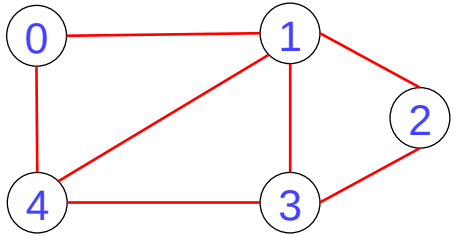
Graphs

Graph Data Structure

- A ***graph*** is a non-linear data structure consisting of nodes and edges
- The nodes are sometimes also referred to as vertices and the edges are lines or arcs that connect any two nodes in the ***graph***
- More formally a ***graph*** can be defined as:

  *A **graph G** consists of a finite set of vertices (or nodes) **V** and set of edges **E** which connect a pair of nodes*
- In the given ***graph***, the set of vertices V = {0, 1, 2, 3, 4} and the set of edges E = {01, 12, 23, 34, 04, 14, 13}



- ***Graph***s are used to solve many real-life problems
  - ***Graph***s are used to represent networks
  - The networks may include paths in a city or telephone network or circuit network
  - ***Graph***s are also used in social networks like linkedIn, Facebook
  - For example, in Facebook, each person is represented with a vertex (or node)
  - Each node is a structure and contains information like person id, name, gender, locale etc.

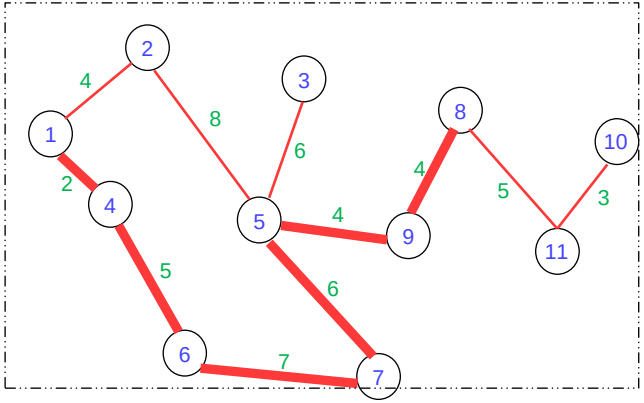
Sample Graph Problems

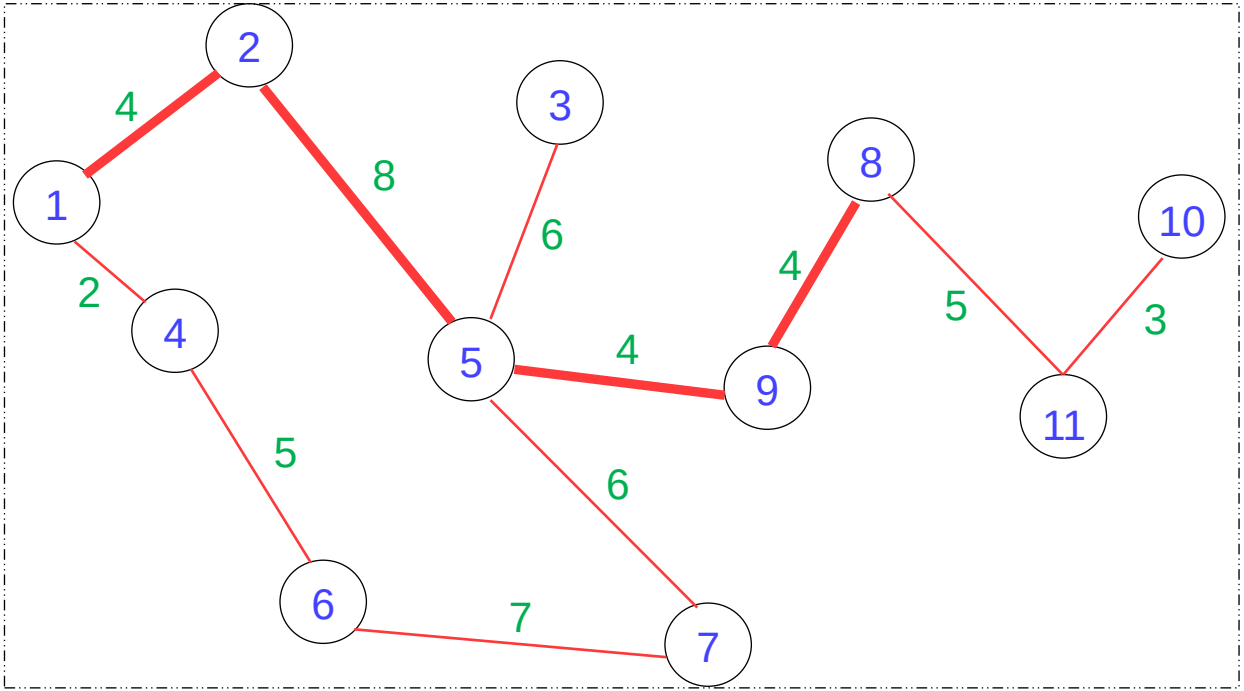
- Path problems
- Connectedness problems
- Spanning tree problems

