Discrete Mathematics

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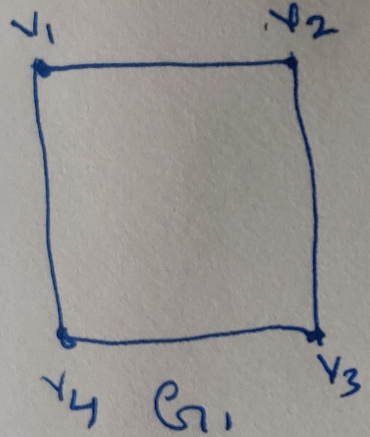
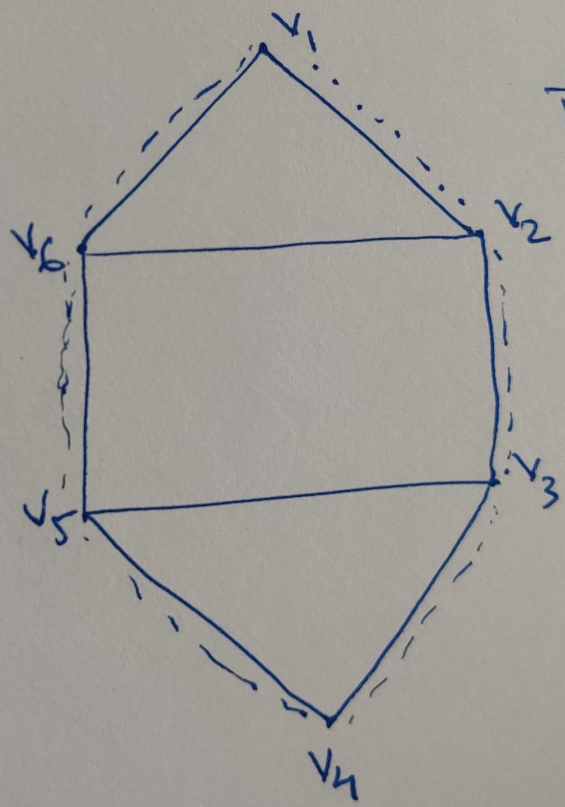
Graph Theory

Hamilton Cycle : In a graph G is a cycle that passes through each vertex exactly once

→ If a graph has a Hamilton Cycle, it is called Hamiltonian graph



G_1



The requirement is to cover all the vertices exactly once. If the edges are not covered its fine.

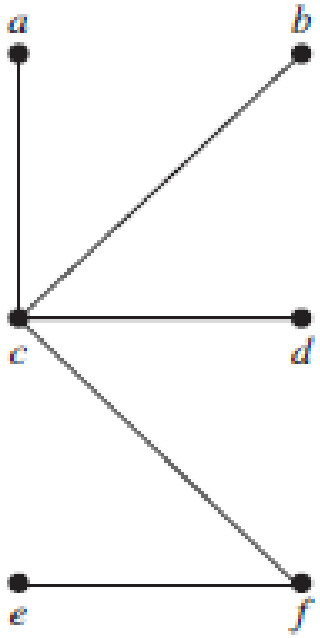
Graph Theory

- A tree is a connected undirected graph with no simple circuits.
- A tree cannot have a simple circuit, a tree cannot contain multiple edges or loops. Therefore any tree must be a simple graph.
- Any connected graph that contains no simple circuits is a tree.
- Graphs containing no simple circuits that are not necessarily connected, these graphs are called forests

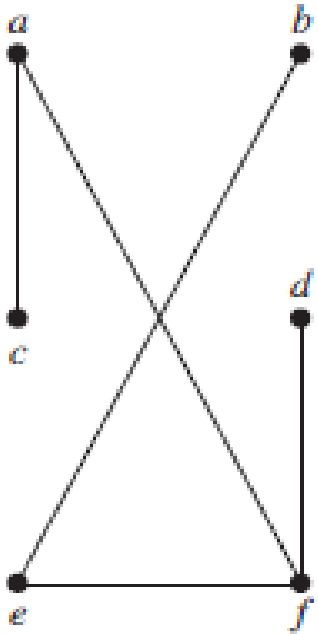
Propositional Logic

- G_1 and G_2 are trees, because both are connected graphs with no simple circuits. G_3 is not a tree because e, b, a, d, e is a simple circuit in this graph.

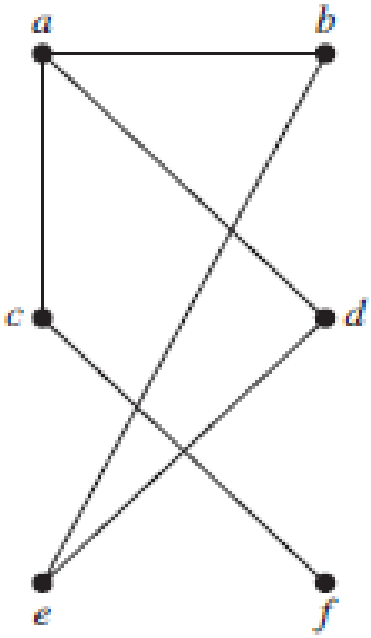
- Finally, G_4 is not a tree because it is not connected.



