Central Limit Theorem - Assignment

(1) X → Random Variable, $\mu = 10$, $\sigma = 4$, Sample size $n = 100$

Probability that sample mean ($\bar{x}$) of these 100 observations is less than 9.

Std for Sample $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

called as Standard error

$$\sigma_{\bar{x}} = \frac{4}{\sqrt{100}} = \frac{4}{10} = \frac{2}{5} = 0.4$$

Z-score to find p-value

$$\frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{9 - 10}{0.4} = \frac{-1}{0.4} = -2.5$$

$\boxed{\text{P-value} = 0.0062}$ → for sample of 100 observations < 9. (less than 1% probability)

(2) Given, elevator capacity = 550kg

Population Mean $\mu = 50$kg, $\sigma = 15$

Sample size (n) = 10, Probability to find these sample of students reach safely to 8th floor through elevator = ?

avg stud weight to reach safely $= \frac{550}{10} = 55$kg

$\bar{x} = 550$kg (< 550kg)

↳ Sample of 10 student should be 550kg (or) < 50kg.

Population $\mu = 50$, Sampling distribution std

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = 4.74$$

$$= \frac{15}{\sqrt{10}} = 4.74$$

$$Z\text{ score} = \frac{\bar{x}-\mu}{\sigma}$$

$$= \frac{55-50}{4.74} = \frac{5}{4.74} = 1.054$$

P-value (area under z-curve) = $\boxed{0.8531}$

$\boxed{85.31\%}$ → that sample of 10 students reach safely to floor 8 with $\mu = 55$ & $\sigma = 4.74$

(3) Given, $\mu = 2.4$, $\sigma = 2.0$ (for '1' passenger)
(Mean tickets purchased by passenger)

$n = 100$ (population) every passengers sample , 250 tickets remained
$\bar{x} = 250$

for ticket per passenger $= \frac{250}{100}$
$= 2.5$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$2 = \frac{\sigma}{\sqrt{100}}$$

Population std $\sigma = 20$ population mean (μ)
$= 100 \times 2.4 = 240$ tickets
(for 1 passenger)
↓
calculating for 100 passengers

$\bar{x} - \mu / \sigma$

$$Z\text{-score} = \frac{250-240}{20}$$

$$= \frac{10}{20} = 0.5$$

$\boxed{\text{P-value} = 0.6915}$

69.15%

(4) Given, avg IQ of soldier $\mu = 96, \sigma = 16$

Sample(n) = 35

Probability that this sample have mean of IQ > 98

$\bar{x} = 98$, standard error $= \frac{\sigma}{\sqrt{n}}$

$= \frac{16}{\sqrt{35}} = 2.704$

$z\text{-score} = \frac{98-96}{2.704} = 0.7396$

P-value = 0.7673

> 98 means Right side area of z-score

= 1 - left side area of z-score

$= 1 - 0.7673 = \boxed{0.2327}$

23.27% that selected sample will cross mean IQ > 98 as the officer wants.

(5) Given, $\mu = 6.0$ inch, $\sigma = 1.0$ inch

a) One man is randomly selected, probability that his head breadth < 6.2 inch.

$\bar{x} = 6.2, \mu = 6.0, \sigma = 1.0$

Standerr $= \frac{\sigma}{\sqrt{n}} = \cancel{\frac{1}{\sqrt{10}}}$

One man, so $z\text{-score} = \bar{x} - \mu / \sigma$

$= \frac{6.2-6}{1.0} = 0.2$

P-value = 0.5793 = $\boxed{57.93\%}$ male have head breadth less than 6.2 inch.

(b) probability that 100 randomly selected men have a mean breadth less than 6.2 inch

$n = 100$ standard error $= \frac{\sigma}{\sqrt{n}} = \frac{1}{\sqrt{100}} = 0.1$

$\bar{x} = 6.2, \mu = 6$

$z\text{-score} = \frac{6.2-6}{0.1} = \frac{0.2}{0.1} = 2$

P-value = 0.9772

97.72% of selected sample have mean head breadth less than 6.2 inch

(6) Production Manager from (b), made decision based on single sample of 100 males, whereas from (a). there are around 43% males who have head breadth greater than 6.2 inch; so instead of manufacturing helmets based on single sample, better to opt for more number of samples to arrive at better decision.

(7) Given, $\mu = 268$ days, $\sigma = 15$ days

$n = 25$ (sample)

probability of sample of 25 women that their lengths of pregnancy have a mean < 260 days

σ std error $= \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{25}} = 3$

$z\text{-score} = \frac{260-268}{3} = \frac{-8}{3} = -2.66$

P-value = 0.0039 = 0.39% (< 1%)

Probability for this sample having pregnancy length less than 260 days is (0.39% < 1%) less than 1%.

(8) 25 women put on special diet before they become pregnant & end up having mean length of pregnancy less than 260 days

from (7), it is so probability is less than 1%

for duration of less than 260 days.

If calculated for single female

$$z\text{-score} = \frac{260-268}{15} = \frac{-8}{15} = -0.533$$

$$P\text{-value} = 0.2981 = 29.81\%$$

29% females have pregnancy length < 260 days

↓

In that, very less percent (0.39%) achieved duration less than 260 days, when taken on sample almost equal to (29% ⇒ (original female probability); difference is diet they have took before pregnancy.

In total females → 29% → (less than 260 days)

↓

25 (women) sample (almost equal to population of probability of women who have pregnancy duration < 260 days

(≃ 0.0039) (probability) → this percent achieved for sample

→ diff apart from other females is diet.

