Discrete Mathematics

Dinesh Naik

Manjunath K Vanahalli

Department of Information Technology,
National Institute of Technology Karnataka, India

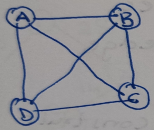
November 11, 2020

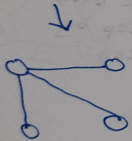
Group Theory

Spanning Tree

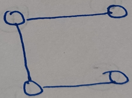
How many minimum edges required to have a connected graph

Spanning tree : It is the minimum number of edges required to connect all the nodes



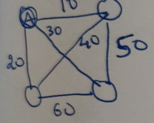


all are connected
⇒ Spanning tree

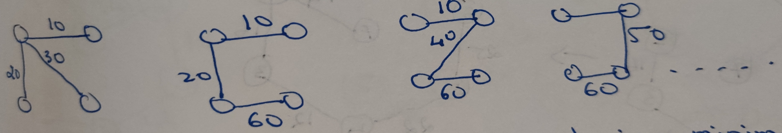


all are connected
⇒ Spanning tree

Group Theory



There are many spanning tree possible.
The minimum spanning tree should be found so that the cost is also minimum.

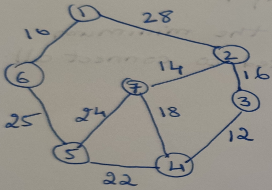


In the above first one is having minimum cost hence minimum spanning tree

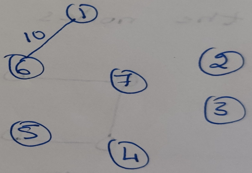
Group Theory

Prim's Algorithm for minimum Spanning tree

Minimum cost Spanning tree



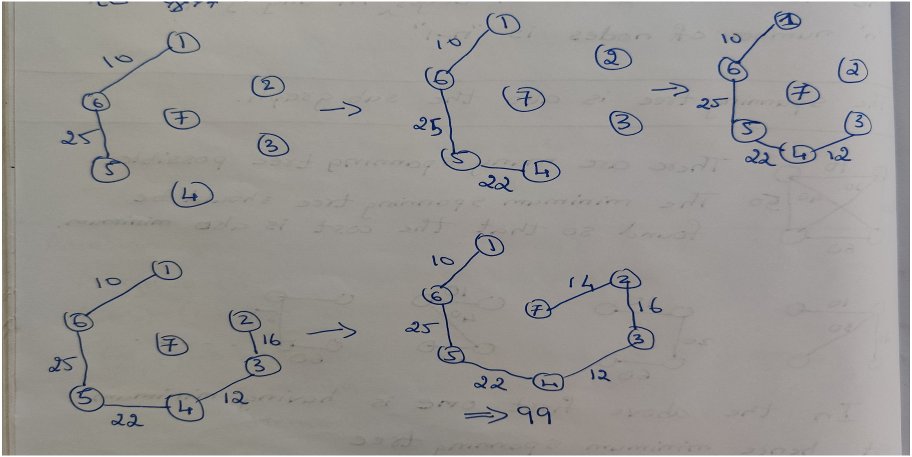
→ First consider the edge with minimum weight



→ construct the tree step by step

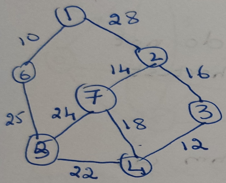
→ Now we need to check the neighbours of 1 & 6 and choose the least weight edge to ~~span~~ connect the tree further

