Probability Assignment

1. **Basic Probability:**

Sample Space $n(S) = 6^2 = 36$

Event → sum of numbers being 'even' and one of die shows '6'

'e' + 'e' = 'even'

$E = (6,2)(6,4)(6,6)$

$n(E) = 3$

$P(E) = n(E)/n(S) = 3/36 = \boxed{1/12}$

2. $n(S) = 6^2 = 36$

Event → probability for sum of numbers less than 7

$E = (1,1)(1,2)(1,3)(1,4)(1,5)(2,2)(2,3)(2,4)(3,3)$

$P(E) = n(E)/n(S) = 9/36 = \boxed{1/4}$

3. Given, fair coin tossed 3 times, $n(S) = 2^3 = 8$

Event = probability that you observe atleast two heads, given you observe 1H

≥ 2 Heads

$P(2H \cap H) = P(H) \times P(2H/1H)$

we need to find $P\left(\frac{2H}{1H}\right) = \frac{P(2H \cap H)}{P(H)} = \frac{P(2H)}{P(H)}$

$= \frac{4/8}{7/8} = \boxed{\frac{4}{7}}$

$\boxed{P(E) = 4/7}$

Sample Space

HHH
HHT
HTH
THH
THT
TTH
HTT
TTT

4. Given, Sample Space $n(S) = 2^2 = 4$

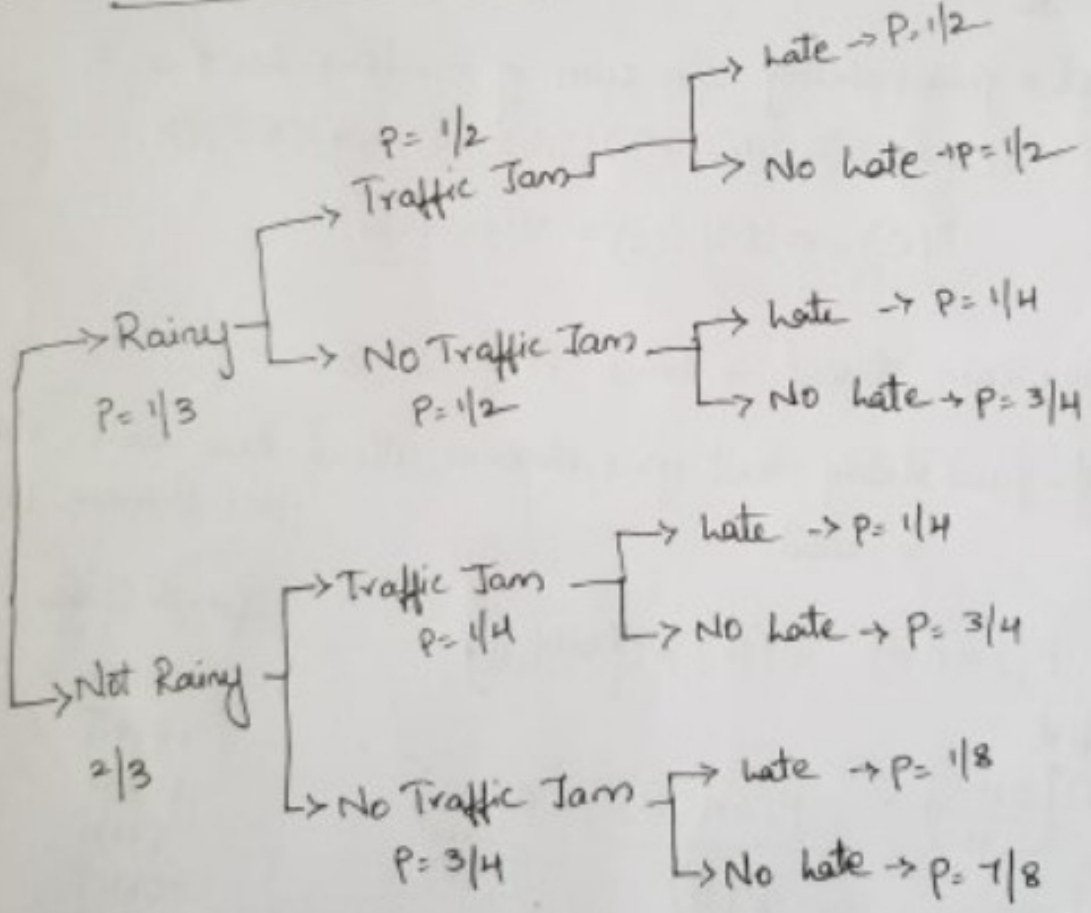
→ (B,B)
(G,G) ✓
(G,B)
(B,G)

