

Issues and Challenges in Learning Foundation Linear Algebra Course with Technology: A Literature Review

Gurwinder Singh
Chitkara University Institute of
Engineering and Technology
Chitkara University
Punjab, India
gurwinder55@chitkara.edu.in

Neha Tuli
Chitkara University Institute of
Engineering and Technology
Chitkara University
Punjab, India
neha.tuli@chitkara.edu.in

Archana Mantri
Chitkara University Institute of
Engineering and Technology
Chitkara University
Punjab, India
archana.mantri@chitkara.edu.in

Abstract — Linear algebra is one of the core subjects for the majors in emerging technologies such as data science, artificial intelligence, robotics and computer graphics. Various solutions had been designed to effectively teach the essence of linear algebra. Teaching such a subject is regarded as a challenging task due to a lack of visualization tools that addresses the essence of linear algebra. In this paper, we analyse the state of technologies used in learning foundation linear algebra course. We analyse features, issues, challenges and review learning material and technology illustrated in the research papers that appear on this issue published from 1993 through 2018. Through this paper, we identify areas for potential research on teaching and learning for current applications in this increasingly important field. This article highlights some of the overall findings of this research concerning student's understanding of these concepts and also areas for future research into the development of application in teaching and learning linear algebra.

Index Terms — linear algebra, applications review, learning issues, application analysis

I. INTRODUCTION

Linear algebra is a field of research of mathematics that is mainly concerned with the study of vector spaces and their linear transformations. Matrices allow finite-dimensional vector spaces and linear maps to be manipulated directly. Consequently, their theory is an integral aspect of linear algebra. Linear algebra courses at university level are generally considered by students as a complex mathematics course. The content is often complex and comprehensive, which can be intimidating to beginners, as opposed to what they were exposed to from previous math courses [1]. This may detach linear algebra from the mathematics concepts previously learned by students. From an educational point of view, algorithms and procedures solve most analysis problems in conventional linear algebra classes without the need of spatial skills [3][5]. Thus, students lack these skills at further stages with application-based approach of the concepts. Basic linear algebra concepts are explained through definitions and theorems with symbolic representation. However, there are

only a few teaching themes that focus on the visual representation of these concepts for developing spatial skills [14]. The typical classroom atmosphere contained learning linear algebra exclusively to the presentation of lessons. It does not satisfy the need for students to learn topics that they will need in later career. Visualization of abstract concepts works as the starting point for idea creation and critical reasoning in linear algebra. Various application had been developed that facilitate the learning of linear algebra with learning material, quizzes, visualization and games [16-35].

II. LITERATURE REVIEW

A study of over fifteen research papers dating from 1993 to 2020 to analyse the issues and challenges along with the recommendations in the teaching of linear algebra. The literature review is depicted in Table 1. that summarizes challenges in learning material and teaching approach and also with recommendations in curriculum and technology. A literature review summarizes that 15 studies focused on spatial skills and importance of visualization for learning linear algebra.

Curriculum study group that aimed to improve the undergraduate linear algebra curriculum with specific recommendations advised the mathematics department for making their course in linear algebra a matrix -oriented and involving technologies specifically software that requires no programming experience [1]. Students face difficulty in learning linear algebra without getting the application phase of it. The investigation highlighted the difficulty in learning abstract concepts and central concepts such as vector space without any intuition [2]. An alternative approach is suggested for learning linear algebra with computer simulations to build mental constructs of abstract concepts such as linear dependence, vector spaces, eigenvalues and eigenvectors and other basic concepts.

Table 1. Review of issues and challenges in learning linear algebra from research studies

