Discrete Mathematics

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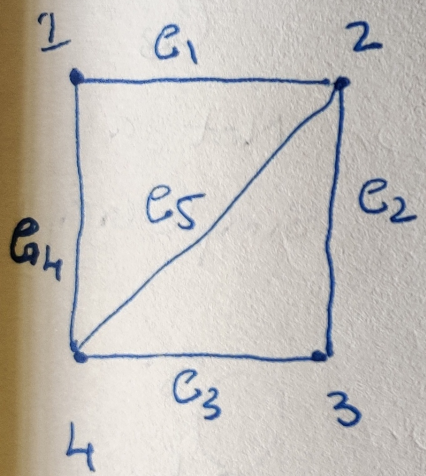
Graph Theory

Graph Theory

A Graph G consists of vertex set V(G), an edge set E(G), and the relation that assosiates with each edge, two vertices called its end points



$V(G) = \{1, 2, 3, 4\}$

$E(G) = \{e_1, e_2, e_3, e_4, e_5\}$

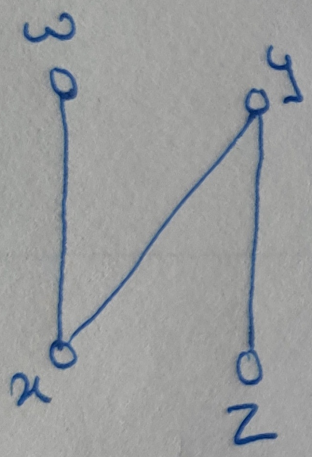
$e_1 \rightarrow \{1, 2\}$ $e_2 \rightarrow \{2, 3\}$ $e_3 \rightarrow \{3, 4\}$

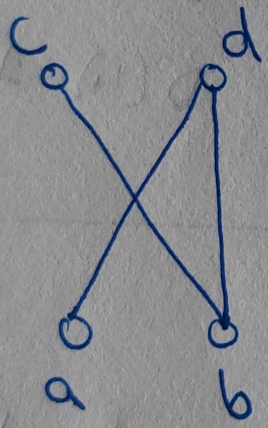
$e_4 \rightarrow \{4, 1\}$ $e_5 \rightarrow \{2, 4\}$

Undirected graph

Graph Theory

Isomorphism: An isomorphism from a simple graph G to a simple graph H is a bijection $f: V(G) \to V(H)$ such that $uv \in E(G)$ iff $f(u)(v) \in E(H)$

