Region based segmentation in presence of intensity inhomogeneity using Legendre polynomials

Suvadip Mukherjee, *Student Member, IEEE,* and Scott T. Acton, *Fellow, IEEE*

***Abstract*—We propose a novel region based segmentation method capable of segmenting objects in presence of significant intensity variation. Current solutions use some form of local processing to tackle intra-region inhomogeneity, which makes such methods susceptible to local minima. In this letter, we present a framework which generalizes the traditional Chan-Vese algorithm. In contrast to existing local techniques, we represent the illumination of the regions of interest in a lower dimensional subspace using a set of pre-specified basis functions. This representation enables us to accommodate heterogeneous objects, even in presence of noise. We compare our results with three state of the art techniques on a dataset focusing on biological/biomedical images with tubular or filamentous structures. Quantitatively, we achieve a 44% increase in performance, which demonstrates efficacy of the method.**

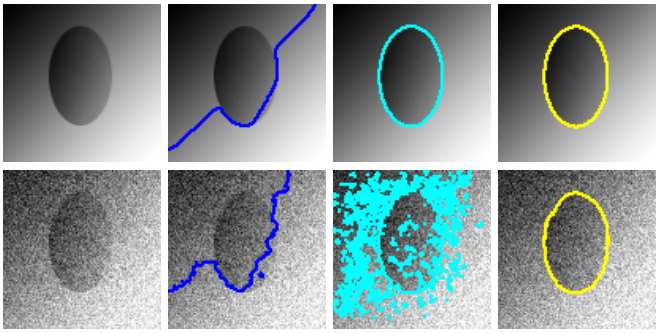
***Index Terms*—segmentation, level set, active contour**

I. Introduction

ACTIVE contours [1]–[4] have gained popularity for image segmentation due to their ability to elastically deform and delineate object boundaries with sub-pixel accuracy. Furthermore, the energy minimization framework, which serves as the basic platform for most active contour based techniques, can be manipulated easily to introduce additional constraints based on shape, appearance etc. to assist segmentation. Geometric active contours are favored when the application requires the propagating curves to be able to adapt to the varying topology of the underlying object by automated splitting or merging.

Broadly, the geometric active contours may be divided in two subgroups. Edge based techniques [2], [5], [6] perform curve evolution geometrically, with the stopping criteria governed by edge dependent features extracted from the image. However, in many applications where the edge information is unreliable, region based techniques have gained popularity. Chan and Vese [3] proposed a level set formulation to minimize the Mumford Shah functional [7] for segmentation. The Chan-Vese framework models the image as a set of constant illumination regions and performs a two-class segmentation by computing the optimal partition which satisfies the constant illumination constraint. The authors also propose a multi-phase variant [8] of their approach to perform multi-class grouping.

The constant illumination assumption is challenged in applications where the signal intensity is inhomogeneous. This is encountered frequently in many medical and biological imaging applications like magnetic resonance (MR) imaging, ultrasound, X-ray, confocal and electron microscopy, etc. While edge based techniques are better suited for heterogeneous images, they are unable to tackle fragmented object boundaries or blurred edges, which occur frequently in medical applications. In this letter we propose an edge independent segmentation approach *Legendre Level Set* (L2S) which is robust to variations in intensity levels. State of the art techniques that tackle inhomogeneity typically require some form of local processing. However, while a global method like Chan-Vese's is insufficient in handling large scale intensity variations, a strictly local approach may lead to undesired segmentation artifacts, especially in presence of noise. Such artifacts are shown in Fig. 1. We aim to eradicate these issues by proposing a generalized solution for region based segmentation in presence of significant intensity variation and additive noise.



(a) Original image (b) Chan-Vese [3] (c) Li et al. [9] (d) L2S

Fig. 1: Segmentation results of Chan-Vese (blue) [3] and LBF model [9] (cyan) vs L2S (yellow) on a simulated image. The second row shows segmentation when a Gaussian noise of std. dev. 0.1 is added.