Req: Probability that other kid also girl
Given one kid is girl

$$P\left(\frac{2G}{1G}\right) = \frac{P(2G \cap G)}{P(G)} = \frac{1/4}{3/4} = 1/3$$

Req. probability = $\boxed{1/3}$

5. Conditional, Joint, Marginal probability:



a) Probability that it is not raining and there is heavy traffic and I am not late

From the diagram $P(E) = \frac{2}{3} \times \frac{1}{4} \times \frac{3}{4} = \boxed{1/8}$

b) Probability that I am late

From the diagram P(late)

(Rainy × T.jam × late) + (Rainy × NO Traffic jam × late) +
(Not Rainy × Traffic jam × late) + (Not Rainy × No Traffic jam × late)

$= \left(\frac{1}{3} * \frac{1}{2} * \frac{1}{2}\right) + \left(\frac{1}{3} * \frac{1}{2} * \frac{1}{4}\right) + \left(\frac{2}{3} * \frac{1}{4} * \frac{1}{4}\right) + \left(\frac{2}{3} * \frac{3}{4} * \frac{1}{8}\right)$

$\frac{1}{12} + \frac{1}{24} + \frac{1}{24} + \frac{1}{16} = \boxed{11/48 = 0.2291}$

c) Given that I arrived late, what is probability that it rained that day?

$P\left(\frac{\text{late}}{\text{rainy}}\right)$ = from the diagram,

$\left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} * \frac{1}{4}\right) = 1/8 + 1/4 = 3/8$

$P(L/R) = 3/8$

$P(\text{Late}) = 11/48$ (from (b)) $P(\text{Rainy}) = 1/3$

We need to find, $P\left(\frac{\text{Rainy}}{\text{late}}\right) = ?$

$$P\left(\frac{R}{L}\right) = \frac{P(R)P(L/R)}{P(L)}$$

$$= \frac{\frac{1}{3} * \frac{3}{8}}{\frac{11}{48}} = \frac{1}{8} \times \frac{48}{11}$$

$= \boxed{\frac{6}{11}}$

(81)

$$P\left(\frac{\text{Rainy}}{\text{late}}\right) = \frac{P(\text{Rainy} \cap \text{late})}{P(\text{late})} \rightarrow \frac{1/8}{11/48} = \boxed{6/11}$$

(Both Traffic jam & No Traffic jam cases occuring late)

$= \left(\frac{1}{3} \times \frac{1}{2} * \frac{1}{2}\right) + \left(\frac{1}{3} \times \frac{1}{2} * \frac{1}{4}\right) * \left(\frac{3}{8}\right) * = \frac{1}{12} + \frac{1}{24} = 3/24 = 1/8$

6 Given total 3 coins

$P = 2/3$ Regular coin < Heads $\rightarrow P = 1/2$; Tails $\rightarrow P = 1/2$

(1 coin is) $P = 1/3$ fake 2 headed coin — Heads only $\rightarrow P = 1$

a) Probability that it lands heads up?

