

Assignment on T-Test

① Given, $\mu = 72$

$$\bar{x} = 69, s = 6.5, n = 25$$

n is small (< 30), sample standard deviation given,
let us calculate t-statistic

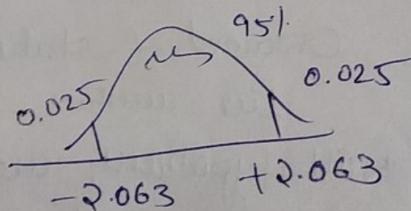
$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{69 - 72}{\frac{6.5}{\sqrt{25}}} = \frac{-3}{\frac{6.5}{5}} = \frac{-3}{1.3} = -2.307$$

Suppose we take at level of significance 5%. (Not mentioned in assuming)

$$t \text{ at } df = n-1 = 25-1 = 24, \alpha/2 = 0.025$$

t-critical values are

$$-2.063, +2.063$$

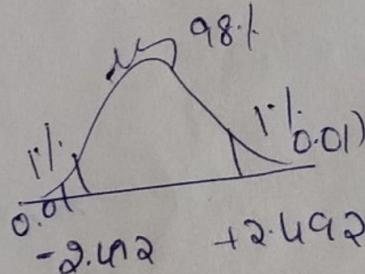


Obtained t-statistic is -2.3 ; outside significant region; so, we don't say aerobic program helped in lowering heart rate.

Suppose we assume at $\alpha = 0.02$ 2% level of significance

$$t \text{-values at } df = 24, \alpha = 0.02 \text{ (0.01)}$$

critical values for region
are ± 2.492



Obtained t-score $= -2.307$ lies within critical values (not outside but within acceptance region)
so, we can say, aerobic program has effective reason for lowering heart rate to 69 beats per minute at 2% level of significance.

(2) Given, $\mu = 15$

$$n = 30, \bar{X} = 17$$

$$S = 5.5$$

Null Hypothesis: $\mu = 15$

Alternate Hypothesis: $\mu > 15$

Two-tailed test at $P < 0.05$

$$t\text{-score} = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}}$$

$$= \frac{17 - 15}{\frac{5.5}{\sqrt{30}}} = \frac{2}{\frac{5.5}{\sqrt{30}}} = \frac{2}{1.01} = 1.98$$

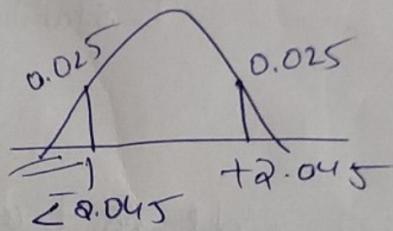
$$\approx t = 1.98 (\approx 2)$$

$$\text{at } df = (30-1) = 29, P < 0.05$$

Obtained 't' statistic ≈ 1.98
lies within 't' values critical

values at $P < 0.05$,

we accept Null Hypothesis that avg lifetime for
particular model of shoe is 15 months; & feel that
sample may due chance it is 17 months



(3)

Given, $\bar{X}_1 = 30$ Relaxation

Control $S_1 = 6.63$

$$n_1 = 15$$

$$\bar{X}_2 = 26$$

$$S_2 = 6.20$$

$$n_2 = 15$$

Two samples, separate groups
Applying t-test

Null Hypothesis: Control & Relaxation groups are same

Alternate Hypothesis: There is difference between Control and
Relaxation groups

$$\text{we have } df = n_1 + n_2 - 2 \\ = 15 + 15 - 2 = 28$$

$$\text{Standard error (SE)} = \sqrt{\frac{(S_1)^2}{n_1} + \frac{(S_2)^2}{n_2}}$$

$$= \sqrt{\frac{(6.63)^2}{15} + \frac{(6.20)^2}{15}}$$

$$= \sqrt{\frac{43.95}{15} + \frac{38.44}{15}}$$

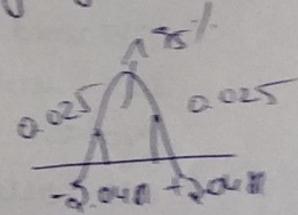
$$= \sqrt{2.93 + 2.56} = \sqrt{5.49} = 2.34$$

$$T\text{-statistic} = \frac{\bar{x}_1 - \bar{x}_2}{SE} = \frac{30 - 26}{2.34}$$

$$= \frac{4}{2.34} = 1.709$$

$df = 28$, t-critical value at 0.05 level of significance

1.709 lies in 't'-values
within Acceptance region



No, change in Control and Relaxation groups, we accept Null hypothesis

(4)

