# Discrete Mathematics

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# Mathematical Induction

**Mathematical induction**, is a technique for proving results or establishing statements for natural numbers. This part illustrates the method through a variety of examples.

## Definition

**Mathematical Induction** is a mathematical technique which is used to prove a statement, a formula or a theorem is true for every natural number.

The technique involves two steps to prove a statement, as stated below –

**Step 1(Base step)** – It proves that a statement is true for the initial value.

**Step 2(Inductive step)** – It proves that if the statement is true for the $n^{th}$ iteration (or number *n*), then it is also true for $(n+1)^{th}$ iteration ( or number *n+1*).

# Mathematical Induction

## How to Do It

**Step 1** – Consider an initial value for which the statement is true. It is to be shown that the statement is true for n = initial value.

**Step 2** – Assume the statement is true for any value of $n = k$. Then prove the statement is true for $n = k+1$. We actually break $n = k+1$ into two parts, one part is $n = k$ (which is already proved) and try to prove the other part.

# Mathematical Induction

Problem 1

$3^n - 1$ is a multiple of 2 for n = 1, 2, ...

**Solution**

**Step 1** – For $n = 1, 3^1 - 1 = 3 - 1 = 2$ which is a multiple of 2

**Step 2** – Let us assume $3^n - 1$ is true for $n = k$ , Hence, $3^k - 1$ is true (It is an assumption)

We have to prove that $3^{k+1} - 1$ is also a multiple of 2

$$3^{k+1} - 1 = 3 \times 3^k - 1 = (2 \times 3^k) + (3^k - 1)$$

The first part $(2 \times 3k)$ is certain to be a multiple of 2 and the second part $(3k - 1)$ is also true as our previous assumption.

Hence, $3^{k+1}-1$ is a multiple of 2.

So, it is proved that $3^n-1$ is a multiple of 2.

# Mathematical Induction

Problem 2

$$1+3+5+\ldots+(2n-1)=n^2 \quad \text{for} \quad n=1,2,\ldots$$

**Solution**

**Step 1** – For $n=1, 1=1^2$ , Hence, step 1 is satisfied.

**Step 2** – Let us assume the statement is true for $n=k$ .

Hence, $1+3+5+\cdots+(2k-1)=k^2$ is true (It is an assumption)

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We have to prove that $1+3+5+\ldots+(2(k+1)-1)=(k+1)^2$ also holds

$$1+3+5+\cdots+(2(k+1)-1)$$

$$=1+3+5+\cdots+(2k+2-1)$$

$$=1+3+5+\cdots+(2k+1)$$

$$=1+3+5+\cdots+(2k-1)+(2k+1)$$

$$=k^2+(2k+1)$$

$$=(k+1)^2$$

So, $1+3+5+\cdots+(2(k+1)-1)=(k+1)^2$ hold which satisfies the step 2.

Hence, $1+3+5+\cdots+(2n-1)=n^2$ is proved.

# Mathematical Induction

Problem 3

Prove that $(ab)^n = a^n b^n$ is true for every natural number $n$

**Solution**

**Step 1** – For $n = 1, (ab)^1 = a^1 b^1 = ab$ , Hence, step 1 is satisfied.

**Step 2** – Let us assume the statement is true for $n = k$ , Hence, $(ab)^k = a^k b^k$ is true (It is an assumption).

We have to prove that $(ab)^{k+1} = a^{k+1} b^{k+1}$ also hold

Given, $(ab)^k = a^k b^k$

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Given, $(ab)^k = a^k b^k$

Or, $(ab)^k(ab) = (a^k b^k)(ab)$ [Multiplying both side by 'ab']

Or, $(ab)^{k+1} = (aa^k)(bb^k)$

Or, $(ab)^{k+1} = (a^{k+1} b^{k+1})$

Hence, step 2 is proved.

So, $(ab)^n = a^n b^n$ is true for every natural number n.

# Mathematical Induction

## Strong Induction

Strong Induction is another form of mathematical induction. Through this induction technique, we can prove that a propositional function, $P(n)$ is true for all positive integers, $n$ , using the following steps –

- **Step 1(Base step)** – It proves that the initial proposition $P(1)$ true.
- **Step 2(Inductive step)** – It proves that the conditional statement $[P(1) \wedge P(2) \wedge P(3) \wedge \cdots \wedge P(k)] \rightarrow P(k+1)$ is true for positive integers $k$ .