Discrete Mathematics

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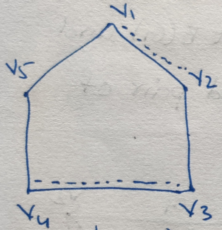
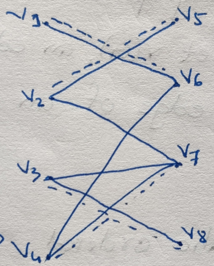
Graph Theory

Matching: A matching in a graph G is a set of ~~non~~ non-loop edges with no shared end points



(1,2)	(1,5)
(4,3)	(2,3)



* The vertices incident to the edges of matching M are saturated by M, the others are unsaturated

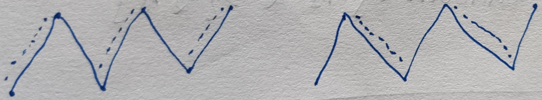
* A perfect matching in a graph is a matching that saturates every vertex

Graph Theory

→ A matching is a set of edges, so its size is the number of edges

→ A maximal matching in a graph is a matching that cannot be enlarged by adding an edge

→ A maximum matching is a matching of maximum size among all matchings in the graph.



Both are maximal

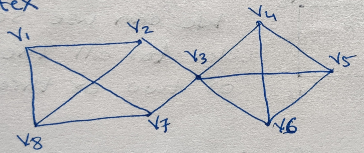
The first one is maximum

Graph Theory

Cuts and Connectivity

→ A separating set or vertex cut of a graph G is set $S \subseteq V(G)$ such that $G-S$ has more than one component

→ The connectivity of G ($K(G)$), is the minimum size of a vertex S such that $G-S$ is disconnected or has only one vertex



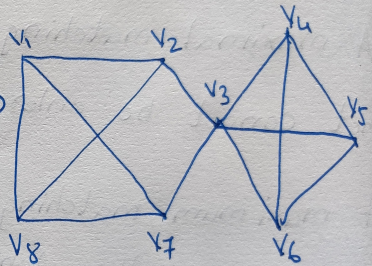
Cut = {3}
cut = {2, 7}
cut = {3, 2, 7}

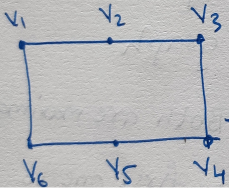
If we get vertex cut any superset is going to be a vertex cut

Graph Theory

The connectivity of G (k(G)), is the minimum size of a vertex set S such that G-S is disconnected or "has only one vertex"

Connectivity is one $\{3\}$ →





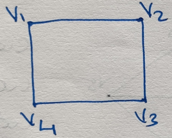
→ Connectivity is 2 → $\{v_2\ v_5\}$

Graph Theory

Vertex Coloring

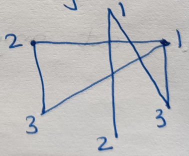
→ A K-coloring of a graph G is a labeling [coloring]

$f: V(G) \to S$, where S is set of labels and $|S| = K$



We can use same color ~~for~~ for all the vertices or two or three or four

A K-coloring is proper if adjacent vertices have different labels



Graph Theory

→ A graph is K-colorable if it has a proper K-coloring. In the previous example Graph is 3-colorable but not 2-colorable

If we are able to color the graph such that no two ~~vertice~~ adjacent vertices are having same color using only K-colors then such a graph is called as K-colorable

Example is 3-colorable, 4 colorable .... but not 2 colorable

(G) is the least K such

Graph Theory

→ The chromatic number $\chi(G)$ is the least K such that G is K-colorable

→ The chromatic number of bipartite graph is 2 $\chi(G) = 2$