# Discrete Mathematics

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August 26, 2020

# Propositional Logic

- Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

$$
\begin{aligned}
(p \wedge q) \rightarrow (p \vee q) &\equiv \neg(p \wedge q) \vee (p \vee q) && \text{by Example 3} \\
&\equiv (\neg p \vee \neg q) \vee (p \vee q) && \text{by the first De Morgan law} \\
&\equiv (\neg p \vee p) \vee (\neg q \vee q) && \text{by the associative and commutative} \\
& && \quad \text{laws for disjunction} \\
&\equiv \mathbf{T} \vee \mathbf{T} && \text{by Example 1 and the commutative} \\
& && \quad \text{law for disjunction} \\
&\equiv \mathbf{T} && \text{by the domination law}
\end{aligned}
$$

# Propositional Logic

- A compound proposition is satisfiable if there is an assignment of truth values to its variables that makes it true (that is, when it is a tautology or a contingency).
- When no such assignments exists, that is, when the compound proposition is false for all assignments of truth values to its variables, the compound proposition is unsatisfiable.
- Note that a compound proposition is unsatisfiable if and only if its negation is true for all assignments of truth values to the variables, that is, if and only if its negation is a tautology.
- When we find a particular assignment of truth values that makes a compound proposition true,we have shown that it is satisfiable; such an assignment is called a solution of this particular satisfiability problem.

# Propositional Logic

- Determine whether the compound propositions $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$ is satisfiable.
- Instead of using a truth table to solve this problem, we will reason about truth values.
- Note that $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$ is true when the three variables p, q, and r have the same truth value.
- Hence, it is satisfiable as there is at least one assignment of truth values for p, q, and r that makes it true.

# Propositional Logic

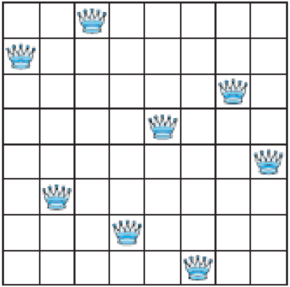
- Determine whether each of the compound propositions $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ is satisfiable.
- Similarly, note that $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ is true when at least one of p, q, and r is true and at least one is false.
- Hence, $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ is satisfiable, as there is at least one assignment of truth values for p, q, and r that makes it true.

# Propositional Logic

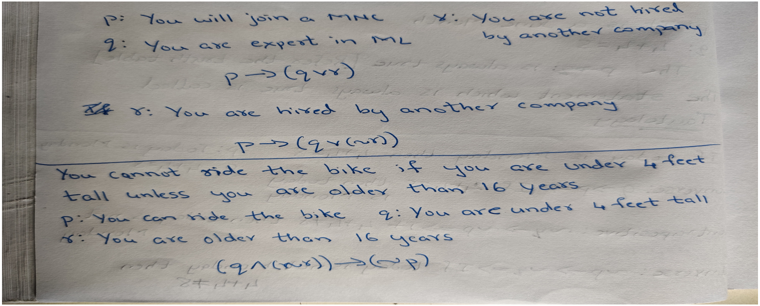
- Determine whether the compound propositions and $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \wedge (p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ is satisfiable.
- Note that for $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \wedge (p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ to be true, $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$ and $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ must both be true.
- For the first to be true, the three variables must have the same truth values, and for the second to be true, at least one of the three variables must be true and at least one must be false.
- However, these conditions are contradictory. From these observations we conclude that no assignment of truth values to p, q, and r makes $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \wedge (p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ true. Hence, it is unsatisfiable.

# The n-Queens Problem

- The n-queens problem asks for a placement of n queens on an n $\times$ n chessboard so that no queen can attack another queen. This means that no two queens can be placed in the same row, in the same column, or on the same diagonal. We display a solution to the eight-queens problem in following Figure.