$P(H) =$ from diagram

$$\left(\frac{2}{3} * \frac{1}{2}\right) + \left(\frac{1}{3} * 1\right)$$

$$= \frac{1}{3} + \frac{1}{3} = \boxed{2/3}$$

b) you pick coin at random, toss it and get heads what is probability that it is two-headed coin?

$$P\left(\frac{\text{2 headed coin}}{\text{Heads}}\right) = \frac{P(\text{2 headed coin})\, P\left(\frac{\text{Head}}{\text{2 headed coin}}\right)}{P(\text{Head})}$$

from the diagram

$$= \frac{\frac{1}{3} * 1}{\frac{2}{3}} = \boxed{\frac{1}{2}}$$

(from (a))

7. Given (a) Coffee $P(Co) = 70\% = 0.7$

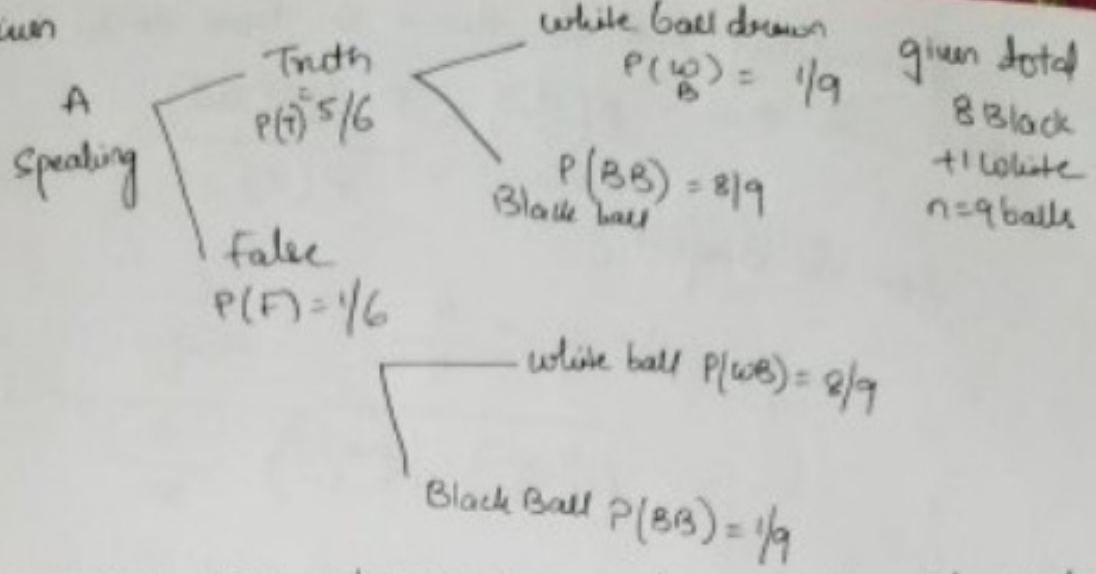
(b) Cake $P(Ca) = 40\% = 0.4$

(c) Both coffee & cake $P(Co \cap Ca) = 20\% = 0.2$

To find $P\left(\frac{Co}{Ca}\right)$ we know that $P\left(\frac{\text{Coffee}}{\text{Cake}}\right) = \frac{P(\text{coffee} \cap \text{Cake})}{P(\text{Cake})}$

$$= \frac{0.2}{0.4} = \boxed{\frac{1}{2} = 0.5}$$

8. Given



Probability that white ball was drawn & he states that white ball was drawn from bag of 9 balls

