Limitations and Countermeasures of PID Controllers

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It is well-known that PID controllers show poor control performances for an integrating process and a large time delay process. Moreover, it cannot incorporate ramp-type set-point change or slow disturbance. We discuss the above-mentioned limitations of the PID controller. Through the analysis of the limitations, simple techniques are developed to avoid these poor control performances and incapabilities. We introduce a control structure composed of two PID controllers to manipulate both the set-point change process and the input disturbance rejection process. A simple feedforward and feedback loop are proposed to treat a ramp-type set point and disturbance (low-frequency set point and disturbance). In addition, an internal feedback loop is introduced to incorporate integrating processes, and a tuning strategy using the model reduction is proposed to efficiently tune the controller for high-order processes. A dead-time compensator is used to manipulate a large time delay process more efficiently.

Introduction

The PID controller has been widely used in industry since it is simple and robust, moreover familiar to the field operator. There are many identification methods to obtain the integrator plus time delay model, the first-order plus time delay model, or second-order plus time delay model for the tuning of the PID controller (Chen (1989), Friman and Waller (1994), Huang and Chou (1994), Huang et al. (1982), Huang and Huang (1993), Jutan and Rodriguez (1984), Lee and Sung (1993), Lee (1989), Lee et al. (1990), process reaction curve method, Sung et al. (1994), Sung et al. (1996)). Also, there are many tuning rules developed for these models such as the TL (Tyreus and Luyben) tuning rule, ZN (Ziegler−Nichols), IMC (internal model control), ITAE (the integral of the time weighted absolute value of the error), Cohen−Coon tuning methods, and a tuning rule using desired trajectory (Cohen and Coon (1953), Lee et al. (1990), Lopez et al. (1967), Morari and Zafiriou (1989), Sung et al. (1995a), Sung et al. (1996), Tyreus and Luyben (1992), Ziegler and Nichols (1942)).

In recent years, PID autotuning methods have been proposed to improve the control performance of the PID controller and to automatically tune the PID controller. After Åström and Hägglund (1984) introduced the original relay feedback identification method, many modified relay feedback methods have been proposed and applied to many industrial processes (Åström et al. (1992), Åström et al. (1993), Chang and Shen (1992), Dumont et al. (1989), Friman and Waller (1994), Hang et al. (1993), Hägglund and Åström (1991), Hwang (1995), Kim (1995), Lee et al. (1993), Lee and Sung (1993), Li et al. (1991), Lin and Yu (1993), Loh et al. (1993), Loh and Vasnani (1994), Luyben (1987), Schei (1992), Shen and Yu (1994), Sung et al. (1995b)) and several autotuning methods using the proportional (P) controller have been proposed to derive the transfer function of the process and tune the PID controller automatically (Chen (1989), Hwang (1995), Jutan and Rodriquez (1984), Lee (1989), Lee et al. (1990), Sung et al. (1994), Sung and Lee (1995), Yuwana and Seborg (1982)). These autotuning theories contribute to improve the control performance and wide applications of the PID controller.

Also, many modified PID controllers are proposed to improve the control performance. For example, the proportional gain can be varied corresponding to the magnitude of the error (Luyben (1989)). The control action becomes more aggressive as the process output deviates from the set point further. Cheung and Luyben (1980) studied a PI controller with proportional and integral gain varied as a function of the error. A nonlinear PI-type algorithm proposed by Jutan (1989) used trends of the past errors or process measurements instead of the derivative term of the error of the linear PID controller. This controller showed better performance than the linear PID controller in terms of sensitivity to measurement noise and ITAE (the integral of the time weighted absolute value of the error). Rugh (1987) designed a nonlinear PID controller with parameters that are continuous functions of the manipulated input corresponding to the operating point. The nonlinear PID controller maintained the same response characteristics at various operating points. Sung et al. (1995a) proposed a modified PID controller to efficiently manipulate both the servo and the regulatory process.

Even though the above-mentioned autotuning methods and modified PID controllers have contributed to the improvement of the control performance and the simplification of the design procedure, they still cannot deal with many types of processes. It is well-known that the PID controller cannot treat large time delay processes efficiently. For a ramp-type set-point change or disturbance, the controller shows offset and the performances of the PID controller are poor for slow set-point tracking or slow disturbance rejection. Moreover, the PID controller shows a large overshoot for a step set-point change in an integrating process. Also, easy and efficient tuning rules for an integrating process and a high-order process are rare.

Therefore, we propose a control structure composed of two PID controllers and internal feedback loops to incorporate various processes (for clarity, refer to Figures 4 and 6). Here, two PID controllers are combined to separately manipulate the set-point tracking and disturbance rejection process, and an internal feedback loop is joined to control an integrating process efficiently. Furthermore, a simple dead time compensator is implemented to treat a large time delay process. A

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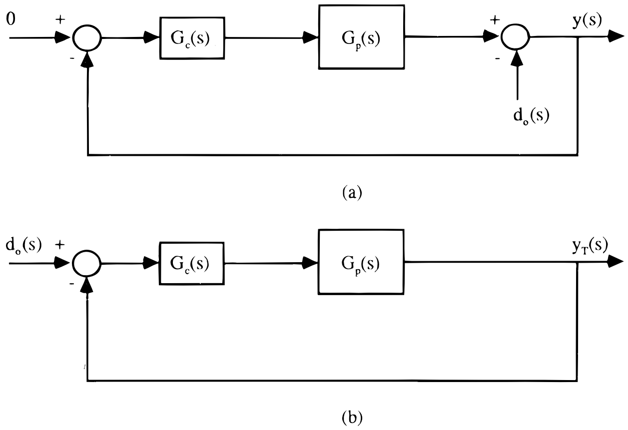

Figure 1. Equivalence of servo process and regulatory processes.

simple feedforward and a feedback loop are introduced to incorporate a ramp-type set-point change or disturbance. A model reduction method is used to tune the PID controller efficiently for a high-order process.

Limitations of PID Controllers and Guidelines for Improvement

1. Difficulties in Controlling Both Set-Point Tracking and Disturbance Rejection Processes. Consider the feedback control structure in Figure 1. Parts a and b of Figure 1 represent an output disturbance rejection process and a set-point tracking process, respectively. If the PID controller works well for the step set-point change or step output disturbance rejection process, it usually shows a poor performance (sluggish action) for the usual disturbance rejection process, because the step input disturbance is more common than the step output disturbance. Even though the disturbance transfer function connecting the process disturbance with the output disturbance is much faster than the process dynamics, if the disturbance itself has slow dynamics, the disturbance effects (output disturbance) on the process output would show slower dynamics. Usually, the controller has been designed based on the very fast disturbance dynamics (that is, step output disturbance or step set-point change) even though its dynamics is usually slower than that of the process. Then, it shows a very sluggish response for the slow disturbance. On the other side, if it is designed to guarantee a good control performance for the step input disturbance, it shows too aggressive control action for the step set-point change.

Proposition 1. Optimal design parameters based on the output disturbance rejection are the same as those based on the set-point tracking if the set-point dynamics is the same as the output disturbance.

Proof. Consider the output disturbance rejection process of Figure 1a. By inspection the following equation can be obtained.

$$y(s) = \frac{-d_o(s)}{1 + G_c(s)\, G_p(s)} \tag{1}$$

Here, $y(s)$ and $d_o(s)$ denote the process output and the output disturbance, and $G_c(s)$ and $G_p(s)$ represent the transfer function of the controller and the process, respectively. Let us define the following variable to show that the disturbance rejection problem is essentially the same as the set-point tracking problem (if the dynamics of the set point is the same as that of the disturbance).

$$y_T(s) = y(s) + d_o(s) \tag{2}$$

Then,

$$y_T(s) = \frac{G_c(s)\, G_p(s)\, d_o(s)}{1 + G_c(s)\, G_p(s)} \tag{3}$$

It should be noted that (3) is the transfer function of the set-point tracking as shown in Figure 1b. Usually, the objective of the controller design is as follows.

$$\min_{\text{parameters}} ||y(t)|| = \min_{\text{parameters}} ||y_T(t) - d_o(t)|| \tag{4}$$

From (1), (3), and (4), we can recognize that optimal design parameters based on the disturbance rejection (left-hand side of (4)) are the same as those based on the set-point tracking problem (right-hand side of (4)) with $y_s(s) = d_o(s)$. Here, $y_s(s)$ denotes the set point.

From proposition 1, the following discussion can be applied to both the set point tracking and disturbance rejection problems. Thus, we would not explain the set-point tracking and the disturbance rejection problem separately.

