1. Maximum Likelihood Estimation (MLE) for Normal Distribution Parameters

→ Given a random sample $(x_1, x_2, x_3, \ldots, x_n)$ from a Normal Distribution with mean μ and variance σ^2, the likelihood function $L(\mu, \sigma^2)$ is given by:

$$L(\mu, \sigma^2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

Taking the natural log, we get the log likelihood function:

$$\ell(\mu, \sigma^2) = -\frac{n}{2}\log(2\pi) - \frac{n}{2}\log(\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2$$

Now, to find the maximum likelihood estimators, we differentiate the log likelihood func^n with respect to μ and σ^2, set the derivatives equal to zero, and solve for μ and σ^2.

$$\frac{d\ell}{d\mu} = \frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i-\mu) = 0$$

$$\frac{d\ell}{d\sigma^2} = \frac{-n}{2\sigma^2} + \frac{1}{2(\sigma^2)^2}\sum_{i=1}^{n}(x_i-\mu)^2 = 0$$

Solving these equations will give us the MLE's for μ and σ^2.

<u>for μ:</u>

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

<u>for σ^2:</u>

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\hat{\mu})^2$$

So, the maximum likelihood estimates for parameters are:

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\hat{\mu})^2$$

2. Maximum likelihood estimator for 'θ'.

The Probability mass func^n (PMF) of a binomial distribution is given by :

$$f(x;\, m, \theta) = {}^{m}C_{x}\, \theta^{x} (1-\theta)^{m-x}$$

likelihood func^n is the product of these probabilities for the given sample:

$$L(\theta) = \prod_{i=1}^{n} \left({}^{m}C_{x_i}\right) \theta^{x_i} (1-\theta)^{m-x_i}$$

taking the natural log, we get the log likelihood func^n:

$$l(\theta) = \sum_{i=1}^{n} \left[\log\left({}^{m}C_{x_i}\right) + x_i \log(\theta) + (m-x_i)\log(1-\theta)\right]$$

To find the maximum likelihood estimator for 'θ', we differentiate the log likelihood func^n w.r.t 'θ', set it equal to 0, and solve for 'θ'.

$$\frac{dl}{d\theta} = \sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta}\right] = 0$$

$$\sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta}\right] = 0$$

$$\sum_{i=1}^{n} = \left[\frac{x_i - \theta m}{\theta(1-\theta)}\right] = 0$$

$$\frac{\sum_{i=1}^{n} x_i}{\theta} - \frac{nm}{\theta} = 0$$

$$\sum_{i=1}^{n} x_i = nm\theta$$

$$\hat{\theta} = \frac{\sum_{i=1}^{n} x_i}{nm}$$

So, the maximum likelihood estimator for 'θ' is

$$\hat{\theta} = \frac{\sum_{i=1}^{n} x_i}{nm}$$