self-paced-periods	0.90 (0.07)	0.84 (0.04)	0.99 (0.05)	0.94 (0.09)
Paced-periods	0.34 (0.18)	0.41 (0.17)	0.40 (0.15)	0.41 (0.05)
Paced-asynchronies	0.92 (0.10)	0.98 (0.10)	0.96 (0.14)	1.08 (0.09)

Values are mean SD of series, number of series best fitted by an ARFIMA (autoregressive fractionally integrated moving average) model, mean sum of ARFIMA weights, and mean α exponent [detrended fluctuation analysis (DFA)]; N, no. of series. Results from experimental series and modeling section are presented. SD in parentheses.

Free-walking series. The ARMA/ARFIMA procedure selected an ARFIMA as best model in all series for normal and fast walking rates, and in these conditions the mean sum of ARFIMA weights was close to 1.0 (i.e., ARFIMA models captured all relative likelihood). The most often selected model was a very parsimonious ARFIMA (1, d , 0). In contrast, for low walking rate, only 6 series over 10 accepted an ARFIMA as best model. An ARMA model was selected for three series, and, for the latest series, the d parameter of the selected ARFIMA model was not significantly different from 0. For the 10 series, the mean sum of ARFIMA weight was only 0.77. Note that the variability of series at low walking rate was more than twice higher than in the two others conditions. However, there were important interparticipant differences, which could be related to ARFIMA results: One participant (*participant 8*) presented a very high variability (SD = 149 ms), but the series was adequately fitted by an ARFIMA model. The others five series that were best fitted by ARFIMA models presented variability similar to that observed at normal and fast waling rate (SD = 26 ± 8 ms). In contrast, the four series that were not fitted by ARFIMA models presented a variability twice higher (SD = 50 ± 10 ms).

In the 26 selected ARFIMA models, the d parameter was positive, evidencing the presence of long-range, persistent correlation in the series. DFA confirmed these results, yielding mean α exponents close to 1.0 (see Table 1). In all cases, the regression in the diffusion plot was perfectly linear, with r^2 close to 1.0. The obtained exponents were identical to those reported by Hausdorff et al. (16). The ANOVA revealed a significant effect of walking rate [$F(2,18) = 5.99$; $P < 0.05$]. Honestly significant difference Tukey post tests showed that the mean α was significantly lower at normal speed than in the two others conditions. This effect was previously reported by Hausdorff et al. (16).

The application of spectral analysis yielded in each condition a log-log power spectrum typical of fractal fluctuation, with a linear, negative slope over the entire range of frequen-

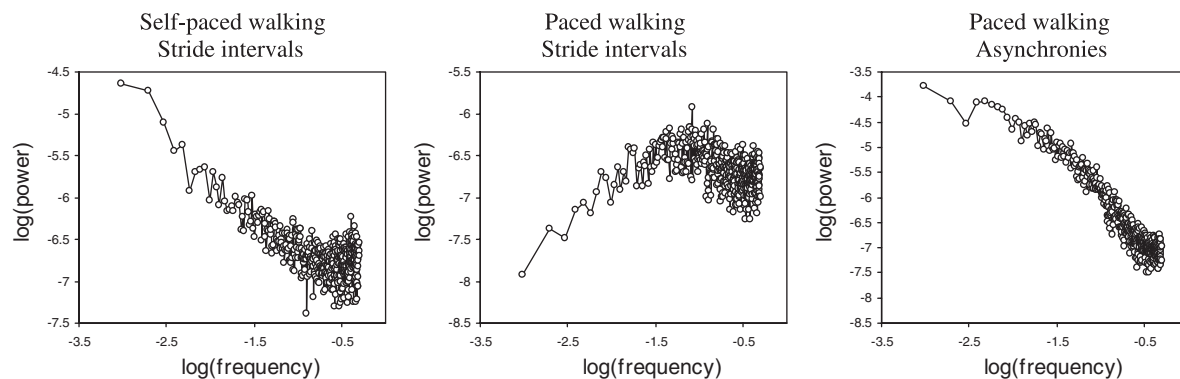


Fig. 3. Mean bilogarithmic power spectra computed by point-by-point averaging over the 10 participants (normal speed).

cies. We present in Fig. 3, *left*, the mean power spectrum, computed by point-by-point averaging over the 10 participants, for the normal speed condition.