Ref No. / Year of publishing	Journal / Conference / Book Name	Publisher	Summary
[1] / 1993	The College Mathematics Journal	Mathematical Association of America	Curriculum is more emphasized on theorem proving and computational oriented. There is a need to utilize computation technology and design syllabus according to disciplines. A Matrix Oriented Course have to be designed.
[2] / 1997	Resources for Teaching Linear Algebra	Mathematical Association of America	Student's face difficulty in learning abstraction and concept and courses are taught too early. There is no connection to prior experience of student knowledge. There is a need to introduce transformation of matrices with the visualization tool.
[3] / 2000	On the Teaching of Linear Algebra	Springer	Curriculum is more emphasized on theorem proving and computational oriented. Teaching pattern summarises that assistive teaching technology is less utilised while teaching linear algebra. Spatial skills need to be enhanced among students using the geometric representation of concepts.
[4] / 2001	The Teaching and Learning of Mathematics at University Level: An ICMI Study	Springer	No visualization of abstract concepts is available to the students. There is a limitation of geometry-based approach in Traditional Teaching without the use of technology. Student-Tutor interaction is subjective thus understanding of subjects is also based upon that. There is a need to introduce 'Engaging Questions' approach in Teaching interaction with instructor for linear algebra course to be effective.
[5] / 2009	International Journal of Mathematical Education in Science and Technology	Taylor & Francis	Curriculum doesn't build relations among concepts such as linear dependence and span. Students struggle at learning abstract concepts as there is less understandably of definitions. There is a need for application-based teaching approach and frameworks to learn linear algebra.
[6] / 2010	Linear Algebra and its Applications	Elsevier	No visualization approach in teaching abstract concepts were discovered in teaching approaches. There is a need to introduce visualization tool such as Maple and Cabri. Visualization and Manipulation systems in linear algebra can enhance the learning experience of students. Teachers should utilize Computation Technology to shift approach to application-based curriculum.
[7] / 2013	Handbook of Research on Educational Communications and Technology	Springer	No research design cycle of task design and analysis of learning material. No decomposition of expert knowledge into enabling students to imagine concepts. Recommended visualization tools to use and activities to re-invent span, linear dependence and other theoretical concepts
[8] / 2013	Technology, Knowledge and Learning	Springer	No clear relationship defined between computational and geometric interpretation. More computational oriented Recommended visualization tools to learn Eigenspace Material should consist of a combination of arithmetic/algebraic/geometric.
[9] / 2017	Journal of Mathematical Behaviour	Elsevier	Geometry focused but no specific relation to application-based problem-solving. Inadequate attention to conceptual understanding. Recommended visualization tools to learn spatially based terminology. Focus on prior knowledge before building concepts
[10] / 2018	Teaching Linear Algebra	Springer	Material is strictly focused on specific approach arithmetic/algebraic/geometric. Recommended visualization tools to use. Material should consist of a combination of arithmetic/algebraic/geometric
[11] / 2019	ZDM	Springer	Less focus on Spatial based terminology and no physical reality-based references in teaching linear algebra. Recommended visualization tools to use in teaching methodology. Geometry based curriculum is suggested to
[12] / 2019	ZDM	Springer	No clear relationship defined between computational and geometric interpretation. Homogeneity of approach adopted by the instructor. Recommended visualization tools to use such as Geometer's Sketchpad Labs. Intuition based model for teaching
[13] / 2019	International Journal of Mathematical Education in Science and Technology	Taylor & Francis	Learning material is generic even for engineering branches. Students show disinterest in lecturing and computation session Visualization tools is recommended as an assistive technology to enhance the learning experience. There is also a recommendation to establish the relationship among visual to abstract vector spaces

[14] / 2019	ZDM	Springer	There is no connection between textbook content by author and research-based curriculum approach. And also, less usage of online material to assist the teaching of complex linear algebra. Dynamic geometry system should be introduced to students in teaching linear algebra.
[15] / 2020	Teaching Mathematics and its Applications	Oxford	There is less focus on intuitive proofs while more computational oriented approach while teaching linear algebra. Matrix should be represented as geometric, tabular and graphical to enhance spatial skills of students which then improves problem-solving ability.

Curriculum reforms are suggested that focus on cognitive flexibility and trans-object level of thinking which omits the computational part and only focuses on the conceptual part. Study also emphasis on violation of ‘necessity principle’ i.e. developing knowledge as a solution to the problem rather than the transmission of information [4]. Evidence revealed lack of essence and foundation of linear algebra among the majority of students. The study analysed the impact of framework helping educators to trace and help students while mentioning the future scope of visualization tool [5].

Visualization software such as CAS Maple and geometric software facilitates and improves the learning procedure of linear algebra. However, there is lack of engagement of educators and students and also less focus on improving analytical skills [6]. Synthetic-geometric mode of thinking is emphasised among students using geometric sketches for teaching basic linear algebra concepts. The study emphasis on visual learning as essential to the understanding of the subject [8] [9]. Study states positive effect on students' attitude through geometric based teaching process for linear algebra [12]. Visualization tools are emphasized in curriculum and a dynamic simulation system is suggested for learning concepts such as linear transformation, linear independence or eigenspace [13]. Following are the few research gaps analysed from the literature review:

- A need has to be met for visualization tool in the field of linear algebra that doesn't have computer programming prerequisite.
- There are no studies based on the literature review that defines a framework for the visualization of basic spatial-based concepts of linear algebra.

- There is significant improvement in learning experience using matrix-oriented visualization approach for teaching linear algebra concepts but it's potential is not yet explored.

