Week 39: Advanced Geometry – Half-Plane Intersection & Rotating Calipers

Topics: - Half-Plane Intersection (HPI) - Convex Polygon Intersection - Rotating Calipers Method - Applications: Smallest Enclosing Rectangle, Diameter of Convex Polygon - Area and Perimeter Calculations of Intersections - Geometry Optimization Problems

Weekly Tips: - Half-plane intersection can be solved with line sorting and deque-based algorithm. - Rotating calipers can find farthest points, polygon diameter, and minimum bounding rectangle. - Convex polygon problems often reduce to geometry + rotating calipers. - Be careful with floating-point precision, use `long double` when necessary. - Always normalize line directions when working with HPI.

Problem 1: Convex Polygon Diameter (Rotating Calipers) Link: CSES Polygon Diameter **Difficulty:** Advanced

C++ Solution with Explanation Comments:

```cpp
#include <bits/stdc++.h>
using namespace std;
struct Point{ long double x,y; };
long double dist2(Point a,Point b){ return (a.x-b.x)*(a.x-b.x)+(a.y-b.y)*(a.y-b.y); }
int main(){
    int n; cin>>n; vector<Point> p(n);
    for(int i=0;i<n;i++) cin>>p[i].x>>p[i].y;
    // Convex polygon assumed (if not, run convex hull first)
    long double ans=0; int j=1;
    for(int i=0;i<n;i++){
        while(dist2(p[i],p[(j+1)%n])>dist2(p[i],p[j])) j=(j+1)%n;
        ans=max(ans,dist2(p[i],p[j]));
    }
    cout<<fixed<<setprecision(6)<<sqrt(ans)<<endl;
}
```

Explanation Comments: - Rotating calipers checks antipodal points on convex polygon. - Maintains two pointers to find max distance efficiently. - O(n) after convex hull computation.

Problem 2: Half-Plane Intersection (HPI) Conceptual Overview: - Represent half-planes as lines + side. - Sort lines by angle. - Use deque to maintain feasible intersection polygon. - Final intersection polygon is convex.

Applications: - Polygon clipping. - Visibility regions. - Linear constraints optimization.

Skeleton Code:

```
// Pseudocode structure for Half-Plane Intersection
struct Line{ Point p; Point dir; double ang; };

// 1. Sort lines by angle
// 2. Use deque to store intersection polygon edges
// 3. Remove lines that make intersection infeasible
// 4. Compute final polygon from deque
```

Applications of Rotating Calipers: - Smallest enclosing rectangle of convex polygon. - Width and thickness of polygon. - Farthest pair of points in convex hull.

End of Week 39 - Master rotating calipers for polygon problems. - Learn Half-Plane Intersection for advanced geometry optimization. - Practice convex polygon problems and intersection calculations.