Discrete Mathematics

Dinesh Naik

Manjunath K Vanahalli

Department of Information Technology,
National Institute of Technology Karnataka, India

September 7, 2020

Graph Theory

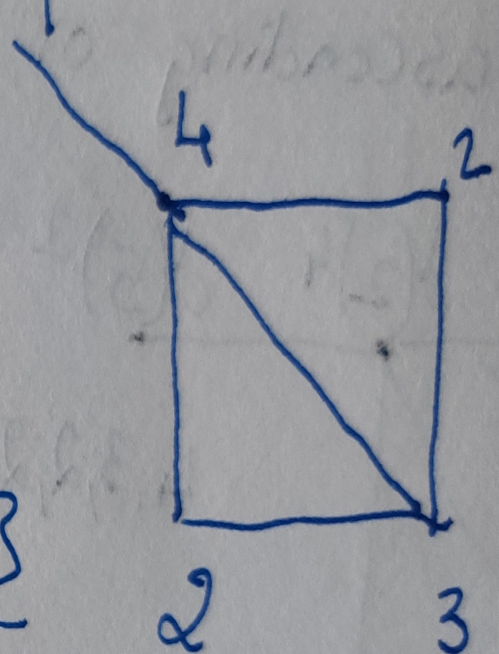
Min degree (δ): Max degree(Δ):

$$\delta \leq \frac{2|E|}{|V|} \leq \Delta$$

$\delta = 1$

$\Delta = 4$

$$\frac{1+4+2+2+3}{5} = 2.4$$



Graph Theory

1] G is a graph with 11 edges and minimum degree is 3. What is the maximum number of vertices

$\delta = 3 \qquad \delta \leq \frac{2|E|}{|V|}$

$|V| \leq \frac{2|E|}{\delta} = \frac{2 \times 11}{3} = 7 \ldots\ldots$

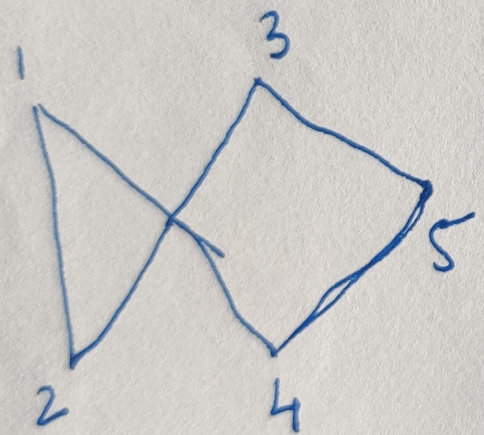
2] G is a graph with 12 vertices and max degree is 4 what is the maximum number of edges.

$\frac{2|E|}{|V|} \leq \Delta$

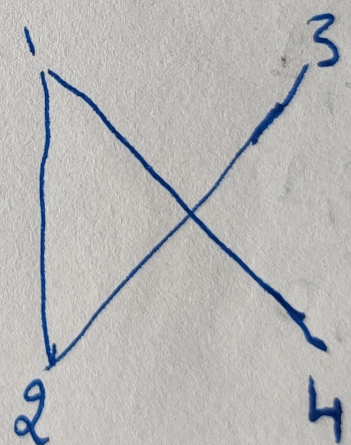
$|E| \leq \frac{\Delta \times |V|}{2} = \frac{4 \times 12}{2} = 24$

Graph Theory

Subgraph: A subgraph of G is a graph H such that $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$, the assignment of endpoints to edges in H is same as in G

