

Discrete Mathematics

Dinesh Naik

Manjunath K Vanahalli

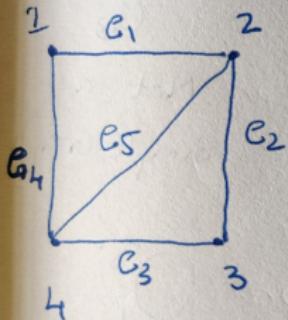
Department of Information Technology,
National Institute of Technology Karnataka, India

September 1, 2020

Graph Theory

Graph Theory

A Graph G consists of vertex set $V(G)$, an edge set $E(G)$, and the relation that associates with each edge, two vertices called its end points



$$V(G) = \{1, 2, 3, 4\}$$

$$E(G) = \{e_1, e_2, e_3, e_4, e_5\}$$

$$e_1 \rightarrow \{1, 2\} \quad e_2 \rightarrow \{2, 3\} \quad e_3 \rightarrow \{3, 4\}$$

$$e_4 \rightarrow \{4, 1\} \quad e_5 \rightarrow \{1, 3\}$$

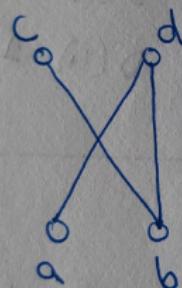
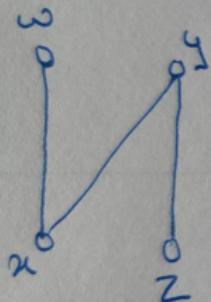
Undirected graph

Graph Theory

Isomorphism: An isomorphism from a simple graph G to a simple graph H is a bijection

$f: V(G) \rightarrow V(H)$ such that $uv \in E(G)$ iff

$$f(u)v \in E(H)$$



$$f: V(G) \rightarrow V(H)$$

$$w \rightarrow c$$

$$x \rightarrow b$$

$$y \rightarrow d$$

$$z \rightarrow a$$

Graph Theory

The number of simple graphs with n -vertices is 2^{nC_2}

$$2^{nC_2} \text{ or } 2^{nC_2} \Rightarrow {}^nC_2$$

out of n vertices \Rightarrow how many ways we can choose 2 vertices

[2 vertices are required for an edge]

Let us consider following
no. of edges

$$\begin{matrix} E_1 & E_2 & E_3 & \dots & E_{nC_2} \\ \downarrow & \downarrow & \downarrow & \dots & \downarrow \\ 2 & 2 & 2 & & 2 \end{matrix}$$

Two because (One with graph with E_1 and another without E_1)

$$= 2 \times 2 \times 2 \dots \times 2^{nC_2 \text{ times}}$$

$$= 2^{nC_2}$$

Graph Theory

$$= 2^{\binom{n}{2}}$$

(i)

2 3

(ii)

2 3

(iii)

2 3

(iv)

2 3

(v)

2 3

(vi)

2 3

(vii)

2 3

(viii)

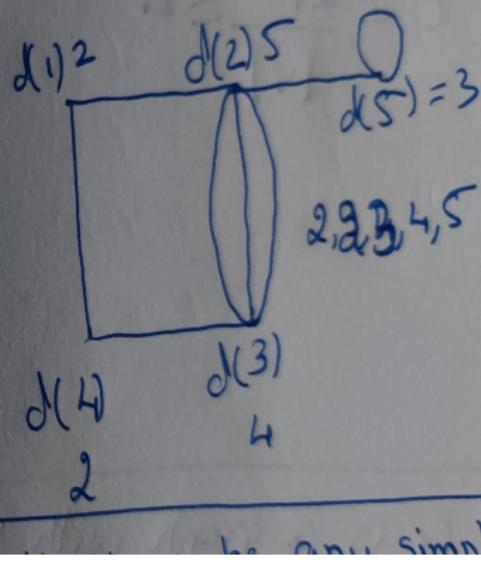
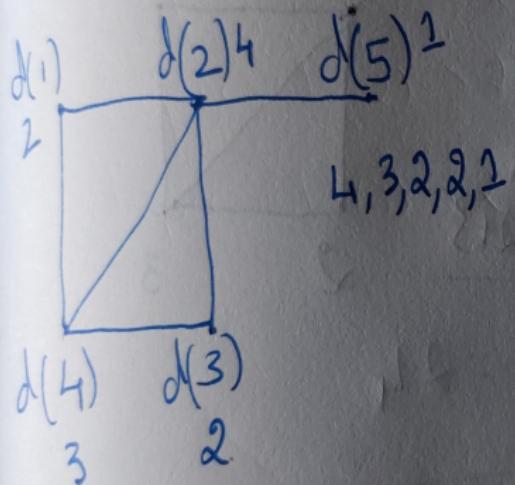
2 3

E_1	E_2	E_3
0	0	0
0	0	1
0	1	0
1	0	0

E_1	E_2	E_3
0	1	1
1	0	1
1	1	0
1	1	1

Graph Theory

Degree Sequence: The arrangement of degrees in non-ascending or non-descending order



be any simple

Graph Theory

For a given degree sequence will there be any simple graph

Ex1: 2, 2, 2, 2



Ex2: 3, 2, 1, 1, 0 (According to hand shaking lemma)

Ex3: 7, 6, 5, 4, 4, 3, 2, 1

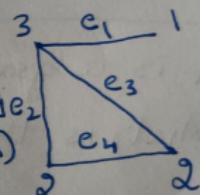
7, 6, 5, 4, 4, 3, 2, 1

7, 6, 5, 3, 3, 2, 1, 0

7, 2, 2, 1, 0, 0

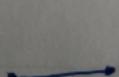
1, 1, 0, 0, 0

If vertex 3 is deleted (Removing a vertex & Edges)



3, 2, 2, 1
2, 1, 0

.



Graph Theory

6, 6, 6, 6, 3, 3, 2, 2

5, 5, 5, 2, 2, 1, 2

5, 5, 5, 2, 2, 2, 1

4, 4, 1, 1, 1, 1

3, 0, 0, 0, 2

3, 1, 0, 0, 0



Graph Theory

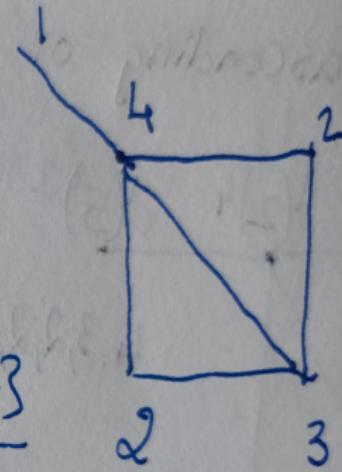
Min degree (δ): Max degree (Δ):

$$\delta \leq \frac{2|E|}{|V|} \leq \Delta$$

$$\delta = 1$$

$$\Delta = 4$$

$$\frac{1+4+2+2+3}{5} = 2.4$$



Graph Theory

1] G is a graph with 11 edges and minimum degree is 3. What is the maximum number of vertices

$$\delta = 3 \quad S \leq \frac{2|E|}{|V|}$$

$$|V| \leq \frac{2|E|}{\delta} = \frac{2 \times 11}{3} = 7 \dots$$

2] G is a graph with 12 vertices and max degree is 4 what is the maximum number of edges.

$$\frac{2|E|}{|V|} \leq \Delta$$

$$|E| \leq \frac{\Delta \times |V|}{2} = \frac{4 \times 12}{2} = 24$$