$v_\pi(s_4) = 0$

Since the equations for $v_\pi(s_2)$ and $v_\pi(s_3)$ are identical, we must have that $v_\pi(s_2) = v_\pi(s_3)$.

Thus, the equations for $v_\pi(s_1)$ and $v_\pi(s_2)$ can be changed to:

$v_\pi(s_1) = \frac{1}{2}(-1 + v_\pi(s_2)) + \frac{1}{2}(-3 + v_\pi(s_2)) = -2 + v_\pi(s_2)$

$v_\pi(s_2) = \frac{1}{2}(-1 + v_\pi(s_1)) + \frac{1}{2}(5 + 0) = 2 + \frac{1}{2}v_\pi(s_1)$

Combining the two latest equations yields

$v_\pi(s_1) = -2 + 2 + \frac{1}{2}v_\pi(s_1) = \frac{1}{2}v_\pi(s_1)$,

which implies $v_\pi(s_1) = 0$. Furthermore, $v_\pi(s_3) = v_\pi(s_2) = 2 + \frac{1}{2}v_\pi(s_1) = 2 + 0 = 2$.

Thus, the state-value function is given by:

$v_\pi(s_1) = 0$

$v_\pi(s_2) = 2$

$v_\pi(s_3) = 2$

$v_\pi(s_4) = 0$

Note. This example serves to illustrate the fact that it is ***possible*** to *directly* solve the system of equations given by the Bellman expectation equation for v_π. However, in practice, and especially for much larger Markov decision processes (MDPs), we will instead use an *iterative* solution approach.