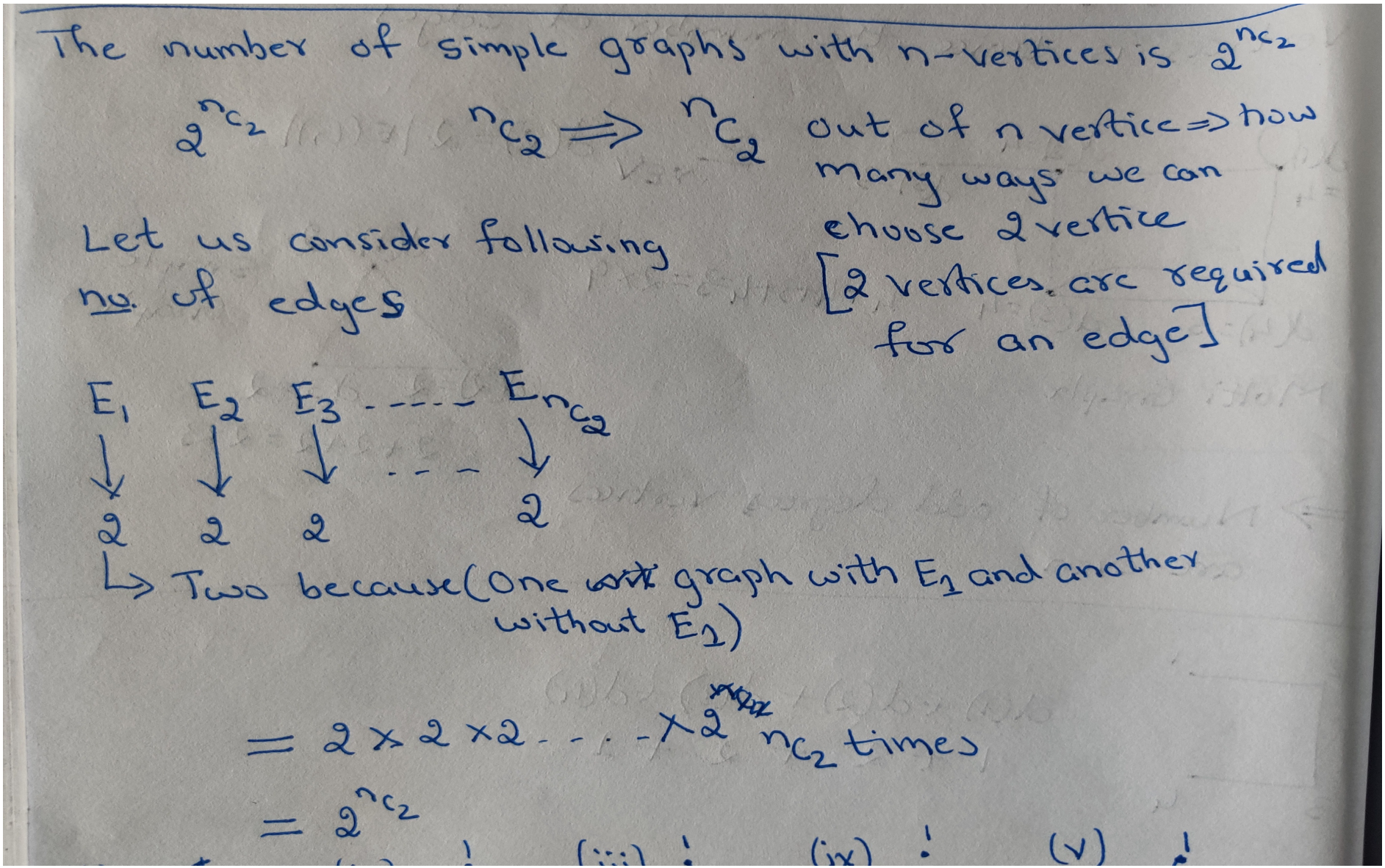




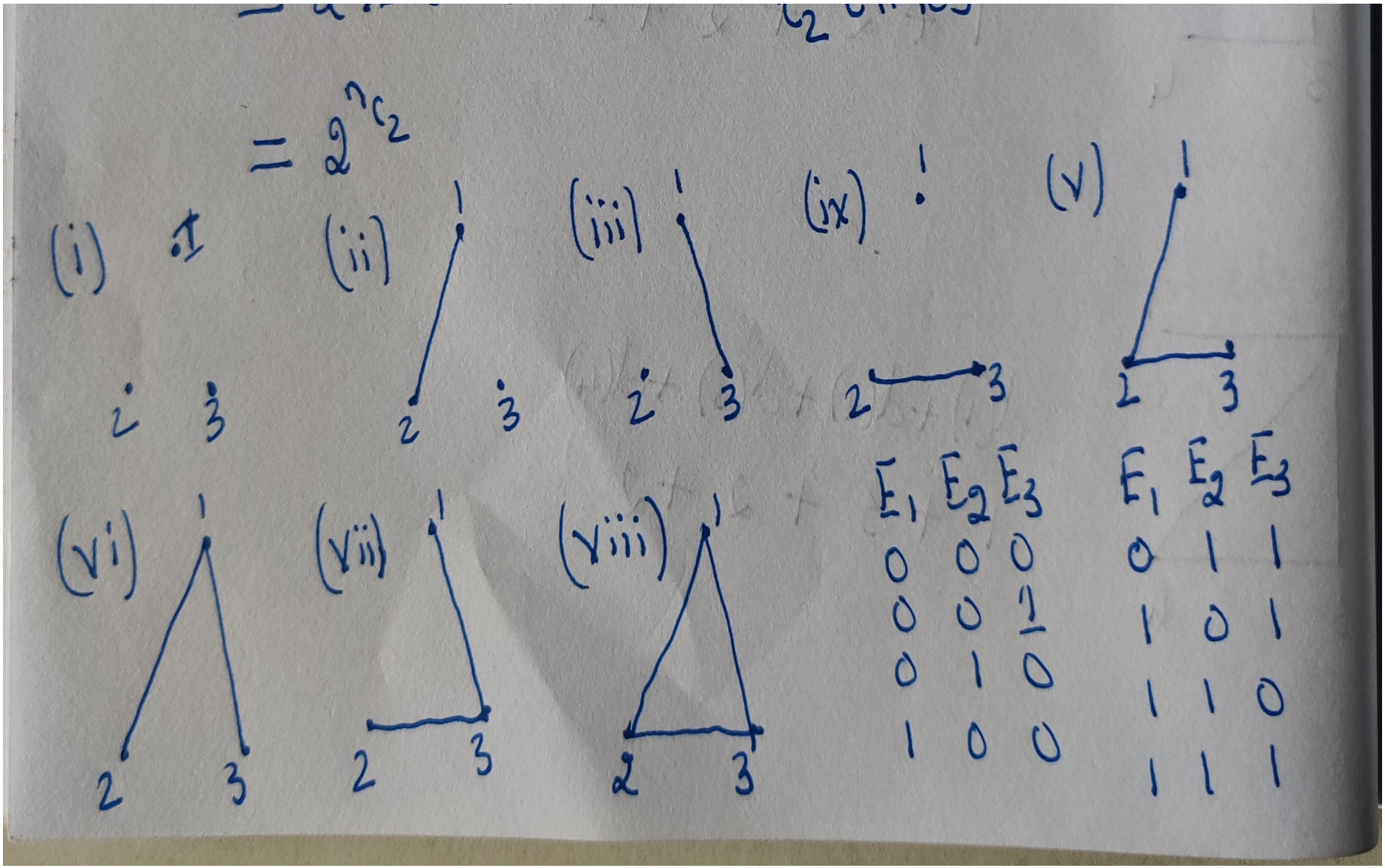
$f: V(G) \to V(H)$

$w \to c$
$x \to b$
$y \to d$
$z \to a$

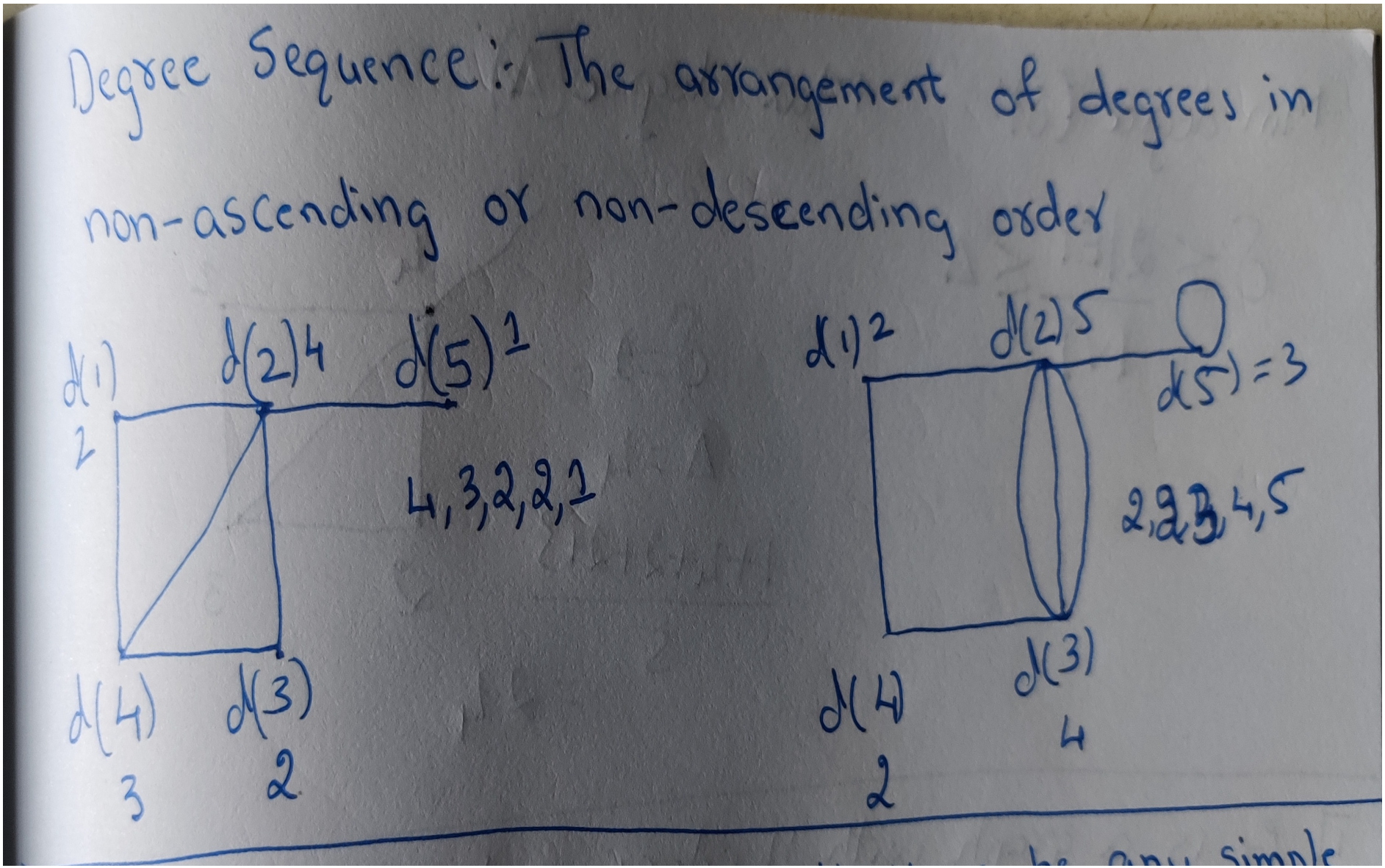
Graph Theory



Graph Theory



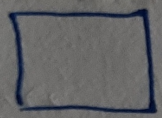
Graph Theory



Graph Theory

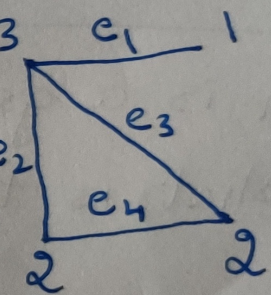
For a given degree sequence will there be any simple graph

Ex1: 2, 2, 2, 2



Ex2: 3, 2, 1, 1, 0 (According to hand shaking lemma)

If vertex 3 is deleted (Removing e2 & Edges)