Consider the following to comprehend closed-loop responses according to the dynamics of the set-point change (or the output disturbance). Assume that the following set point (or disturbance) affects the process output

$$d_o(s) = \frac{1}{(\tau_{dis} s + 1)} \left\{ \frac{\text{dis}}{s} \right\} \tag{5}$$

where τ_{dis} determines the dynamics of the set point. Then, (3) becomes

$$y_T(s) = \frac{1}{(\tau_{dis} s + 1)} \left\{ \frac{G_c(s)\, G_p(s)\, \text{dis}}{(1 + G_c(s)\, G_p(s))s} \right\} \tag{6}$$

Here, it should be noted that (5) is the filtered step set point and (6) is the filtered process output corresponding to the step set-point change by a low-pass filter. That is,

$$y_T(s) = \frac{1}{(\tau_{dis} s + 1)} y_{step}(s) \tag{7}$$

where $y_{step}(s)$ denotes the process response for the step set-point change dis/s. Therefore, from (5) and (6), we can realize that the filtered error $d_o(s) - y_T(s)$ has a larger phase lag and a smaller amplitude ratio than the original error dis/s − $y_{step}(s)$ because the set-point dynamics (or output disturbance) is a low-pass filter.

$$\angle\{d_o(j\omega) - y_T(j\omega)\} = -\arctan(\tau_{dis}\omega) + \angle\{(\text{dis}/j\omega) - y_{step}(j\omega)\} \tag{8}$$

$$|d_o(j\omega) - y_T(j\omega)| = \frac{|(\text{dis}/j\omega) - y_{step}(j\omega)|}{\sqrt{1 + (\tau_{dis}\omega)^2}} \tag{9}$$

Therefore, we can conclude that, when τ_{dis} is large, the controller tuned optimally based on the step set-point

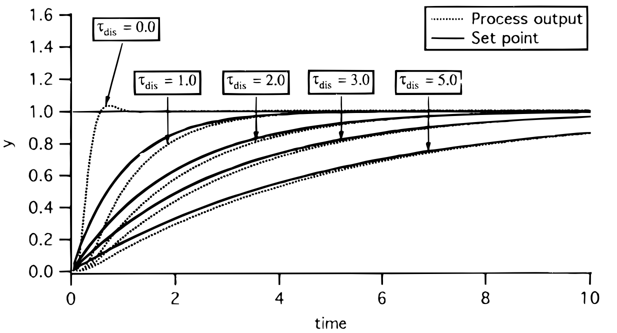

Figure 2. Effects of the set-point dynamics when the PID controller is tuned for the step set-point change.

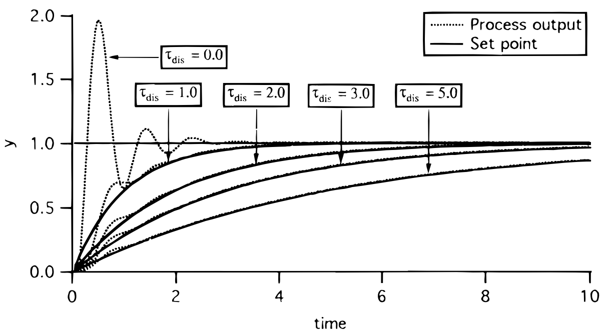

Figure 3. Effects of the set-point dynamics when the PID controller is tuned for the step input disturbance rejection.

change shows a sluggish control action in rejecting the error of the slow set-point change process because the error $dis/s - y_{step}(s)$ is filtered to a slower error $d_o(s) - y_T(s)$.

Consider the control results of Figures 2 and 3 to demonstrate the effects of τ_{dis} more easily. Figure 2 shows that the control results of the PID controller tuned for the step set-point change problem ($\tau_{dis} = 0$) are too sluggish for the slow set-point change because of the low-pass dynamics of the set point (or output disturbance). The larger is τ_{dis}, the poorer is the control performance in terms of the offset rejection. Figure 3 depicts that control results of the PID controller tuned to reject the input step disturbance (that is, with the assumption that the dynamics of the process is the same as that of the disturbance) take a too aggressive control action to the step set-point change or the step output disturbance ($\tau_{dis} = 0$). However, since these oscillatory responses for the step set-point change ($\tau_{dis} = 0$) are filtered by the low-pass filter (disturbance dynamics), a good control action for slow set-point tracking or slow disturbance rejection can be guaranteed as shown in Figure 3.

In the simulations, the process transfer function is

$$G_p(s) = \frac{\exp(-0.15s)}{(s+1)^2} \tag{10}$$

and the PID tuning rule of Table 1 is used to tune the PID controller. The tuning rule shows almost the same tuning results as those of the optimal tuning parameters in the viewpoint of the integral of the time weighted absolute error (ITAE) (Sung et al. (1996)). They obtained extensive optimal tuning parameters by solving an optimization problem with the ITAE as the objective function. Then, they developed the tuning rule by fitting the optimal tuning parameters (for details, refer to Sung et al. (1996)). This tuning rule is based on the

Table 1. PID Tuning Rule for the Second-Order Plus Time Delay Model

Set-Point Change

$$k_m k_c = -0.04 + \left\{0.333 + 0.949\left(\frac{\theta_m}{\tau_m}\right)^{-0.983}\right\}\xi_m, \qquad \xi_m \le 0.9$$

$$k_m k_c = -0.544 + 0.308\left(\frac{\theta_m}{\tau_m}\right) + 1.408\left(\frac{\theta_m}{\tau_m}\right)^{-0.832}\xi_m, \qquad \xi_m > 0.9$$

$$\frac{\tau_i}{\tau_m} = \left\{2.055 + 0.072\left(\frac{\theta_m}{\tau_m}\right)\right\}\xi_m, \qquad \frac{\theta_m}{\tau_m} \le 1$$

$$\frac{\tau_i}{\tau_m} = \left\{1.768 + 0.329\left(\frac{\theta_m}{\tau_m}\right)\right\}\xi_m, \qquad \frac{\theta_m}{\tau_m} > 1$$

$$\frac{\tau_m}{\tau_d} = \left\{1 - \exp\left(-\frac{\left(\frac{\theta_m}{\tau_m}\right)^{1.060}\xi_m}{0.870}\right)\right\}\left\{0.55 + 1.683\left(\frac{\theta_m}{\tau_m}\right)^{-1.090}\right\}$$

Disturbance Rejection

$$k_m k_c = -0.67 + 0.297\left(\frac{\theta_m}{\tau_m}\right)^{-2.001} + 2.189\left(\frac{\theta_m}{\tau_m}\right)^{-0.766}\xi_m, \qquad \frac{\theta_m}{\tau_m} < 0.9$$

$$k_m k_c = -0.365 + 0.260\left(\frac{\theta_m}{\tau_m} - 1.4\right)^2 + 2.189\left(\frac{\theta_m}{\tau_m}\right)^{-0.766}\xi_m, \qquad \frac{\theta_m}{\tau_m} \ge 0.9$$

$$\frac{\tau_i}{\tau_m} = 2.212\left(\frac{\theta_m}{\tau_m}\right)^{0.520} - 0.3, \qquad \frac{\theta_m}{\tau_m} < 0.4$$

$$\frac{\tau_i}{\tau_m} = -0.975 + 0.910\left(\frac{\theta_m}{\tau_m} - 1.845\right)^2 + \left\{1 - \exp\left(-\frac{\xi_m}{0.15 + 0.33\left(\frac{\theta_m}{\tau_m}\right)}\right)\right\} \times \left\{5.25 - 0.88\left(\frac{\theta_m}{\tau_m} - 2.8\right)^2\right\}, \qquad \frac{\theta_m}{\tau_m} \ge 0.4$$

$$\frac{\tau_m}{\tau_d} = -1.9 + 1.576\left(\frac{\theta_m}{\tau_m}\right)^{-0.530} + \left\{1 - \exp\left(-\frac{\xi_m}{-0.15 + 0.939\left(\frac{\theta_m}{\tau_m}\right)^{-1.121}}\right)\right\} \times \left\{1.45 + 0.969\left(\frac{\theta_m}{\tau_m}\right)^{-1.171}\right\}$$

following second-order plus time delay model and the ideal PID controller.

$$G_p(s) = \frac{k_m \exp(-\theta_m s)}{\tau_m^2 s^2 + 2\tau_m \xi_m s + 1} \tag{11}$$

$$G_c(s) = k_c\left(1 + \frac{1}{\tau_i s} + \tau_d s\right) \tag{12}$$

Here, k_c denotes the proportional gain, τ_i and τ_d denote integral time and derivative time, respectively, and k_m, τ_m, ξ_m, and θ_m represent static gain, time constant, damping factor, and time delay, respectively.

In summary, if the controller is well tuned for the step set-point change or the step output disturbance rejection, the controller shows poor control results for the input step disturbance (or slow output disturbance) or slow set-point tracking, and vice versa. The optimally tuned PID controller for the set-point tracking process

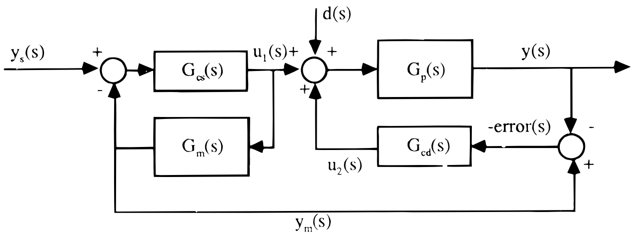

Figure 4. Proposed control structure to manipulate the servo and the regulatory problems separately.

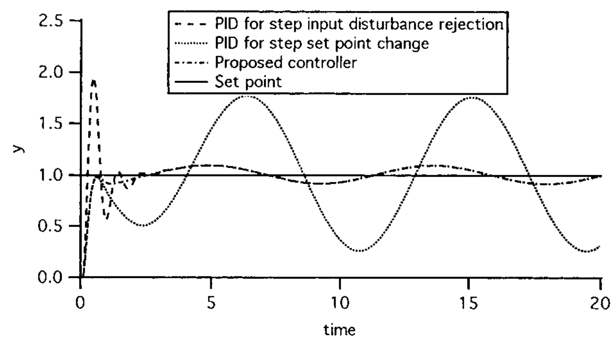

Figure 5. Controls results of the proposed method and the conventional PID controller.

is also optimal for the disturbance rejection process if the dynamics of the set-point change is the same as that of the disturbance.

