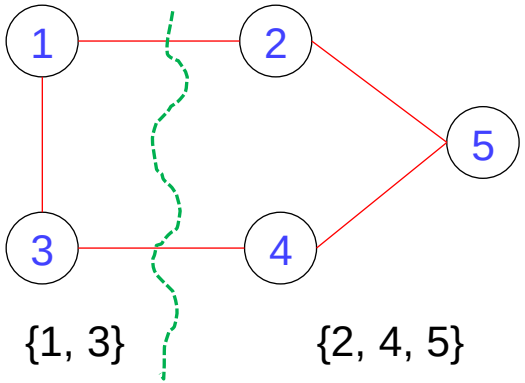
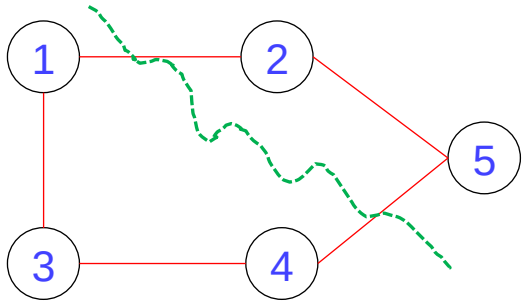
# Prim's Algorithm

# Cuts in a Graph

- The notion of a spanning tree is defined only for an undirected graph
- A cut in graph ***G*** is a partition of the vertex set into two parts
- How many number of different cuts are possible in a graph ***G = (V, E)***?
  - $2^{V-1} - 1$
- Edges which have one end point in one side of the partition and the other end point in the other side of the partition are called ***edges in the cut*** or ***edges of the cut***

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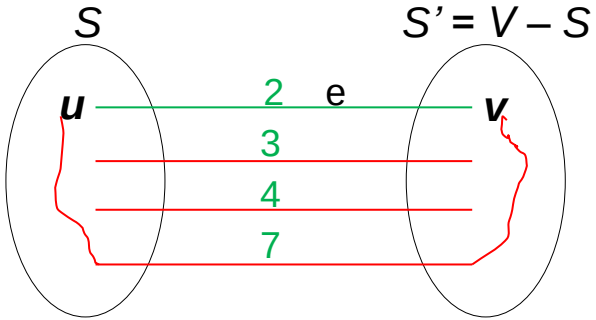
# Cuts and MST

- Suppose we take a cut **(*S, S'*)** in the graph ***G***
- **Assumption:** Edge lengths are distinct
- **Claim:** For any cut **(*S, S'*)** in a graph ***G***, the minimum edge in the cut belongs to the minimum spanning tree (MST)
- **Note:** There can be more than one edge in a cut which is a part of the MST, but the minimum edge in the cut will belongs to the MST

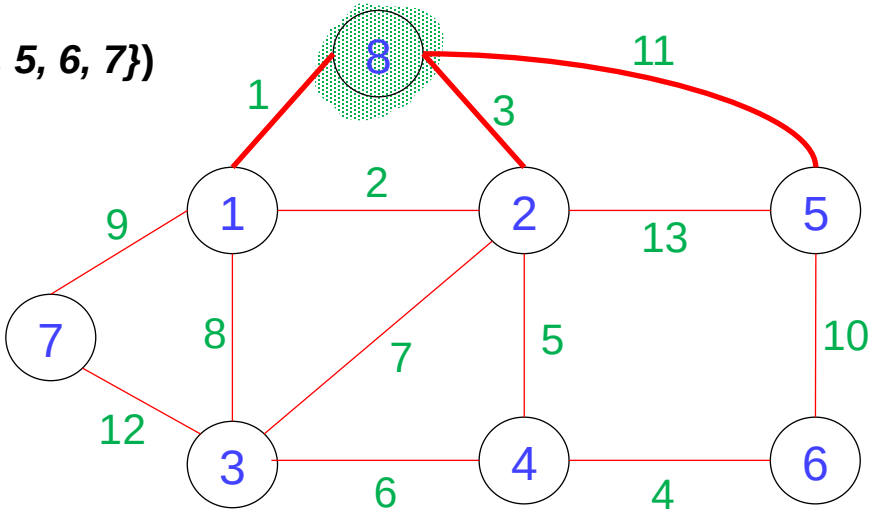



- **Proof** *by Contradiction*
- Let ***T*** be a MST which doesn't contain edge ***e***
- Add ***e*** to ***T*** → Cycle will be formed
  - Addition of ***e*** to ***T*** forms a cycle ***C***
- ***C*** contains atleast one edge of the cut other than ***e***
  - ***C*** contains an edge more than the cost of ***e***
  - By removing this edge from ***T*** ∪ ***{e}***, we get a lighter tree