II. Background and Motivation

The piecewise constant model of Chan-Vese [3] models the object foreground and background by the scalars c_1 and c_2. In a level set framework, the Chan-Vese energy functional can be written as

$$\mathcal{E}(\phi, c_1, c_2) = \int_{\Omega} |f(\mathbf{x}) - c_1|^2 H_\epsilon(\phi) d\mathbf{x} + \int_{\Omega} |f(\mathbf{x}) - c_2|^2 \left(1 - H_\epsilon(\phi)\right) d\mathbf{x} \quad (1)$$

where $f(\mathbf{x})$ is the image defined over the domain Ω. The level set function ϕ is defined to be positive inside the zero level set and negative outside it. Zero level sets of ϕ represents the object boundary. H_ϵ denotes the regularized version of the heaviside function. The ϕ that minimizes (1) creates segments

S. Mukherjee and S.T. Acton are with the Department of ECE, University of Virginia, Charlottesville, VA, 22901, USA.
E-mail: sm5vp,acton@virginia.edu. Phone: 434-982-2003, FAX: 434-924-8818.

This work was supported in part by NSF under 1062433.

in a manner such that the foreground and background are best approximated by the intensity levels c_1 and c_2.

As mentioned previously, this model is incapable of handling spatially varying illumination (Fig. 1). A solution was proposed in [8], where the authors replaced the scalars c_1, c_2 in (1) with smooth functions, the smoothness enforced by minimizing the total variation. While the solution is attractive, this piecewise smooth model is computationally expensive, since it requires simultaneous solution of two PDEs at each stage of the level set evolution.

Recently, Li et al. [9] introduced a region scalable model to localize the energy functional. The region localization is controlled by the scale of a Gaussian kernel, which is manually tuned for optimal performance. Efforts have been made to incorporate the region statistics for segmentation [10], [11]. These methods are robust to initialization and relatively less sensitive to noise. However, Lankton et al. [12] demonstrated that global statistics may not be the best resort for segmenting inhomogeneous objects. Instead, the authors generalize the local region based methods, by proposing a generic energy functional capable of performing segmentation using different region based criteria. However, one downside of their approach is that it requires additional local computation, thus increasing the risk of being stuck within local minima. Feng et al. [13] proposed a method for tomographic reconstruction by using a low order parametric model to represent object texture. However, the algorithm is tailored for tomographic reconstruction and is difficult to generalize. Recently a method was proposed to model the foreground and background by a linear function [14]. This approach is an improvement over the model of Chan-Vese, but does not accommodate nonlinear illumination change.

From the above discussion we observe that a majority of these approaches rely on local information only. While localizing the segmentation energy is essential in dealing with inhomegeneity, a generic global framework is also necessary to avoid the local minima problem. We propose to model the foreground and background illumination by a set of Legendre basis functions. This model allows the region intensities to be represented in a lower dimensional subspace, thereby permitting smooth approximation. Low dimensional signal representation has been used in a slightly different context in the literature, primarily to accomodate shape priors for segmentation [15], [16]. However, although shape based information assists segmentation, such techniques require an atlas of preregistered objects, which may be unavailable for general purpose segmentation. We further show that our model (called L2S) is computationally simple, since we achieve a stable, closed form solution at each iteration, which allows faster processing.

III. Methodology

The traditional Chan-Vese functional (1) can be reformulated and generalized by replacing the scalars c_1 and c_2 by two smooth functions $c_1^m(\mathbf{x})$ and $c_2^m(\mathbf{x})$. These functions are used to model the intensity in the two regions separated by the zero level set curves of ϕ. The essence of our approach is embedded in computing these functions. To preserve the smoothness and flexibility of the functions, we represent them as a linear combination of a few Legendre basis functions. Mathematically, we can write $c_1^m(\mathbf{x}) = \sum_k \alpha_k \mathcal{P}_k(\mathbf{x})$ and $c_2^m(\mathbf{x}) = \sum_k \beta_k \mathcal{P}_k(\mathbf{x})$. Here $\mathcal{P}_k$ is a multidimensional Legendre polynomial, which can be written as the outer product of the one dimensional counterparts. The 2-D polynomial is computed as $\mathcal{P}_k(x,y) = p_k(x)p_k(y)$, $\mathbf{x} = (x,y) \in \Omega \subset [-1,1]^2$. p_k is a one dimensional Legendre polynomial of degree k defined as

$$p_k(x) = \frac{1}{2^k} \sum_{i=0}^{k} \binom{k}{i} (x-1)^{k-i}(x+1)^i \tag{2}$$

The highest degree of the 1D bases is denoted by m. Hence, for the 2D case, we would represent the regions by a linear combination of a set of $(m+1)^2$ 2D Legendre basis functions.