We propose a combined control structure composed of two PID controllers tuned to work well in the step set-point change process and the step input disturbance rejection process, respectively. The proposed control structure is shown in Figure 4. Here, $G_m(s)$ and $G_p(s)$ represent the model and the process, $d(s)$ denotes the input disturbance, and two PID controllers, $G_{cs}(s)$ and $G_{cd}(s)$, are well tuned for the step set-point change process and the step input disturbance process, respectively. It should be noted that the set-point tracking and the disturbance rejection are separated. We can obtain the following equations by inspection with the assumption of the perfect model.

$$y_m(s) = G_m(s)\, u_1(s) \tag{13}$$

$$y(s) = G_p(s)\, (u_1(s) + u_2(s) + d(s)) \tag{14}$$

$$\mathrm{error}(s) = G_p(s)\, (u_2(s) + d(s)) \tag{15}$$

From (15), we recognize that the controller $G_{cd}(s)$ only participates to reject the input disturbance. Also, if the disturbance is zero, the remaining equation is (13) so that the remaining process becomes the set-point tracking process controlled by only the controller $G_{cs}(s)$. Therefore, the proposed structure can manipulate the set-point change and the disturbance rejection process, separately. Consider the simulation results of Figure 5, where both the set-point change and the input disturbance enter at $t = 0$. The process and disturbance are

$$G_p(s) = \frac{\exp(-0.15s)}{(s+1)^2} \tag{16}$$

$$d(s) = 5 \sin(3.606t/5) \tag{17}$$

where the ultimate frequency of the process is 3.606. The conventional PID controller tuned for the step set-point change shows a very sluggish action for the slow disturbance, and the conventional PID controller for the step input disturbance rejection shows too aggressive control action in the beginning of the step set-point change. Contrarily, the proposed controller shows a good control performance in both cases.

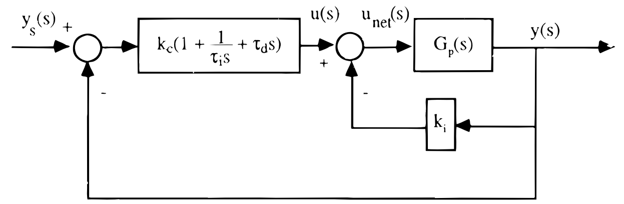

Figure 6. Proposed control strategy to control the integrating process.

2. Difficulties in Controlling the Integrating Process. Until now, simple identification methods to derive the transfer function of the integrating process and corresponding tuning rules are rare. Moreover, for the step set-point change process, it is difficult to efficiently control the integrating process using only the PID controller due to the integrator of the process. Relay feedback identification methods can be used to identify the ultimate data of the integrating process, and then a modified Ziegler–Nichols tuning method can be used to tune the PID controller for the integrating process (Åström and Hägglund (1984), Åström et al. (1992), Sung et al. (1995b)). This method considers only one point of the Nyquist plot of the process to tune the PID controller so that the tuning result can be poor. To derive the process transfer function of the integrating process, an autotune variation (ATV) method can be used (Luyben (1987), Li et al. (1991)). However, the method needs several relay feedback tests so that the identification work is tedious and requires a long identification time.

We propose a modified PID control strategy combined with an internal feedback loop to control the integrating process of single integrator efficiently. In this paper, a P controller is used as an internal feedback loop. The P controller plays an important role in converting the integrating process to an open-loop stable process. In addition, we propose a simple identification method and tuning rule to tune the proposed controller. The proposed method shows almost the same control results as the optimally tuned PID controller for the disturbance rejection process and better control results than the optimally tuned PID controller for the set-point change process.

2.1. Proposed Control Structure and Tuning Rule for the Integrating Process. The proposed control strategy is shown in Figure 6. Here, k_i plays an important role in converting the integrating process to an open-loop stable process. To identify the process of single integrator, we use the following simple integrating model.

$$G_m(s) = \frac{k \exp(-\theta s)}{s(\tau s + 1)} \tag{18}$$

From Figure 6 and (18), we can obtain the following differential equations and the corresponding transfer functions using the second-order Taylor series expansion for the approximation of the time delay term (which is used in the derivation of (21)).

$$\tau \frac{d^2y(t)}{d^2t} + \frac{dy(t)}{dt} = ku_{net}(t - \theta) \tag{19}$$

$$u_{net}(t) = u(t) - k_i y(t) \tag{20}$$

$$\frac{y(s)}{u(s)} = \frac{k \exp(-\theta s)}{\tau s^2 + s + kk_i \exp(-\theta s)}$$

$$\approx \frac{k \exp(-\theta s)}{\tau s^2 + s + kk_i - kk_i\theta s + \dfrac{kk_i\theta^2 s^2}{2}} \tag{21}$$

$$= \frac{\dfrac{k}{kk_i}\exp(-\theta s)}{\left(\dfrac{\tau}{kk_i} + \dfrac{\theta^2}{2}\right)s^2 + \left(\dfrac{1}{kk_i} - \theta\right)s + 1} \tag{22}$$

$$\equiv \frac{k_m \exp(-\theta_m s)}{\tau_m^2 s^2 + 2\tau_m \xi_m s + 1} \tag{23}$$

where

$$\tau_m = \sqrt{\frac{\tau}{kk_i} + \frac{\theta^2}{2}}$$

$$\xi_m = \left(\frac{1}{kk_i} - \theta\right)/(2\tau_m)$$

$$k_m = \frac{1}{k_i}$$

$$\theta_m = \theta \tag{24}$$

Therefore, we can obtain the second-order plus time delay model as the overall transfer function of $y(s)$ with respect to $u(s)$ using the P controller only if the model (18) is used to identify the considered integrating process. Then, the simple tuning rule of Table 1 for the second-order plus time delay model can be directly used to tune the PID controller only if the internal feedback loop gain (k_i) can be determined.

We solved the following optimization problem for extensive cases of k, τ, and θ to obtain the design equation for the determination of k_i.

$$\min_{k_i} \left[\int_0^\infty t|y_s - y(t)|\, dt\right] \tag{25}$$

$$\tau \frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} = ku(t - \theta) - kk_i y(t - \theta) + k\{\text{disturbance}\} \tag{26}$$

$$u(t) = k_c(y_s - y(t)) + \frac{k_c}{\tau_i}\int_0^\infty (y_s - y(t))\, dt + k_c\tau_d \frac{d(y_s - y(t))}{dt} \tag{27}$$

$$y(t) = u(t) = 0, \qquad t \le 0 \tag{28}$$

where the parameters of the PID controller of (27) are tuned by the integral of the time weighted absolute value of the error (ITAE) tuning method (Table 1) and $y_s = 1$ and disturbance $= 0$ for the step set-point change process and $y_s = 0$ and disturbance $= 1$ for the regulatory process. With consideration of the dimensionless grouping, we fitted the obtained optimal data and made the following design equations:

Step set-point change process:

$$k_i = \frac{0.076 \exp\left(-\dfrac{\tau}{1.1\theta}\right) + 0.112}{k\theta} \tag{29}$$

Step input disturbance rejection process:

$$k_i = 0.2/k\theta \tag{30}$$

Therefore, we can easily tune the proposed controller (PID controller and P controller of the internal feedback loop) using (29) or (30) and the tuning method of Table 1.

For the disturbance rejection process ($y_s = 0.0$), the proposed controller is the same as the PID controller. That is

$$u_{net}(t) = -(k_c + k_i)y(t) + \frac{k_c}{\tau_i}\int_0^\infty -y(t)\, dt - k_c\tau_d \frac{dy(t)}{dt} \tag{31}$$

However, the P controller of the proposed method plays an important role in suppressing overshoot by reducing the proportional error term in the set-point change process, that is

$$u_{net}(t) = k_c\left(y_s - \frac{(k_c + k_i)}{k_c}y(t)\right) + \frac{k_c}{\tau_i}\int_0^\infty (y_s - y(t))\, dt - k_c\tau_d \frac{d(y_s - y(t))}{dt} \tag{32}$$

and the tuning parameters of the proposed controller can be directly obtained for servo and disturbance rejection processes without any simulation or optimization.

In Table 2 and Figure 7, the performances of the proposed control strategy and the optimally tuned PID controller are compared. Here, the parameters of the PID controller are tuned by solving the optimization problem which has the integral of the time weighted absolute error (ITAE) as the objective function. Since the proposed controller has the same structure as the PID controller for the regulatory process as shown in (31) and the tuning rule of Table 1 is almost optimal, the control results of the proposed method are almost the same as that of the optimally tuned PID controller for the disturbance rejection process. The proposed controller shows a much smaller overshoot than the optimally tuned PID controller.

In summary, we derived a simple and efficient control structure by converting the integrating process to the open-loop stable process using the additional P controller as an internal feedback loop. Even though the proposed tuning rule is simple and algebraic equations,

Table 2. Control Results of the Proposed and the Optimal PID Controllers[a]

		set-point change			disturbance rejection		
		$\tau = 0.050$	$\tau = 0.200$	$\tau = 2.000$	$\tau = 0.050$	$\tau = 0.200$	$\tau = 2.000$
optimal-PID	k_c	4.810	4.625	11.011	5.472	5.798	20.634
	τ_i	0.630	0.958	2.813	0.523	0.620	0.760
	τ_d	0.101	0.201	0.636	0.114	0.210	0.452
	ITAE	0.236	0.332	0.405	0.074	0.114	0.051
proposed	k_i	0.863	0.713	0.560	1.000	1.000	1.000
	k_c	2.475	2.240	2.083	3.950	4.551	17.125
	τ_i	1.010	1.251	1.635	0.434	0.531	0.708
	τ_d	0.125	0.279	2.176	0.133	0.254	0.486
	ITAE	0.073	0.106	0.194	0.091	0.117	0.070

[a] $\theta = 0.2$.

