Discrete Mathematics

Dinesh Naik

Manjunath K Vanahalli

Department of Information Technology,
National Institute of Technology Karnataka, India

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Sets

A *set* is an unordered collection of distinct objects, called *elements* or *members* of the set. A set is said to *contain* its elements. We write $a \in A$ to denote that a is an element of the set A. The notation $a \notin A$ denotes that a is not an element of the set A.

- It is common for sets to be denoted using uppercase letters. Lowercase letters are usually used to denote elements of sets.
- The set V of all vowels in the English alphabet can be written as V = {a, e, i, o, u}.

Sets

- The set O of odd positive integers less than 10 can be expressed by O $= \{1, 3, 5, 7, 9\}$.
- The set of positive integers less than 100 can be denoted by $\{1, 2, 3, \ldots, 99\}$.
- The set O of all odd positive integers less than 10 can be written as O $= \{x \mid x$ is an odd positive integer less than $10\}$,

Sets

- These sets, each denoted using a boldface letter, play an important role in discrete mathematics:
- N = {0, 1, 2, 3,..}, the set of all natural numbers
- Z = {..., -2, -1, 0, 1, 2,...}, the set of all integers
- Z^+ = {1, 2, 3,...}, the set of all positive integers
- Q = {p/q | p $\in$ Z, q $\in$ Z, and q $\neq$ 0}, the set of all rational numbers
- R, the set of all real numbers

Sets

- R^+, the set of all positive real numbers
- C, the set of all complex numbers.

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Sets

Two sets are *equal* if and only if they have the same elements. Therefore, if A and B are sets, then A and B are equal if and only if $\forall x(x \in A \leftrightarrow x \in B)$. We write $A = B$ if A and B are equal sets.

The sets $\{1, 3, 5\}$ and $\{3, 5, 1\}$ are equal, because they have the same elements. Note that the order in which the elements of a set are listed does not matter. Note also that it does not matter if an element of a set is listed more than once, so $\{1, 3, 3, 3, 5, 5, 5, 5\}$ is the same as the set $\{1, 3, 5\}$ because they have the same elements.

Sets

- THE EMPTY SET There is a special set that has no elements. This set is called the empty set, or null set, and is denoted by $\emptyset$.
- The empty set can also be denoted by$\{\ \}$
- That is, we represent the empty set with a pair of braces that encloses all the elements in this set
- A set with one element is called a singleton set.
- A common error is to confuse the empty set $\emptyset$ with the set $\{\emptyset\}$, which is a singleton set.
- The single element of the set $\{\emptyset\}$ is the empty set itself

Sets

- Sets can be represented graphically using Venn diagrams
- In Venn diagrams the universal set U, which contains all the objects under consideration, is represented by a rectangle
- Inside this rectangle, circles or other geometrical figures are used to represent sets
- Venn diagrams are often used to indicate the relationships between sets.

Sets

- Draw a Venn diagram that represents V, the set of vowels in the English alphabet
- We draw a rectangle to indicate the universal set U, which is the set of the 26 letters of the English alphabet.
- Inside this rectangle we draw a circle to represent V. Inside this circle we indicate the elements of V with points

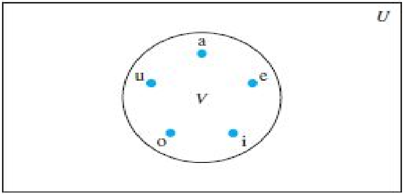

Figure 1. Venn diagram for the set of vowels.

Sets

The set A is a *subset* of B, and B is a *superset* of A, if and only if every element of A is also an element of B. We use the notation $A \subseteq B$ to indicate that A is a subset of the set B. If, instead, we want to stress that B is a superset of A, we use the equivalent notation $B \supseteq A$. (So, $A \subseteq B$ and $B \supseteq A$ are equivalent statements.)

When we wish to emphasize that a set A is a subset of a set B but that $A \neq B$, we write $A \subset B$ and say that A is a **proper subset** of B. For $A \subset B$ to be true, it must be the case that $A \subseteq B$ and there must exist an element x of B that is not an element of A. That is, A is a proper subset of B if and only if

Sets

Let S be a set. If there are exactly n distinct elements in S where n is a nonnegative integer, we say that S is a *finite set* and that n is the *cardinality* of S. The cardinality of S is denoted by $|S|$.

Let A be the set of odd positive integers less than 10. Then $|A| = 5$.

Let S be the set of letters in the English alphabet. Then $|S| = 26$.

Because the null set has no elements, it follows that $|\emptyset| = 0$.

We will also be interested in sets that are not finite.

A set is said to be *infinite* if it is not finite.