***Contradiction in MST !!!***

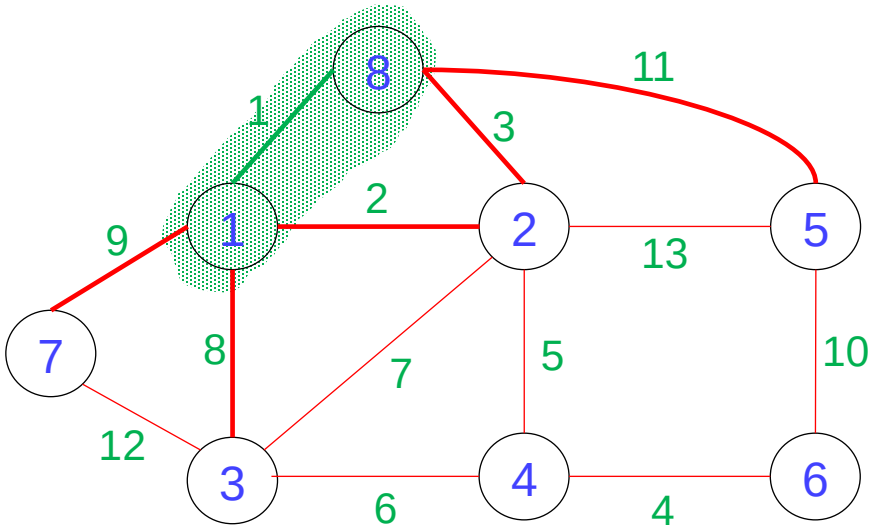
# Example

- Start with any vertex, suppose ***8***
  - It is called as ***root*** vertex
- The first cut is (***root, V – {root}***) = (***{8}, {1, 2, 3, 4, 5, 6, 7}***)
- Which are the ***edges of the cut***?
  - (***8, 1, 1***), (***8, 2, 3***), (***8, 5, 11***) [(***u, v, c***)]
- Which edge has to be a part of the MST then?
  - (***8, 1, 1***)
- Now what is cut we are going to consider?
  - (***{8, 1}, {2, 3, 4, 5, 6, 7}***)



# Example

- Which are the ***edges of the cut***?
  - (***1, 7, 9***), (***1, 3, 8***), (***1, 2, 2***), (***8, 2, 3***), (***8, 5, 11***)
- Which edge has to be a part of the MST then?
  - (***1, 2, 2***)
- Now what is cut we are going to consider?
  - ***({8, 1, 2}, {3, 4, 5, 6, 7})***

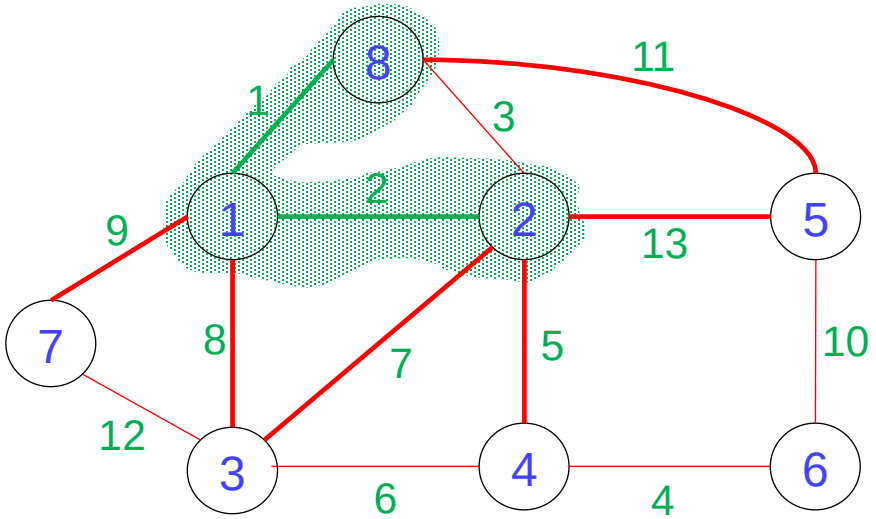


# Example

- Which are the ***edges of the cut***?
  - (***1, 7, 9***), (***1, 3, 8***), (***2, 3, 7***), (***2, 4, 5***), (***2, 5, 13***), (***8, 5, 11***)

- Which edge has to be a part of the MST then?
  - (***2, 4, 5***)

- Now what is cut we are going to consider?
  - **(*{8, 1, 2, 4}, {3, 5, 6, 7}*)**

