

1. let ~~$x[n] = X(e^{j\omega})$~~ and ~~$y[n] = Y(e^{j\omega})$~~

$$Y(e^{j(\omega-\theta)}) = e^{j\omega n} \cdot Y(e^{j\omega})$$

$$\text{So } \frac{1}{2\pi} \cdot \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) \cdot Y(e^{j\omega}) \cdot e^{j\omega n} d\omega$$

does $x[n] y[n] \leftrightarrow \frac{1}{2\pi} [X(\Omega) Y(\Omega)]$

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\Omega) e^{j\Omega n} d\Omega$$

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) Y(e^{j\omega}) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} e^{j\omega n} X(e^{j\omega}) \cdot \frac{1}{2\pi} e^{j\omega n} Y(e^{j\omega})$$

$$= x[n] \cdot y[n]$$

2. $(s[n])^2 = U[n] \quad s[n-h_0] \cdot s[n-h_0] = U[n-h_0]$

$$\text{So } x[n] \cdot h[n] = U[n] + 2U[n-1] - U[n-2] + U[n-3] + U[n-1] + 2U[n-2] - U[n-3] + U[n-4]$$

$$= U[n] + 3U[n-1] + U[n-2] + U[n-4]$$