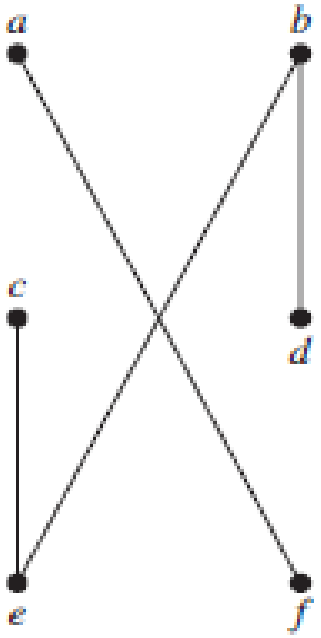
G_1



G_2



G_3



G_4

Graph Theory

- A rooted tree is a tree in which one vertex has been designated as the root and every edge is directed away from the root.

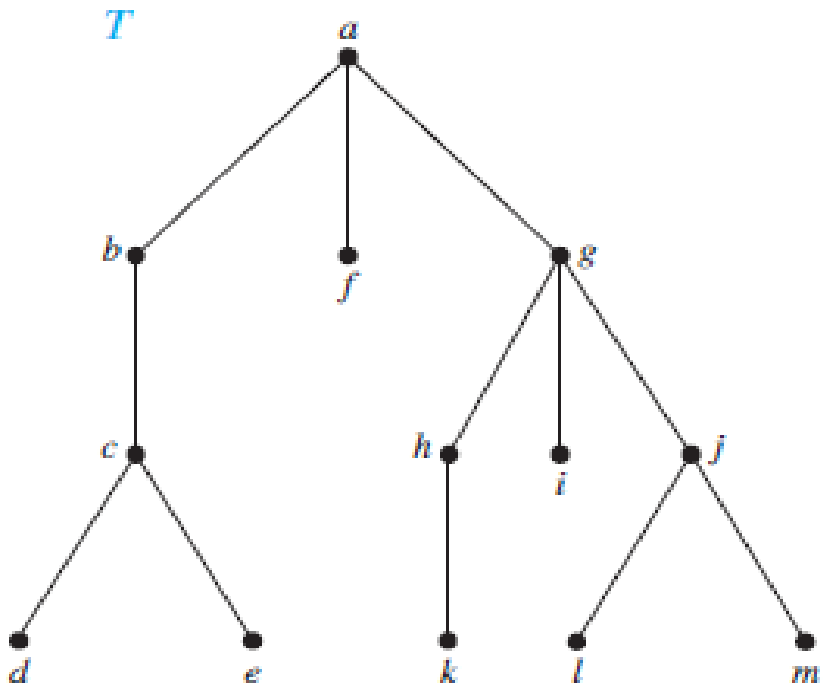


Figure 1. A rooted tree T.

Graph Theory

- If v is a vertex in T other than the root, the parent of v is the unique vertex u such that there is a directed edge from u to v
- When u is the parent of v, v is called a child of u.
- Vertices with the same parent are called siblings.
- A vertex of a rooted tree is called a leaf if it has no children. Vertices that have children are called internal vertices.
- The ancestors of a vertex other than the root are the vertices in the path from the root to this vertex, excluding the vertex itself and including the root (that is, its parent, its parent's parent, and so on, until the root is reached). The descendants of a vertex v are those vertices that have v as an ancestor.

Graph Theory

- If *a* is a vertex in a tree, the subtree with *a* as its root is the subgraph of the tree consisting of a and its descendants and all edges incident to these descendants.

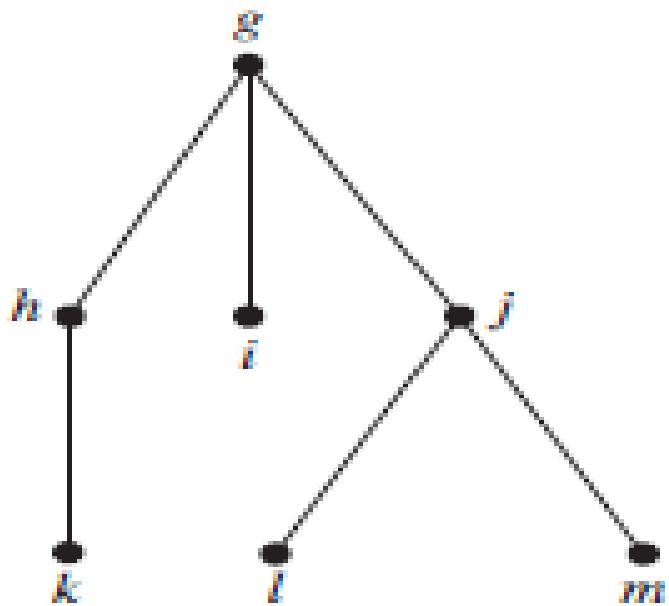


Figure 2. A rooted tree T.

Graph Theory

- A rooted tree is called an m-ary tree if every internal vertex has no more than m children.
- The tree is called a full m-ary tree if every internal vertex has exactly m children.
- An m-ary tree with $m = 2$ is called a binary tree.

Graph Theory

- An ordered rooted tree is a rooted tree where the children of each internal vertex are ordered. Ordered rooted trees are drawn so that the children of each internal vertex are shown in order from left to right.
- In an ordered binary tree (usually called just a binary tree), if an internal vertex has two children, the first child is called the left child and the second child is called the right child.
- The tree rooted at the left child of a vertex is called the left subtree of this vertex, and the tree rooted at the right child of a vertex is called the right subtree of the vertex
- A tree with n vertices has n - 1 edges.