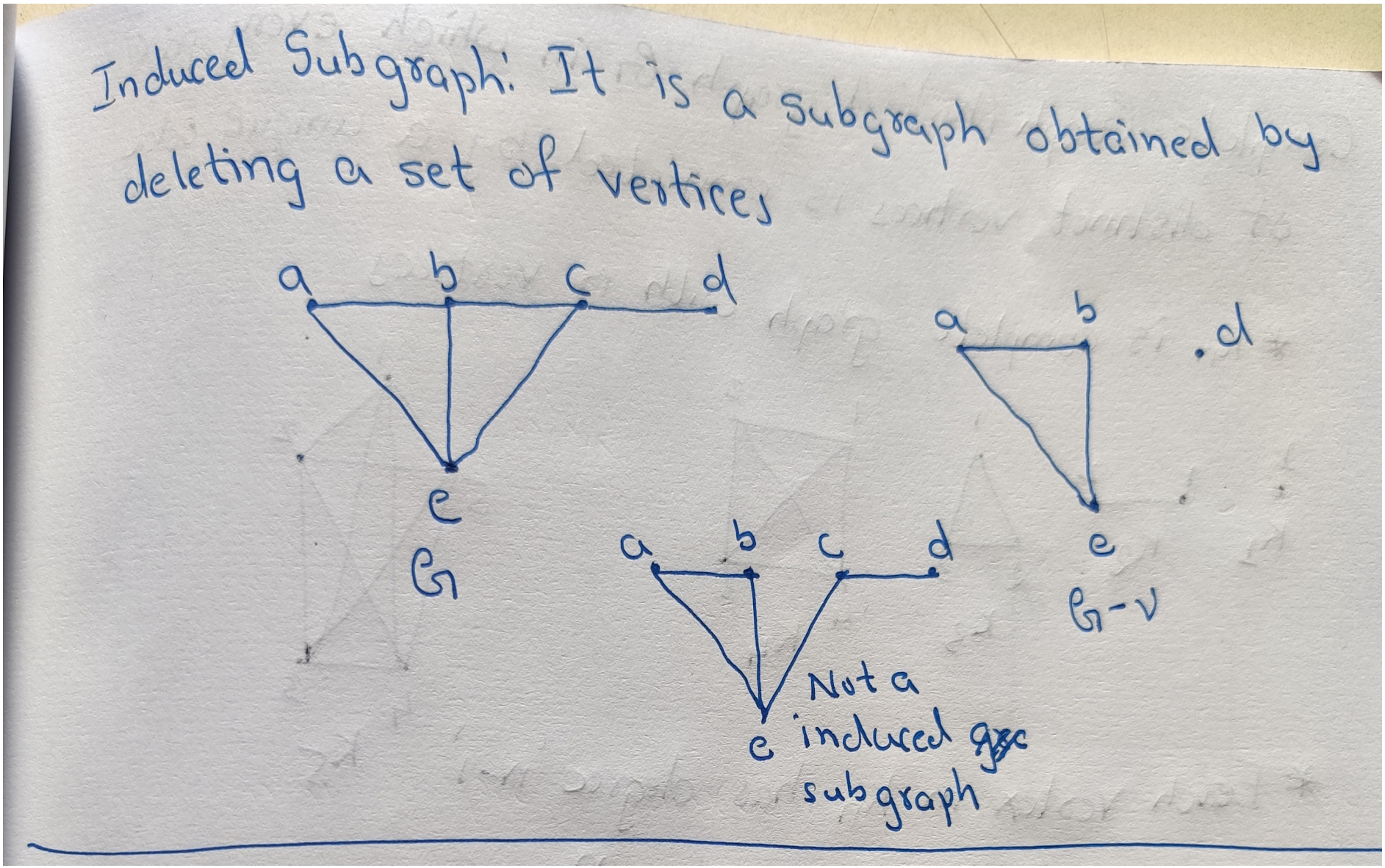


G



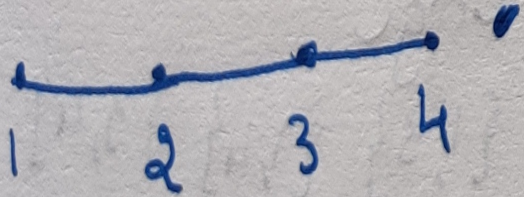
H

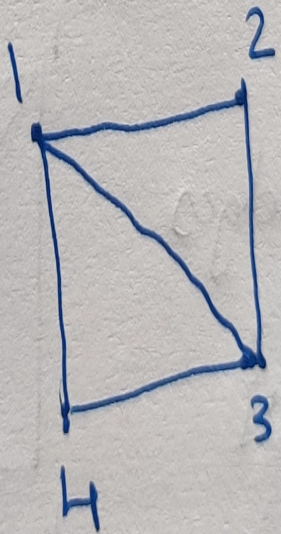
Graph Theory



Graph Theory

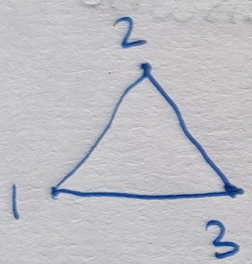
Path: A path is a simple graph whose vertices can be ordered so that two vertices are adjacent if and only if they are consecutive in the list