A. Choice of basis function

Our primary objective is to perform segmentation in the presence of an inhomogeneous intensity field. In many applications (e.g. phlebotomy via ultrasound), it is valid to assume that the intra-region inhomogeneity field (in homogeneous tissue, for example) is smoothly varying. Therefore, we seek a smooth representation of the region approximating functions. The set of Legendre polynomials provide an efficacious platform for such representation, since the basis functions are smoothly changing themselves. Legendre polynomials have found application in certain aspects of image processing such as designing illumination models for object tracking [17] and generating shape signatures for supervised segmentation [18]. Using Legendre polynomials to model intensity based appearance allows the estimating functions to vary spatially, yet the variation is constrained by the inherent smoothness of the polynomials. A demonstrative example of nine 2D Legendre functions are shown in Fig. 2. Note that the zero degree ($m = 0$) version of the Legendre polynomial reduces to the traditional Chan-Vese model, thus presenting a generalized framework.

B. Optimization of the energy functional

Let us denote $\mathbb{P}(\mathbf{x}) = (\mathcal{P}_0(\mathbf{x}), \ldots, \mathcal{P}_N(\mathbf{x}))^T$ as the vector of Legendre polynomials. $A = (\alpha_0, \ldots, \alpha_N)^T$ and $B = (\beta_0, \ldots, \beta_N)^T$ are the coefficient vectors for the two regions. $N = (m+1)^2$ is the total number of basis functions. We can now rewrite the modified version of (1) in matrix form as

$$\begin{aligned}\mathcal{E}^m(\phi, A, B) = &\int_\Omega |f(\mathbf{x}) - A^T\mathbb{P}(\mathbf{x})|^2 m_1(\mathbf{x})d\mathbf{x} + \lambda_1||A||_2^2 \\ &+ \int_\Omega |f(\mathbf{x}) - B^T\mathbb{P}(\mathbf{x})|^2 m_2(\mathbf{x})d\mathbf{x} + \lambda_2||B||_2^2 \\ &+ \nu \int_\Omega \delta_\epsilon(\phi)\frac{\nabla\phi}{|\nabla\phi|}d\mathbf{x}\end{aligned} \tag{3}$$

In (3) the last term introduces smoothness in the zero level curve, which is regulated by the parameter ν. Let us also denote $m_1(\mathbf{x}) = H_\epsilon(\phi)$ and $m_2(\mathbf{x}) = 1 - m_1(\mathbf{x})$. The non negative regularizing parameters λ_1, λ_2 can be selected using cross validation techniques to avoid over-fitting. The energy functional (3) is optimized using alternating minimization. In

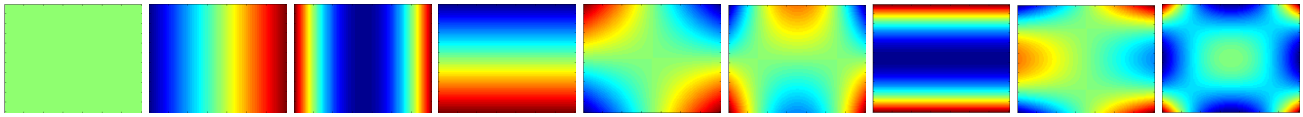


Fig. 2: The set of nine 2D Legendre basis functions. The 2D polynomials are computed from 1D functions of degree 2.

