Discrete Mathematics

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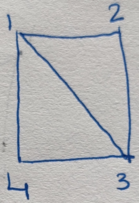
Graph Theory

A clique in a graph is a set of pairwise adjacent vertices

clique with two edges

{1,2,3} and {1,3,4}



Clique with single edge

{1,3} {1,4} {3,4}

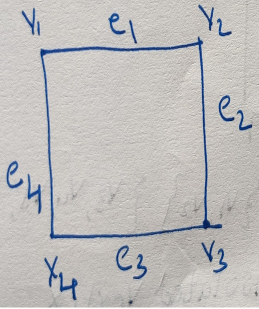
{1,2} {2,3}

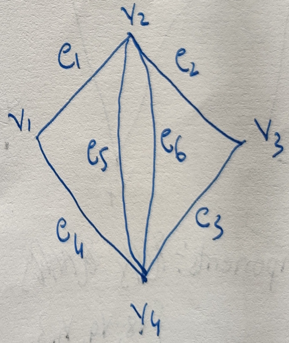
Max clique is the maximum subset of vertices with which we can form an biggest complete graph

In above example: max clique is K_3

Graph Theory

Eulerian Graph: A graph is Eulerian if it has a closed trail connecting all edges





Graph Theory

* We call a closed trail a circuit

* An Eulerian circuit or Eulerian trial in a graph is circuit or a trial connecting all the edges

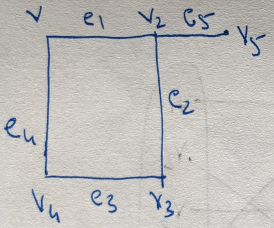
* If every vertex of a graph G has degree atleast 2 then G contains a cycle

* A graph G is Eulerian if and only if it is connected and its all vertices have even degrees

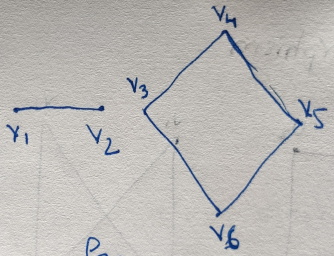
Graph Theory

⇒ A graph G is connected if it has a u,v path whenever $u,v \in V(G)$ [otherwise G is disconnected]

* G has u,v path then 'u' is connected to 'v'



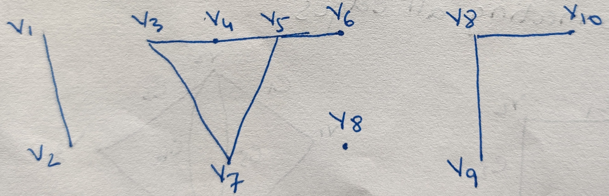
G_1



G_2

Graph Theory

Component: The component of a graph is a sub-graph in which any two vertices are connected to each other by paths, and which is connected to no additional vertices in the graph

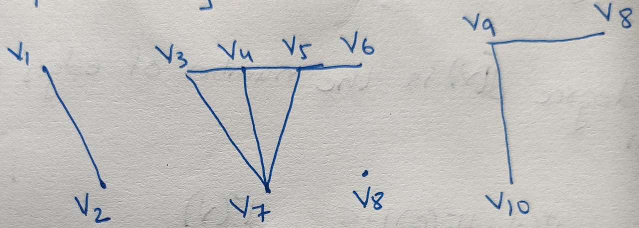


Components:- $\{v_8\}$ $\{v_1, v_2\}$ $\{v_3, v_4, v_5, v_6, v_7\}$ $\{v_8, v_9, v_{10}\}$

↳ isolated vertex

Graph Theory

→ Adding an edge decrease the number of components by 0 or 1.



→ Deleting an edge increase the number of components by 0 or 1

Graph Theory

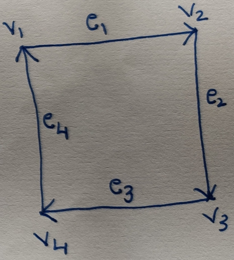
A directed graph or digraph G: G is a graph consisting of a vertex set V(G), an edge set E(G) and function assigning each edge of an ordered pair of vertices

*The first vertex of the ordered pair is the tail of edge, and the second is head

$G(V, E)$

$e_1 \to (v_1, v_2)$ $e_2 \to (v_2, v_3)$ $V(G) = \{v_1, v_2, v_3, v_4\}$

$e_3 \to (v_3, v_4)$ $e_4 \to (v_4, v_1)$ $E(G) = \{e_1, e_2, e_3, e_4\}$



Graph Theory

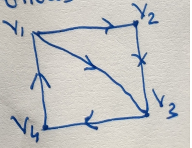
The directed graph is again divided into two graphs

→ Simple directed graph

→ Multi directed graph

Path: A path is a simple digraph whose vertices can be linearly ordered so that there is an edge with tail 'u' and head 'v' if and only if 'v' immediately follows 'u' in the vertex ordering

$v_1 \to v_2 \to v_3 \to v_4$



Graph Theory

Vertex degrees: Let v be a vertex in a diagraph.

→ The out-degree $d^+(v)$ is the number of edges with tail 'v'

→ The in-degree $d^-(v)$ is the number of edges with head 'v'

$$\sum_{v \in V(G)} d^+(v) = |E(G)| = \sum_{v \in V(G)} d^-(v)$$