Path problems: Path Finding

- Path between ***1*** and ***8***
- Path length is **20**

- Another Path Between ***1*** and ***8***
- Path length is **28**



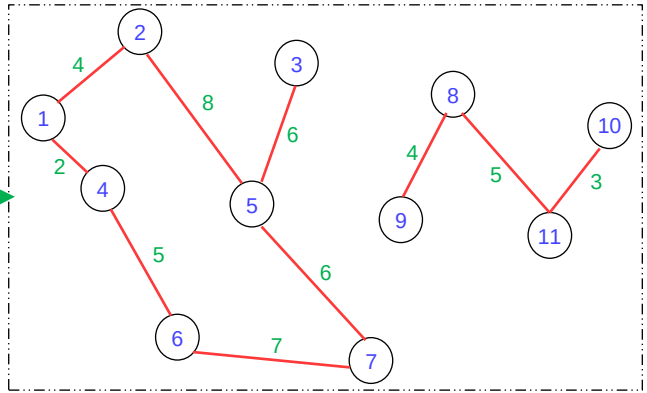


Vertices represent cities and edges represent highways, edge weights are distances or driving times, depending on the context, path length may either be the number of edges on the path or the sum of the weights of the edges on the path

No Path, Connected Graph

- Since a graph may have more than one path between two vertices, we may be interested in finding a path with a particular property
- For example, find a path with minimum length

- ***Example of no path:*** no path between ***2*** and ***9***



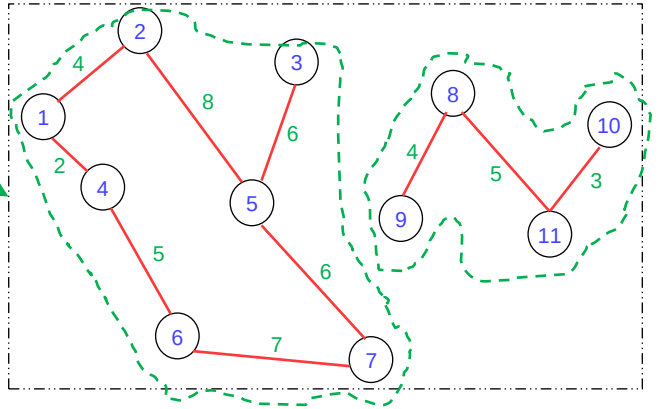
Not connected graph

Connected graph

- Undirected graph
- There is a path between every pair of vertices
- Determine whether an undirected graph is connected