the first step, to find optimal A and B, we perform $\frac{\partial \mathcal{E}^m}{\partial A} = 0$ and $\frac{\partial \mathcal{E}^m}{\partial B} = 0$. A closed form solution $\hat{A}$ and $\hat{B}$ is obtained as

$$\hat{A} = \left[K + \lambda_1 \mathbb{I}\right]^{-1} P \tag{4}$$

$$\hat{B} = \left[L + \lambda_2 \mathbb{I}\right]^{-1} Q \tag{5}$$

$[.]$ denotes a matrix. The $N \times 1$ vectors P and Q are obtained as $P = \int_\Omega \mathbb{P}(\mathbf{x}) f(\mathbf{x}) m_1(\mathbf{x}) d\mathbf{x}$ and $Q = \int_\Omega \mathbb{P}(\mathbf{x}) f(\mathbf{x}) m_2(\mathbf{x}) d\mathbf{x}$ respectively. $[K]$ and $[L]$ are $N \times N$ Gramian matrices [19], whose $(i,j)^{th}$ entry are obtained as

$$[K]_{i,j} = \left\langle \sqrt{m_1(\mathbf{x})}\mathcal{P}_i(\mathbf{x}), \sqrt{m_1(\mathbf{x})}\mathcal{P}_j(\mathbf{x}) \right\rangle \tag{6}$$

$$[L]_{i,j} = \left\langle \sqrt{m_2(\mathbf{x})}\mathcal{P}_i(\mathbf{x}), \sqrt{m_2(\mathbf{x})}\mathcal{P}_j(\mathbf{x}) \right\rangle \tag{7}$$

$0 \leq i,j \leq N$. $\langle , \rangle$ denotes the inner product operator. With the updated coefficient vectors, we can now minimize (3) with respect to ϕ borrowing techniques from variational calculus. The curve evolution is performed by numerically solving the following partial differential equation:

$$\frac{\partial \phi}{\partial t} = \left[-|f(\mathbf{x}) - \hat{A}^T \mathbb{P}(\mathbf{x})|^2 + |f(\mathbf{x}) - \hat{B}^T \mathbb{P}(\mathbf{x})|^2\right] \delta_\epsilon(\phi) + \nu \delta_\epsilon(\phi) \text{div}\left(\frac{\nabla \phi}{|\nabla \phi|}\right) \tag{8}$$

$\delta_\epsilon(\phi)$ is a regularized version of the Dirac delta function. We solve (8) using gradient descent and initializing $\phi|_{t=0} = \phi_0$ and $\frac{\delta_\epsilon(\phi)}{|\nabla \phi|} \frac{\partial \phi}{\partial \hat{n}} = 0$ at the domain boundary.

C. Discussion

The surface approximate for foreground and background are obtained by computing $\hat{A}^T \mathbb{P}(\mathbf{x})$ and $\hat{B}^T \mathbb{P}(\mathbf{x})$. Since the coefficient vectors are available in closed form, it makes our algorithm fast and effective. The amount of intensity variation is governed by the coefficient vectors which are computed automatically. However, computing the coefficient vectors require a matrix inversion step. Here we show that the matrices $[K]$ and $[L]$ are invertible when the heaviside function is suitably regularized.

Since $[K]$ is a Gramian matrix, it is full rank iff the polynomials $\sqrt{m_1(\mathbf{x})}\mathcal{P}_i(\mathbf{x})$, $(i = 1, \ldots, N)$ are linearly independent [19]. Since the polynomials $\mathcal{P}_i(\mathbf{x})$ are linearly independent themselves, it is easy to show that the linear independence holds if $0 < m_1(x) < 1$. A similar argument holds for analyzing the invertibility of $[L]$.

In [3], the authors propose a regularized version of the heaviside function as $H_\epsilon(y) = \frac{1}{2}\left(1 + \frac{2}{\pi}\tan^{-1}\left(\frac{y}{\epsilon}\right)\right)$. By this definition, the functions $m_1(\mathbf{x})$ and $m_2(\mathbf{x})$ are bounded in $(0,1)$, which makes the matrices invertible.

However, inverting the above mentioned matrices may still be prone to numerical error when $\sqrt{m_i(\mathbf{x})}$ is small. The regularizing constants λ_1 and λ_2 contribute to make these matrices well conditioned. Furthermore, the regularization terms are necessary to avoid over-fitting. In most situations, we find that only a few (typically 16) 2-D Legendre functions are sufficient to model the region intensity. However, image noise may lead to over-fitting of the polynomials to the image segments, which may disrupt segmentation as the propagating level set may settle at a local minima. The scalars λ_1, λ_2 produce a damping effect by constraining the $\mathbb{L}_2$ norm of the bases coefficients, thereby favoring interior regions approximated by smooth functions.

