Discrete Mathematics

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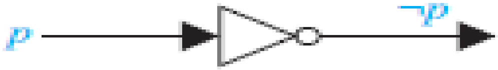
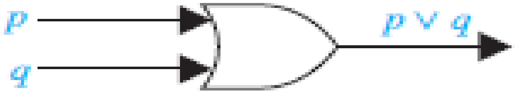
Propositional Logic



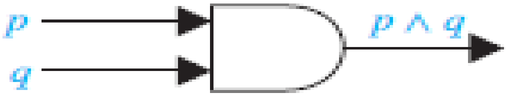
Inverter

Propositional Logic



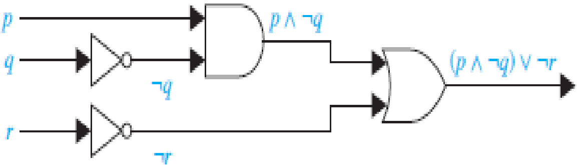
OR gate

Propositional Logic

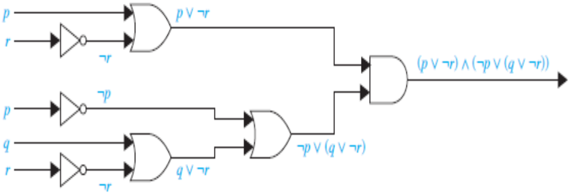


AND gate

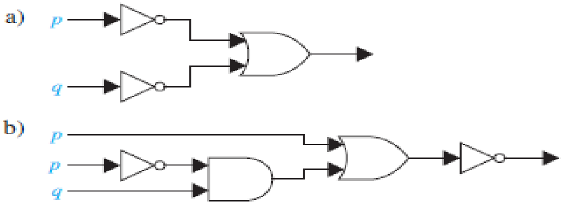
Propositional Logic



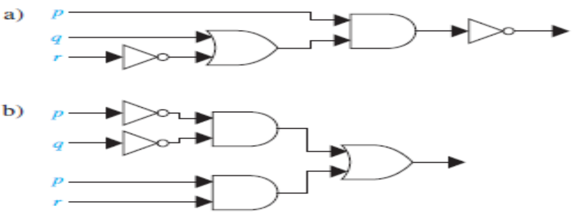
Propositional Logic



Propositional Logic



Propositional Logic



Propositional Logic

- An important type of step used in a mathematical argument is the replacement of a statement with another statement with the same truth value.
- Methods that produce propositions with the same truth value as a given compound proposition are used extensively in the construction of mathematical arguments.

Propositional Logic

- A compound proposition that is always true, no matter what the truth values of the propositional variables that occur in it, is called a **tautology**.
- A compound proposition that is always false is called a **contradiction**.
- A compound proposition that is neither a tautology nor a contradiction is called a **contingency**.
- Tautologies and contradictions are often important in mathematical reasoning.

Propositional Logic

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Propositional Logic

- Tautologies and contradictions are often important in mathematical reasoning.
- Examples of tautologies and contradictions using just one propositional variable.
- Consider the truth tables of $p \lor \neg p$ and $p \land \neg p$,
- $p \lor \neg p$ is always true, it is a tautology. $p \land \neg p$ is always false, it is a contradiction

TABLE 1 Examples of a Tautology and a Contradiction.

p	$\neg p$	$p \lor \neg p$	$p \land \neg p$
T	F	T	F
F	T	T	F

Propositional Logic

- Compound propositions that have the same truth values in all possible cases are called logically Demo equivalent.
- The compound propositions p and q are called logically equivalent if $p \leftrightarrow q$ is a tautology. The notation $p \equiv q$ denotes that p and q are logically equivalent
- The symbol $\equiv$ is not a logical connective, and $p \equiv q$ is not a compound proposition but rather is the statement that $p \leftrightarrow q$ is a tautology
- Symbol $\Leftrightarrow$ is sometimes used instead of $\equiv$ to denote logical equivalence.

Propositional Logic

TABLE 2 **De Morgan's Laws.**
$\neg(p \wedge q) \equiv \neg p \vee \neg q$
$\neg(p \vee q) \equiv \neg p \wedge \neg q$

- Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent

TABLE 3 **Truth Tables for $\neg(p \vee q)$ and $\neg p \wedge \neg q$.**

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Propositional Logic

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Propositional Logic

- Show that p $\rightarrow$ q and $\neg$p $\vee$ q are logically equivalent. (This is known as the conditional disjunction equivalence.)

TABLE 4 Truth Tables for $\neg p \vee q$ and $p \rightarrow q$.

p	q	$\neg p$	$\neg p \vee q$	$p \rightarrow q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Propositional Logic

$(p \rightarrow q) \equiv \sim p \vee q$

p	q	$\sim p$	r: $p \rightarrow q$	s: $\sim p \vee q$	$r \leftrightarrow s$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Propositional Logic

$$\underbrace{(p \rightarrow q) \wedge (p \rightarrow r)}_{s} \equiv \underbrace{(p) \rightarrow (q \wedge r)}_{t}$$

p	q	r	$p \rightarrow q$	$p \rightarrow r$	$(p \rightarrow q) \wedge (p \rightarrow r)$	$q \wedge r$	$p \rightarrow (q \wedge r)$	$s \leftrightarrow t$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	F	T	F	F	F	T
T	F	F	F	F	F	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	F	T	T
F	F	T	T	T	T	F	T	T
F	F	F	T	T	T	F	T	T