# Discrete Mathematics

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# Group Theory

## Algebraic structure

A non-empty set 'S' is called an algebraic structure with respect to binary operation '$*$' if $(a*b) \in S$ $\forall\, a,b \in S$ i.e '$*$' is a closure operation of S.

$S = \{1, -1\}$

$* = \{\times\}$

$(S, *) \rightarrow$ whether it is going to be algebraic structure or not

$$\left.\begin{aligned} 1 \times -1 &= -1 \\ 1 \times 1 &= 1 \\ -1 \times -1 &= 1 \end{aligned}\right\}$$

Belongs to set, the set is closed under the operation. Hence algebraic structure

# Group Theory

$S=\{\phi,\{a\},\{b\},\{a,b\}\}$

$* \rightarrow \cup$ (Union)

$(S,\cup) \rightarrow$

$\phi \cup \{a\} \rightarrow \{a\}$

$\phi \cup \{b\} \rightarrow \{b\}$

$\{a\} \cup \{b\} \rightarrow \{a,b\}$

$(R,+)$ Set of Real numbers are closed under the operation '+'

$(N,\times)$ Set of Natural numbers are closed under the operation '$\times$'

# Group Theory

Semi-group: An algebraic Structure $(S,*)$ is called a semi-group if $(a*b)*c = a*(b*c)\ \forall\ a,b,c \in S$
that is '$*$' is associative on '$S$'

$(N,+)$? $\rightarrow$ First chect for Algebraic structure $\rightarrow$ Yes

Next Semi-group: $\rightarrow$ If any three ~~numb~~ Natural numbers are taken $\rightarrow$ $(a+b)+c = a+(b+c)$
if equal then Semi-group

# Group Theory

$(N, *) \rightarrow$ First it is algebraic structure

if $\rightarrow (a \times b) \times c = a \times (b \times c)$

if equal then semi-group.

$(Z, -) \rightarrow$ It is algebraic structure

$$(a-b)-c = a-(b-c)$$
$$(1-2)-3 = 1-(2-3)$$
$$-1-3 = 1-(-1)$$
$$-4 \neq 2$$

Not equal, hence it is only algebraic structure but not semi-group.

# Group Theory

Moniod: A Semigroup $(S,*)$ is called a moniod if there exists an element $e \in S$ such that

$$(a*e) = (e*a) = a, \ \forall a \in S$$

The element e is called identity element of S with respect to $*$

$(N, \times) \rightarrow$ It is closed, Hence Algebraic structure $\rightarrow$ It is associative, Hence it is Semi-group $\rightarrow$ $a \times e = e \times a$, $e = 1$, Hence it is moniod

$(N, +) \rightarrow$ Algebraic structure $\rightarrow$ Semi-group $\rightarrow$ $a + e = e + a$, $e = 0$

0 is not present in Natural numbers Hence it is not moniod

In sets 0 is included in set of Natural numbers

# Group Theory

Groups: A moniod $(S, *)$ with identity element 'e' is called a group if to each element $a \in S$, there exists an element $b \in S$, such that $(a*b) = (b*a) = e$ the b is called 'inverse of an element a, denoted by $a^{-1}$

$(Z, +) \rightarrow a + \underbrace{(-a)}_{\text{Inverse}} = 0$ (Identity)

Set of integers

# Group Theory

Abelian group (Commutative Group)

A group $(G, *)$ is said to be abelian if

$(a*b) = (b*a) \ \forall a, b \in G$

$(Z, +) \rightarrow$ We have seen it that it is a group

$a+b = b+a$, It is commutative

Hence it is a Abelian group