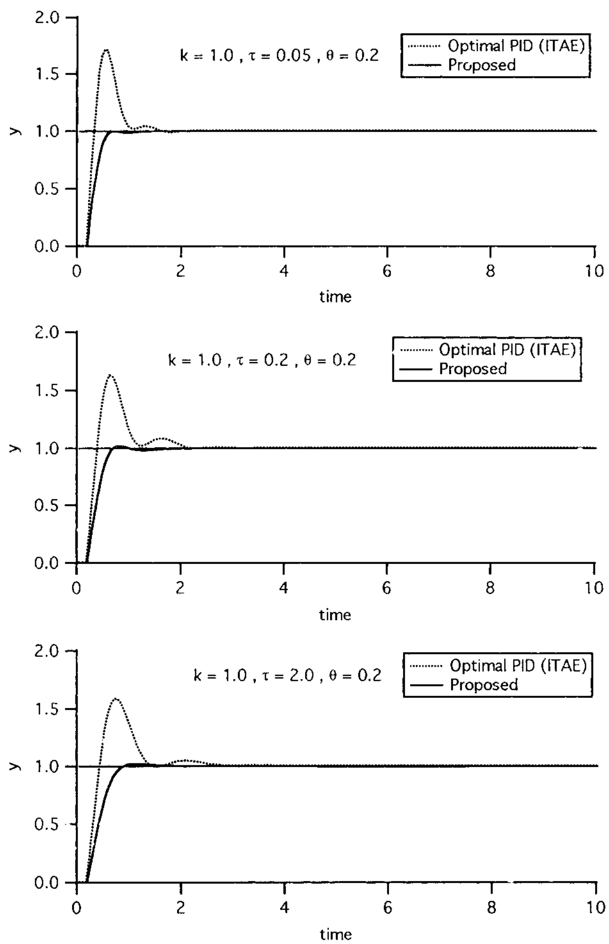

Figure 7. Control results of the proposed method and the optimally tuned PID controller.

the method shows almost the same responses as those of the optimally tuned PID controller for the input disturbance rejection problem and better performance than the optimally tuned PID controller for the set-point change process.

2.2. Theory for On-Line Process Identification for the Integrating Process. To identify the process, the process should be excited by a test signal and then the model is estimated from the measured process data sets. We want to use the proportional (P) controller as the test signal generator to identify the integrating process. That is, as shown in Figure 8, a test signal to activate the integrating process is generated by the P controller as Yuwana and Seborg's (1982) method (P control method), and we would obtain the model of (18) using the measured data sets.

The following differential equation and transfer function (here, the second-order Taylor series expansion is used to approximate time delay term) can be derived from Figure 8.

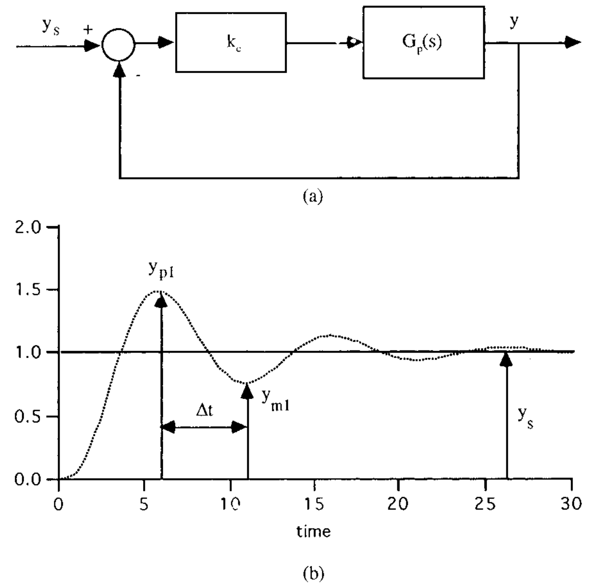

Figure 8. Activated process by the P controller for the identification work of the integrating process.

$$\tau \frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} = kk_c(y_s - y(t - \theta)) \tag{33}$$

$$y(s) = \frac{kk_c y_s(s)}{\tau s^2 + s + kk_c \exp(-\theta s)}$$

$$\approx \frac{y_s(s)}{\left(\frac{\tau}{kk_c} + \frac{\theta^2}{2}\right)s^2 + \left(\frac{1}{kk_c} - \theta\right)s + 1} \tag{34}$$

$$\equiv \frac{y_s(s)}{\tau_c^2 s^2 + 2\tau_c \xi_c s + 1} \tag{35}$$

where

$$\tau_c = \sqrt{\frac{\tau}{kk_c} + \frac{\theta^2}{2}} \tag{36}$$

$$\xi_c = \left(\frac{1}{kk_c} - \theta\right)/(2\tau_c) \tag{37}$$

Here, first, we would estimate τ_c, ξ_c, and k from the measured process data sets, and then τ, θ, and k can be directly calculated.

Table 3. Identification Results of the Proposed Identification Method for Example (i)

process			identified model			
k	τ	θ	k_c	k	τ	θ
1.000	0.100	0.200	3.000	1.000	0.122	0.176
1.000	0.200	0.200	3.000	1.000	0.214	0.186
1.000	1.000	0.200	2.000	1.000	1.004	0.200
1.000	5.000	0.200	1.000	1.000	4.995	0.202

The closed-loop response $y(s)$ for the step set-point change $y_s(s)$ can be easily approximated by using well-known properties of the second-order plus time delay process for the step input. That is, assume that an underdamped closed-loop response is obtained by the P controller with a step-type set-point change as shown in Figure 8, and then τ_c and ξ_c can be obtained from the following equations easily (Coughanowr and Koppel (1965), Yuwana and Seborg (1982)):

$$H = \frac{y_s - y_{m1}}{y_{p1} - y_s} \tag{38}$$

$$\xi_c = -\frac{\ln(H)}{\sqrt{\pi^2 + \{\ln(H)\}^2}} \tag{39}$$

$$\tau_c = \frac{\Delta t\sqrt{1 - \xi^2}}{\pi} \tag{40}$$

Here, H and Δt denote overshoot and half-period, respectively. The gain k is obtained from the following equation.

$$k = \frac{y_s}{\int_0^\infty u(t)\,dt} \tag{41}$$

where $u(t)$ and $y(t)$ denote the output of the P controller and the process, respectively. Using (38), (39), (40), and (41), we can estimate τ_c, ξ_c, and k and then the model parameters τ and θ are calculated from the following (42) and (43) derived from (36) and (37).

$$\tau = kk_c\left(\tau_c^2 - \frac{\theta^2}{2}\right) \tag{42}$$

$$\theta = \frac{1}{kk_c} - 2\tau_c\xi_c \tag{43}$$

Therefore, we can identify the process easily using the control results of the P controller.

To show the performances of the proposed identification method for the PID controller autotuning, consider the following examples.

(i) Process has the same structure as the proposed model structure.

$$G_p(s) = \frac{k\exp(-\theta s)}{s(\tau s + 1)} \tag{44}$$

Identified models are shown in Table 3. From the simulation results, we recognize that the proposed method produces acceptable model parameters.

(ii) Process has a different structure from the proposed model structure.

$$G_p(s) = \frac{\exp(-0.2s)}{s(s + 1)^2} \tag{45}$$

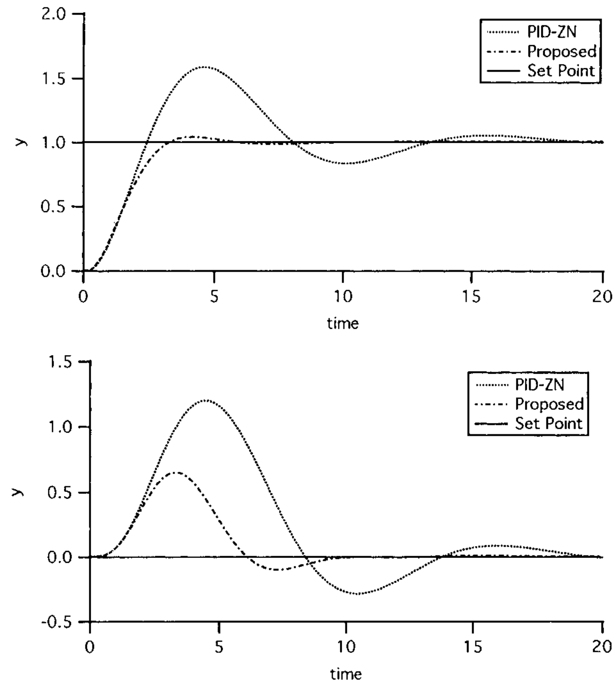

Figure 9. Control results of the proposed method and the conventional PID controller.

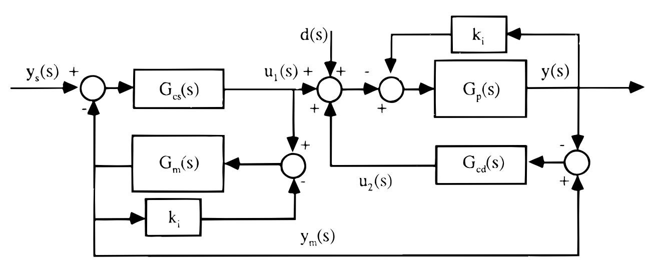

Figure 10. Proposed control strategy with the consideration of the integrating process.

