



## Normalization

# Third Normal Form

- A relation schema  $R$  is in **third normal form (3NF)** if for all:

$$\alpha \rightarrow \beta \text{ in } F^+$$

at least one of the following holds:

- $\alpha \rightarrow \beta$  is trivial (i.e.,  $\beta \in \alpha$ )
  - $\alpha$  is a superkey for  $R$
  - Each attribute  $A$  in  $\beta - \alpha$  is contained in a candidate key for  $R$   
(**NOTE:** each attribute may be in a different candidate key)
- If a relation is in BCNF, it is in 3NF (since in BCNF one of the first two conditions above must hold)
  - Third condition is a minimal relaxation of BCNF to ensure dependency preservation (will see why later)

# 3NF Example

- Consider a schema:

*dept\_advisor(s\_ID, i\_ID, dept\_name)*

- With function dependencies:

$i\_ID \rightarrow dept\_name$   
 $s\_ID, dept\_name \rightarrow i\_ID$

- Two candidate keys =  $\{s\_ID, dept\_name\}, \{s\_ID, i\_ID\}$
- We have seen before that *dept\_advisor* is **not** in BCNF
- R*, however, is in 3NF
  - $s\_ID, dept\_name$  is a superkey
  - $i\_ID \rightarrow dept\_name$  and  $i\_ID$  is NOT a superkey, but:
    - $\{dept\_name\} - \{i\_ID\} = \{dept\_name\}$  and
    - $dept\_name$  is contained in a candidate key

# Redundancy in 3NF

- Consider the schema  $R$  below, which is in 3NF
  - $R = (J, K, L)$
  - $F = \{JK \rightarrow L, L \rightarrow K\}$
  - And an instance table:

$J$	$L$	$K$
$j_1$	$l_1$	$k_1$
$j_2$	$l_1$	$k_1$
$j_3$	$l_1$	$k_1$
null	$l_2$	$k_2$

- What is wrong with the table?
  - Repetition of information
  - Need to use null values (e.g., to represent the relationship  $l_2, k_2$ , where there is no corresponding value for  $J$ )

# Goals of Normalization

- Let  $R$  be a relation scheme with a set  $F$  of functional dependencies
- Decide whether a relation scheme  $R$  is in “good” form
- In the case that a relation scheme  $R$  is not in “good” form, need to decompose it into a set of relation scheme  $\{R_1, R_2, \dots, R_n\}$  such that:
  - Each relation scheme is in good form
  - The decomposition is a lossless decomposition
  - Preferably, the decomposition should be dependency preserving

# How Good is BCNF?

- There are database schemas in BCNF that do not seem to be sufficiently normalized
- Consider a relation

*inst\_info* (*ID*, *child\_name*, *phone*)

- Where an instructor may have more than one phone and can have multiple children
- Instance of *inst\_info*

<i>ID</i>	<i>child_name</i>	<i>phone</i>
99999	David	512-555-1234
99999	David	512-555-4321
99999	William	512-555-1234
99999	William	512-555-4321

- There are no non-trivial functional dependencies and therefore the relation is in BCNF
- Insertion anomalies, i.e., if we add a phone 981-992-3443 to 99999, we need to add two tuples:  
 (99999, David, 981-992-3443)  
 (99999, William, 981-992-3443)

# Higher Normal Forms

- Therefore, it is better to decompose *inst\_info* into:

- *inst\_child*:

<i>ID</i>	<i>child_name</i>
99999	David
99999	William

- *inst\_phone*:

<i>ID</i>	<i>phone</i>
99999	512-555-1234
99999	512-555-4321

- This suggests the need for higher normal forms, such as Fourth Normal Form (4NF), which we shall see later

# Functional-Dependency Theory Roadmap

- We now consider the formal theory that tells us which functional dependencies are implied logically by a given set of functional dependencies
- We then develop algorithms to generate lossless decompositions into BCNF and 3NF
- We then develop algorithms to test if a decomposition is dependency-preserving



# Closure of a Set of Functional Dependencies

- Given a set  $F$  set of functional dependencies, there are certain other functional dependencies that are logically implied by  $F$ 
  - If  $A \rightarrow B$  and  $B \rightarrow C$ , then we can infer that  $A \rightarrow C$
  - etc.
- The set of **all** functional dependencies logically implied by  $F$  is the **closure** of  $F$
- We denote the *closure* of  $F$  by  $F^+$

# Closure of a Set of Functional Dependencies

- We can compute  $F^+$ , the closure of  $F$ , by repeatedly applying **Armstrong's Axioms**:
  - **Reflexive rule**: If  $\beta \subseteq \alpha$ , then  $\alpha \rightarrow \beta$
  - **Augmentation rule**: If  $\alpha \rightarrow \beta$ , then  $\gamma\alpha \rightarrow \gamma\beta$
  - **Transitivity rule**: If  $\alpha \rightarrow \beta$ , and  $\beta \rightarrow \gamma$ , then  $\alpha \rightarrow \gamma$
- These rules are:
  - **Sound**: Generate only functional dependencies that actually hold, and
  - **Complete**: Generate all functional dependencies that hold