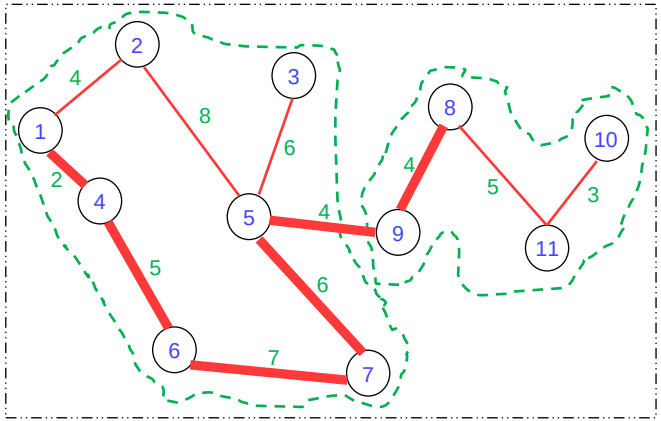
Connected components

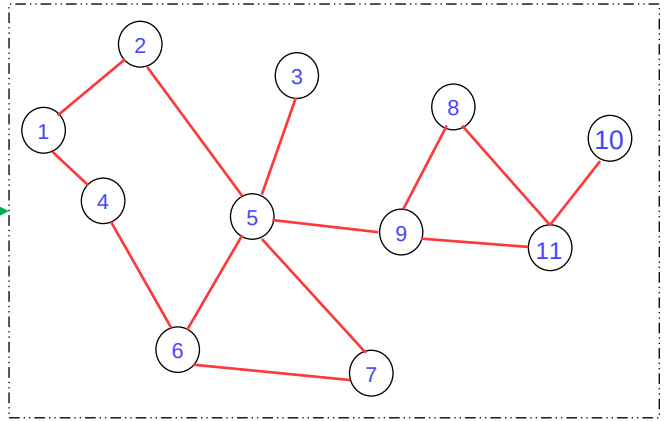
- A maximal subgraph that is connected
  - Cannot add vertices and edges from original graph and retain connectedness
- A connected graph has exactly 1 component



Not a Component, Communication Network

- Determine connected components of an undirected graph



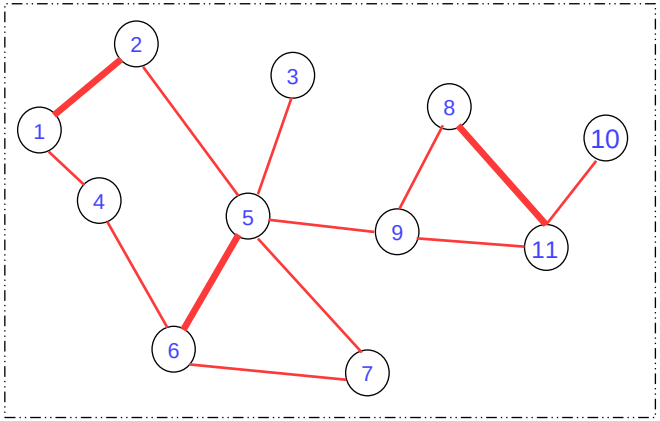


Communication Network

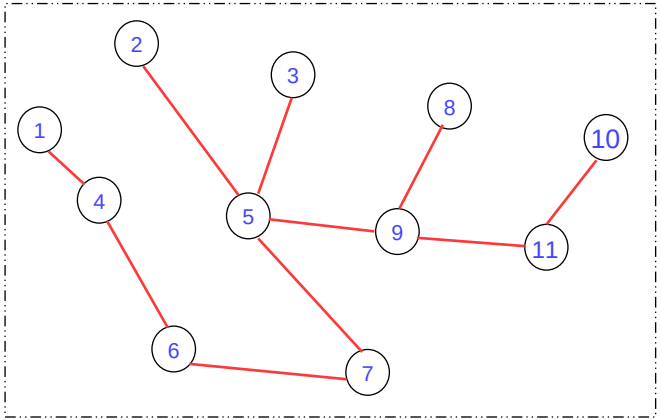
- Each edge is a link that can be constructed (i.e., a feasible link)
- Is the network connected?
  - Can we communicate between every pair of cities?
- Find the components
- Want to construct smallest number of feasible links so that resulting network is connected

Cycles and Connectedness

- Removal of an edge that is on a cycle does not affect connectedness

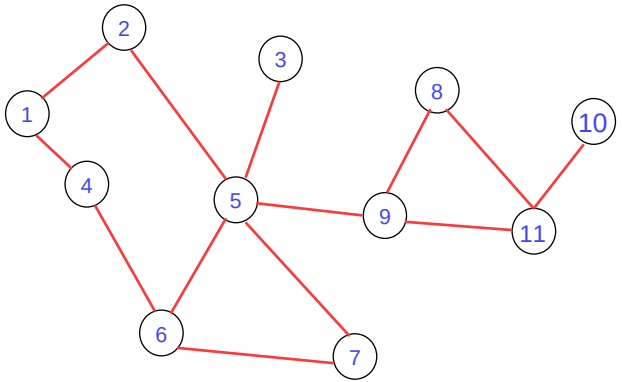


- Connected subgraph with all vertices and minimum number of edges has no cycles



Graph Parameters

- Order of the graph: The number of vertices in the graph
- Size of the graph: The number of edges in the graph
- Degree of a vertex of a graph: Number of edges incident to the vertex
- For directed graph: InDegree and OutDegree