Metronomic walking series: stride intervals. ARMA/ARFIMA modeling selected an ARFIMA as best model for all series, and the mean sum of ARFIMA weights was over 0.90 in all conditions. The mean d parameter was negative in all conditions: -0.44 ± 0.35 , -0.33 ± 0.30 , and -0.38 ± 0.33 for slow, normal, and fast walking rate, respectively. The d parameters were, in most cases, significantly different from zero, except for one series at slow speed and two series at normal speed. We obtained only three positive estimates: one at low speed, and two at fast speed. These results suggested that stride interval series cannot be considered as uncorrelated, but, rather, contained, in most cases, antipersistent correlations. This was confirmed by DFA, yielding mean α comprised between 0.34 and 0.41 (see Table 1). The mean r^2 value for the linear regression was $0.93 (\pm 0.06)$. The ANOVA revealed no significant effect of walking rate [$F(18,2) = 0.80$, $P = 0.46$].

Spectral analyses yielded in all conditions typical log-log power spectra, with a positive linear slope in the low-frequency region, and a negative linear slope in the high-frequency region (see Fig. 3, *middle*).

Metronomic walking series: asynchronies. The ARMA/ARFIMA procedure selected an ARFIMA model for all series. The most often selected model was a very simple ARFIMA (1, d , 0), and the d parameter was positive for all series. The mean sum of weights for ARFIMA models was above 0.96 in all conditions. DFA yielded α close to 1.0 in all conditions, and the mean r^2 value for the linear regression was $0.98 (\pm 0.01)$. The ANOVA revealed no significant effect of walking rate [$F(18,2) = 0.68$, $P = 0.52$].

The application of spectral analysis yielded in each condition a log-log power spectrum typical of fractal fluctuation, with a linear, negative slope over the entire range of frequencies (Fig. 3, *right*).

DISCUSSION

Preliminary discussion. The first important result is the statistical demonstration of the presence of long-range dependence in stride intervals during self-paced walking, at least for normal and fast speed conditions. This result was attested by the ARFIMA/ARMA procedure that selected an ARFIMA model for all series. The results are not so convincing in

slow-speed condition. This cannot be interpreted as a decrease of correlation in the series, as suggested by DFA results. With the exception of one participant, the series that were not recognized as containing fractional integration presented variability twice higher than the others. Scafetta et al. (22) suggested that participants could have difficulties to maintain a stable baseline speed at low walking rate. This is important because fractal analyses suppose that the system under study is in steady state and maintains the same scaling regime over the whole observation. Further studies, using a more rigorous definition of experimental conditions and a better control of individual speeds, could allow checking whether long-range dependencies were also present at slow speed.

Note that the application of inferential tests, in the domain of fractal analysis, is recent, and this method sometimes led to rejection of the long-range dependence hypothesis in situations in which classical methods tended to support the presence of fractal fluctuations (31). The present results allow the consideration, with some confidence, of the set of previous papers suggesting the presence of $1/f$ fluctuations in stride interval series (14–17).

Contrary to Hausdorff et al. (16), we clearly showed that, in the metronomic condition, stride interval series contained antipersistent dependence. All methods gave convergent results in this regard: ARFIMA modeling provided estimates for the d parameter that were negative in most cases, DFA yielded mean fractal exponents inferior to 0.5, and the log-log power spectrum presented a positive linear trend typical of antipersistent dependence. Note that some preliminary evidences for this result were previously reported: using the same data set, and applying a slightly different version of DFA, Hausdorff et al. (15) briefly evoked the possible presence of antipersistent correlations in metronomic series, and Scafetta et al. (23), using a different method (Hölder exponent spectra), reported some cases of antipersistent dependence in metronomic series.