These key points are the basis of research and review of existing market application that offer to teach linear algebra. Next section comprises of reviews of application existing in the market based upon the features provided.

III. APPLICATIONS REVIEW

This section discusses the findings of the applications available on the internet and systematic app reviews.

A. Methods

Systematic app review has been conducted for applications related to linear algebra. This research comprises of twenty market application from Android Play Store searched with the keyword ‘linear algebra’, ‘learn linear algebra’ and ‘matrix solver’. Top twenty user rated application were selected to analyse its features in teaching linear algebra and issues and challenges regarding the same.

Five experts were given twenty application to review with minimum time spent of 20 minutes per application. Their reviews were categorised in features of application and issues that they find with regards to teaching theme from Section 2.

B. Application Overview

In the light of the literature review, the section discusses the findings concerning the three questions the research was intended to address: What application exist in market that can be suggested for a first-year linear algebra undergraduate course? How does that application enable user to learn linear algebra? Does the application provide matrix-oriented visualization of abstract concepts?

Table 2. Review of market application for Teaching Linear Algebra

Ref No.	Application Name	Features	Issues	App ID	Developer
[16]	Linear Algebra	Application consists of matrix solver for various operations and summary of fundamental theorems of linear algebra.	No visualization of concepts No step by step explanation of computation	com.aswdc_lineara lgebra	Darshan Institute of Engineering & Technology
[17]	Linear Algebra Calculator	Linear Algebra calculator for computing matrix operations and linear system solutions	No explanation of concepts No visualization tool and application-based example for concepts	com.ttyrovou.linea ralgebra.full	oogee
[18]	Linear Algebra: Practice Tests and Flashcards	Application includes 60 practice tests and diagnostic test	No computation calculator and visualization tool available Application is practice question based only	com.varsitytutors.l earningtools.linear algebra	Varsity Tutors LLC

[19]	Linear Algebra Course App	This app consists of vector visualization, vector operations, matrix operations while connected online. This is an extended module of Wolfram Alpha for linear algebra.	Works only in online mode as a module is connected to wolfram alpha	com.wolfram.android.linearalgebra	Wolfram Group
[20]	Linear Algebra	Linear Algebra calculator for computing linear operations and operation with matrices.	No explanation of concepts No visualization tool and application-based example for concepts	mathapp.linearalgebra	Spiros Diplaris
[21]	3000 Solved Problems in Linear Algebra	Application consists of solved problems and video lectures of Linear Algebra.	Only web view of various PDF of books in the application No matrix operation calculator, no visualization tool	com.newandromo.dev26743.app651980	MasterpieceApps
[22]	Linear Algebra Courses	Video Lecture available for Linear Algebra Courses	No visualization of concepts No step by step explanation of computation	aplus.linearalgebracourses	APLUS
[23]	LINEAR ALGEBRA PLUS CALCULATOR	Explanation of fundamental topics of linear algebra and basic matrix calculator	No matrix-oriented visualization of concepts No step by step explanation of computation	com.acme.algebraonline_1	Áscar Montero Fernández
[24]	Matrix operations	Advance matrix operations calculator and explanation of fundamental topics of linear algebra	No visualization of the concepts provided	com.highermathematics.linearalgebra.free	HMLA
[25]	Linear algebra	Learning material for Linear Algebra such as Videos and written content	No visualization tool as well as no matrix operation calculator was provided	com.do_apps.catalog_1075	Kirill Sidorov
[26]	Linear Algebra	Step by step matrix solver, row echelon, reduced row, matrix inverse finder.	No learning material available No explanation of concepts No visualization tool and application-based example for concepts	org.test.la	Arash Gholami
[27]	Maths: linear algebra course	Video lectures and courses of linear algebra	No learning material and matrix operation calculator available	com.devh.algebre	Dev H
[28]	Linear Equation Solver	Solving systems for linear equations.	No visualization tool and application-based example for concepts	com.kasun.easysolutions	K. Dev
[29]	Linear Algebra FREE	Only Matrix Calculator	No learning material available No explanation of concepts No visualization tool and application-based example for concepts	com.matrixcalculator.app	Artem Grinko
[30]	Linear Algebra Calculator - Eigenvalues And More!	Application consists of learning material for basic concepts of linear algebra and matrix solver for various operations	No matrix-oriented visualization of concepts No step by step explanation of computation	comfymobile.edu.matrixhelper	Kay Studio
[31]	Linear Algebra PRO	Matrix operation calculator	No learning material available No explanation of concepts No visualization tool and application-based example for concepts	com.KayStudio.android.linearalgebracalculator	Artem Grinko
[32]	Linear Equation Solver & Matrix Solver	Mathematical app used to solve system of linear equation up to five variables.	No explanation of concepts No visualization tool and application-based example for concepts	comfymobile.edu.matrixhelper.noads	Akash Pradeep
[33]	Quinoa Linear Algebra Toolkit	Only basic calculator for matrix operations available	No explanation of concepts No visualization tool and application-based example for concepts	com.AppShare2021.linear_equation_solver	Samuel Sweet
[34]	Math & Science Tutor - Algebra, Calculus, Physics	Video lectures and courses of linear algebra, calculus and other topics	No computation calculator of matrix operation. No visualization of topics	com.spaceroostudios.quinoa	Math Tutor DVD, LLC
[35]	Matrices and Determinants	Explanation of fundamental topics of linear algebra and matrices.	Only theory of the topics is given. No computation calculator for matrix operation. No visualization of topics	com.mathtutordvd.mathtutor.mathtutor	Learner's Series

