$p\circ g = p$ $\quad g : \mathrm{Aut}\,\tilde{X} \to \tilde{X}$

$\{g\in \mathrm{Aut}(\tilde{X}) : p\circ g = p\} = \mathrm{Deck}(\tilde{X}/X) \overset{\text{universal}}{\sim} \pi_1(X)$

$\forall p$ 万有

$$\begin{tikzcd} \tilde{X} \arrow[r] \arrow[dr] & P^1 \arrow[d] \\ & P^1 \end{tikzcd}$$

$\boxed{P^1 \to P^1}$

Aut

P^1's uni. cov. :

$? : \tilde{P}^1 \to P^1 \to$ ctns

$(\tilde{P}^1, ?)$ 是 P^1的 covering space

$\pi : P^1 \to P^1$

$X = \tilde{X}/\mathrm{Aut}(\tilde{X}, \pi)$

$\pi : \tilde{X} \to X$

$\displaystyle \bigcup_{x\in X} \bigsqcup_{\alpha\in D_x} V_\alpha \xleftarrow{\pi^{-1}} U_x$ $\quad X$

$\pi^{-1}(U_x)$

$V_\alpha \leftarrow$ 同胚

$D_x = \{d_1, \dots, d_{n_x}\}$

$\pi_1(T^2) \cong \mathbb{Z}\times\mathbb{Z}$

Fuchsian $\quad \Gamma \hookrightarrow \mathrm{Aut}(D)$

$\tilde{X}/\Gamma = X$, $\Gamma = \{e\}$

$$\begin{array}{ccc} \tilde{X} & & X \\ P' & \longleftrightarrow & P' \\ C & \longleftrightarrow & C \\ C & \longrightarrow & C/\mathrm{sos} \\ C & \longrightarrow & T^2 \end{array}$$

$$D \overset{X}{\longleftrightarrow} \text{other}$$

Def: 1.8.12 (Finite type) A Riemann surface is of finite type if it is iso to a cpt surface from which finite points removed.

non-hyperbolic $\rightarrow$ finite type

hyperbolic $\rightarrow$ 1. genus 0 + ≥ 3 points removed

2. genus 1 + ≥ 1 points removed

$(\chi = 2-2g-r<0)$ 3. genus ≥ 2

Def 1.8.13.

of Aut D;

of PSL_2

$X = D/\Gamma$

Pro 1.8.14.

Then $X = D/\Gamma$

Riemann surf

ce is of
surface

removed
moved

Def 1.8.13. A Fuchsian group is a discrete subgroup of Aut D; A Kleinian group is a discrete subgroup of $PSL_2\mathbb{C}$

$X = D/\Gamma$ Γ: Fuchsian group

Prop 1.8.14. If $\Gamma \subsetneq \mathrm{Aut}\, D$ is torsion free

Then $X = D/\Gamma$ has a unique structure of a Riemann surface $D \to X$ local iso.

Pf: ① $\forall z \in D$, $\exists U$. $z \in U$. s.t $P|_U : U \to X$ is inj

$\Rightarrow$ If $\gamma \in \Gamma$ s.t $\gamma(z) \in U$, then γ is id

Pf: $\exists \gamma_i$ $\gamma_i(z) \to z$ $\underline{\gamma_i(z) \neq z}$ $\forall z$, then $d(z, \gamma_i(z))$

converge subsequence $\{\gamma_{ij}\}$ $\delta = \lim_{j\to\infty} \gamma_{ij}$ $\delta(z) = z$

Γ discrete γ_{ij} constant $\gamma_{ij} = \delta \Rightarrow$ inj

② AIM: $[z_1] \neq [z_2]$ i.e. $\underline{z_2 \notin \Gamma z_1}$ $\exists U_1 \ni z_1, U_2 \ni z_2$ s.t. $P(U_1) \cap P(U_2) = \phi$

$\exists \gamma_i \in \Gamma$ $\underline{w_i \to z_1}$, s.t. $\gamma_i(w_i) \to z_2$ cpt neibor K_2

$\exists N, \forall i > N$ $\gamma_i(w_i) \in K_2$

$S: \{\gamma \mid \gamma(z_1) \in K_2\}$ cpt $\rho(\gamma_i(z_1), \gamma_i(w_i)) = \rho(z_1, w_i) \to 0$

$\gamma_i(w_i) \to \gamma_i(z_1)$ $\{\gamma_i\} \subset S$ $\underline{\exists \gamma_{ij} \to \delta}$

Γ δ $\delta(z_1) = \lim_{j\to\infty} \gamma_{ij}(z_1)$ $\rho(\gamma_{ij}(z_1), \gamma_{ij}(w_{ij})) =$

$\gamma_{ij}(w_{ij}) \to z_2$ $\rho(z_1, w_{ij}) \to 0$

$\lim \gamma_{ij}(z_1) = z_2$

$\delta(z_1) = z_2$ $\Rightarrow\!\!\!\!/$ X is T_2

Finally, $\forall [z] \in X$, find U_z $p|_{U_z} : U_z \to X$ is bij.

coordinate chart $\varphi_z : p(U_z) \to U_z$ is the inverse

If $p(U_{z_1}) \cap p(U_{z_2}) \neq \phi$ $\ni x$

$\exists y_1 \in U_{z_1}, y_2 \in U_{z_2}$ $\gamma \in \Gamma$ $p(y_1) = p(y_2) = x$

$y_2 = \gamma(y_1)$

$\varphi_{z_2} \circ \varphi_{z_1}^{-1} : y_1 \to \gamma(y_1)$. $\gamma \in \mathrm{Aut}\, D$ $\therefore$ analytic

Def

of

of

$X =$

Prol.

Then

Riema

1.1.1. $X \simeq \mathbb{P}^1$ or $\mathbb{C}$ or $\mathbb{D}$

$\uparrow$

$\boxed{1.1.2 \Rightarrow 1.1.3}$ Chapter 1

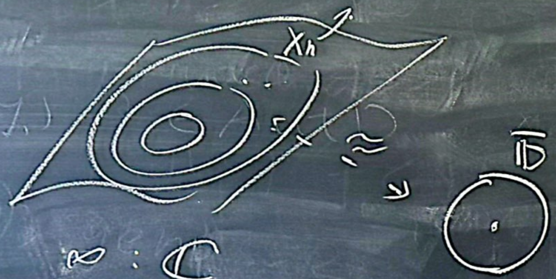
Thm 1.1.2

$\boxed{X \text{ connected, noncompact, } H^1(X,\mathbb{R})=0 \Rightarrow X \simeq \mathbb{C} \text{ or } \mathbb{D}.}$

1.2–1.7 $\longrightarrow$ 1.1.2

1.1.2 $\longrightarrow$ 1.1.3

$\longrightarrow$ 1.1.1



$X_n \nearrow$

$\infty : \mathbb{C}$

有 : $\mathbb{D}$

Def 1.8.13. A Fuchsian ... of Aut $\mathbb{D}$; A Kleinian ... of $PSL_2\mathbb{C}$

$\tilde{X}$

$\mathbb{P}^1 \longleftrightarrow \mathbb{P}^1$

$\mathbb{C} \longleftrightarrow \mathbb{C}$

$\mathbb{C} \longrightarrow \mathbb{C}/_{\{0\}}$

$\mathbb{C} \longrightarrow T^2$

$\mathbb{D}$ Riemann $\longleftarrow X \longrightarrow T^2$

$\mathbb{D} \longrightarrow$ other