Group Theory

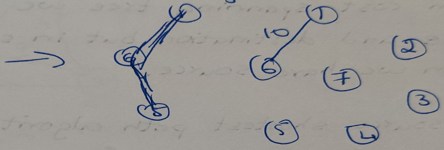


Group Theory

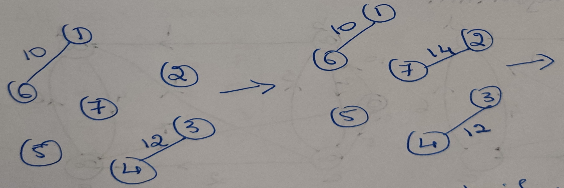
Kruskal's Algorithm

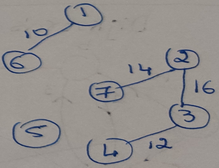


→ First consider the edge with minimum weight



→ In Kruskal's algorithm we have search for 2nd least edge

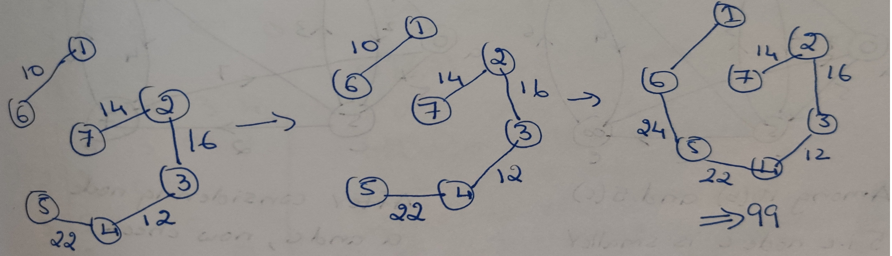




but if we added that

Group Theory

The next minimum is 18 but if we added that then there will be cycle but a tree should not contain a cycle. Hence go for next minimum



Group Theory

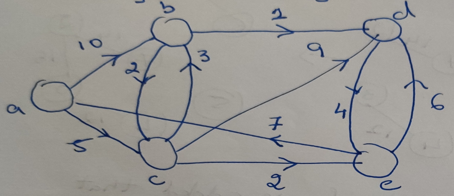
The problem of shortest path is different from minimum cost spanning tree.

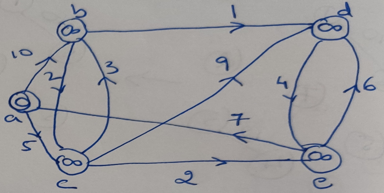
In minimum cost spanning tree we do not have source and destination but in shortest path problem we have source.

In single source shortest path algorithm we have to find the shortest path from a single source node to other nodes.

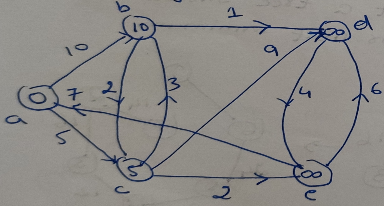
Group Theory

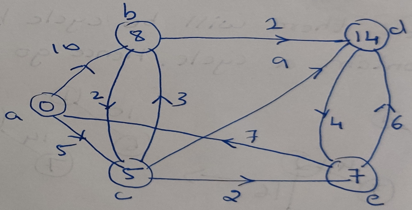
Dijkstra's Algorithm





Considering 'a' as source

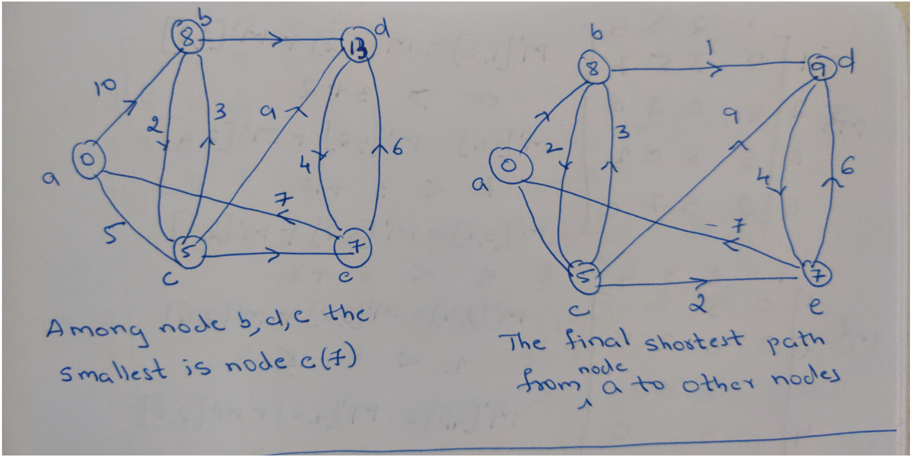




Among 10(b) and 5(c)
5 i.e node c is smaller
So start with node c

After considering node
a and c, now check
next smallest node

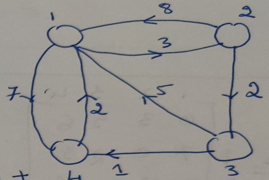
Group Theory



Group Theory

All pair shortest path algorithm → Floyd Warshall

$$M^0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{array}$$



$$M^1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & \infty & 0 \end{bmatrix} \end{array}$$