Analysis of twenty applications has been formulated in the Table 2. Experts reviewed the applications on the following metrics: Does the application explain basic concepts of linear algebra? Does it have visualization module of matrices? Is there a calculator available for matrix operations? Does the application provide any practice questions with video

lectures/books? The detailed findings of the apps have been discussed in Section 4.

IV. FINDINGS AND DISCUSSION

A. Application Review Analysis

The analysis showed that there were only a small number of apps have been developed. Nonetheless, we mention that eventually, further application development efforts will come within the future. Our research revealed the key uses of the

creation of smartphone apps and functionality is to accomplish the fundamental principles of linear algebra.

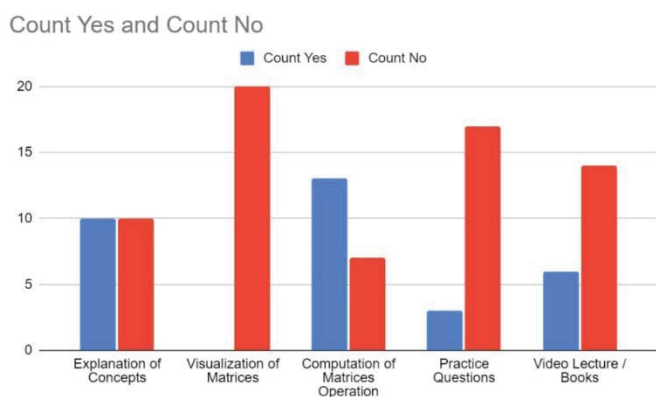


Fig 1. Analysis of 20 Mobile Applications

From the analysis of Fig 1. it has been observed that about 50% (10 out of 20) of the applications explain basic concepts of linear algebra while none of them provided visualization tool for matrix which must factor analysed from literature review. None of the applications provided the framework for visualization of matrix-oriented concepts. More than half of the selected applications provide a calculator for matrix operations which is a necessary tool while learning conceptual part with complex computation. One-third of the application included additional learning material such as practice questions, video lectures and books. The three key points analysed from the literature review were not addressed in any of the application. A need still has to be met for a matrix-oriented visualization framework and research on its impact on teaching-learning experience.

B. Drawbacks of the study

There are a few drawbacks to the research discussed in this paper that are important to mention. First, all available applications may not be addressed by the parameters (search strings/keywords) chosen to be appropriate for this analysis. Second, related apps created or made available in the app stores after November 20, 2020, are not included in this report. Third, the evaluation data was collected using the review and notice-collect-think method using the qualitative method. Qualitative analysis is subjective and relies in part on the talents, experience, and knowledge of analysts.

V. CONCLUSION AND FUTURE SCOPE

In this paper, we reviewed applications that can be used to learn and explore concepts of linear algebra. The work aims to provide in-depth research and review of in the learning and teaching applications for students and teachers.

Research gaps from the literature review were extracted regarding the issues and challenges faced in learning foundation course of linear algebra. Then systematic review of available applications was conducted by five subject experts based on research gaps and evaluation metrics. The

review study explored twenty applications, their features and learning material issues.

Given current attempts to undertake empirically-based studies, there are still many fields of study in learning linear algebra that demand work. In the research literature, the attempt to systematically document industry needs in ways that will guide research on the teaching and learning of linear undergraduate algebra has not received much attention. Emerging fields such as data science, machine learning, robotics heavily rely on linear algebra. Understanding the mathematical content in more advanced courses is becoming more and more vital for many in the industry. For the industry, mathematical content and visualization skills for application of advanced courses is becoming increasingly important. It can be extremely useful to inform the research of industry experts who closely document how linear algebra is applied in these fields for qualitative research.

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