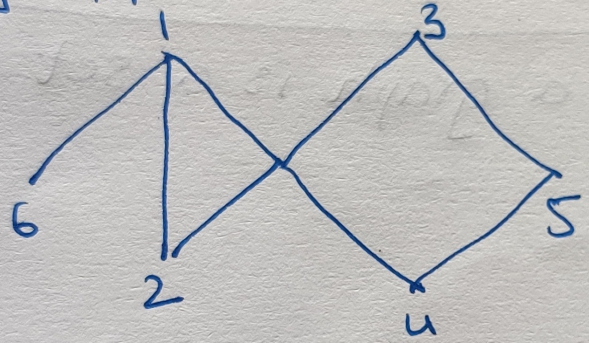


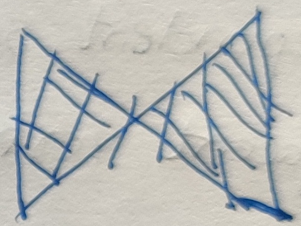


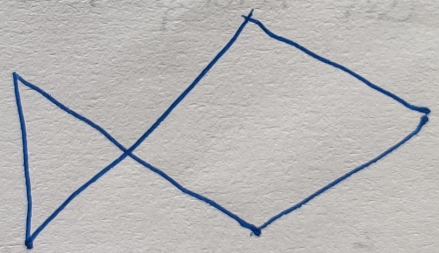
Graph Theory

Cycle: Cycle is a graph with an equal number of vertices and edges, whose vertices can be placed around a circle so that two vertices are adjacent if and only if they appear consecutively eg. along the circle