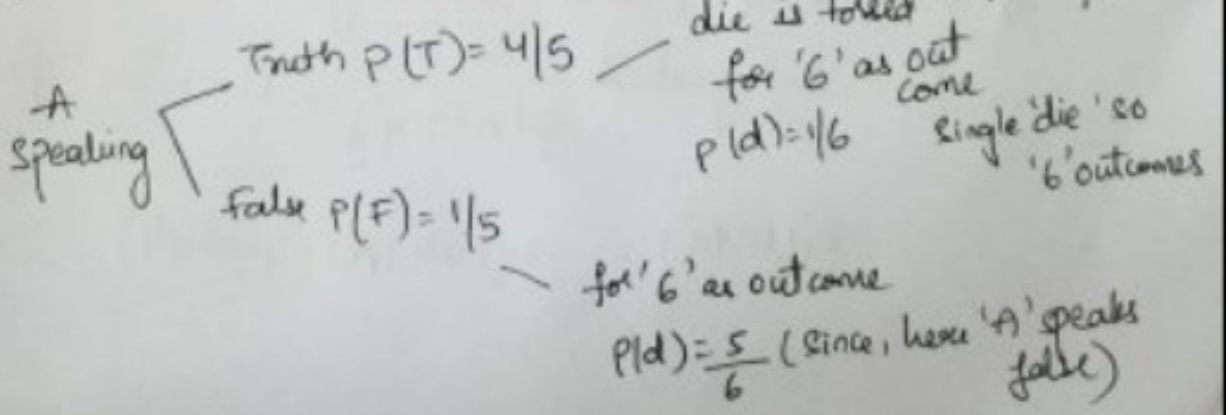
For that, we have to find probability of white ball drawn when he says truth, $P\left(\frac{\text{white}}{\text{Truth}}\right) = \frac{P(WB)}{P(T)}$ - ?

$$P\left(\frac{WB}{T}\right) = \frac{P(WB)\,P\left(\frac{T}{WB}\right)}{P(T)}$$

from the diagram

$$= \frac{\left(\frac{1}{9}\right)\left(\frac{5}{6}\right)}{\left(\frac{5}{6} * \frac{1}{9}\right) + \left(\frac{1}{6} * \frac{8}{9}\right)} = \frac{\frac{5}{54}}{\left(\frac{5}{54} + \frac{8}{54}\right)} = \boxed{\frac{5}{13}}$$

9. Given,



Probability that die shows 6, there to be '6', 'a' speaks truth, for that $P\left(\frac{d}{T}\right) = \frac{P(d)P(T|d)}{P(T)}$

from the diagram,

$$= \frac{\frac{1}{6} * \frac{4}{5}}{\left(\frac{4}{5}\times\frac{1}{6}\right) + \left(\frac{1}{5}\times\frac{5}{6}\right)} = \frac{\frac{4}{30}}{\frac{9}{30}} = \boxed{4/9}$$

Q. 10. Given,

$P(m \cap Sci) = 40\% = 0.4$

$P(M) = 60\% = 0.6$

$P\left(\frac{Sci}{Math}\right) = ?$

we know that, $P\left(\frac{Sci}{math}\right) = \frac{P(Sci \cap Math)}{P(math)}$

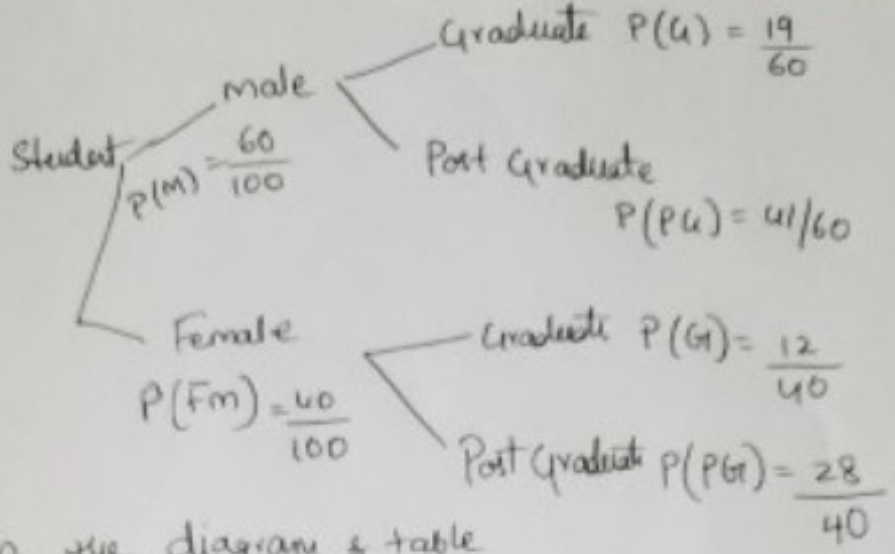
$$= \frac{0.4}{0.6} = \boxed{2/3}$$

11.