IV. Experimental results

A. Dataset and parameter selection

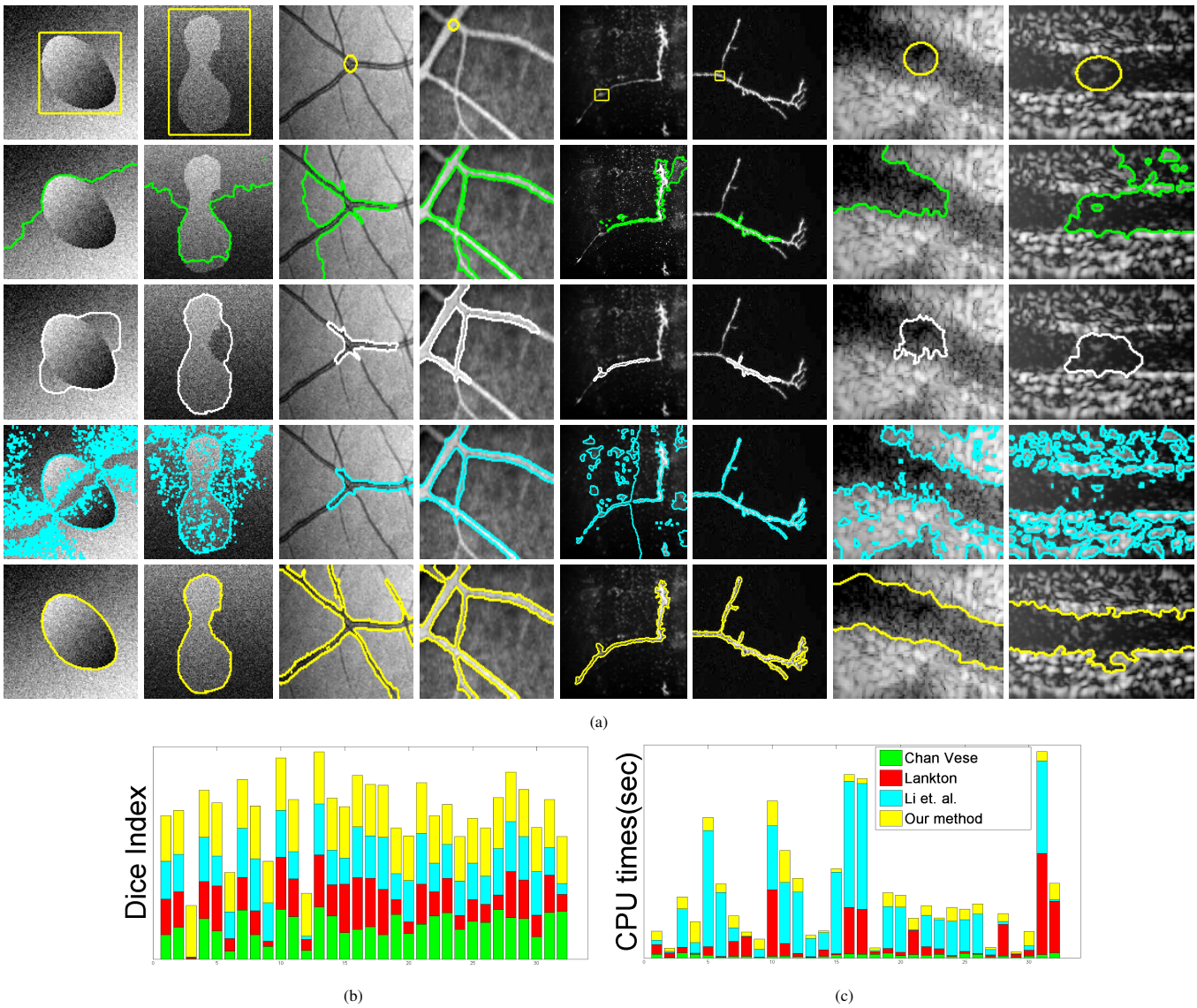
To demonstrate the efficacy of the proposed method, we have performed experiments on a dataset of 32 images. The dataset consists of a set of synthetic images with added noise and simulated intensity inhomogeneity, a set of biomedical images consisting of blood vessels using magnetic resonance angiogram (MRA), neurons and dendritic spines imaged by confocal microscope and finally, a set of ultrasound images of human blood vessels.

