Discrete Mathematics

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Propositional Logic

- We can form some new conditional statements starting with a conditional statement $p \to q$.
- The proposition $q \to p$ is called the converse of $p \to q$.
- The contrapositive of $p \to q$ is the proposition $\neg q \to \neg p$.
- The proposition $\neg p \to \neg q$ is called the inverse of $p \to q$.

Propositional Logic

- Because conditional statements play such an essential role in mathematical reasoning, a variety of terminology is used to express $p \rightarrow q$.

- You will encounter most if not all of the following ways to express this conditional statement:

"if p, then q"	"p implies q"
"if p, q"	"p only if q"
"p is sufficient for q"	"a sufficient condition for q is p"
"q if p"	"q whenever p"
"q when p"	"q is necessary for p"
"a necessary condition for p is q"	"q follows from p"
"q unless $\neg p$"	"q provided that p"

Propositional Logic

Find the contrapositive, the converse, and the inverse of the conditional statement

"The home team wins whenever it is raining."

Solution: Because "q whenever p" is one of the ways to express the conditional statement $p \rightarrow q$, the original statement can be rewritten as

"If it is raining, then the home team wins."

Consequently, the contrapositive of this conditional statement is

"If the home team does not win, then it is not raining."

The converse is

"If the home team wins, then it is raining."

The inverse is

"If it is not raining, then the home team does not win."

Only the contrapositive is equivalent to the original statement. ◄

Propositional Logic

→ Implication and contrapositive both are equal

→ Converse and Inverse both are equal

Propositional Logic

~~Th~~ Truth Table

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$q \rightarrow p$	$\neg q \rightarrow \neg p$	$\neg p \rightarrow \neg q$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	F	T	F	T	F
F	F	T	T	T	T	T	T

Propositional Logic

Let p and q be propositions. The *biconditional statement* $p \leftrightarrow q$ is the proposition "p if and only if q." The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Biconditional statements are also called *bi-implications*.

TABLE 6 The Truth Table for the Biconditional $p \leftrightarrow q$.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Propositional Logic

The truth table for $p \leftrightarrow q$ is shown in Table 6. Note that the statement $p \leftrightarrow q$ is true when both the conditional statements $p \to q$ and $q \to p$ are true and is false otherwise. That is why we use the words "if and only if" to express this logical connective and why it is symbolically written by combining the symbols $\to$ and $\leftarrow$. There are some other common ways to express $p \leftrightarrow q$:

"p is necessary and sufficient for q"
"if p then q, and conversely"
"p iff q." "p exactly when q."

The last way of expressing the biconditional statement $p \leftrightarrow q$ uses the abbreviation "iff" for "if and only if." Note that $p \leftrightarrow q$ has exactly the same truth value as $(p \to q) \wedge (q \to p)$.

Propositional Logic

Let p be the statement "You can take the flight," and let q be the statement "You buy a ticket." Then $p \leftrightarrow q$ is the statement

"You can take the flight if and only if you buy a ticket."

This statement is true if p and q are either both true or both false, that is, if you buy a ticket and can take the flight or if you do not buy a ticket and you cannot take the flight. It is false when p and q have opposite truth values, that is, when you do not buy a ticket, but you can take the flight (such as when you get a free trip) and when you buy a ticket but you cannot take the flight (such as when the airline bumps you). ◀

Propositional Logic

Construct the truth table of the compound proposition

$$(p \vee \neg q) \rightarrow (p \wedge q).$$

TABLE 7 The Truth Table of $(p \vee \neg q) \rightarrow (p \wedge q)$.

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Propositional Logic

Implication

p: You get 80 marks in exam

q: You will get A grade

$p \rightarrow q$: If you get 80 marks in exam then you will get A grade

Propositional Logic

The entire sentence will have different meaning in English and the same sentence will have different meaning in logic

→ In logic both are different statements
Both are independent statement. Both are not related.
The value depends upon the truth table

Implication

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

It is a June Month

Propositional Logic

p: It is a June Month

q: $4+4=8$

The $p \rightarrow q$: is always true [Refer the truth table]

The statement which is always true is called <u>Tautology</u>

If today is Monday then $4+4=8$

p: Today is Monday

q: $4+4=8$

Converse: $q \rightarrow p \Rightarrow$ If $4+4=8$ then today is Monday

Contrapositive: $\sim q \rightarrow \sim p \Rightarrow$ If $4+4 \neq 8$ then today is not Monday

Inverse: $\sim p \rightarrow \sim q \Rightarrow$ If today is not Monday then $4+4 \neq 8$

Propositional Logic

Bi Conditional ($\leftrightarrow$)

p	q	$(p \rightarrow q)$	$\wedge (q \rightarrow p)$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

In English $\Rightarrow$ if and only if (iff)

$\Rightarrow$ it is necessary and sufficient

Propositional Logic

p: You will join a MNC

q: You are expert in ML

r: You are not hired by another company

$$p \rightarrow (q \vee r)$$

~~If~~ r: You are hired by another company

$$p \rightarrow (q \vee (\sim r))$$

You cannot ride the bike if you are under 4 feet tall unless you are older than 16 years

p: You can ride the bike

q: You are under 4 feet tall

r: You are older than 16 years

$$(q \wedge (\sim r)) \rightarrow (\sim p)$$

Propositional Logic

You will not recieve mail when the drive is full

p: You will recieve mail   q: The drive is full

$(\sim p)$ when (q)

$(q \rightarrow (\sim p))$