

Computer Vision-IT813

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Concept of Histogram Equalization

What is PMF?

PMF stands for probability mass function. As its name suggests, it gives the probability of each number in the data set or you can say that it basically gives the count or frequency of each element.

How PMF is calculated

We will calculate PMF from two different ways. First from a matrix, because in the next tutorial, we have to calculate the PMF from a matrix, and an image is nothing more than a two dimensional matrix.

Then we will take another example in which we will calculate PMF from the histogram.

Consider this matrix.

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 7 | 5 | 6 |
| 7 | 2 | 3 | 4 | 5 |
| 0 | 1 | 5 | 7 | 3 |
| 1 | 2 | 5 | 6 | 7 |
| 6 | 1 | 0 | 3 | 4 |

Now if we were to calculate the PMF of this matrix, here how we are going to do it.

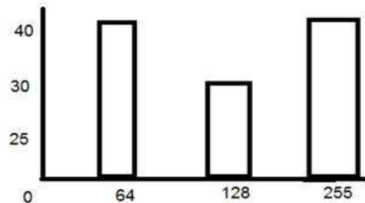
At first, we will take the first value in the matrix , and then we will count, how much time this value appears in the whole matrix. After count they can either be represented in a histogram, or in a table like this below.

PMF

| | | |
|---|---|--------|
| 0 | 2 | $2/25$ |
| 1 | 4 | $4/25$ |
| 2 | 3 | $3/25$ |
| 3 | 3 | $3/25$ |
| 4 | 2 | $2/25$ |
| 5 | 4 | $4/25$ |
| 6 | 3 | $3/25$ |
| 7 | 4 | $4/25$ |

Note that the sum of the count must be equal to total number of values.

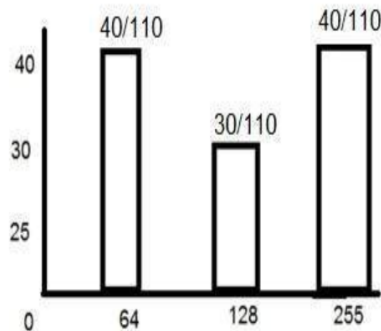
Calculating PMF from histogram



The above histogram shows frequency of gray level values for an 8 bits per pixel image.

Now if we have to calculate its PMF, we will simply look at the count of each bar from vertical axis and then divide it by total count.

So the PMF of the above histogram is this.



Another important thing to note in the above histogram is that it is not monotonically increasing. So in order to increase it monotonically, we will calculate its CDF.

What is CDF?

CDF stands for cumulative distributive function. It is a function that calculates the cumulative sum of all the values that are calculated by PMF. It basically sums the previous one.

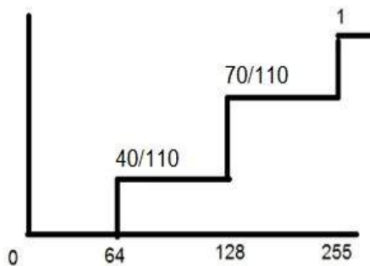
How it is calculated?

We will calculate CDF using a histogram. Here how it is done. Consider the histogram shown above which shows PMF.

Since this histogram is not increasing monotonically, so will make it grow monotonically.

We will simply keep the first value as it is, and then in the 2nd value , we will add the first one and so on.

Here is the CDF of the above PMF function.



Now as you can see from the graph above, that the first value of PMF remain as it is. The second value of PMF is added in the first value and placed over 128. The third value of PMF is added in the second value of CDF , that gives $110/110$ which is equal to 1.

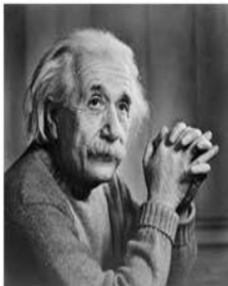
And also now, the function is growing monotonically which is necessary condition for histogram equalization.

Histogram Equalization

Histogram equalization is used to enhance contrast. It is not necessary that contrast will always be increase in this. There may be some cases were histogram equalization can be worse. In that cases the contrast is decreased.

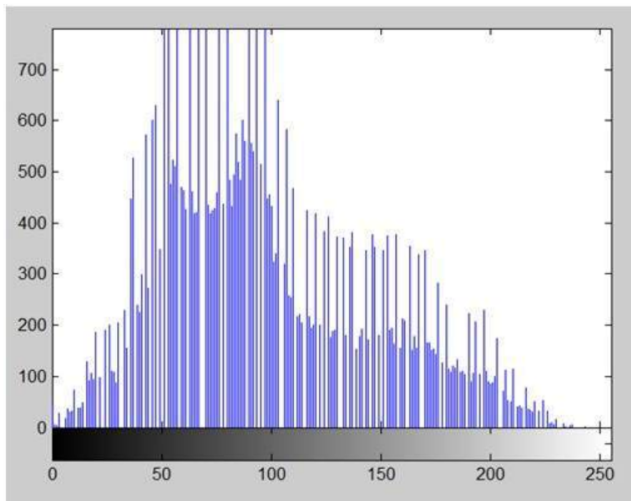
Lets start histogram equalization by taking this image below as a simple image.