The identified model is $k = 1.000$, $\tau = 1.486$, and $\theta = 0.804$, and control performances are shown in Figure 9. Here, the PID controller is tuned by the Ziegler–Nichols tuning relation based on the continuous cycling identification method. Simulation results show that the proposed strategy can control the integrating process efficiently.

To control the integrating process, we proposed a modified PID control strategy combined with a P controller as an internal feedback loop and a simple identification strategy for the tuning of the proposed controller. Even though the proposed control strategy is simple and guarantees on-line identification and an easy controller tuning, the method shows a better performance than the optimally tuned PID controller for the step set-point change process and almost the same performances as those of the optimally tuned PID controller for the regulatory process. However, the k_c value of the P controller to identify the process should be chosen appropriately to guarantee an underdamped response. This is the main shortcoming of the proposed identification method and existing P control identification methods for the tuning of the PID controller (Chen (1989), Hwang (1995), Jutan and Rodriquez (1984), Lee (1989), Lee et al. (1990), Sung et al. (1994), Yuwana and Seborg (1982)). We can add the P controller as the internal feedback loop to the control structure of Figure 4, as shown in Figure 10, to control the integrating process more efficiently.

3. Difficulties in Treating Ramp-Type (Very Slow) Set-Point Tracking or Disturbance Rejection. The type of the desired trajectory of the process output can be ramp, and very low frequency or ramp-type disturbance can affect the process output. However, the PID controller cannot manipulate the ramp-type set-point change or disturbance because it has only one integrator. Consider the following ramp-type set point.

$$y_s(s) = \frac{1}{s}\left\{\frac{y_{set}}{s}\right\} \tag{46}$$

Then, the closed-loop response becomes

$$y(s) = \frac{1}{s}\left\{\frac{G_c(s)\, G_p(s)\, y_{set}}{(1 + G_c(s)\, G_p(s))s}\right\} \tag{47}$$

Here, it should be noted that (46) is the integral of the step set-point change and (47) is the integral of the process output corresponding to the step set-point change. That is

$$y(s) = \frac{1}{s} y_{step}(s) \tag{48}$$

where $y_{step}(s)$ denotes the process response for the step set-point change y_{set}/s. Therefore, $y_s(s) - y(s)$ has an additional phase lag $-90°$ and a smaller amplitude ratio than $y_{set}/s - y_{step}(s)$ as follows so that the controller tuned for the step set-point change process would show sluggish responses.

$$\angle\{y_s(j\omega) - y(j\omega)\} = -\pi/2 + \angle\{(y_{set}/j\omega) - y_{step}(j\omega)\} \tag{49}$$

$$|y_s(j\omega) - y(j\omega)| = \frac{|(y_{set}/j\omega) - y_{step}(j\omega)|}{\omega} \tag{50}$$

Further, with a simple algebraic calculation, the following equation can be obtained for the general time invariant linear system except the integrating system:

$$\lim_{s\to 0} \{(y_{set}/s) - y_{step}(s)\} = \text{positive} \tag{51}$$

Therefore, from the final value theorem with (51) we can know that the offset cannot be rejected.

This fact can be recognized more clearly by considering (52). Here (52) is obtained by subtracting (48) from (46).

$$\text{error}_{ramp} = \frac{\text{error}_{step}}{s} = \text{IE} \tag{52}$$

$$\text{error}_{ramp} = y_s(s) - y(s) \tag{53}$$

$$\text{error}_{step} = y_{set}/s - y(s) \tag{54}$$

As shown in (52), the error of the ramp set-point change process is the integral of the error (IE) of the step set-point change process, and the following equation should be satisfied in the steady state when the nonintegrating process is controlled by the PID controller with the step set-point (y_{set}) change.

$$y_{set} = k\frac{k_c}{\tau_i}\int_0^{\infty} \text{error}_{step}\, dt = k\frac{k_c}{\tau_i}\text{IE} \tag{55}$$

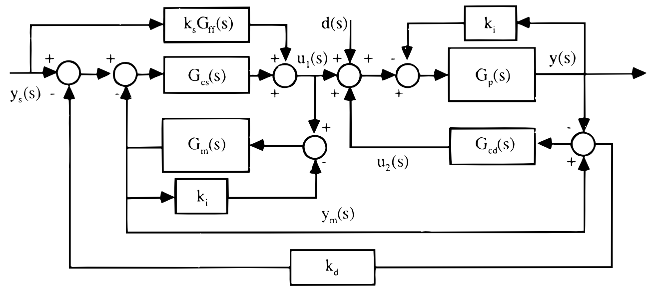

Figure 11. Proposed control strategy with the consideration of the ramp-type external signal.

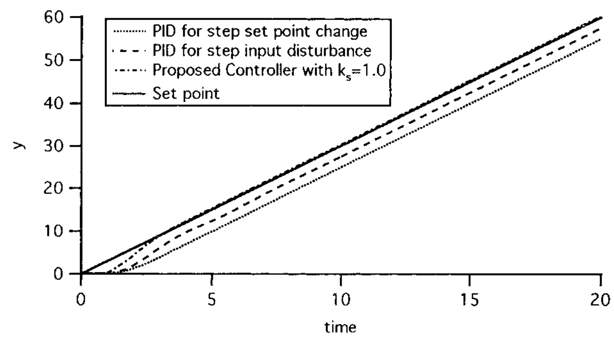

Figure 12. Control results of the proposed method and the conventional PID controller for the ramp-type set-point change process.

where k denotes the static gain of the process. In (55), in the nonintegrating process the IE cannot be zero since the controller gain cannot be an infinite value because of the stability problem. Therefore, from (52) and (55), the offset (error_{ramp}) cannot be rejected using only the PID controller when the process has a ramp-type set point or disturbance.

Here, usually, the controller gain k_c/τ_i of the PID controller for the step input disturbance rejection is much larger than that of the PID controller for the step set-point change so that the offset of the former PID controller is much smaller than that of the latter PID controller. In fact, if the PID controller is tuned for the step input disturbance and the process is a low order and has a small time delay, the offset in the ramp-type set-point change process is almost negligible. However, as the ratio of the time delay to the time constant grows, the offset gets bigger.

We add two simple terms to the control structure of Figure 10 to manipulate a ramp-type or a very slow set-point change (or disturbance) as shown in Figure 11. Here $G_{ff}(s)$ is

$$G_{ff}(s) = 1/G_m^+(s) \tag{56}$$

where $G_m^+(s)$ denotes the minimum-phase part of the model. The feedforward term $G_{ff}(s)$ incorporates the type of set point, and the remaining error is rejected by $G_{cs}(s)$. k_d incorporates the type of the disturbance, and the remaining error can be rejected by $G_{cd}(s)$. In Figure 11, k_s and k_d should be chosen by user according to the assumed type. That is, k_s and k_d should be chosen as a value between 0 and 1 according to the assumed dynamics. Of course, as the type is closer to 1, k_s and k_d should be nearly 1.

Consider the simulation results of Figure 12, where the proposed strategy shows a good tracking performance but the PID controller cannot reject offset for the ramp set-point change. In the simulation the process

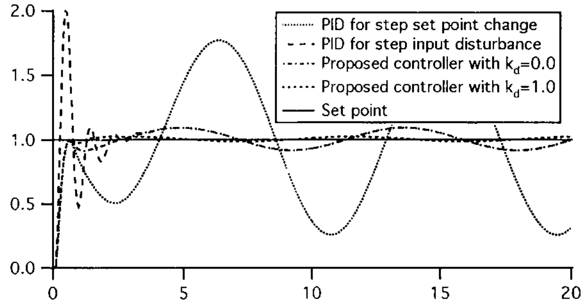

Figure 13. Control results of the porposed method and the conventional PID controller for the slow disturbance rejection process.

and the model are

$$G_p(s) = \frac{\exp(-s)}{(s+1)^2} \tag{57}$$

and the set point is

$$y_s(s) = 3/s^2 \tag{58}$$

Moreover, as shown in Figure 13, the proposed strategy with $k_d = 1.0$ shows a better performance than the conventional PID controllers and the proposed strategy with $k_d = 0.0$ because the effects of the very slow disturbance are similar to those of the ramp-type disturbance. In this simulation, the disturbance is

$$d(t) = 5 \sin(3.606t/5) \tag{59}$$

and the process (model) is

$$G_p(s) = \frac{\exp(-0.15s)}{(s+1)^2} \tag{60}$$

4. Difficulties in Controlling the Large Time Delay Process. In general, the control action is purposed to guarantee faster closed-loop response than open-loop response. However, if the time delay is much larger than the time constant of the process, this time delay term plays as a bottleneck for the fast closed-loop response. Consider the following process and the simulation results of Figure 14.

$$G_p(s) = \frac{\exp(-3s)}{(s+1)} \tag{61}$$

If we want to obtain a faster closed-loop response than an open-loop response as shown in Figure 14a, we cannot help choosing an exponentially decreasing control action as shown in Figure 14b because the usual process is a low-pass filter. However, the PID controller can use only the feedback error (which is constant) so that the control action is inevitably an increasing function during the time delay. Therefore, the structure itself of the PID controller has the limitation to control the large time delay process efficiently. In the case of the oscillatory desired trajectory, we reach the same conclusion. The time invariant feedback controllers may show the same difficulties since the feedback information is only a constant value during the time delay. Therefore, it is inevitable that a predictive control strategy should be used to improve the control performance for the large time delay process.