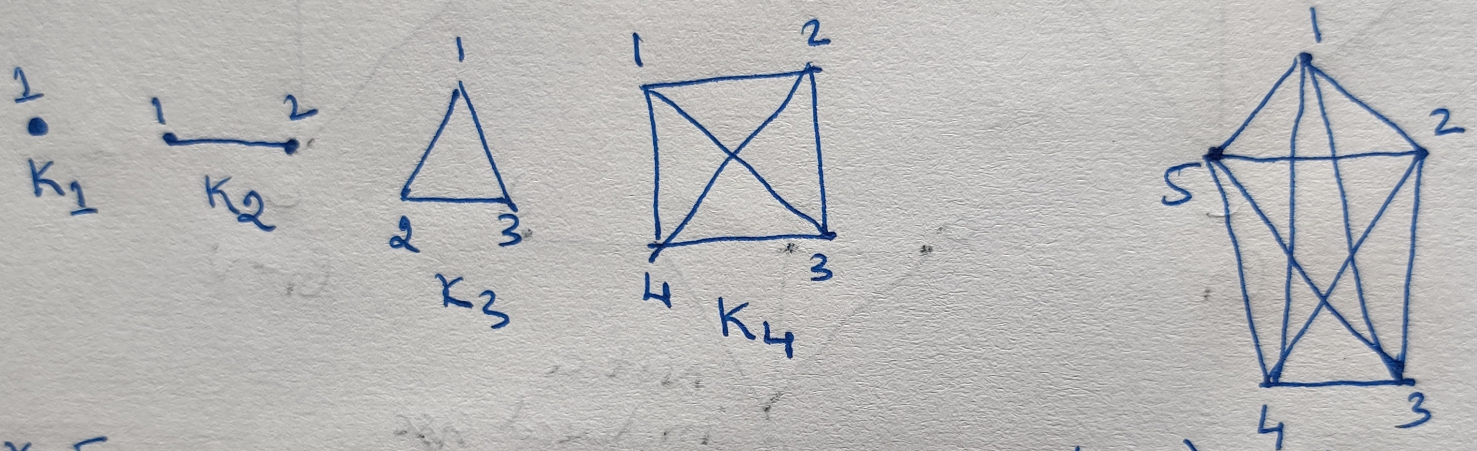




Graph Theory

Complete Graph (K_n): A graph G in which every pair of distinct vertices is connected by a unique edge

* K_n is complete graph with n vertices



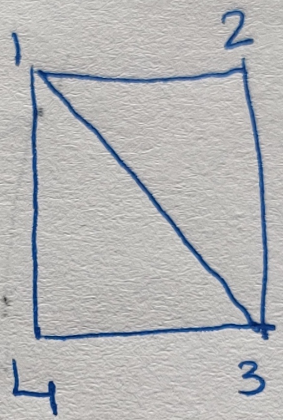
* Each vertex in K_n has degree 'n-1'
* Number of Edges in K_n is ${}^nC_2$

Graph Theory

A clique in a graph is a set of pairwise adjacent vertices

clique with two edges

{1,2,3} and {1,3,4}



clique with single edge

{1,3} {1,4} {3,4}

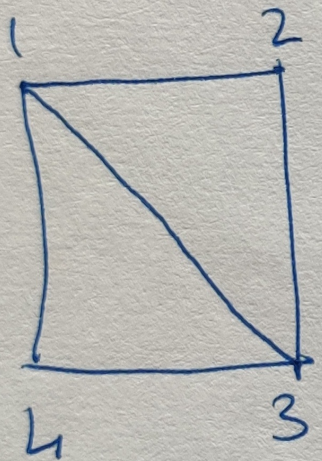
{1,2} {2,3}

Max clique is the maximum subset of vertices with which we can form an biggest complete graph

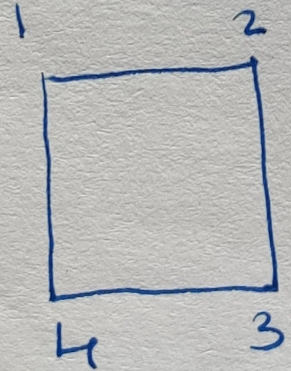
In above example: max clique is K_3

Graph Theory

An independent set in a graph is a set of pairwise non-adjacent vertices



{2,4} is an independent set

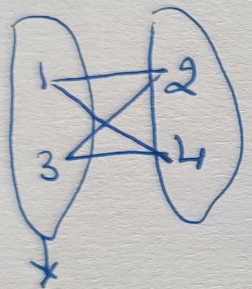


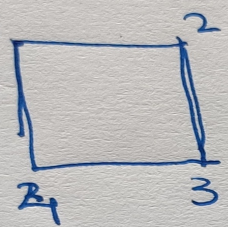
{1,3}

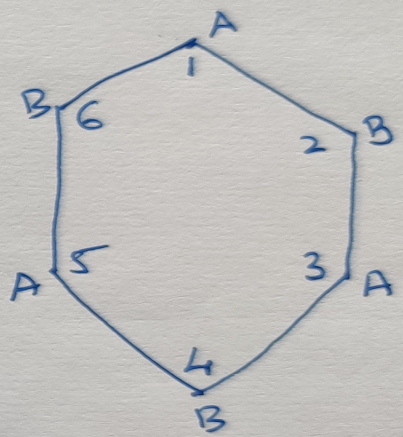
{2,4}

Graph Theory

Bipartite graph: A graph G whose vertices can be divided into two disjoint sets U and V such that every edge connects a vertex in U to one in V [U and V are independent sets]







If every vertices is adjacent to all the vertice in another set then it is complete bipartite graph $K_{2,2}$