As expected, the series of asynchronies presented positive serial dependence, and the application of ARMA/ARFIMA modeling showed that these correlations were fractal in nature. The simultaneous presence of persistent dependence in asynchronies and antipersistent dependence in stride intervals is consistent, with asynchronies being the integration of stride intervals (see Eq. 1), and stride intervals the differentiation of asynchronies (2). Note that, if stride interval series were purely random, as argued by Hausdorff et al. (16), then asynchronies

should automatically display Brownian motion (with α exponents close to 1.5). As such, the presence of $1/f$ noise in asynchrony series (with α exponents close to 1.0) is a supplementary evidence for the presence of antipersistence in stride interval series.

Most importantly, in contrast with the conclusions of Hausdorff et al. (16) and of some subsequent studies based on the same data set (4, 32), dependencies did not disappear in metronomic conditions. The present analyses show that stride intervals and asynchronies present a complex pattern of serial dependence, and the signatures reported in Fig. 3 constitute constraining criteria for testing candidate models. The following section aims at checking whether the SCPG model (32) could account for the present results.

Modeling. The SCPG model belongs to a family of gait models that hypothesize that locomotion is regulated through an intraspinal network of neurons called a central pattern generator (CPG), capable of producing a syncopated output (3). Hausdorff et al. (16) developed a stochastic version of a CPG for capturing the fractal properties of gait. More recently, Ashkenazy et al. (1) extended this model for accounting for developmental changes in gait dynamics. West and Scafetta (32) proposed another extension, the SCPG model, assuming that gait dynamics are regulated by a stochastic correlated CPG, coupled to a van der Pol nonlinear oscillator. We retained this model because it includes a source of fractal fluctuation (the stochastic correlated CPG) that is supposed to be at work in all walking conditions. The intention to follow a given walking rate or the external pacing of a metronome is modeled by a forcing function whose strength can be modulated by a single parameter.

In this model, the motion of the lower limb is modeled by a forced van der Pol oscillator obeying the following equation:

$$\ddot{x} = \mu\dot{x} - \lambda\dot{x}^2 - \omega_i^2 x + A \sin(\omega_0 t) \quad (5)$$

where x represents position and the dot notation differentiation with respect to time. In this equation, μ is the linear damping coefficient, and λ the nonlinear van der Pol damping coefficient. ω_i is the virtual inner frequency of the oscillator during the i th cycle. ω_0 and A are the frequency and the strength of the external driver, respectively. The model assumes that each cycle of the oscillator is initiated with a new virtual inner frequency ω_i , produced by the stochastic CPG.

The series of virtual inner frequencies ω_i is centered around the driver frequency:

$$\omega_i = \omega_0 + \gamma \delta_i \quad (6)$$

where γ is a constant, and δ_i represents the successive deviations from mean frequency. δ_i are successively extracted from a linear Markov process δ_j , generated by a first-order autoregressive equation:

$$\delta_j = \alpha \delta_{j-1} + \beta \varepsilon_j \quad (7)$$

where $0 < \alpha < 1$ is a constant, and ε_j a white noise process of variance β^2 . This Markov chain represents the CPG, conceived as a network of neural centers (1). Each element in this chain is supposed to represent a neural center that fires an impulse with a particular intensity that would induce a particular virtual

frequency (32). Neighboring neural centers are correlated because they are likely to be influenced by similar factors (1). The Markov chain generated by Eq. 7 is composed of “correlated zones” with a typical size r :

$$r = -1/\log \alpha \quad (8)$$

The successive values of δ_i in Eq. 7 are activated by a random walk along the chain, whose jump sizes follow a Gaussian distribution of width ρ . In this process, correlations within the δ_i series are assumed to increase with the size of correlation within the Markov chain (r), and to decrease as the width ρ of the distribution of the jumps increases.

West and Scafetta (32) suggested that this model could give account for both self-paced and metronomic conditions. Considering that the intensity A of the forcing component is related to voluntary action to try to follow a particular cadence, they proposed to set this parameter to a low value ($A = 1$) for modeling self-paced walking at normal speed. Metronomic conditions were taken into account by a simple increase of A , up to 4 for normal speed walking.