Here, it is Paired Sample t-test

Pairs	1	2	3	4	5	6	7	8	9	10	11	12	13
Control group	38	40	35	36	35	32	31	30	28	26	24	21	18
Relaxation group	35	32	30	34	30	32	28	27	22	22	18	17	14

D → 3 8 5 2 5 0 3 3 6 4 6 4 1

14	15												
34	22												
25	21												

Mean of differences = $\frac{3+8+5+2+5+0+3+3+6+4+6+4+1}{14} = \frac{60}{14} = 4$

D → 9	1	3	4	5	6	7	8	9	10	11	12	13	14
D ²	1	2	25	4	25	0	9	9	56	16	56	16	1

we have, Null hypothesis: Mean of differences of Control and Relaxation group = 0 ($D=0$)

Alternate hypothesis: $H_1(D) \neq 0$

D	Standard Deviation														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
D	3	8	5	2	5	0	3	3	6	4	6	4	1	9	1
D^2	9	64	25	4	25	0	9	9	36	16	36	16	1	81	1
Mean of $D=4$	16	1	4	1	16	1	1	4	0	4	0	9	25	9	
$(D-\text{mean})^2$	16	1	4	1	16	1	1	4	0	4	0	9	25	9	

$$\begin{aligned}
 & (4-3)^2 + (4-8)^2 + (4-5)^2 + (4-2)^2 + (4-5)^2 + (4-0)^2 + (4-3)^2 + \\
 & \quad (4-3)^2 + (4-6)^2 + \\
 & \quad 1 + 16 + 1 + 4 + 1 + 16 + 1 + 1 + 4 + 0 + 4 + 0 + 9 + 25 + 9 + (4-4)^2 \\
 & \quad + (4-6)^2 + (4-4)^2 + (4-1)^2 + (4-9)^2 \\
 & \quad + (4-1)^2
 \end{aligned}$$

$$\begin{aligned}
 \text{mean of Std i.e. } (D-\text{mean})^2 &= 1+16+1+4+1+16+1+1+4+0+4+0+9+25+9 \\
 & \quad / 15 - 1 \\
 & = 92 / 15 = 6.13 \quad 92 / 14 = 6.57 = \sqrt{6.57} \\
 & \quad = 2.56
 \end{aligned}$$

$$\text{Standard error} = \frac{\text{Mean of Std}}{\sqrt{n}}$$

$$= \frac{2.56}{\sqrt{15}} = \frac{2.56}{3.87} = 0.66$$

$$\text{Calculating t-static} = \frac{\text{Mean of } D}{\text{Standard error}} = \frac{4}{0.66} = 6.06$$

t-critical value at 0.05, assume two tailed test

t-stat $t = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$
 $t = \frac{18 - 16}{2.05/\sqrt{10}} = 3.086$
 Obtained t-statistic is 3.086 > 2.01
 lies outside critical line in Rejection Region
 Hence, we reject null hypothesis
 There is difference in Pop means of Control and Relaxation groups based on Training.

⑤ Given, $n=10$, $\alpha=0.05$, $\bar{X}=18$, $\mu=16$

Null Hypothesis: $H_0: \mu = 16$ (complaints per month)

Alternative Hypothesis: $H_1: \mu \neq 16$

$$t\text{-statistic} = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}} = \frac{18 - 16}{\frac{2.05}{\sqrt{10}}} = 3.16$$

$$= \frac{18 - 16}{2.05} = \frac{2}{2.05} = \frac{2}{0.648} = 3.086$$

At $\alpha=0.05$ (assuming), $df=9$ ($10-1$)

t-critical values are ± 2.26

Obtained 't'-statistic at $\alpha=0.05$
 is 3.086 lies within critical values

is Acceptance Region & hence

we accept Null Hypothesis i.e avg complaints received per month is 16 & 18 complaints received is by chance on some day/month.