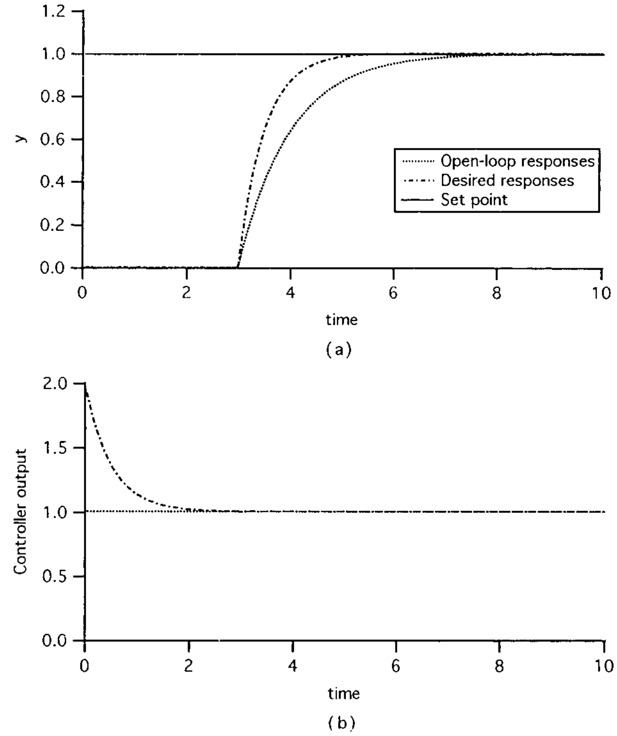

Figure 14. Typical controller output to achieve good control results for the large time delay process.

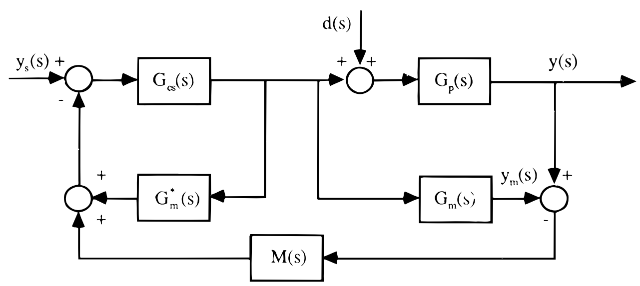

Figure 15. Control structure of the Smith predictor.

Until now, in the continuous time system, the Smith predictor ($M(s) = 1.0$) or modified Smith predictor ($M(s) \neq 1$) of Figure 15 is usually applied to improve the control performance in the large time delay process (Horowitz (1983), Hocken et al. (1983), Huang et al. (1990), Palmor and Powers (1985), Smith (1957), Watanabe et al. (1983)). In the control structure of the Smith predictor, the feedback loop is corrupted by the effects of the disturbance (the modeling error). That is, the characteristic equation of the Smith predictor in Figure 15 for the disturbance rejection and the set-point change is

$$1 + G_{cs}(s)\, G_m^*(s) + G_{cs}(s)\, M(s)\, (G_p(s) - G_m(s)) = 0 \tag{62}$$

Here, because the controller $G_{cs}(s)$ may be tuned by large gains to guarantee the good set-point tracking performance, the stability of the Smith predictor may be sensitive to the modeling error $G_p(s) - G_m(s)$ (Palmor (1980)).

To analyze the input disturbance rejection performance of the Smith predictor, consider Figure 15. With the assumption of the perfect model, the following

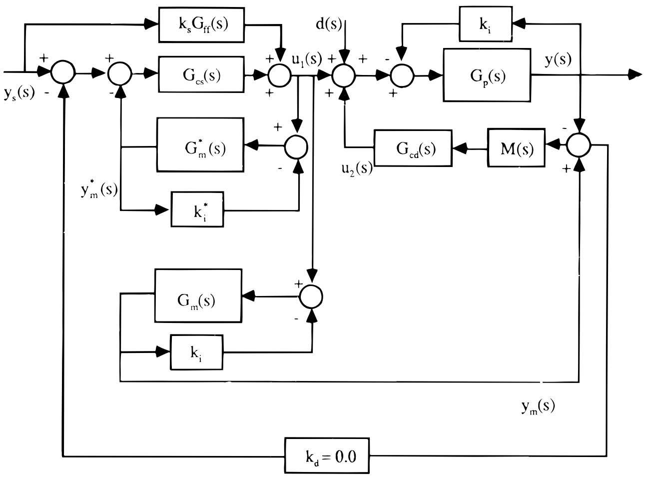

Figure 16. Proposed control strategy with the consideration of the large time delay process.

equation can be easily obtained for the unit step input disturbance rejection process

$$y(s) = \left(\frac{G_p(s) + G_p(s)G_c(s)(G_m^*(s) - G_p(s))}{1 + G_m^*(s)G_c(s)}\right)\frac{1}{s} \quad (63)$$

and assume that the $G_c(s)$ has almost infinite gains, then

$$y(s) \approx \left(\frac{G_p(s) + G_p(s)\ G_c(s)\ G_m^*(s)(1 - \exp(-\theta s))}{G_m^*(s)\ G_c(s)}\right)\frac{1}{s} \quad (64)$$

$$= \left(\frac{\exp(-\theta s)}{G_c(s)} + G_p(s)\ (1 - \exp(-\theta s))\right)\frac{1}{s} \quad (65)$$

$$\approx \frac{G_p(s)}{s} - \frac{G_p(s)\exp(-\theta s)}{s} \quad (66)$$

Here, the objective of the controller is usually to derive $y(s)$ to zero as fast as possible. However, as shown in (66), even though the gains of the controller are infinite and the model is perfect, the $y(s)$ can go to zero after the time required to reach the steady state in the open-loop state. Therefore, the Smith predictor ($M(s) = 1.0$) shows a large settling time for the input disturbance.

Consider the proposed control structure of Figure 16 for a good control performance and robustness for the large time delay process. Here, we choose $k_d = 0.0$ to remove the effects of the modeling error to the controller $G_{cs}(s)$. $M(s)$ is a dynamical time delay compensator of the disturbance to improve the disturbance rejection performance. Here, we recommend the Huang et al. (1990) method to determine $M(s)$. For the integrating process, we can recognize through a simple calculation that k_i^* should be chosen as $k_i \exp(-(\theta - \theta^*)s)$ to achieve $y_m^*(s) = \exp((\theta - \theta_m^*)s)y_m(s)$; here θ_m^* is the time delay of $G_m^*(s)$. The proposed predictor has the following characteristic equation for the set-point change and for the disturbance rejection, respectively.

$$1 + G_{cs}(s)\ G_m^*(s) = 0 \quad (67)$$

$$1 + G_{cd}(s)\ M(s)\ G_p(s) = 0 \quad (68)$$

Therefore, the proposed predictor can guarantee the closed-loop stability only if (67) and (68) are stable. It should be noted that the proposed method uses two differently tuned PID controllers to separately manipulate the servo and disturbance (modeling error) rejection problem. The model of the disturbance or modeling error cannot be almost defined so that the net predicted future time of $M(s)$ is much smaller than the time delay of $G_m(s)$ to improve the robustness. Therefore, $G_{cs}(s)$ can be tuned by large gains while guaranteeing the stable closed-loop response because we know $G_m^*(s)$ (which has a small time delay) exactly and since $G_{cd}(s)$ is tuned for $M(s)\ G_m(s)$ (which has a large time delay); $G_{cd}(s)$ has small gains so that the proposed predictor guarantees better robustness to the modeling error than the Smith predictor.

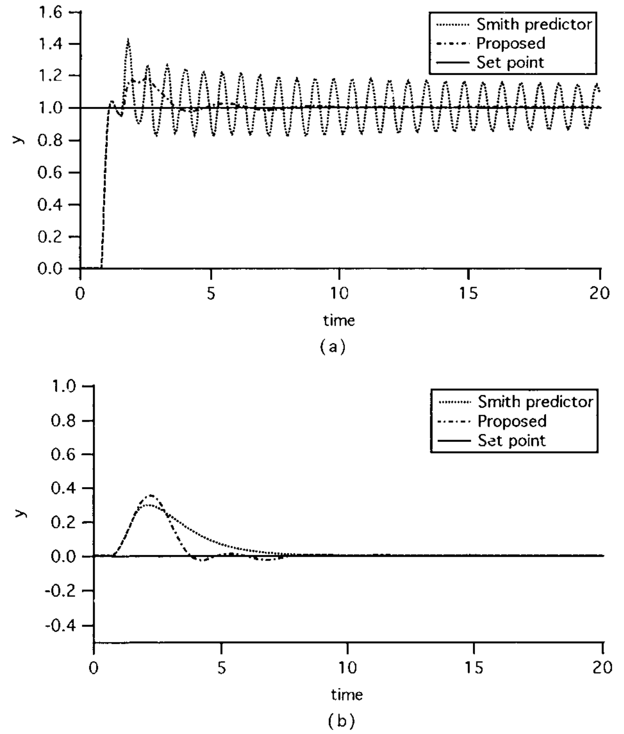

Figure 17. Control results of the proposed method and the Smith predictor.

It is desirable to tune the Smith predictor and the proposed predictor as simple as possible. That is, the existing simple PID tuning rule is recommended to tune the proposed predictor. If $G_m^*(s)$ has no time delay term and is a low-order process, tuning parameters obtained by the usual PID tuning rules such as Ziegler−Nichols (ZN), internal model control (IMC), the integral of the time weighted absolute error (ITAE), and Cohen−Coon methods can be so big or infinite that it is difficult to determine how big tuning parameters are appropriate to incorporate the modeling error. Therefore, we recommend that the process $G_m^*(s)$ has some time delay and the remaining time delay is used to predict so that automatically tuning parameters can be calculated by the usual tuning methods for the set-point change process.