As proposed by West and Scafetta (32), we set $A = 1$ for modeling self-paced walking. The other parameters of the forced van der Pol model were set to $\mu = \lambda = 1$, and $\omega_0 = 2\pi$. To generate stride interval series with similar variability than that observed in empirical series, we set the parameters of the CPG to $r = 25$, $\rho = 25$, $\beta = 0.3$, and $\gamma = 0.02$. Then we tried to determine the appropriate A value to apply for accounting to metronomic conditions.

Keeping fixed all parameters in the model, we successively assigned to A all integer values from 0 to 12. In each case, we simulated 100 series of asynchronies series (1,024 data points each), using a four-stage Runge-Kutta algorithm. Asynchronies were measured as the time lag between the maximal position of the forced oscillator and the maximal position of the sine function $y = \sin \omega_0 t$. We report in Fig. 4 the evolution of the mean standard deviation of asynchrony series with increasing A . For low A values, asynchronies presented a high variability, showing that coupling is not sufficient for stabilizing stride interval series. Variability decreased quickly in the range

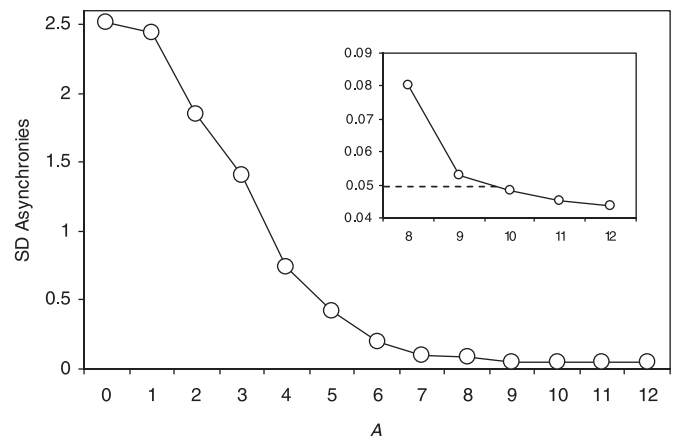


Fig. 4. Evolution of the standard deviation (SD) of simulated asynchronies series, for A (strength of external driver) values increasing from 0 to 12. One hundred series of 1,024 points were generated for each A values. *Inset*: graph focuses on the evolution of variability for A , ranging from 8 to 12, with an appropriate rescaling of the ordinate scale. The dashed line indicates the mean SD experimentally obtained for normal speed.

$A = 2$ to $A = 8$, and synchronization began to be efficient for $A = 9$. A close examination of the results indicated that variability similar to that experimentally observed was obtained for $A = 10$.

We computed sets of 100 series, with $A = 1$ for simulating self-paced condition (periods), and $A = 10$ for metronomic condition (periods and asynchronies), and we submitted these series to the above-mentioned analyses. Results are detailed in Table 1 (*right* column). As can be seen, ARMA/ARIMA modeling detected fractional integration in most series, and DFA yielded exponents similar to those obtained from experimental series. We present in Fig. 5 the mean power spectra, in log-log coordinates, computed from 10 randomly selected simulated series in each set. These spectra present similar shapes than those obtained from corresponding experimental series.

Final discussion and conclusion. The SCPG model was able to generate series reproducing the experimentally evidenced signatures. Qualitatively, the simulated power spectra presented similar shapes than their experimental counterparts, and fractal exponents fall in similar ranges. Furthermore, the ARFIMA/ARMA procedure attested for the presence of genuine long-range dependence in simulated series.

Some discrepancies can be noticed, however, that could require further refinements in the model. Notably, the experimental power spectrum for stride interval in self-paced condition presented a typical flattening in high frequency (see Fig. 3) that was not accounted for by the model. Concerning fractal exponents, the model tends to yield higher estimates for asynchronies.

