Hypothesis Testing- Assignment

(1) Given, from

(a) Null hypothesis → Grades remain same

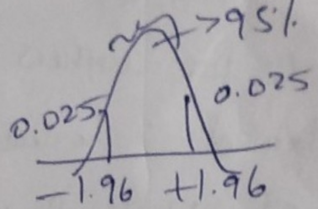
Alternate hypothesis → Grades changed

(b) Standard error → Sampling distribution $\sigma_{SD} = \frac{\sigma_{pop}}{\sqrt{n}} = \frac{\sigma_{sample}}{\sqrt{n}}$

Single sample standard deviation is 0.65 $= \frac{0.65}{\sqrt{256}} = \frac{0.65}{16} = 0.04$

(c) Setting critical regions and what would be at 0.05 alpha level

$z\text{-score} = \frac{\bar{x} - \mu}{\sigma}$



$= \frac{2.85 - 2.75}{0.04}$

$= \frac{0.1}{0.04} = 2.5$

$2.5 > +1.96$

(d) So, we reject Null Hypothesis that grades remain same because obtained z-score which lies outside & greater than interval, i.e is critical region.

(2) Given,

(μ) Sample mean = 52

(S) " Standard deviation = 4.50

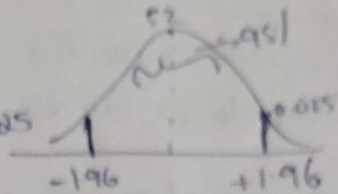
Sample size (100) n = 100

Null hypothesis → Bookstore says, avg cost of its textbook is 52.

Alternate Hypothesis → Statistic Students say cost is higher

$\bar{x}$ - Mean from Random Sample = 52.80

Hypothesis test at 5% level of significance



$$z\text{-score} = \frac{x - \mu}{\frac{s}{\sqrt{n}}}$$

$$= \frac{52.80 - 52}{\frac{4.50}{\sqrt{100}}}$$

$$= \frac{0.80}{0.45} = 1.78$$

1.78 lies within (± 1.96) (at 5% level of significance)

Obtained z-value lies within confidence interval, so we accept Null Hypothesis, i,e avg cost of textbook is Rs 52

(3) Given, Mean(μ) = 34

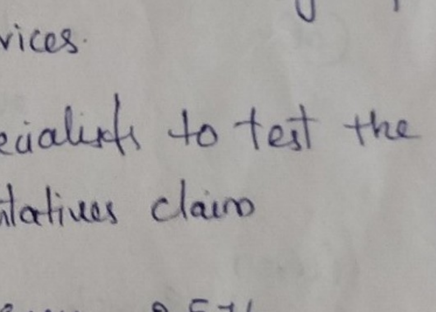
Standard deviation = 8

Null hypothesis → factory representation state that they lowered discharge into rivers by improving filtration devices.

Alternate hypothesis → Environment specialists to test the factory representatives claim

1% level of significance, z-score = 2.576

Sample size(n) = 50, $\bar{x}$ = 32.5



Calculate z-score

$$= \frac{32.5 - 34}{\frac{8}{\sqrt{50}}}$$

$$= -1.5/8/07.07 = -1.5/1.13 = -1.327$$

Calculated z-value (-1.327) lies within 99% (1% level of significance) (b/n -2.576 to +2.576) values,

So, we accept Null Hypothesis at given sample size, 1% level of significance, at given standard deviation that factory representatives using filters to lessen the discharge into rivers.

(4) Given,

<u>Null hypothesis</u>: pop proportion of traveler's check buyers who buy atleast \$2500 in checks when sweepstakes prizes are offered atleast 10% higher than proportion of buyers when no sweepstakes are on.

$$\mu_1 - \mu_2 \leq 10\%$$

$$\mu_1 - \mu_2 \leq 0.1$$

Alternate hypothesis : $\mu_1 - \mu_2 > 0.1$

With Sweep Stakes: $n1 = 300$, $X1 = 120$, $S1 = 0.53$

No Sweepstakes: $n2 = 700$, $X2 = 140$, $S2 = 0.20$

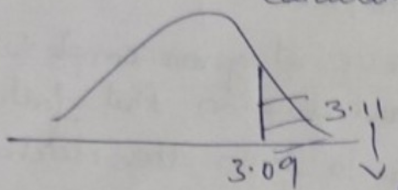
we have prop formula for calculating z-score

$$z\text{-score} = \frac{P_1 - P_2 - D}{\sqrt{\frac{P_1(1-P_1)}{n1} + \frac{P_2(1-P_2)}{n2}}}$$

$$= \frac{\frac{120}{300} - \frac{140}{700} - 0.1}{\sqrt{\frac{\left(\frac{120}{300}\right)\left(\frac{180}{300}\right)}{300} + \left(\frac{140 \times 560/700}{700}\right)}}$$

$$= 3.118$$

For one-tailed test



calculating z at 99% confidence

$\alpha = 0.1\%$
0.001
z-value = 3.09

Obtained z-score is outside critical region, hence we reject Null hypothesis

(5) Given, Sample size (n) = 100

		Expected votes for equal popularity
Hig	= 41	25
Rear	= 19	25
white	= 24	25
Clar	= 16	25

$\chi^2 = 14.96$, df = 3 < 0.05

Observed valued

Expected by each party) = 100/4 = 25

$$\chi^2 = \Sigma \frac{(O-E)^2}{E} = \frac{(41-25)^2 + (19-25)^2 + (24-25)^2 + (16-25)^2}{E}$$

$$= \frac{256 + 36 + 1 + 81}{25}$$

$$= 256/25 + 36/25 + 1/25 + 81/25$$

$$= 10.24 + 1.44 + 0.04 + 3.24 = 14.96$$

df = n = 4 - 1 = 3

At 3 df, chi-square with 0.05 critical value, it is 7.815

for 0.01, it is 13.277

Obtained Chi-Square is greater than original chi-square values hence, it is not possible for above data to be equally popular