$M^0[2,3] = M^0[2,1] + M^0[1,3]$

$2 < 8 + \infty$

$M^0[2,4] = M^0[2,1] + M^0[1,4]$

$\infty > 8 + 7$

$M^0[3,2] = M^0[3,1] + M^0[1,2]$

$\infty > 5 + 3$

$M^0[3,4] = M^0[3,1] + M[1,4]$

$1 < 5 + 2$

$M^0[4,2] = M^0[4,1] + M^0[1,2]$

$\infty > 2 + 3$

$M^0[4,3] = M^0[4,1] + M^0[1,3]$

$\infty \quad 2 + \infty$

$$M^1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & & \\ 5 & & 0 & \\ 2 & & & 0 \end{bmatrix} \end{array}$$

Group Theory

$$M^2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & 5 & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & 7 & 0 \end{bmatrix} \end{array}$$

$$M^2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & & \\ 8 & 0 & 2 & 15 \\ & 8 & 0 & \\ & 5 & & 0 \end{bmatrix} \end{array}$$

$$M'[1,3] = M'[1,2] + M'[2,3]$$
$$\infty > 3+2$$
$$M'[1,4] = M'[1,2] + M'[2,4]$$
$$7 < 3+15$$
$$M'[3,1] = M'[3,2] + M'[2,1]$$
$$5 < 8+8$$
$$M'[3,4] = M'[3,2] + M'[2,4]$$
$$1 < 8+15$$
$$M'[4,1] = M'[4,2] + M'[2,1]$$
$$2 < 5+8$$
$$M'[4,3] = M'[4,2] + M'[2,3]$$
$$\infty > 5+2$$

Group Theory

$$M^3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 5 & 6 \\ 2 & 7 & 0 & 2 & 3 \\ 3 & 5 & 8 & 0 & 1 \\ 4 & 2 & 5 & 7 & 0 \end{array}$$

$$M^2[1,2] = M^2[1,3] + M^2[3,2]$$
$$3 < 5 + 8$$
$$M^2[1,4] = M^2[1,3] + M^2[3,4]$$
$$7 > 5 + 1$$
$$M^2[2,1] = M^2[2,3] + M^2[3,1]$$
$$8 > 2 + 5$$
$$M^2[2,4] = M^2[2,3] + M^2[3,4]$$
$$15 > 2 + 1$$
$$M^2[4,1] = M^2[4,3] + M^2[3,1]$$
$$2 < 7 + 5$$
$$M^2[4,2] = M^2[4,3] + M^2[3,2]$$
$$5 < 7 + 8$$

$$M^3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & & 5 & \\ 2 & & 0 & 2 & \\ 3 & 5 & 8 & 0 & 1 \\ 4 & & & 7 & 0 \end{array}$$

Group Theory

$$M^4 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & 5 & 6 \\ 5 & 0 & 2 & 3 \\ 3 & 6 & 0 & 1 \\ 2 & 5 & 7 & 0 \end{bmatrix} \end{array}$$

$$M^3[1,2] = M^3[1,4] + M^3[4,2]$$
$$3 < 6+5$$
$$M^3[1,3] = M^3[1,4] + M^3[4,3]$$
$$5 < 6+7$$
$$M^3[2,1] = M^3[2,4] + M^3[4,1]$$
$$7 > 3+2$$
$$M^3[2,3] = M^3[2,4] + M^3[4,3]$$
$$2 < 3+7$$
$$M^3[3,1] = M^3[3,4] + M^3[4,1]$$
$$5 > 1+2$$
$$M^3[3,2] = M^3[3,4] + M^3[4,2]$$
$$8 > 1+5$$

$$M^4 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & & & 6 \\ & 0 & & 3 \\ & & 0 & 1 \\ 2 & 5 & 7 & 0 \end{bmatrix} \end{array}$$