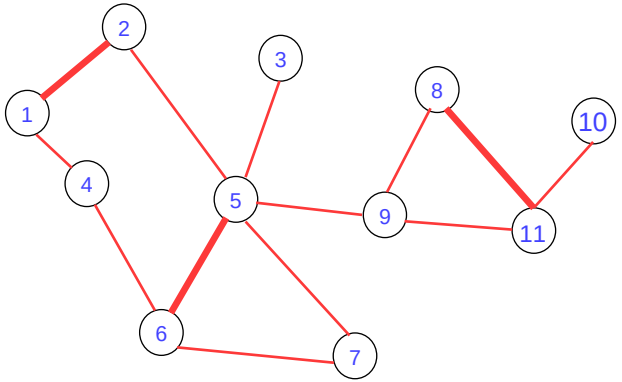


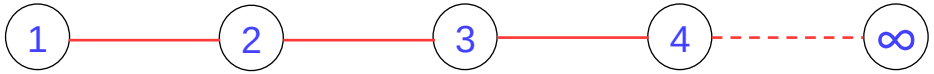
Order of the graph: 11
Size of the graph: 13
Degree of a vertex of a graph: 3 [Vertex 9]
1 [Vertex 3]
2 [Vertex 2]
5 [Highest degree vertex]

Graph Type: Finite vs. Infinite Graph

- A graph G = (V, E) is finite in case the number of vertices and edges in the graph is finite in number

- A graph G = (V, E) is said to infinite in case the number of edges and vertices in the graph is infinite in number



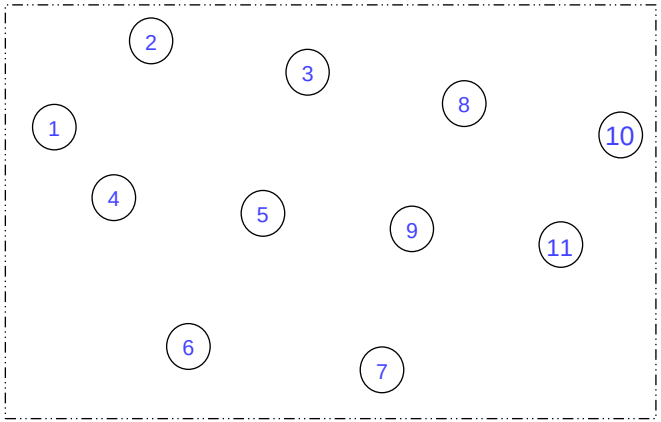


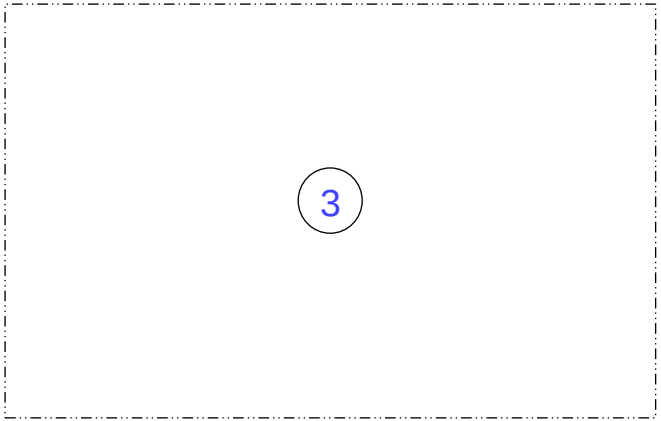
Example: A graph constructed by using all non-negative integers as vertices and connecting vertex i to vertex (i+1) with an edge

Graph Type: Null Graph, Trivial Graph

- A graph whose edge set is empty is called as a null graph
- In other words, a null graph does not contain any edges in it

- A graph having only one vertex in it is called as a trivial graph. It is the smallest possible graph

