Flex:

Call option $T = 1$ yr.
Accident happens when $S_T > K$
"strike"

K will be determined later!

"K will be the price of stocks in 3-months"

"Transportation" Asian

Accident happens if oil price goes above \$ K — too expensive

Accident happens if Annual Average Oil price goes above K — Cheaper

"σ" is reduced

$\underline{\underline{\sigma}}$ is $\underline{\underline{\text{reduced}}}$

Exchange Option

Insurance pays $\underline{\text{Max}(S_1 - S_2, \underline{\underline{0}}) \text{ at } T}$

S_1 ? ? S_2 ✓

Rainbow Option. which $\underline{\underline{\text{to bu}}}$y

$\underline{\text{Pays Max}(S_1, S_2) \text{ at } \cancel{E}\ T}$

$\text{Max}(\underbrace{\text{JPM}, \text{GS}, \text{ML}, \ldots\ldots}) \text{ at } T$

$$\boxed{\text{Max}\left(\frac{1}{S_1}, \frac{1}{S_2}, \ldots, \frac{1}{S_n}\right) \text{ at } T.}$$

Lookback (No regret option)

There $\underline{\underline{\text{is regret}}}$

When to $\underline{\underline{\text{buy}}}$

Lookback pays $S_{max} - S_{min}$ in the lookback window

$$Max(S_1 - S_2, 0) = Max(S_1, S_2) - S_2$$

exchange = Rainbow + short S_2

exchange + S_2 = Rainbow

more expensive

$S_T = 150$

I am hedged

I have to pay you \$110

I have the stock = 150

$$\begin{array}{r} 150 \\ -110 \\ \hline 40 \end{array}$$

return the borrowed money with $r = 0\%$

$S_T = 100$

I have ... pay you \$60

stock = 100

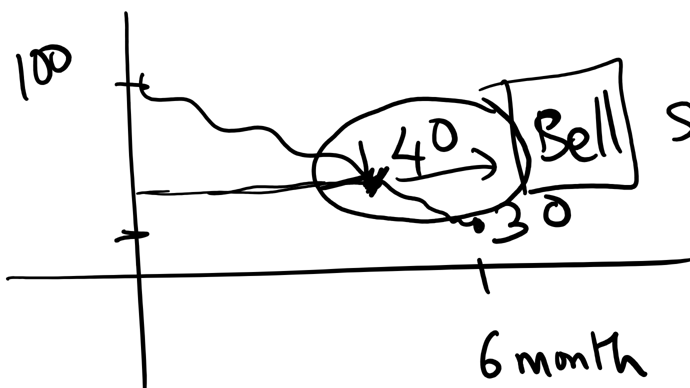
$S_T = 100$ { stock = 100
-60
40 ✓

return with $r = 0\%$

$\boxed{S_T = 30}$ { I have to pay you 0
S_T = 30 ✗ not okay!!

return 40 ?

$S_T = 30$



stock @ 40

Does this work.

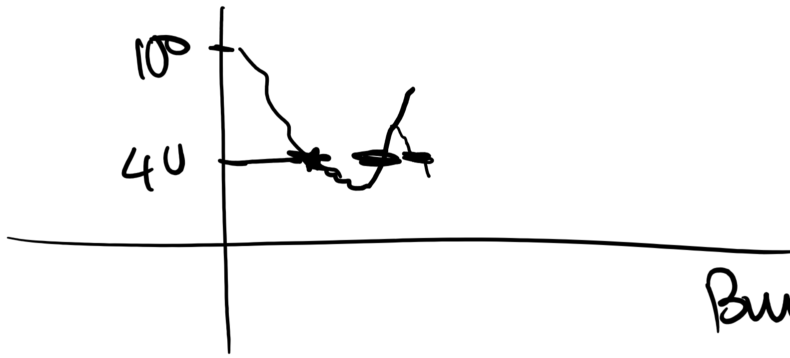
$S_T = 30$ I have to pay you 0
S_T sold @ 40 40 ✓
return with $r = 0\%$

okay

'Microstructure problem' Cannot sell large stock orders @ 40.

Take \$60

mot.



Take \$60

Borrow $\underline{40+\epsilon}$

100

Buy Stock $-\underline{100}$

Put @ K=40

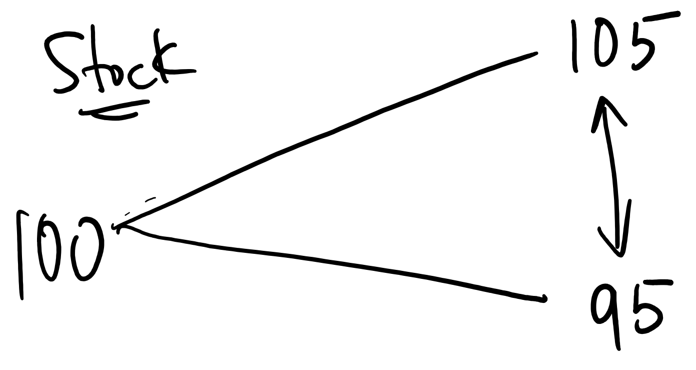
Call @ K=40

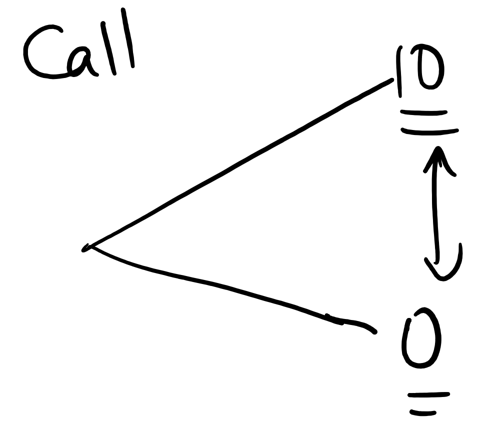
Put @ K=40

Call @ K=40

I sold $\underline{K=95}$ Call to you

What is the Hedging Scheme?





$$-C + \text{Buy}\left(\frac{10-0}{105-95}\right)\text{ Shares}$$

Borrow

$$-C + \text{Buy } \frac{10}{10} = -C + 1 \text{ Shares}$$

$$-C + 1\text{share}$$

$$-10 + 105 = 95$$

$$\parallel$$

$$0 + 95 = 95$$

$$(-C + 1\text{share}) = 95e^{-r(0.5)}$$

$$100 = 95$$

$$C = 5$$

Increase tree steps

Accounting equations at every node

Accounting equations at every node

$\Downarrow$

"I am okay" at every node

$\Downarrow$ $n \to \infty$

Diff. Equation (no probability)

$\Downarrow$

B.S diff equation (prob. free)

$$\text{BS} \quad \boxed{\frac{\partial f}{\partial t} + r \cdot S \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = r f}$$

f: price of derivative.

$$\boxed{f = E^{Q}\left[f(S_T) \mid \ldots \right]}$$

Q = "Special Probability" (Feynman Kac)

"Determistic" $\Longleftrightarrow$ "Probabilistic"

Comment:- If you replace Stocks with bonds.

Then "probability" is important

"Accounting Eqns" will need prob.

HJM framework =) remove the prob again!