Discrete Mathematics

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August 18, 2020

Propositional Logic

Construct the truth table of the compound proposition

$$(p \vee \neg q) \rightarrow (p \wedge q).$$

TABLE 7 The Truth Table of $(p \vee \neg q) \rightarrow (p \wedge q)$.

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Propositional Logic

Implication

p: You get 80 marks in exam

q: You will get A grade

$p \rightarrow q$: If you get 80 marks in exam then you will get A grade.

Propositional Logic

The entire sentence will have different meaning in English and the same sentence will have different meaning in logic

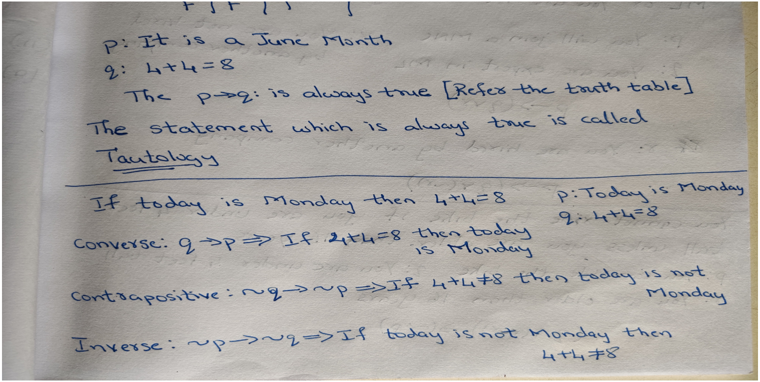
→ In logic both are different statements
Both are independent statement. Both are not related.
The value depends upon the truth table

Implication

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

→ This is a June Month

Propositional Logic



p: It is a June Month

q: 4+4=8

The $p \to q$: is always true [Refer the truth table]

The statement which is always true is called <u>Tautology</u>

If today is Monday then 4+4=8

p: Today is Monday

q: 4+4=8

Converse: $q \to p \Rightarrow$ If 4+4=8 then today is Monday

Contrapositive: $\sim q \to \sim p \Rightarrow$ If $4+4 \neq 8$ then today is not Monday

Inverse: $\sim p \to \sim q \Rightarrow$ If today is not Monday then $4+4 \neq 8$

Propositional Logic

Bi Conditional ($\leftrightarrow$)

p	q	$(p \to q) \wedge (q \to p)$	
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

In English $\Rightarrow$ "if and only if" (iff)
$\Rightarrow$ "it is necessary and sufficient"

Propositional Logic

p: You will join a MNC

q: You are expert in ML

r: You are not hired by another company

$$p \to (q \vee r)$$

~~If~~ r: You are hired by another company

$$p \to (q \vee (\sim r))$$

You cannot ride the bike if you are under 4 feet tall unless you are older than 16 years

p: You can ride the bike

q: You are under 4 feet tall

r: You are older than 16 years

$$(q \wedge (\sim r)) \to (\sim p)$$

Propositional Logic

You will not recieve mail when the drive is full

p: You will recieve mail   q: The drive is full

$(\sim p)$ when (q)

$(q \rightarrow (\sim p))$

Propositional Logic

TABLE 8
Precedence of Logical Operators.

Operator	*Precedence*
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

Figure 1. Precedence of Logical Operators

Propositional Logic

We can construct compound propositions using the negation operator and the logical operators defined so far. We will generally use parentheses to specify the order in which logical operators in a compound proposition are to be applied. For instance, $(p \vee q) \wedge (\neg r)$ is the conjunction of $p \vee q$ and $\neg r$. However, to reduce the number of parentheses, we specify that the negation operator is applied before all other logical operators. This means that $\neg p \wedge q$ is the conjunction of $\neg p$ and q, namely, $(\neg p) \wedge q$, not the negation of the conjunction of p and q, namely $\neg(p \wedge q)$. (It is generally the case that unary operators that involve only one object precede binary operators.)

Propositional Logic

Another general rule of precedence is that the conjunction operator takes precedence over the disjunction operator, so that $p \vee q \wedge r$ means $p \vee (q \wedge r)$ rather than $(p \vee q) \wedge r$ and $p \wedge q \vee r$ means $(p \wedge q) \vee r$ rather than $p \wedge (q \vee r)$. Because this rule may be difficult to remember, we will continue to use parentheses so that the order of the disjunction and conjunction operators is clear.

Propositional Logic

Finally, it is an accepted rule that the conditional and biconditional operators, $\rightarrow$ and $\leftrightarrow$, have lower precedence than the conjunction and disjunction operators, $\wedge$ and $\vee$. Consequently, $p \rightarrow q \vee r$ means $p \rightarrow (q \vee r)$ rather than $(p \rightarrow q) \vee r$ and $p \vee q \rightarrow r$ means $(p \vee q) \rightarrow r$ rather than $p \vee (q \rightarrow r)$. We will use parentheses when the order of the conditional operator and biconditional operator is at issue, although the conditional operator has precedence over the biconditional operator. Table 8 displays the precedence levels of the logical operators, $\neg$, $\wedge$, $\vee$, $\rightarrow$, and $\leftrightarrow$.

Propositional Logic

- Logic has many important applications to mathematics, computer science, and numerous other disciplines.
- Statements in mathematics and the sciences and in natural language often are imprecise or ambiguous.
- Logic is used in the specification of software and hardware, because these specifications need to be precise before development begins
- Propositional logic and its rules can be used to design computer circuits, to construct computer programs, to verify the correctness of programs, and to build expert systems.
- Logic can be used to analyze and solve many familiar puzzles.

Propositional Logic