Despite these local imperfections, our results confirm that the SCPG model is able to give a satisfying account of both self-paced and metronomic walking, just through the adjustment of a single parameter. Note that we applied a stronger coupling than West and Scafetta (32). These authors suggested that a coupling parameter of ~ 4 could be sufficient for accounting for the effect of external pacing. The present results (see Fig. 4) show that a stronger coupling is necessary, without questioning the overall relevancy of the model.

This model integrates some biologically plausible ingredients, and especially the concept of a stochastic CPG (1, 16, 32). As such, it supports a number of working hypotheses, testable by both simulation and experimentation. For instance, Ashkenazy et al. (1) suggested that this model was able to account for the changes in gait time series from

childhood to adulthood, assuming that neural maturation is associated with the range ρ of the Brownian process that activates the nodes of the finite-size correlated chain of frequencies. Based on the biological significance of SCPG parameters, further studies focusing on the elderly or patients suffering from neurodegenerative syndromes could allow identification of the source of gait disorders, and especially of the genuine extinguishing of long-range correlation in stride series observed with these patients (14). Note that, recently, Gates et al. (10) developed a simpler model, based on biomechanical principles, that seemed able to adequately simulate stride series and the diverse patterns of serial correlations observed in gait experiments. Nevertheless, this model was only tested through DFA, and further analyses are necessary to check whether this model only mimics fractal behavior or generates genuine long-range dependence.

In conclusion, the present study confirms, on the basis of an inferential test, the presence of long-range correlations in stride interval series collected in self-paced walking. This result is important, because previous works, in our opinion, only suggested the presence of such fractal fluctuations without providing a definitive statistical proof. As evoked in the Introduction, the fractal nature of walking has important implications, because such fluctuations are conceived as the natural outcome of complex, nonequilibrium dynamic systems, rather than being the output of a simple homeostatic process (16). The breakdown of fractality with aging and neurodegenerative pathology suggests that long-range dependence plays an important role in the adaptability and the flexibility of the system (14).

Basing on the association of multiple analyses and the derivation of a new variable (asynchronies), we oppose the result of Hausdorff et al. (16), suggesting a breakdown in fractality in metronomic conditions. In their original paper, the authors explained this breakdown through the hypothesis of supraspinal influences that could override the normally present long-range correlations (16). In other words, the intervention of attentional and intentional processes focused on external pacing was supposed to induce a kind of “oversimplification” of the system, yielding the extinguishing of long-range correlation in stride interval series. The present results and our simulation study disregard this oversimplification hypothesis: the intrinsic complexity of the system is still at work in metronomic conditions, but expresses differently in overt performance.

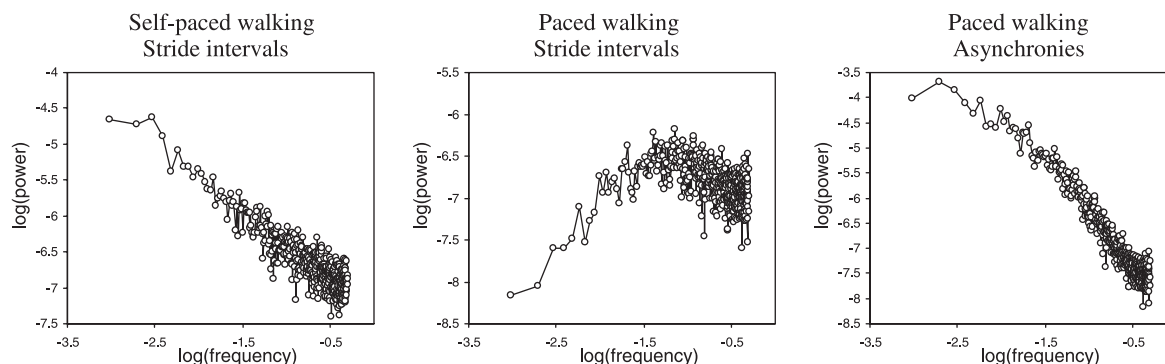


Fig. 5. Simulated series. Mean bilogarithmic power spectra computed by point-by-point averaging over 10 randomly selected series.

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