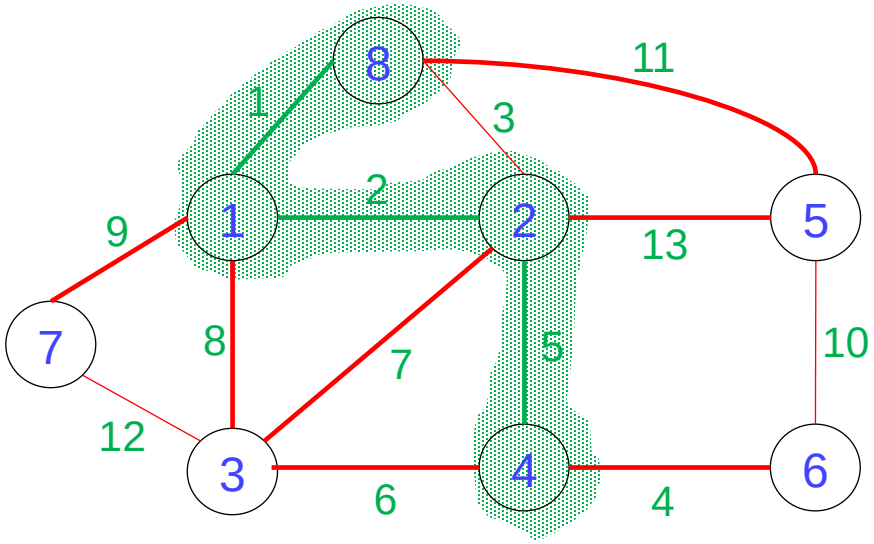


# Example

- Which are the ***edges of the cut***?
  - (***1, 7, 9***), (***1, 3, 8***), (***2, 3, 7***), (***4, 3, 6***), (***4, 6, 4***), (***2, 5, 13***), (***8, 5, 11***)

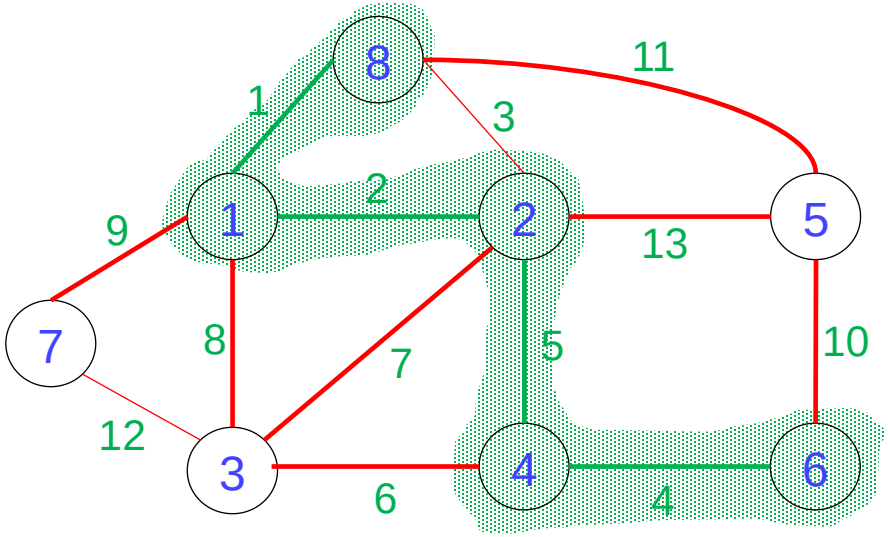
- Which edge has to be a part of the MST then?
  - (***4, 6, 4***)

- Now what is cut we are going to consider?
  - **(*{8, 1, 2, 4, 6}, {3, 5, 7}*)**



# Example

- Which are the ***edges of the cut***?
  - (***1, 7, 9***), (***1, 3, 8***), (***2, 3, 7***), (***4, 3, 6***), (***5, 6, 10***), (***2, 5, 13***), (***8, 5, 11***)
- Which edge has to be a part of the MST then?
  - (***4, 3, 6***)
- Now what is cut we are going to consider?
  - **(*{8, 1, 2, 4, 6, 3}, {5, 7}*)**

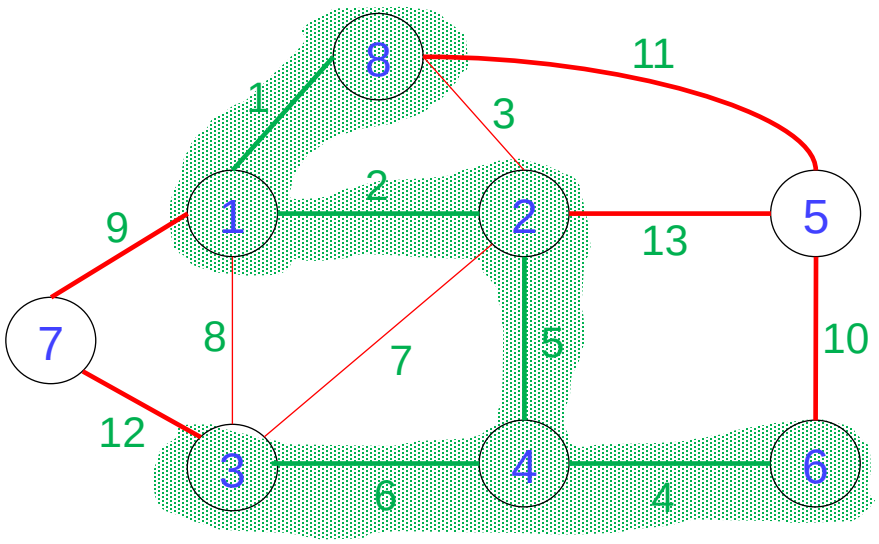


# Example

- Which are the ***edges of the cut***?
  - (***1, 7, 9***), (***3, 7, 12***), (***5, 6, 10***), (***2, 5, 13***), (***8, 5, 11***)