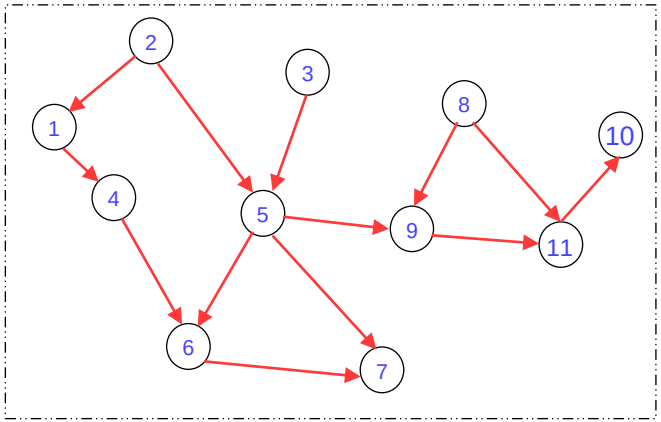


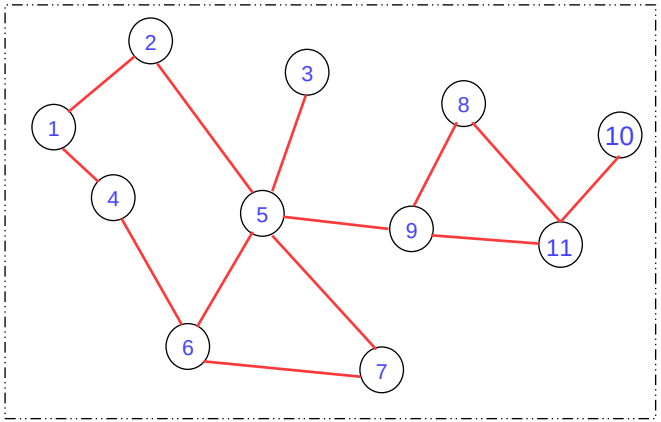


Graph Type: Undirected vs. Directed Graph

- A graph in which all the edges are undirected is called as a non-directed graph
- In other words, edges of an undirected graph do not contain any direction

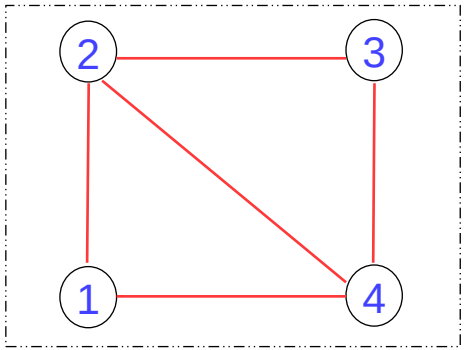
- A graph in which all the edges are directed is called as a directed graph
- In other words, all the edges of a directed graph contain some direction
- Directed graphs are also called as digraphs

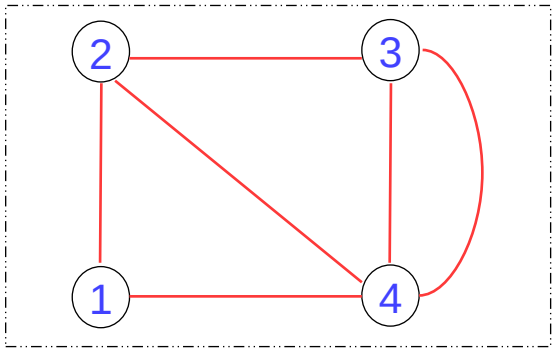


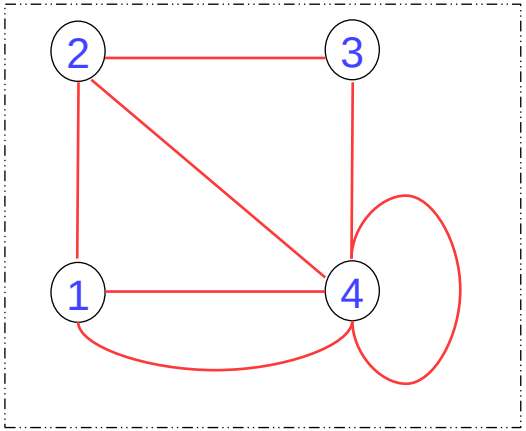


Graph Type: Simple, Multi-, and Pseudo Graph

- A graph having no self loops and no parallel edges in it is called as a simple graph
- A graph having no self loops but having parallel edge(s) in it is called as a multi graph
- A graph having parallel edges and self loop(s) in it is called as a pseudo graph





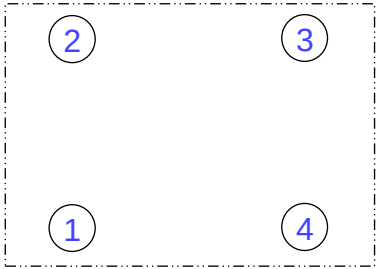


Graph Type: Simple, Multi-, and Pseudo Graph

Type	Edges	Multiple Edges Allowed ?	Loops Allowed ?
Simple Graph	undirected	No	No
Multigraph	undirected	Yes	No
Pseudograph	undirected	Yes	Yes
Directed Graph	directed	No	Yes
Directed Multigraph	directed	Yes	Yes