	Graduate	Post Graduate	Total
Male	19	41	60
FeMale	12	28	40
Total	31	69	100

a) Probability that a randomly selected individual is Male and graduate $P(M \cap G)$ → It is Joint Probability

$P(M \cap G) = P(Male) \cap P(Graduate)$



from the diagram & table

$$P(M \cap G) = \frac{19}{100} = \boxed{0.19}$$

b) Probability that randomly selected individual is Male?
→ Marginal Probability

$$P(\text{Male}) = 60/100 = 0.6$$

c) Probability that randomly selected individual being graduate
P(Graduate) only graduate individuals irrespective of (gender), so, a Marginal probability

from diagram's table $P(G) = 31/100 = 0.31$

d) Probability that randomly selected person is female given that selected person is post-Graduate.
This is Conditional Probability, asking for female who is post-graduate

$$P\left(\frac{\text{female}}{\text{Post Graduate}}\right) = P\left(\frac{F}{PG}\right) = \frac{28}{40}$$

↑ from diagram

12. Given, Bayes Theorem

Submit Fraud fillings (0.1) $P(F.F)=0.1$

- Algorithm to detect Fraud Positive $P\left(\frac{Pos}{F.F}\right) = 92\% = 0.92$
- Negative $P\left(\frac{Neg}{FraF}\right) = 8\% = 0.08$

No Fraud (0.9) $P(NFF)=0.9$

- Positive $P\left(\frac{Pos}{No\ Fraud}\right) = 10\% = 0.1$
- Negative $P\left(\frac{Neg}{No\ Fraud}\right) = 90\% = 0.9$

we observe, algorithm test returns fraud result, probability that this company truly did fraud in their filings, for that;

$$P\left(\frac{Fraud\ Filing}{Pos}\right) = \frac{P(Fraud\ Fili)\ P\left(\frac{Pos}{F.F}\right)}{P(Pos)}$$

$$= \frac{0.1 \times 0.92}{(0.1 \times 0.92) + (0.9 \times 0.1)}$$

$$= \frac{0.092}{(0.092 + 0.09)} = \frac{0.092}{0.182} = \boxed{0.505}$$

Probability is 50% that company did Fraud in Filings.

14. Given,

Population

- swine flu effected $\rightarrow P(SFe) = 0.0001$ (1 in 10,000 people)
  - Tested Positive $P\left(\frac{Pos}{SFe}\right) = 99\% = 0.99$
  - Tested Negative $P\left(\frac{Neg}{SFe}\right) = 0.1$
- swine flu not effected $P(NSFe) = 0.9999$
  - Tested Positive $P\left(\frac{Pos}{NSFe}\right) = 0.01$ (1%)
  - Tested Negative $P\left(\frac{Neg}{NSFe}\right) = 0.99$

we need to find, whether the person has swineflu
A person can have swineflu when tested +ve

$$P\left(\frac{SFe}{Pos}\right) = \frac{P(SFe)\,P\left(\frac{Pos}{SFe}\right)}{P(Pos)}$$

$$= \frac{0.0001 \times 0.99}{(0.0001 \times 0.99) + (0.9999 \times 0.01)}$$

$$= \frac{0.000099}{(0.000099 + 0.009999)}$$

$$= \frac{0.000099}{0.0100}$$

$$= 0.0099$$

$$\simeq 0.01$$

Probability of having effected with swineflu when tested +ve is **0.01 or 1%**