

If this is not entirely clear to you, please take the time now to plug in the values to derive the equation from the video. Then, you are encouraged to repeat the same process for the other states.

Notes on Solving the System of Equations

In the video, we mentioned that you can directly solve the system of equations:

$$v_{\pi}(s_1) = \frac{1}{2}(-1 + v_{\pi}(s_2)) + \frac{1}{2}(-3 + v_{\pi}(s_3))$$

$$v_{\pi}(s_2) = \frac{1}{2}(-1 + v_{\pi}(s_1)) + \frac{1}{2}(5 + v_{\pi}(s_4))$$

$$v_{\pi}(s_3) = \frac{1}{2}(-1 + v_{\pi}(s_1)) + \frac{1}{2}(5 + v_{\pi}(s_4))$$

$$v_{\pi}(s_4) = 0$$

Since the equations for $v_{\pi}(s_2)$ and $v_{\pi}(s_3)$ are identical, we must have that $v_{\pi}(s_2) = v_{\pi}(s_3)$.

Thus, the equations for $v_{\pi}(s_1)$ and $v_{\pi}(s_2)$ can be changed to:

$$v_{\pi}(s_1) = \frac{1}{2}(-1 + v_{\pi}(s_2)) + \frac{1}{2}(-3 + v_{\pi}(s_2)) = -2 + v_{\pi}(s_2)$$

$$v_{\pi}(s_2) = \frac{1}{2}(-1 + v_{\pi}(s_1)) + \frac{1}{2}(5 + 0) = 2 + \frac{1}{2}v_{\pi}(s_1)$$

Combining the two latest equations yields

$$v_{\pi}(s_1) = -2 + 2 + \frac{1}{2}v_{\pi}(s_1) = \frac{1}{2}v_{\pi}(s_1),$$

which implies $v_{\pi}(s_1) = 0$. Furthermore,

$$v_{\pi}(s_3) = v_{\pi}(s_2) = 2 + \frac{1}{2}v_{\pi}(s_1) = 2 + 0 = 2.$$

Thus, the state-value function is given by:

$$v_{\pi}(s_1) = 0$$

$$v_{\pi}(s_2) = 2$$

$$v_{\pi}(s_3) = 2$$

$$v_{\pi}(s_4) = 0$$