(9) Given, $\mu = 172$ pounds, $\sigma = 29$

a) Probability that 1 randomly selected adult male will weigh more than 190 pounds?

$$z\text{-score} = \frac{\bar{x} - \mu}{\sigma}$$

$$= \frac{190 - 172}{29} = \frac{18}{29} = 0.6206$$

P-value = 0.7324

we, want greater than 190 pounds, so 1 − 0.7324

$= \boxed{0.2676 \simeq 26.76\%}$

Probability that any male adult would weigh > 190 pounds.

b) Probability that 25 randomly selected adult males will have mean weight of more than 190 pound?

$n = 25$, std error $= \frac{\sigma}{\sqrt{n}} = \frac{29}{\sqrt{25}} = 5.8$

$$z\text{-score} = \frac{190 - 172}{5.8} = \frac{18}{5.8} = 3.1034$$

P-value = 0.9990

we, need greater than 190 Pounds of adultmales p-value, so

$1 - 0.990 = \boxed{0.001 = 0.1\%}$

c) maximum capacity of elevator = 4750 pounds

$n = 25$, for each person on avg in elevator $\bar{x} = \frac{4750}{25} = 190$ Pounds

For, elevator to be overload with weight, the weight of male adults in sample should be greater than 190 pounds.

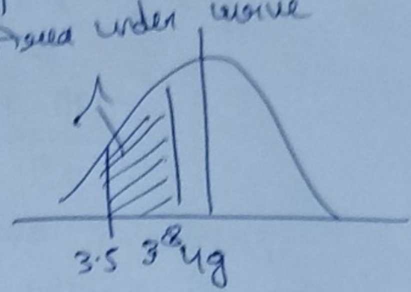
Same as (b) problem

→ Req probability = $\boxed{0.1\% \text{ (or) } 0.001}$

(10.) Given $\mu = 4.0g$, $\sigma = 1.5g$

$n = 50$ $\quad \bar{x}_1 = 3.5$, $\bar{x}_2 = 2.8$
↓
Avg impurity

Std err $= \frac{\sigma}{\sqrt{n}} = \frac{1.5}{\sqrt{50}} = 0.2121$



Z-score for 3.5gms of impurity in chemical product

$$\frac{3.5-4.0}{0.2121} = \frac{-0.5}{0.2121}$$

Z-score $= -2.35$

P-value $= 0.0094$

Z-score for 3.8gm of impurity in chemical product

$$\frac{3.8-4.0}{0.2121} = \frac{-0.2}{0.2121} = -0.942$$

P-value $= 0.1736$

for probability of that avg amount of impurity in these 50 batches is between 3.5g and 3.8gm is

$$(0.1736 - 0.0094)$$
$$= \boxed{0.1642 = 16.42\%}$$

(11) Given, $\mu = 23.1$ years, $\sigma = 3.1$ years

$n = 6$, probability that 6 randomly selected student graduates had mean age > 27 years at

Std error $= \frac{\sigma}{\sqrt{n}} = \frac{3.1}{\sqrt{6}} = 1.265$

$$\frac{27-23.1}{1.265} = \frac{3.9}{1.265} = 3.083$$

P-value $= 0.9990$

For probability > 24 years

1 - 0.9990 = 0.001

= 0.1%

(very less % probability of students graduating at age > 24 years)

(12) Given μ = \$21.50, σ = \$2.22

n = 8 checks

probability that avg of 8 checks is between \$20 and \$23

$$\text{std error} = \frac{\sigma}{\sqrt{n}} = \frac{2.22}{\sqrt{8}} = 0.7848$$

z-score for \$20

$$\frac{20 - 21.50}{0.7848} = \frac{-1.5}{0.7848} = -1.911$$

P-value = 0.0281

z-score for \$23

$$\frac{23 - 21.50}{0.7848} = \frac{1.5}{0.7848} = \cancel{1.395} \; 1.911$$

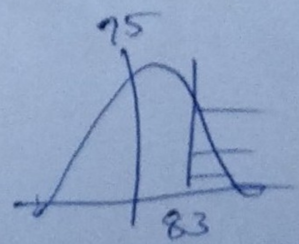
P-value = 0.9719

For req. probability avg amount spent on food b/w \$20 & \$23

is 0.9719 - 0.0281 = $\boxed{0.9438 = 94.38\%}$

(13) Given, μ = 75, σ = 5

a) probability that randomly selected students was atleast 83? (mean ≥ 83)



$$z\text{-score} = \frac{83 - 75}{5} = 1.6$$

P value = 0.9452

Right side of curve (≥ 83) = 1 - 0.9452 = $\boxed{0.0548 = 5.48\%}$

b) Probability that average grade of 5 randomly selected students was atleast 83? (≥ 83)

$n = 5$ $\quad$ stderror $= \frac{\sigma}{\sqrt{n}} = \frac{5}{\sqrt{5}} = 2.236$

$z\text{-score} = \frac{83-75}{2.236} = \frac{8}{2.236} = 3.57$

p_value = 0.9999 (almost)

greater than 83, so 1-0.9999

$\boxed{\begin{array}{l}\text{P\_value} = 0.0001 \\ = 0.01\%\end{array}}$

(14) Given, $\mu = 28.3$ years, $\sigma = 2.3$ years

Ages of baseball players being normally distributed, probability that avg age of 10 randomly selected players is less than 27 years

$n = 10$, $\quad$ Stderror for sample $= \frac{\sigma}{\sqrt{n}}$

z-score for less than 27 years

$= \frac{2.3}{\sqrt{10}} = \frac{2.3}{3.162} = 0.7273$

$z\text{-score} = \frac{27-28.3}{0.7273} = \frac{-1.3}{0.7273} = -1.7874$

$\boxed{\begin{array}{l}\text{P-value} = 0.0375 \\ \quad 3.75\%\end{array}}$ that this sample has avg age less than 27 years.