- Which edge has to be a part of the MST then?
  - (***1, 7, 9***)

- Now what is cut we are going to consider?
  - **(*{8, 1, 2, 4, 6, 3, 7}, {5}*)**

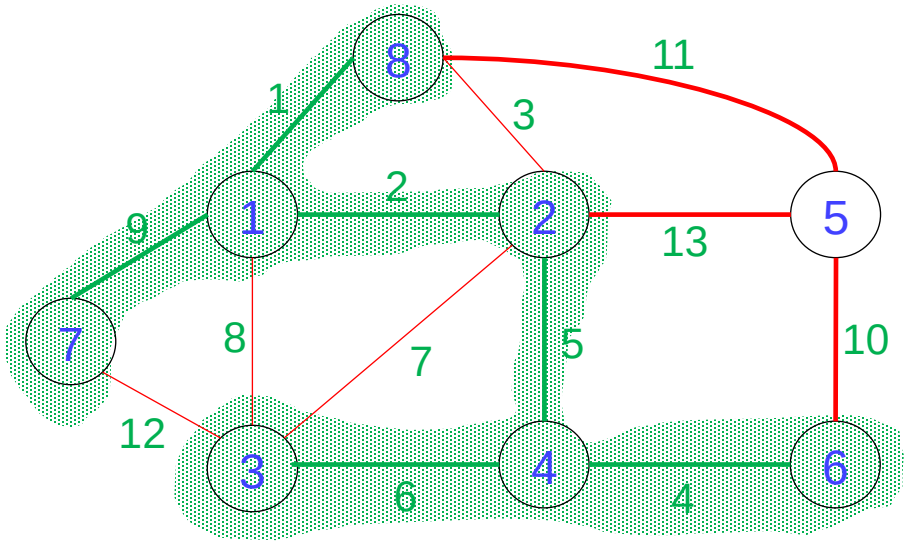


# Example

- Which are the ***edges of the cut***?
  - (***5, 6, 10***), (***2, 5, 13***), (***8, 5, 11***)

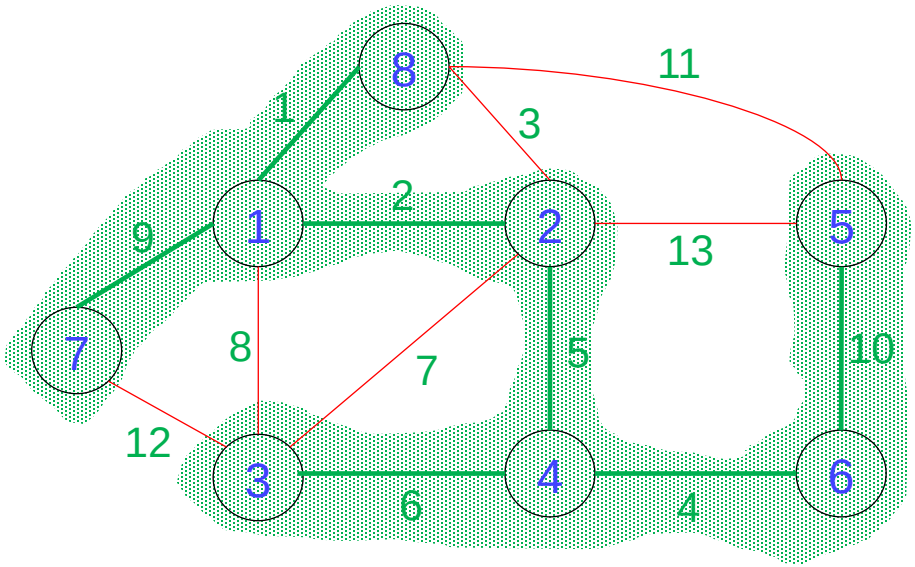
- Which edge has to be a part of the MST then?
  - (***5, 6, 10***)

- Now what is cut we are going to consider?
  - **(*{8, 1, 2, 4, 6, 3, 5, 7}, {}*)**



# Example

- What is the minimum spanning tree?



# Next Lecture

## Prim's Algorithm

# Thank you for your attention...

Any question?

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