# Example of $F^+$

- $R = (A, B, C, G, H, I)$   
 $F = \{ A \rightarrow B$   
 $\quad A \rightarrow C$   
 $\quad CG \rightarrow H$   
 $\quad CG \rightarrow I$   
 $\quad B \rightarrow H \}$
- Some members of  $F^+$ 
  - $A \rightarrow H$ 
    - By transitivity from  $A \rightarrow B$  and  $B \rightarrow H$
  - $AG \rightarrow I$ 
    - By augmenting  $A \rightarrow C$  with  $G$ , to get  $AG \rightarrow CG$  and then transitivity with  $CG \rightarrow I$
  - $CG \rightarrow HI$ 
    - By augmenting  $CG \rightarrow I$  to infer  $CG \rightarrow CGI$ , and augmenting of  $CG \rightarrow H$  to infer  $CGI \rightarrow HI$ , and then transitivity

# Procedure for Computing $F^+$

- To compute the closure of a set of functional dependencies  $F$ :

$F^+ = F$

**repeat**

**for each** functional dependency  $f$  in  $F^+$

        apply reflexivity and augmentation rules on  $f$

        add the resulting functional dependencies to  $F^+$

**for each** pair of functional dependencies  $f_1$  and  $f_2$  in  $F^+$

**if**  $f_1$  and  $f_2$  can be combined using transitivity

**then** add the resulting functional dependency to  $F^+$

**until**  $F^+$  does not change any further

- NOTE:** We shall see an alternative procedure for this task later

# Closure of Functional Dependencies

- Additional derived rules:
  - **Union rule:** If  $\alpha \rightarrow \beta$  holds and  $\alpha \rightarrow \gamma$  holds, then  $\alpha \rightarrow \beta\gamma$  holds
  - **Decomposition rule:** If  $\alpha \rightarrow \beta\gamma$  holds, then  $\alpha \rightarrow \beta$  holds and  $\alpha \rightarrow \gamma$  holds
  - **Pseudotransitivity rule:** If  $\alpha \rightarrow \beta$  holds and  $\gamma\beta \rightarrow \delta$  holds, then  $\alpha\gamma \rightarrow \delta$  holds
- The above rules can be inferred from Armstrong's axioms

# Closure of Attribute Sets

- Given a set of attributes  $a$ , define the **closure** of a **under**  $F$  (denoted by  $a^+$ ) as the set of attributes that are functionally determined by  $a$  under  $F$
- Algorithm to compute  $a^+$ , the closure of  $a$  under  $F$

```
result := a;  
while (changes to result) do  
  for each  $\beta \rightarrow \gamma$  in  $F$  do  
    begin  
      if  $\beta \subseteq \text{result}$  then  $\text{result} := \text{result} \cup \gamma$   
    end
```

# Example of Attribute Set Closure

- $R = (A, B, C, G, H, I)$
- $F = \{A \rightarrow B$   
 $A \rightarrow C$   
 $CG \rightarrow H$   
 $CG \rightarrow I$   
 $B \rightarrow H\}$
- $(AG)^+$ 
  1.  $result = AG$
  2.  $result = ABCG$                        $(A \rightarrow C \text{ and } A \rightarrow B)$
  3.  $result = ABCGH$                      $(CG \rightarrow H \text{ and } CG \subseteq AGBC)$
  4.  $result = ABCGHI$                    $(CG \rightarrow I \text{ and } CG \subseteq AGBCH)$
- Is  $AG$  a candidate key?
  - Is  $AG$  a super key?
    - Does  $AG \rightarrow R? \equiv \text{Is } R \supseteq (AG)^+$
  - Is any subset of  $AG$  a superkey?
    - Does  $A \rightarrow R? \equiv \text{Is } R \supseteq (A)^+$
    - Does  $G \rightarrow R? \equiv \text{Is } R \supseteq (G)^+$
    - In general: Check for each subset of size  $n - 1$

# Uses of Attribute Closure

There are several uses of the attribute closure algorithm:

- Testing for superkey:
  - To test if  $\alpha$  is a superkey, we compute  $\alpha^+$  and check if  $\alpha^+$  contains all attributes of  $R$
- Testing functional dependencies
  - To check if a functional dependency  $\alpha \rightarrow \beta$  holds (or, in other words, is in  $F^+$ ), just check if  $\beta \subseteq \alpha^+$
  - That is, we compute  $\alpha^+$  by using attribute closure, and then check if it contains  $\beta$
  - Is a simple and cheap test, and very useful
- Computing closure of  $F$ 
  - For each  $\gamma \subseteq R$ , we find the closure  $\gamma^+$ , and for each  $S \subseteq \gamma^+$ , we output a functional dependency  $\gamma \rightarrow S$



# Next Lecture

## Normalization

# Thank you for your attention...

Any question?

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