Our algorithm requires specification of a few parameters, namely the Legendre polynomial degree m and the regularizing constants λ_1 and λ_2 in (3). We experimentally verified that the intensity variation in the images can be adequately modeled by using 1-D Legendre polynomials of (highest) degree three. We found that the algorithm is relatively robust to the selection of this value, but a higher degree polynomial typically requires inversion of a larger matrix, which makes computation significantly more expensive. To estimate the value of λ_1 and λ_2, we perform a *leave one out* cross validation on each of the four catagories in our dataset. The cross validation is performed over the values of $\{0, 1, \ldots, 100\}$ in multiples of 2. For simplicity, we have chosen $\lambda_1 = \lambda_2$ for every experiment. The particular value which yields the highest average Dice coefficient for each dataset is chosen for experimentation.

Automated selection of the contour smoothness parameter ν in (3) is non-trivial. Typically, $0 < \nu < 1$, where a higher value produces smoother contour. As a rule of thumb, one may wish to set ν to a relatively higher value if the noise level in the image is high. For our experiments, we observe that the set of ultrasound images and the simulated noisy images require larger values of ν. For all these images, we select $\nu = 0.6$. For the less noisy images, ν is typically set in the range 0.05 to 0.2.

B. Performance Evaluation

To evaluate the performance of L2S, we compare our approach with those of Chan-Vese [3], Lankton et. al. [12] and Li et. al. [9]. We use the freely available CREASEG [20] tool to evaluate the performance. Visual segmentation results for two images from each category in our dataset are presented in Fig. 3(a) for demonstration. Results of our algorithm are shown in yellow, in the fifth row. The Dice coefficient [21] is



(a)

(b)

(c)

Fig. 3: (a) The top row shows eight images from our dataset. The yellow polygon indicates level set initialization. The second, third and fourth rows show segmentation output of Chan-Vese [3] (green), Lankton et. al. [12] (white) and C. Li et. al. [9] (cyan). The fifth row shows segmentation results using L2S (yellow). Segmentation accuracy as calculated via the Dice index and computational time for each method are plotted in (b) and (c) respectively.

used to quantify the results of segmentation. The Dice index $\mathcal{D} \in [0, 1]$ between the experimentally determined segmentation R_1 and the ground truth R_2 is given by $\mathcal{D}(R_1, R_2) = \frac{2Area(R_1 \cap R_2)}{Area(R_1)+Area(R_2)}$. A Dice value closer to 1 indicates superior performance. The quantitative performance is shown in Fig. 3(b). We also present the computation time in terms of CPU cycles (Fig. 3(c)).

We observe that over this entire dataset, L2S yields an average Dice score of 0.9, compared to $0.62, 0.57$ and 0.7 for the methods described in [3], [12] and [9] respectively. Computationally, our method outperforms [12] and [9] on average. It may be noted that the apparent low convergence time of [3] often is a result of convergence at local minima.

V. Conclusion

A novel framework for segmentation in presence of significant intra-region illumination variation is presented. Qualitative and quantitative results and comparison with the state of the art techniques suggest robustness of our approach. Here we have focused on bi-level segmentation, although extension to a multi-level framework appears straightforward. Also, our formulation allows easy incorporation of *a priory* shape information, which may enhance performance in select cases. However, like most level set methods, L2S is somewhat biased towards contour initialization. Recently, there have been efforts to make such algorithms robust against initial curve placement [21]. Incorporating L2S to such a framework would be an interesting future research topic. Using Legendre polynomials for region intensity approximation provides an elegant solution. However, without further analysis it is difficult to comment on the optimality of this choice of basis. Effectiveness of other polynomials such as splines or wavelets [22] needs further investigation. In select cases, it may also be possible to learn a compact set of bases for representation. We wish to investigate the effectiveness of these approaches in the future.

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