However, in the Smith predictor, it should be noted that the addition of some time delay to $G_m^*(s)$ can result in more oscillatory response because the feedback loop is corrupted by the modeling error. Therefore, in the Smith predictor, $G_m^*(s)$ should have some time delay only to tune the PID controller and the time delay of $G_m^*(s)$ should be removed for the implementation.

Figure 17 compares the Smith predictor ($M(s) = 1.0$) with the proposed method ($M(s) = 1.0$) for the set-point tracking and the disturbance rejection performance. Figure 17a shows a poor robustness of the Smith predictor to the modeling error compared with the proposed method. In this case the model and process are as follows:

$$G_m^*(s) = \frac{\exp(-0.1s)}{s^2 + 2s + 1} \tag{69}$$

$$G_p(s) = \frac{\exp(-0.85s)}{s^2 + 2.5s + 1} \tag{70}$$

$$G_m(s) = \frac{\exp(-0.7s)}{s^2 + 2s + 1} \tag{71}$$

Figure 17b shows the step input disturbance rejection performances of the Smith predictor and the proposed method under the assumption of the perfect model. Even though the perfect model is provided, the proposed method using the low gain controller $G_{cd}(s)$ shows a smaller settling time than that of the Smith predictor using the high gain controller $G_{cs}(s)$. The step input disturbance rejection performance of the proposed method is much better than that of the Smith predictor ($M(s) = 1.0$) for a smaller time delay process. The proposed predictor considers the set-point tracking problem and the disturbance (modeling error) rejection problem separately so that it shows a better robustness to the modeling error than the Smith predictor.

It should be noted that, if the model is perfect and the disturbance dynamics is very fast (that is, almost step output disturbance), the Smith predictor would show a much better disturbance rejection performance. Therefore, if this assumption can be reasonable for the considered process, we choose $k_d = M(s)$ and $G_{cd}(s) = 0.0$. In this case, the proposed control structure is the same as that of the Smith predictor. However, in practice, this assumption is very restrictive, and it is dangerous to use the high gain controller. Moreover, the disturbance dynamics is slower and the disturbance rejection performance of the Smith predictor is more and more degraded because the Smith predictor is designed based on the step output disturbance.

In conclusion, we recommend the proposed control structure with two differently tuned PID controllers to manipulate the large time delay process.

5. Limitations When a Given Model Is High Order. Many process identification methods inherently produce a high-order model or many frequency data sets like Melo and Friedly's (1992) identification method or relay feedback identification method (Åström and Hägglund (1984)), and identification methods using Laguerre series (Dumont et al. (1989), Mäkilä (1991)). Especially, if the considered control system has an internal feedback loop, the outer overall transfer function of the system is inevitably high order. For example, in the cascade control structure, the transfer function connecting the outer control output with the process output is high order. In this study, the control system including an internal feedback loop to incorporate the integrating process is also high order. More and more, present control systems have a hierarchy structure so that the upper level controller should be designed to control the high-order process efficiently.

If the given model is high-order plus time delay or many frequency process data sets are given, simple tuning rules except the Ziegler–Nichols tuning rule are rare. However, the performance of the Ziegler–Nichols one is frequently poor. For example, for the underdamped process, the tuning rule frequently shows a large overshoot and oscillatory responses for the set-point change process because it considers only one point in the Nyquist plot (which is usually ultimate information). Frequently, even though this design problem for the high-order process can be treated easily, operators discard the model and obtain a low-order plus time delay model again or use the Ziegler–Nichols tuning rule.

In fact, a high-order plus time delay model or frequency process data sets can be easily approximated by the second-order plus time delay or the first-order plus time delay model. In addition, since the number of the tuning parameters of the PID controller is only 3, improvements by considering the full high-order model rather than an approximated second-order plus time delay model may be small. The second-order plus time delay model can represent various types of the process. However, until now, simple tuning rules for the second-order plus time delay model are rare.

Recently, Sung et al. (1996) proposed a good tuning rule for the second-order plus time delay model. The tuning rule is shown in Table 1. Its performance is almost the same as that of the optimal ITAE (the integral of the time weighted absolute value of the error) tuning rule within $0.05 < \theta_m/\tau_m < 2.0$ and $0.3 < \xi_m < 6.0$. Therefore, only if an approximated second-order plus time delay model can be obtained, the PID controller tuning is easy.

The desirable model reduction method for the PID controller tuning should be simple and produce the first-order plus time delay or the second-order plus time delay model without simulation or a complex numerical technique. Levy (1959) proposed a model reduction method using frequency data sets. The same approaches were reworked to improve the computation efficiency (Sanathanan and Koerner (1963), Payne (1970), Whitfield (1986)). Usually, only a low-order plus time delay model can be used to tune the PID controller, and if the reduced model is low order, then the time delay plays an important role in improving the accuracy of the approximated low-order model. However, their reduced models do not have the time delay, so that, in the case of a low-order model for the tuning of the PID controller, the accuracy of the approximated model can be very poor.

Therefore, to consider the time delay explicitly, we recommend the following model reduction method, especially, to tune the PID controller. Assume that the reduced second-order plus time delay model is as follows:

$$G_{r-m}(s) = \frac{k_m \exp(-\theta_m s)}{{\tau_m}^2 s^2 + 2\tau_m \xi_m s + 1} \tag{72}$$

Then the static gain of the reduced model is obtained by (73)

$$k_m = G_m(0) \tag{73}$$

and ξ_m and τ_m are estimated to satisfy (74)

$$|G_m(j\omega)| \approx |G_{r-m}(j\omega)| = \left|\frac{k_m \exp(-j\theta_m\omega)}{1 - \tau_m^2\omega^2 + j2\tau_m\xi_m\omega}\right| = \frac{k_m}{\sqrt{\{1 - \tau_m^2\omega^2\}^2 + \{2\tau_m\xi_m\omega\}^2}} \quad (74)$$

By using the least-squares methods, ξ_m and τ_m satisfying (74) can be directly estimated from (75) and (76) (Levy (1959))

$$\tau_m^4|G_m(j\omega_i)|^2\omega_i^4 + (4\tau_m^2\xi_m^2 - 2\tau_m^2)\omega_i^2|G_m(j\omega_i)|^2 = k_m^2 - |G_m(j\omega_i)|^2 \quad (75)$$

$$0 < \omega_0 < \omega_1 < ... < \omega_i < ... \le \omega_u \quad (76)$$

and additionally the following phase lag equation for the second-order plus time delay model is used to estimate the time delay of the reduced model.

$$\theta_m = \frac{\pi + \arctan 2(-2\tau_m\xi_m\omega_u, 1 - \tau_m^2\omega_u^2)}{\omega_u} \quad (77)$$

Here, subscript r-m and m denote “reduced-model” and “model”, respectively. ω_u represents the ultimate frequency of the process model, and ω_i’s are located with an equal space between 0 and ω_u. We consider the frequency below ω_u in (76) and (77) because the controller works in this frequency region. (75) is the same approach as Levy’s (1959). Though the improved methods (Sanathanan and Koerner (1963), Payne (1970), Whitfield (1986)) can be used, we use Levy’s (1959) method for simplicity. (77) can be easily derived from the phase lag equation of the second-order plus time delay process. The same procedure can be done to obtain the reduced first-order plus time delay model. However, it should be emphasized that the first-order plus time delay cannot well approximate an underdamped process or a high-order process whose poles are concentrated on one point. In this case, the second-order plus time delay model is strongly recommended in the viewpoint of accuracy.

Moreover, it should be noted that, if the model has unstable zeros, the obtained reduced model can be unstable even though the model is stable because small unstable zeros of the model can affect pole positions of the reduced model. We recommend the use of the equivalent time delay to approximate unstable zeros. Consider the following unstable zeros part of the model.

$$(-z_1^{-1}s + 1)(-z_2^{-1}s + 1)...(-z_n^{-1}s + 1) = 1 - (z_1^{-1} + z_2^{-1} + ... z_n^{-1})s + ... \quad (78)$$

and the equivalent time delay can be expanded using Taylor series expansion

$$\exp(-\theta_{equivalent}s) = 1 - \theta_{equivalent}s + ... \quad (79)$$

From the comparison of (78) and (79), the following equivalent time delay can be defined to approximate unstable zeros.

$$\theta_{equivalent} = z_1^{-1} + z_2^{-1} + ... + z_n^{-1} \quad (80)$$

The equivalent time delay is the same as the equivalent

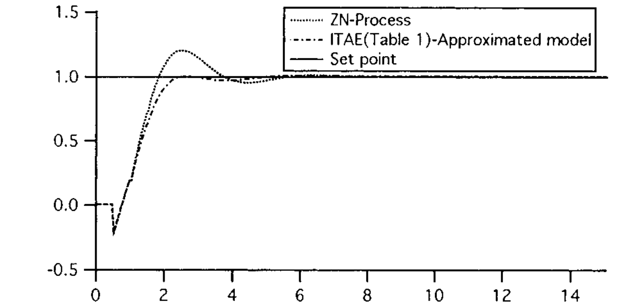

Figure 18. Tuning results of the ITAE rule with the approximated model and the Zn rule with the model.

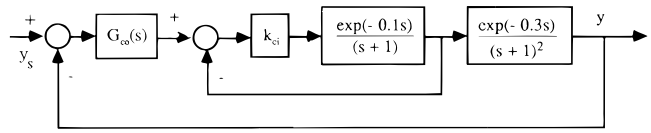

Figure 19. Simulated cascade control process.

dead time of Matsubara (1965). Then we can continue the model reduction with the modified model which has the equivalent time delay term instead of unstable zeros.

