SUNIR SINGH
102117209
3CS8

_/_/_

PARAMETER ESTIMATION

Q) Let $(X_1, X_2, \dots X_n)$ be a random sample of size 'n' taken from a normal population w/ parameters θ_1, mean; Variance $= \theta_2$. Find max. likelihood estimates of these parameters.

$$MLE(\theta_1) = \text{joint pdf } (x_1, x_2, \dots x_n \mid \theta_1)$$

$$MLE(\theta_2) = \text{joint pdf } (x_1, x_2, \dots x_n \mid \theta_2)$$

$$MLE(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i - \theta_1)^2}{2\theta_2}}$$

Take logarithm on both sides.

$$\ln(\theta_1, \theta_2) = \sum_{i=1}^{n} \left[-\ln(\sqrt{2\pi\theta_2}) - \frac{(x_i - \theta_1)^2}{2\theta_2} \ln(e) \right]$$

$$= -n\ln(\sqrt{2\pi\theta_2}) - \frac{\sum x_i^2}{\theta_2^2} - \frac{n\theta_1^2}{\theta_2^2} + \frac{\theta_1 \sum x_i}{\theta_2}$$

diff wrt θ_1

$$-\frac{2n\theta_1}{2\theta_2} + \frac{1}{\theta_2}\sum x_i = 0$$

$$\left[\theta_1 = \frac{\sum x_i}{n}\right]$$

(P.T.O)

$\Rightarrow \theta_1 = \bar{x}$ sample mean of y

diff. wrt θ_2

$$= \frac{-n}{\sqrt{2\pi n}} \cdot \sqrt{2\pi} \times \frac{1}{2\sqrt{\theta_2}} + \frac{\sum x_i^2}{2\theta_2^2} + \frac{n\theta_1^2}{2\theta_2^2} - \frac{\theta_1 \sum x_i}{\theta_2^2}$$

$$\Rightarrow \frac{-n\theta_2 + \sum x_i^2 + n\bar{x}^2 - 2\bar{x}\sum x_i}{2\theta_2^2} = 0$$

$$\theta_2 = E(x_i^2) + E^2(x) + 2E^2(x)$$

$$= E(x_i^2) - E^2(x)$$

$\theta_2 =$ variance (x)

$\theta_1 =$ mean (x)

(P.T.O.)

___/___/___

Let $X_1, X_2, \ldots X_n$ be a random sample from $B(m, \theta)$ dist. where $\theta \in (0,1)$ is unknown and m is a known +ve integer. Get θ using M.L.E.

$$\text{Pdf} = {}^{n}C_x\, \theta^x (1-\theta)^{n-x}$$

$$L(\theta \mid x \ldots x_m) = \text{joint pdf } (x_1 \ldots x_n \mid \theta)$$

$$= \prod_{i=1}^{n} {}^{m}C_{x_i}\, \theta^{x_i} (1-\theta)^{m-x_i}$$

Take log both sides

$$\Rightarrow \sum_{i=1}^{n} \left[\ln ({}^{m}C_{x_i}) + x_i \ln\theta + (m-x_i) \ln(1-\theta) \right]$$

diff. w.r.t θ

$$= \frac{\Sigma x_i}{\theta} + \frac{nm - \Sigma x_i}{1-\theta} (-1) = 0$$

$$\Rightarrow \frac{\Sigma x}{nm} = \theta$$

$$\theta = \frac{\bar{x}}{m}$$

$\Rightarrow \bar{x} \Rightarrow$ mean of sample