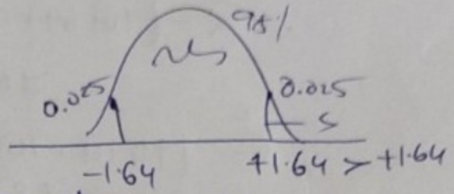
(7) Given:

Null Hypothesis: Height remains same (five years ago and now)
$\mu = 145$

Alternate hypothesis: Height increased > 145
$\mu > 145$

$\alpha = 0.05$,

Single Critical region - Tailed? > 145



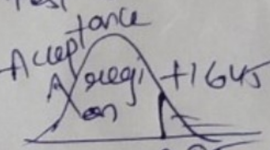
So, greater than +1.64 it is rejection region

$n = 200$, $\bar{x} = 147$, $\mu = 145$, $\sigma = 20$

$$Z\text{-score} = \frac{147 - 145}{\frac{20}{\sqrt{200}}}$$

$$= 1.414$$

For single tailed hypothesis test



$1.414 < 1.645 \rightarrow$ Accept Null hypothesis here

Z-score does not fall in rejection region, so, we cannot reject Null hypothesis; so most of times, the mean height is 145 five years ago and now, 147 is due to some chance by time.

(8) Given, $\mu = 145$, $\sigma = 100$, $\bar{x} = 147$

Null Hypothesis: $\mu = 145$ (145 pods on pea plants)

Alternate hypothesis: $\mu > 147$ (Significant increase in pods on ~~pl~~ pea plants by new planting technique)

To find, test statistic, z-score / t-score, we need sample size (n)

$$Z\text{-score} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \quad (n \text{ is not known})$$

(9) a) Null Hypothesis: Receiving 72 ounces of cheese $\mu = 72$

Alternate Hypothesis: $\mu < 72$ (less cheese than you deserve)

Measurements of cheese: 70, 69, 73, 68, 71, 69, 71 ounces

$\bar{X} = (70+69+73+68+71+69+71)/7-1$

$= 70.14$

$\sigma^2 = (70.14-70)^2 + (70.14-69)^2 + (70.14-73)^2 + (70.14-68)^2 + (70.14-71)^2 + (70.14-69)^2 + (70.14-71)^2 / 7-1$

$= (0.14)^2 + (1.14)^2 + (-2.86)^2 + (2.14)^2 + (-0.86)^2 + (1.14)^2 + (-0.86)^2 / 7-1$

$= 0.0196 + 1.29 + 8.17 + 4.57 + 0.73 + 1.29 + 0.73 / 7-1$

$\sigma^2 = 16.79/6$

$\sigma^2 = 2.79$

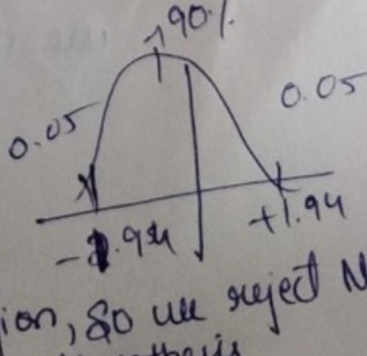
$\sigma = \sqrt{2.79} = 1.67$

Here, it is $n < 30$, sample mean & standard deviation, we calculate t-statistic score

b) $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \dfrac{70.14 - 72}{\frac{1.67}{\sqrt{7}}} = \dfrac{-1.86}{\frac{1.67}{2.64}}$

$= \dfrac{-1.86}{0.63} = -\cancel{2.951} = -2.952$

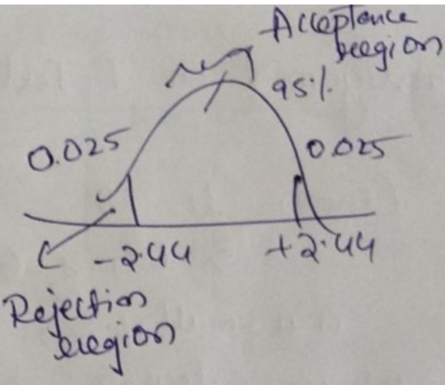
c) α at 10% level
0.1
from t-table at $df = n-1 = 7-1 = 6$
we have critical values as $-1.94, +1.94$
t value -2.952, lies in critical region, so we reject Null Hypothesis



$\alpha = 5\%$ level $= 0.05$

At $df = 6$, 0.025

t-value $= -2.44$ to $+2.44$



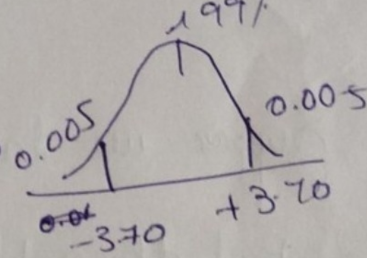
Obtained t-statistic -2.952 lies in critical Rejection Region & hence we reject Null Hypothesis at $\alpha = 5\%$.

$\alpha = 1\%$ level of significance

0.01, $\alpha = 0.005$

t at $df = 6$, $\alpha = 0.005$

t-score values are -3.70 & $+3.70$



Obtained t-statistic -2.952 lies within 1% level of significance, we cannot reject Null Hypothesis; we do accept Null Hypothesis at 1% level of significance thus we receive 12 ounces of cheese only.