Image

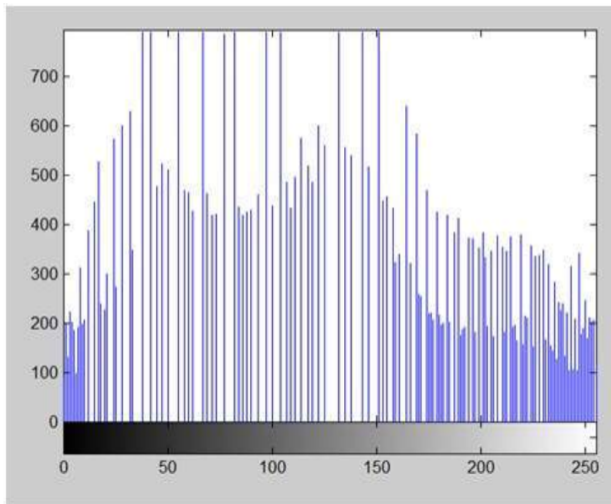


Histogram of this image

The histogram of this image has been shown below.

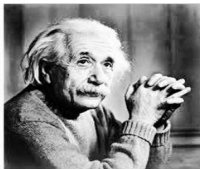


Histogram Equalization histogram

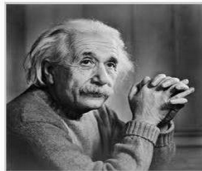


Comparing both the histograms and images

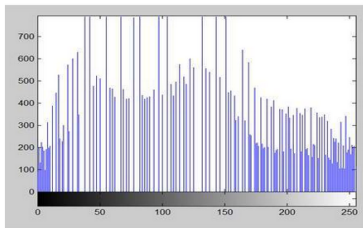
New Image



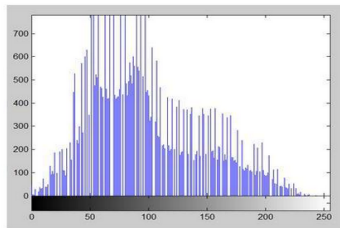
Old image



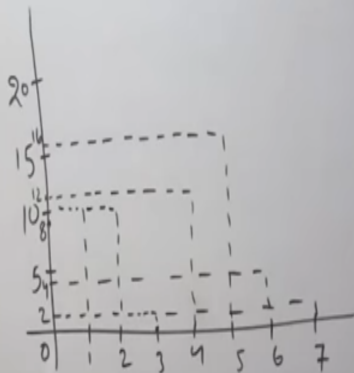
New Histogram



Old Histogram



| Gray level (r_k) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|-------------------------|---|----|----|---|----|----|---|---|
| No. of pixels (p_k) | 8 | 10 | 10 | 2 | 12 | 16 | 4 | 2 |



| Gray level (r_k) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|-------------------------|---|----|----|---|----|----|---|---|
| No. of pixels (p_k) | 8 | 10 | 10 | 2 | 12 | 16 | 4 | 2 |

Total no. of pixels = $8 + 10 + 10 + 2 + 12 + 16 + 4 + 2 = 64$.

| r_k | p_k | p_m | C_m | $L = 7 \times C_m$ | Round off | No. of pixels |
|-------|-------|-------|-------|--------------------|-----------|---------------|
| | | | | | | |
| | | | | | | |
| | | | | | | |
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| | | | | | | |
| | | | | | | |
| | | | | | | |

Q. Perform histogram equalization on the given 8x8 image. The gray level distribution of the image is given below -

| Gray level (r_k) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|-------------------------|---|----|----|---|----|----|---|---|
| No. of pixels (p_k) | 8 | 10 | 10 | 2 | 12 | 16 | 4 | 2 |

$$\text{Total no. of pixels} = 8 + 10 + 10 + 2 + 12 + 16 + 4 + 2 = 64$$

| r_k | p_k | p_m | C_m | $L = 7 \times C_m$ | Round off | No. of pixels |
|-------|-------|---------|---------|--------------------|-----------|---------------|
| 0 | 8 | 0.125 | 0.125 | 0.875 | 1 | 8 |
| 1 | 10 | 0.15625 | 0.28125 | 1.96875 | 2 | 10 |
| 2 | 10 | 0.15625 | 0.4375 | 3.0625 | 3 | 12 |
| 3 | 2 | 0.03125 | 0.46875 | 3.28125 | 3 | 12 |
| 4 | 12 | 0.1875 | 0.65625 | 4.59375 | 5 | 12 |
| 5 | 16 | 0.25 | 0.90625 | 6.34375 | 6 | 16 |
| 6 | 4 | 0.0625 | 0.96875 | 6.78125 | 7 | 6 |
| 7 | 2 | 0.03125 | 1 | 7 | 7 | 2 |