Ex3: 7, 6, 5, 4, 4, 3, 2, 1

~~7~~, 6, 5, 4, 4, 3, 2, 1

~~6~~, ~~4~~, 3, 3, 2, ~~1~~, 0

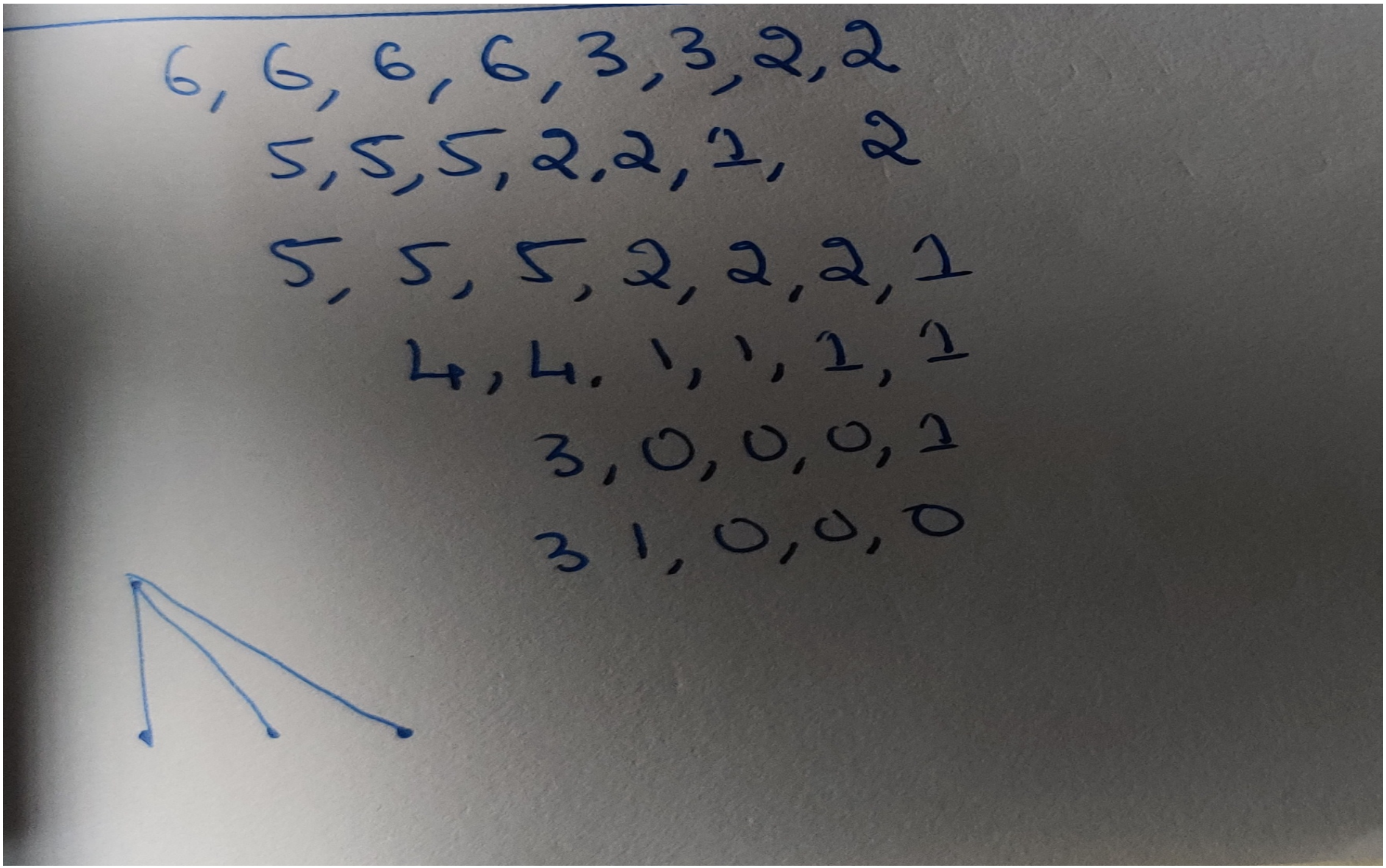
~~5~~, 2, 2, ~~1~~, 0, 0

1, 1, 0, 0, 0

~~3~~, 2, 2, 1

1, ~~1~~, 0

Graph Theory

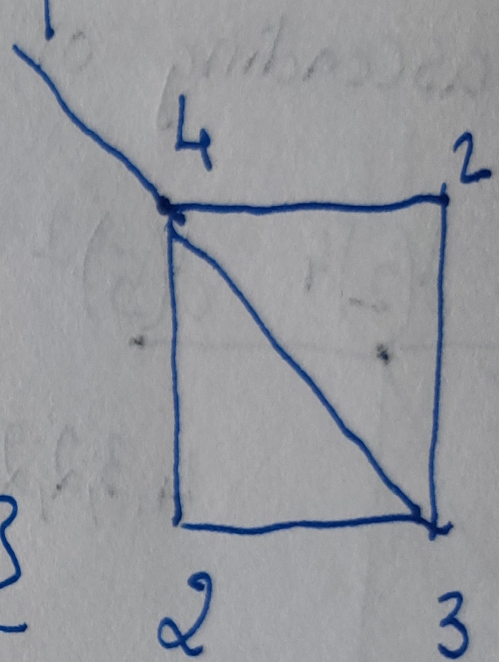


Graph Theory

Min degree (δ):

Max degree(Δ):

$$\delta \leq \frac{2|E|}{|V|} \leq \Delta$$



$\delta = 1$

$\Delta = 4$

$$\frac{1+4+2+2+3}{5} = 2.4$$

Graph Theory

1] G is a graph with 11 edges and minimum degree is 3. What is the maximum number of vertices

$\delta = 3 \qquad \delta \leq \frac{2|E|}{|V|}$

$|V| \leq \frac{2|E|}{\delta} = \frac{2 \times 11}{3} = 7 \ldots\ldots$

2] G is a graph with 12 vertices and max degree is 4 what is the maximum number of edges.

$\frac{2|E|}{|V|} \leq \Delta$

$|E| \leq \frac{\Delta \times |V|}{2} = \frac{4 \times 12}{2} = 24$