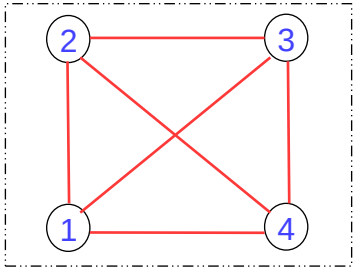
Graph Type: Null vs. Complete Graph

- Null graph is a modified version of a trivial graph
- A graph G = (V, E) is said to a null graph in case there is ***N*** number of vertices exist but no edge exists that connects them



- Order of G = ***N***
- Size of G = 0

- A graph G = (V, E) is said to be a complete graph in case it is also a simple graph
- With this ***N*** number of vertices must be attached to each of other vertices using the edges

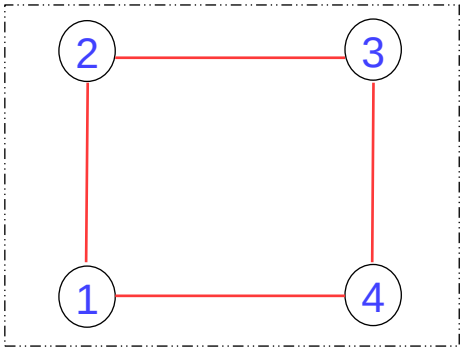


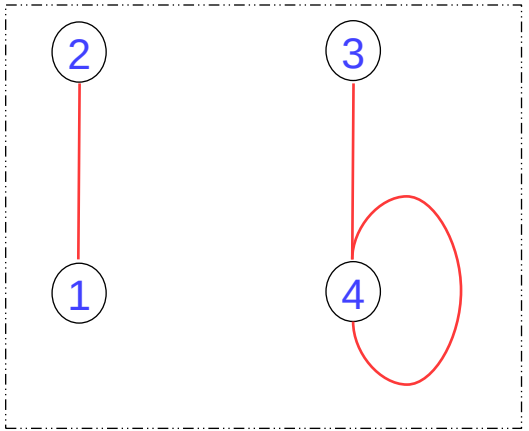
- It is also known as a full graph and the degree of each vertex must be ***N - 1***

Graph Type: Connected vs. Disconnected Graph

- A graph in which we can visit from any one vertex to any other vertex is called as a connected graph
- In connected graph, at least one path exists between every pair of vertices

- A graph in which there does not exist any path between at least one pair of vertices is called as a disconnected graph

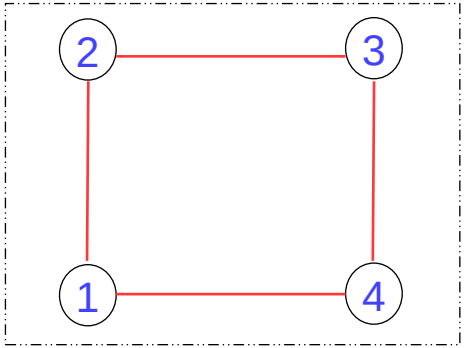


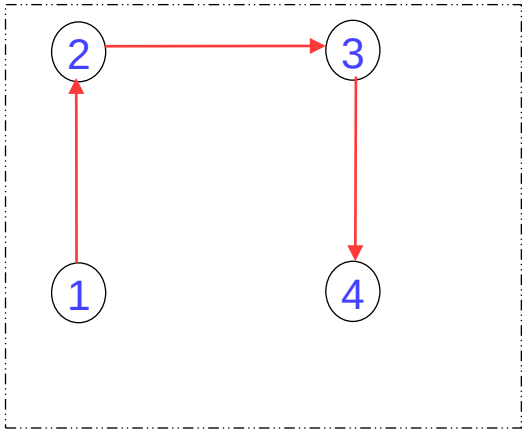


Graph Type: Cyclic vs. Directed Acyclic Graph

- A graph containing at least one cycle in it is called as a cyclic graph
- A graph G = (V, E) is said to be a cyclic graph when one can reach its own while traversal, i.e., if Vertex 1, 2, 3, and 4 are vertices in the graph then, there always exist edges connecting (1, 2) and (2, 3), (3, 4), and (4, 1)

- A Directed Acyclic Graph's also known as DAG
- These are the graphs with directed edges but they do not contain any cycle
- Vertices also hold some data and as it is directed thus edges are represented using an ordered pair of vertices





Graph Type: Others

- Regular Graph
- Cycle Graph
- Bipartite Graph
- Planar Graph
- Pseudo Graph
- Euler Graph
- Hamiltonian Graph
- Bipartite Graph
- Labelled Graph
- Subgraph
- Vertex Labeled Graph

Next Lecture

Graphs and Its Representations

Thank you for your attention...

Any question?

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