Discrete Mathematics

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Propositional Logic

Let p and q be propositions. The *conjunction* of p and q, denoted by $p \wedge q$, is the proposition "p and q." The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

Figure 1. Conjunction

Propositional Logic

TABLE 2 The Truth Table for the Conjunction of Two Propositions.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Figure 2. Conjunction Truth Table

Propositional Logic

- Find the conjunction of the propositions p and q where p is the proposition "Rebecca's PC has more than 16 GB free hard disk space" and q is the proposition "The processor in Rebecca's PC runs faster than 1 GHz."
- Solution: The conjunction of these propositions, $p \wedge q$, is the proposition "Rebecca's PC has more than 16 GB free hard disk space, and the processor in Rebecca's PC runs faster than 1 GHz." This conjunction can be expressed more simply as "Rebecca's PC has more than 16 GB free hard disk space, and its processor runs faster than 1 GHz." For this conjunction to be true, both conditions given must be true. It is false when one or both of these conditions are false.

Propositional Logic

Let p and q be propositions. The *disjunction* of p and q, denoted by $p \vee q$, is the proposition "p or q." The disjunction $p \vee q$ is false when both p and q are false and is true otherwise.

Figure 3. Disjunction

Propositional Logic

TABLE 3 The Truth Table for the Disjunction of Two Propositions.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Figure 4. Disjunction Truth Table

Propositional Logic

- Rebecca's PC has at least 16 GB free hard disk space, or the processor in Rebecca's PC runs faster than 1 GHz
- This proposition is true when Rebecca's PC has at least 16 GB free hard disk space, when the PC's processor runs faster than 1 GHz, and when both conditions are true. It is false when both of these conditions are false, that is, when Rebecca's PC has less than 16 GB free hard disk space and the processor in her PC runs at 1 GHz or slower.

Propositional Logic

Let p and q be propositions. The *exclusive or* of p and q, denoted by $p \oplus q$ (or p XOR q), is the proposition that is true when exactly one of p and q is true and is false otherwise.

Figure 5. Exclusive OR

Propositional Logic

TABLE 4 The Truth Table for the Exclusive Or of Two Propositions.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Figure 6. Exclusive OR Truth Table

Propositional Logic

- Let p and q be the propositions that state "A student can have a salad with dinner" and "A student can have soup with dinner," respectively. What is $p \oplus q$, the exclusive or of p and q?
- The exclusive or of p and q is the statement that is true when exactly one of p and q is true. That is, $p \oplus q$ is the statement "A student can have soup or salad, but not both, with dinner." Note that this is often stated as "A student can have soup or a salad with dinner," without explicitly stating that taking both is not permitted.

Propositional Logic

Let p and q be propositions. The *conditional statement* $p \rightarrow q$ is the proposition "if p, then q." The conditional statement $p \rightarrow q$ is false when p is true and q is false, and true otherwise. In the conditional statement $p \rightarrow q$, p is called the *hypothesis* (or *antecedent* or *premise*) and q is called the *conclusion* (or *consequence*).

Figure 7. Conditional Statement

Propositional Logic

TABLE 5 The Truth Table for the Conditional Statement $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Figure 8. Conditional Statement OR Truth Table

Propositional Logic

- The statement p $\rightarrow$ q is called a conditional statement because p $\rightarrow$ q asserts that q is true Assessment on the condition that p holds. A conditional statement is also called an implication.
- Let p be the statement "Maria learns discrete mathematics" and q the statement "Maria will find a good job." Express the statement p $\rightarrow$ q as a statement in English.
- "If Maria learns discrete mathematics, then she will find a good job."
- "Maria will find a good job when she learns discrete mathematics."

Propositional Logic

- We can form some new conditional statements starting with a conditional statement $p \to q$.
- The proposition $q \to p$ is called the converse of $p \to q$.
- The contrapositive of $p \to q$ is the proposition $\neg q \to \neg p$.
- The proposition $\neg p \to \neg q$ is called the inverse of $p \to q$.

Propositional Logic

- Because conditional statements play such an essential role in mathematical reasoning, a variety of terminology is used to express p $\rightarrow$ q.

- You will encounter most if not all of the following ways to express this conditional statement:

"if p, then q"
"if p, q"
"p is sufficient for q"
"q if p"
"q when p"
"a necessary condition for p is q"
"q unless $\neg p$"
"p implies q"
"p only if q"
"a sufficient condition for q is p"
"q whenever p"
"q is necessary for p"
"q follows from p"
"q provided that p"

Propositional Logic

Implication : $p \rightarrow q$

Converse : $q \rightarrow p$

Contrapositive : $\neg q \rightarrow \neg p$

Inverse : $\neg p \rightarrow \neg q$

Propositional Logic

→ Implication and contrapositive both are equal

→ Converse and Inverse both are equal

Propositional Logic

Truth Table

p	q	$\neg p$	$\neg q$	$p \to q$	$q \to p$	$\neg q \to \neg p$	$\neg p \to \neg q$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	F	T	F	T	F
F	F	T	T	T	T	T	T