

Discrete Mathematics

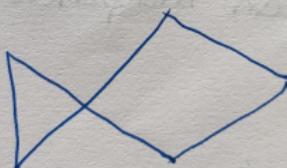
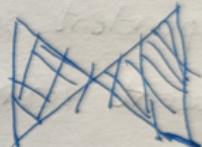
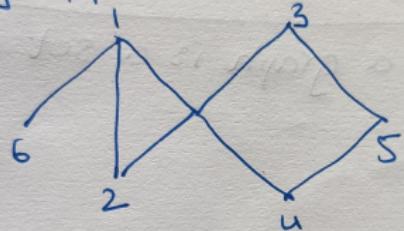
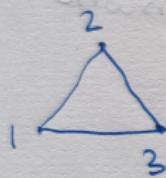
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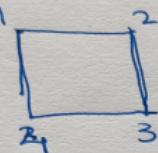
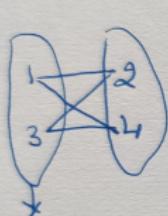
Graph Theory

Cycle: Cycle is a graph with an equal number of vertices and edges, whose vertices can be placed around a circle so that two vertices are adjacent if and only if they appear consecutively along the circle.

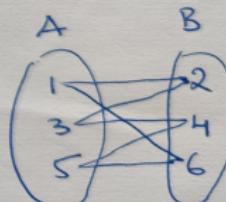
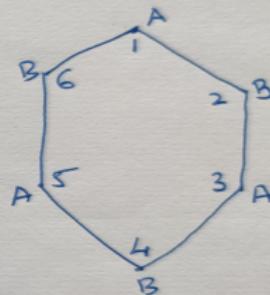


Graph Theory

Bipartite graph: A graph G_1 whose vertices can be divided into two disjoint sets U and V such that every edge connects a vertex in U to one in V [U and V are independent sets]



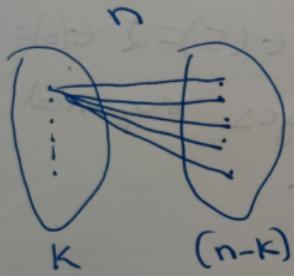
If every vertex is adjacent to all the vertices in another set then it is complete bipartite graph $K_{2,2}$



Graph Theory

The maximum number of edges in bipartite graph with n -vertices

→ the maximum number of edges possible in a bipartite graph with T vertices



$$K(n-k)$$

$$\frac{d}{dk} (kn - k^2)$$

$$\frac{d}{dk} (kn - k^2) = n - 2k$$

$$\frac{d^2}{dk^2} (kn - k^2) < 0 \quad \frac{d}{dk} (n-2k) = -2 < 0$$

$$n/2$$

$$\text{when } T=7 \text{ edges} \rightarrow n=2k \\ n/2 = 7/2 = 3 \dots \quad k = n/2$$

$$3 \times 4 = 12$$

Graph Theory

\Rightarrow If G_i has a uv -path
Then the distance from u to v is the least
length of a uv -path

* If G_i has no path from u to v , $d(u,v) = \infty$

$$\begin{array}{lll} d(1,2)=1 & d(2,3)=1 & d(3,5)=2 \\ d(1,3)=2 & d(2,4)=2 & d(3,6)=3 \\ d(1,4)=2 & d(2,5)=2 & d(4,5)=2 \\ d(1,5)=2 & d(2,6)=2 & d(4,6)=2 \\ d(1,6)=2 & d(3,4)=1 & d(5,6)=1 \end{array}$$



\Rightarrow the diameter of graph G_i is $\max_{u,v \in V(G_i)} d(u,v)$

\Rightarrow the eccentricity of vertex u , $E(u) = \max_{v \in V(G_i)} d(u,v)$

\Rightarrow the radius of graph G_i is $\min_{u \in V(G_i)} E(u)$

\rightarrow The diameter of graph is 3.

The analysis is from any vertex we can reach any other vertex in the distance less than or equal to 3.

\rightarrow The eccentricity of vertex

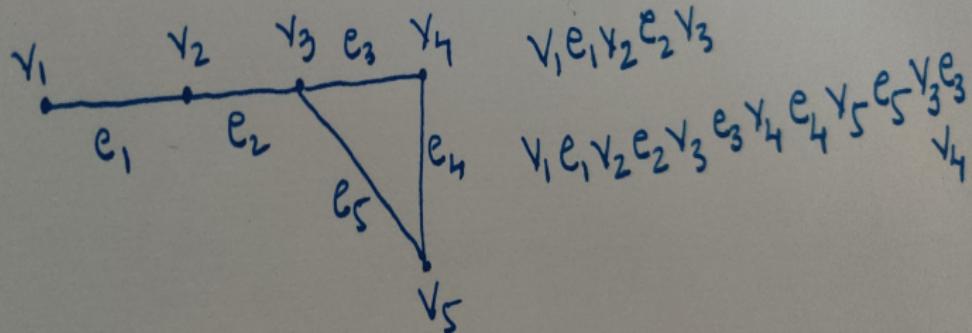
$$E(1)=2 \quad E(2)=2 \quad E(3)=3 \quad E(4)=2 \quad E(5)=2 \quad E(6)=3$$

It is the maximum number of edges required from a vertex to any other vertex

\rightarrow Radius of graph is = 2

Graph Theory

Walk: It is a list $v_0e_1v_1 \dots e_kv_k$ of vertices and edges such that, for $1 \leq i \leq k$, the edge e_i has end points v_{i-1} and v_{i+1}



Graph Theory

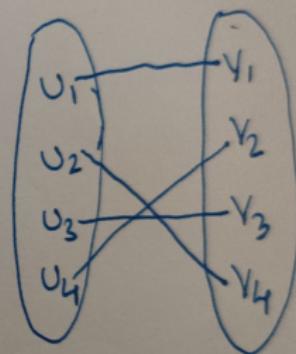
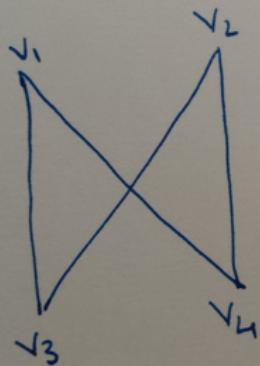
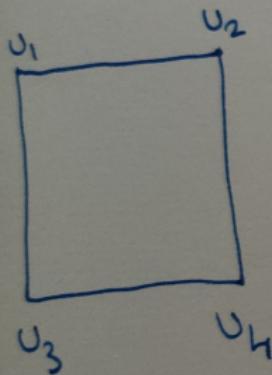
Trail: It is a walk with no repeated edge.

$v_1e_1v_2e_2v_3 \rightarrow$ Trail $v_1e_1v_2e_2v_3e_3v_4e_4v_5e_5v_3e_3v_4$
Walk but Not a Trail

A uv -walk } the first vertex ' u '
A uv -trail } the last vertex ' v '

Graph Theory

Isomorphism



u_1, u_2 u_1, u_3
 v_1, v_4 v_1, v_3

u_2, u_4 u_3, u_4
 v_4, v_2 v_3, v_2