# The n-Queens Problem



p: You will join a MNC

q: You are expert in ML

r: You are not hired by another company

$$p \to (q \vee r)$$

~~If~~ r: You are hired by another company

$$p \to (q \vee (\sim r))$$

You cannot ride the bike if you are under 4 feet tall unless you are older than 16 years

p: You can ride the bike

q: You are under 4 feet tall

r: You are older than 16 years

$$(q \wedge (\sim r)) \to (\sim p)$$

# Graph Theory

- Graphs are discrete structures consisting of vertices and edges that connect these vertices.
- There are different kinds of graphs, depending on whether edges have directions, whether multiple edges can connect the same pair of vertices, and whether loops are allowed.
- Using graph models, we can determine whether it is possible to walk down all the streets in a city without going down a street twice
- we can find the number of colors needed to color the regions of a map.

# Graph Theory

- A graph G = (V, E) consists of V, a nonempty set of vertices (or nodes) and E, a set of edges. Each edge has either one or two vertices associated with it, called its endpoints. An edge is said to connect its endpoints.
- The set of vertices V of a graph G may be infinite
- A graph with an infinite vertex set or an infinite number of edges is called an infinite graph
- A graph with a finite vertex set and a finite edge set is called a finite graph.

# Graph Theory

- This computer network can be modeled using a graph in which the vertices of the graph represent the data centers and the edges represent communication links

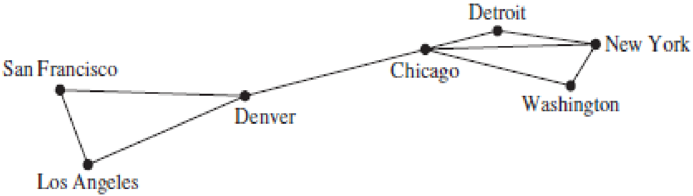



**FIGURE 1** **A computer network.**

# Graph Theory

- A graph in which each edge connects two different vertices and where no two edges connect the same pair of vertices is called a simple graph.

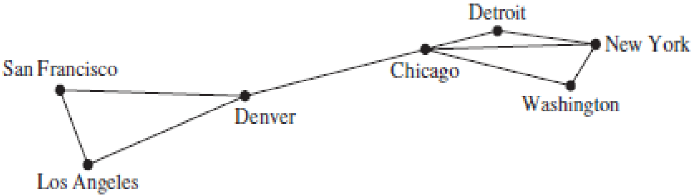



**FIGURE 1** **A computer network.**

# Graph Theory

- Graphs that may have multiple edges connecting the same vertices are called multigraphs.

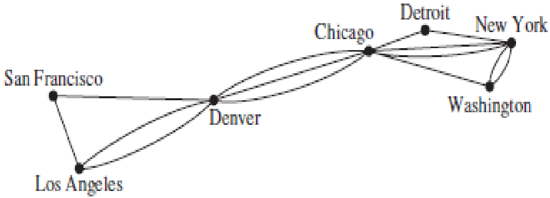



**FIGURE 2** A computer network with multiple links between data centers.

# Graph Theory

- Sometimes a communications link connects a data center with itself, perhaps a feedback loop for diagnostic purposes.
- To model this network we need to include edges that connect a vertex to itself. Such edges are called loops
- So far the graphs we have introduced are undirected graphs. Their edges are also said to be undirected.

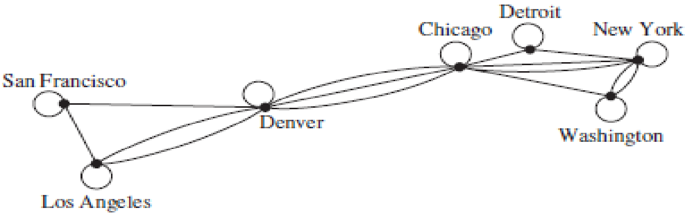



**FIGURE 3** **A computer network with diagnostic links.**

# Graph Theory

- A directed graph (or digraph) (V, E) consists of a nonempty set of vertices V and a set of directed edges (or arcs) E. Each directed edge is associated with an ordered pair of vertices. The directed edge associated with the ordered pair (u, v) is said to start at u and end at v.

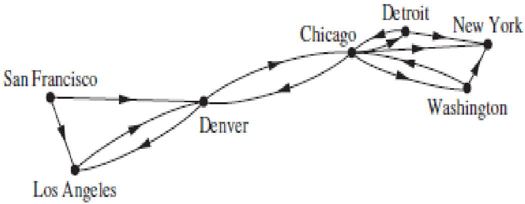



**FIGURE 4** A communications network with one-way communications links.