Logarithm: Definition, Properties, and Examples

Definition of Logarithm >>

A logarithm is the inverse operation of exponentiation. It helps to determine the exponent to which a base number must be raised to obtain another number.

The logarithm of a number x with base b is written as :

$$\mathbf{Log_b x = y}$$

which means,

by=xb^y = xby=x

where:

- b is the base (must be greater than 0 and not equal to 1),
- is the number (must be positive),
- Y is the exponent or logarithm of x to the base b.

For example:

$$\log_2 8 = 3 \text{ , since } 2^3 = 8.$$

Common Types of Logarithms

1. **Common Logarithm (Base 10)**:

   logx = log10x

   Example : log 1000 = 3 since $10^3 = 1000$.

2. **Natural Logarithm (Base eee)**:

   lnx=logex\ln x = \log_e xlnx=logex

where e≈2.718e \approx 2.718e≈2.718 (Euler's number).
Example: ln⁡e=1\ln e = 1lne=1, since e1=ee^1 = ee1=e.

3. **Binary Logarithm (Base 2)**:

$\log_2 x$

Example: $\log_2 16 = 4$ since $2^4 = 16$.

- ## Properties of Logarithms :

Logarithms follow several important properties that make calculations easier.

1. **Product Rule**:

$\log_b(xy) = \log_b x + \log_b y$

Example: $\log_2(8 \times 4) = \log_2 8 + \log_2 4 = 3 + 2 = 5$ since $2^5 = 32$.

2. **Quotient Rule**:

$$\log\left(\frac{x}{y}\right) = log_b x - log_b y$$

Example: $log_2(\frac{8}{2}) = log_2 8 - log_2 2 = 3 - 1 = 2$.

3. **Power Rule**:

$log_b(x^n) = n\log_b x$

Example: $\log_2(8^2) = 2\log_2 8 = 2 \times 3 = 6$.

4. **Change of Base Formula**:

$\log_b x = log_c / \ log\ c^b$

Example: log2⁡10=log10⁡/log2⁡≈10.301≈3.32\log_2 10 = \frac{\log 10}{\log 2} \approx \frac{1}{0.301} \approx 3.32log2 10=log2log10≈0.3011≈3.32.

5. **Logarithm of 1**:

$\log_b 1 = 0$

since any number raised to the power 0 is 1.

6. **Logarithm of the Base**:

$\log_b b = 1$

since $b^1 = b$.

Examples of Logarithm Applications >>

1. **<u>Solving Equations</u> :**
   Find x in $3^x=81$
   $X = \log_3 81$
   Since $3^4 = 81$ we get $x = 4$.
2. **<u>Growth and Decay Problems</u> :**
   Logarithms are used in exponential growth (population growth, bacteria growth) and decay (radioactive decay, depreciation).
3. **<u>Computing Large Numbers</u>** :
   In engineering and physics, logarithms help simplify complex calculations involving very large or small numbers.
4. **<u>Computer Science</u>** :
   Logarithms are essential in algorithms (e.g., binary search runs in $O(\log_n)$ time complexity).

: <u>Conclusion</u> :

Logarithms are a fundamental mathematical tool that simplify complex calculations involving exponentiation. Their properties make them useful in various fields such as science, engineering, finance, and computing.