Consider the following nonminimum phase process.

$$G_p(s) = \frac{(-0.3s + 1)\exp(-0.5s)}{(s + 1)^2} \quad (81)$$

This process can be approximated using the equivalent time delay

$$G(s) = \frac{\exp(-0.8s)}{(s + 1)^2} \quad (82)$$

Tuning results of the ITAE (the integral of the time weighted absolute value of the error) tuning rule (Table 1) based on approximated model (82) and the Ziegler–Nichols method based on the process (81) are compared in Figure 18. The ITAE tuning rule based on the approximated model shows superior tuning results to the Ziegler–Nichols method based on the process because the approximation results in a small degradation of the good performance of the ITAE tuning rule.

As an example of the application of the model reduction method, consider a cascade control of Figure 19. If we use $k_{ci} = 8.0$ as the internal feedback loop controller gain, then the transfer function of $y(s)$ to the outer loop controller $G_{co}(s)$ is as follows.

$$G_o(s) = \frac{8.0\exp(-0.4s)}{(s + 1 + 8.0\exp(-0.1s))(s + 1)^2} \quad (83)$$

The obtained reduced model of the overall transfer function $G_o(s)$ using the model reduction method and the corresponding $G_{co}(s)$ for the set-point change case are as follows.

$$G_{r-m}(s) = \frac{8\exp(-0.4297s)}{9(0.9756s^2 + 1.9908s + 1)} \quad (84)$$

$$G_{co}(s) = 2.7292\left(1 + \frac{1}{2.0469s} + 0.5495s\right) \quad (85)$$

Simulation results are shown in Figure 20. The ITAE tuning rule (Table 1) using the model reduction shows a better performance than the Ziegler–Nichols tuning

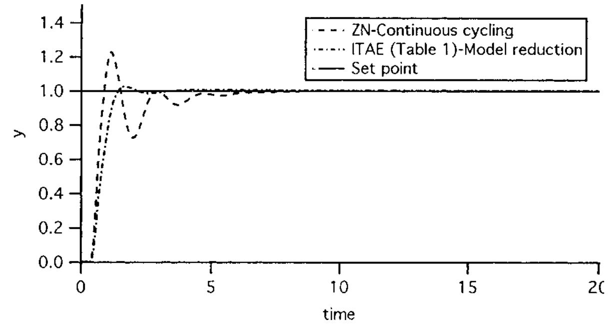

Figure 20. Tuning results of the ITAE rule with the reduced model and the ZN rule with the model for a cascade control process.

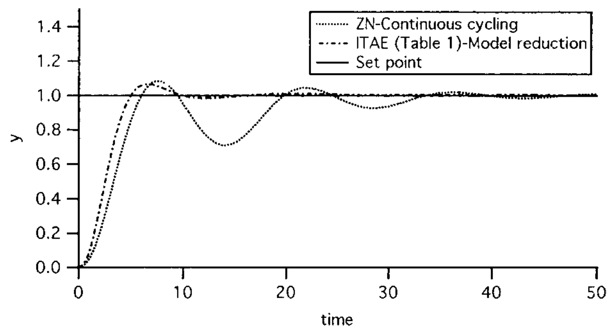

Figure 21. Tuning results of the ITAE rule with the reduced model and the Zn rule with the model for an unstable process.

rule based on the continuous cycling method because of a small degradation of superiorities of the ITAE tuning rule in the model reduction.

As another example, consider an unstable process stabilized by the internal feedback loop. If the process is unstable, an additional feedback loop is usually used to stabilize the unstable process and then the PID controller can be used to control the stabilized process. Consider the following unstable process

$$G_p(s) = \frac{1}{(s+1)^3(-10s+1)} \tag{86}$$

and this unstable process can be stabilized by the internal feedback loop ($k_i = -3.0$) of Figure 6. Then the overall transfer function is

$$G_o(s) = \frac{1}{(s+1)^3(-10s+1) - 3.0} \tag{87}$$

and the following reduced model is obtained by the model reduction method and the corresponding PID controller:

$$G_{r-m}(s) = \frac{-0.5\exp(-1.2860s)}{10.485s^2 + 2.2808s + 1} \tag{88}$$

$$G_c(s) = -1.8104\left(1 + \frac{1}{2.3406s} + 4.4538s\right) \tag{89}$$

Here, the damping factor is about 0.3522 so that the stabilized process is severely underdamped. Therefore, as shown in the simulation result of Figure 21, the Ziegler–Nichols tuning rule shows very poor control results compared with the good control performance of the ITAE tuning rule (Table 1) based on the model reduction.

Discussions

We propose several simple control strategies to overcome disadvantages of the conventional PID controller in a unified structure by improving previous control strategies and introducing new control concepts. Even though the proposed methods are very simple and explicit, it shows good control performances and robustness to modeling errors. Also, many functions contained in this study are independent of each other. Therefore, it can be much simplified by simply removing unnecessary functions for a given process. The tuning of the proposed control strategy is simple and explicit so that it is possible to automatically tune the proposed controller provided that an on-line process identification method is available.

The proposed structure is the same as that of predictive control strategies such as dynamic matrix control (DMC), model algorithmic control (MAC), and generalized predictive control (GPC). Predictive control strategies are usually designed to manipulate the step output disturbance so that they show very sluggish responses for the step input disturbance. However, Lee et al. (1994) improved the disturbance rejection performance by introducing a first-order—integrated—disturbance model. Their method shows a very good disturbance rejection performance only if the time constant of the disturbance model is appropriately chosen. Also, predictive control strategies explicitly incorporate the types of set-point change, various constraints, and interactions in multiinput, multioutput (MIMO) systems.

Physical model-based control strategies such as generic model control (GMC), global linearizing control (GLC), and control methods using a Wiener-type model can explicitly incorporate nonlinearities of nonlinear processes (for example, pH processes, high-purity distillation column, polymerization processes using a continuous stirred tank reactor (CSTR)). As a result, predictive control strategies and model-based control strategies can provide a superior control performance than the PID control strategy.

However, it should be emphasized that the PID control strategy is very simple and robust to modeling errors. Also, low-order plus time delay model is sufficient to tune the PID controller. Contrarily, model-based control strategies require rigorous physical models, and predictive control strategies require a nonparametric model such as step response or finite impulse response. If a process is low-order plus time delay such as the first-order plus time delay or the second-order plus time delay, the proposed PID control strategy with a dead-time compensator can achieve almost the same nominal control performance as that of predictive control strategies. Also, it is not necessary to use more complicated model-based control strategies for mild nonlinear processes.

In summary, the proposed PID control strategy is strongly recommended in controlling unconstrained and mild nonlinear single-input, single-output systems (including integrating and large time delay processes). Since the proposed method cannot handle constraints and nonlinearities systematically, predictive control strategies and model-based control strategies should be used to control processes involving hard constraints and strong nonlinear processes, respectively. The proposed dead-time compensator and the proposed control strategy to simultaneously treat both the servo and regulatory processes can be extendible to multiinput, multioutput (MIMO) systems. However, it cannot treat interactions systematically so that predictive control strategies are recommended in controlling multiinput, multioutput (MIMO) systems involving strong interactions.

Conclusions

We introduce a control structure to manipulate servo and regulatory processes separately. An internal feedback loop is added to the control structure to treat the integrating process with a simple tuning strategy using the model reduction method. The control strategy for the integrating process shows a better control performance than the optimally tuned PID controller for a simulated set-point change process. Additional feedforward and feedback loops are implemented to incorporate ramp-type or very slow set-point tracking (disturbance rejection). We also propose a new dead-time compensator to incorporate the large time delay process more efficiently. The proposed predictor shows a better robustness to the modeling error than the Smith predictor because the proposed control structure can treat servo and regulatory problems separately. Moreover, the model reduction method is applied to tune the PID controller for a high-order model, especially the process including an internal feedback loop.

Nomenclature

$d(s)$ = input disturbance
$d_o(s)$ = output disturbance
$\mathrm{error}_{\mathrm{step}}(s)$ = error in the step set-point change process
$\mathrm{error}_{\mathrm{ramp}}(s)$ = error in the ramp-type set-point change process
$G_c(s)$ = transfer function of the controller
$G_{cs}(s)$ = transfer function of the controller for the set-point change
$G_{cd}(s)$ = transfer function of the controller for the disturbance rejection
$G_{ff}(s)$ = transfer function of the feedforward loop
$G_m(s)$ = transfer function of the model
$G_m^*(s)$ = transfer function of the model without time delay
$G_m^+(s)$ = minimum-phase part of the model
$G_{r-m}(s)$ = transfer function of the reduced model
$G_o(s)$ = overall process transfer function
$G_p(s)$ = transfer function of the process
H = overshoot
IE = integral of the error
k = static gain of the process
k_c = proportional gain of the PID controller
k_d = gain of the feedback loop for the ramp-type disturbance
k_i = gain of the internal feedback loop
k_m = static gain of the model
k_s = gain of the feedforward loop for the ramp-type set-point change
t = time
$u(s)$ = controller output
$y(s)$ = process output
$y_m(s)$ = model output
y_{set}, $y_s(s)$ = set point
$y_{step}(s)$ = response of the process for the step set-point change

Greek Letters

Δt = half-period of the step response
θ = time delay of the process
θ_m = time delay of the model
τ = time constant of the process
τ_d = derivative time of the PID controller
τ_i = integral time of the PID controller
τ_m = time constant of the model
ω = frequency
ω_u = ultimate frequency
ξ_m = damping factor of the model

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