Computer Vision-IT416

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Image Stitching

Image 1

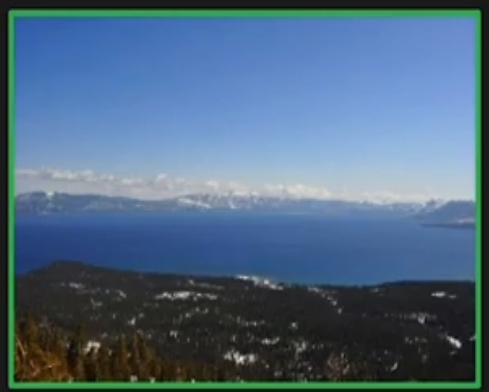


Image 2

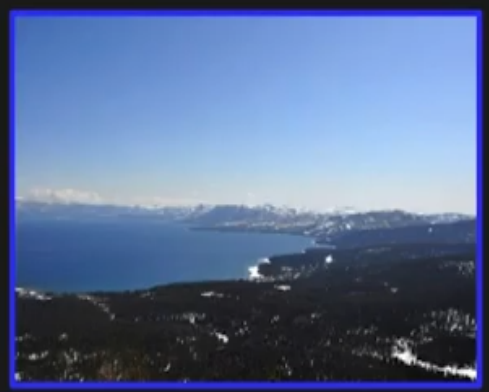


Image 3

How would you align these images?

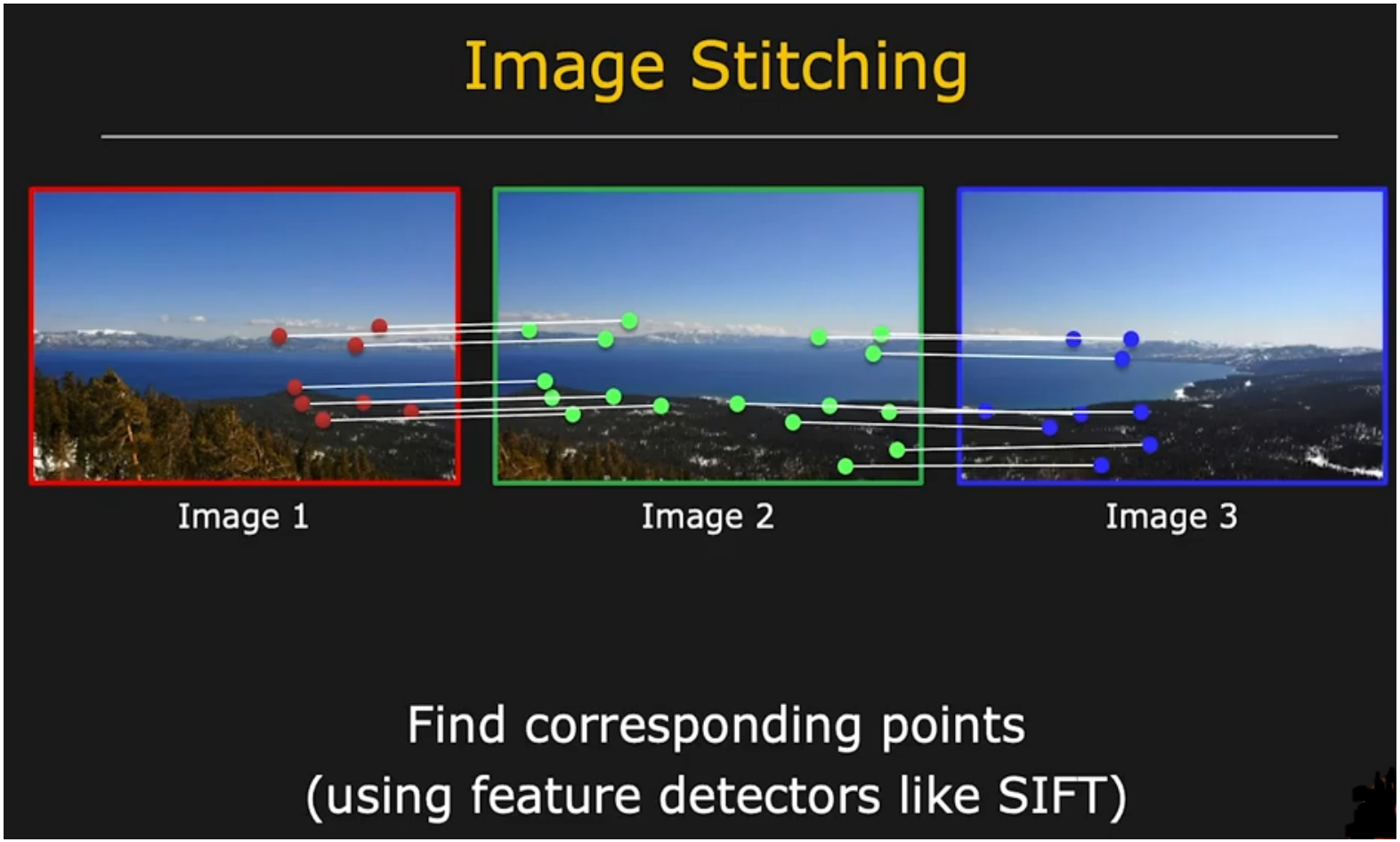


Image Stitching
Image 1
Image 2
Image 3
Find corresponding points
(using feature detectors like SIFT)

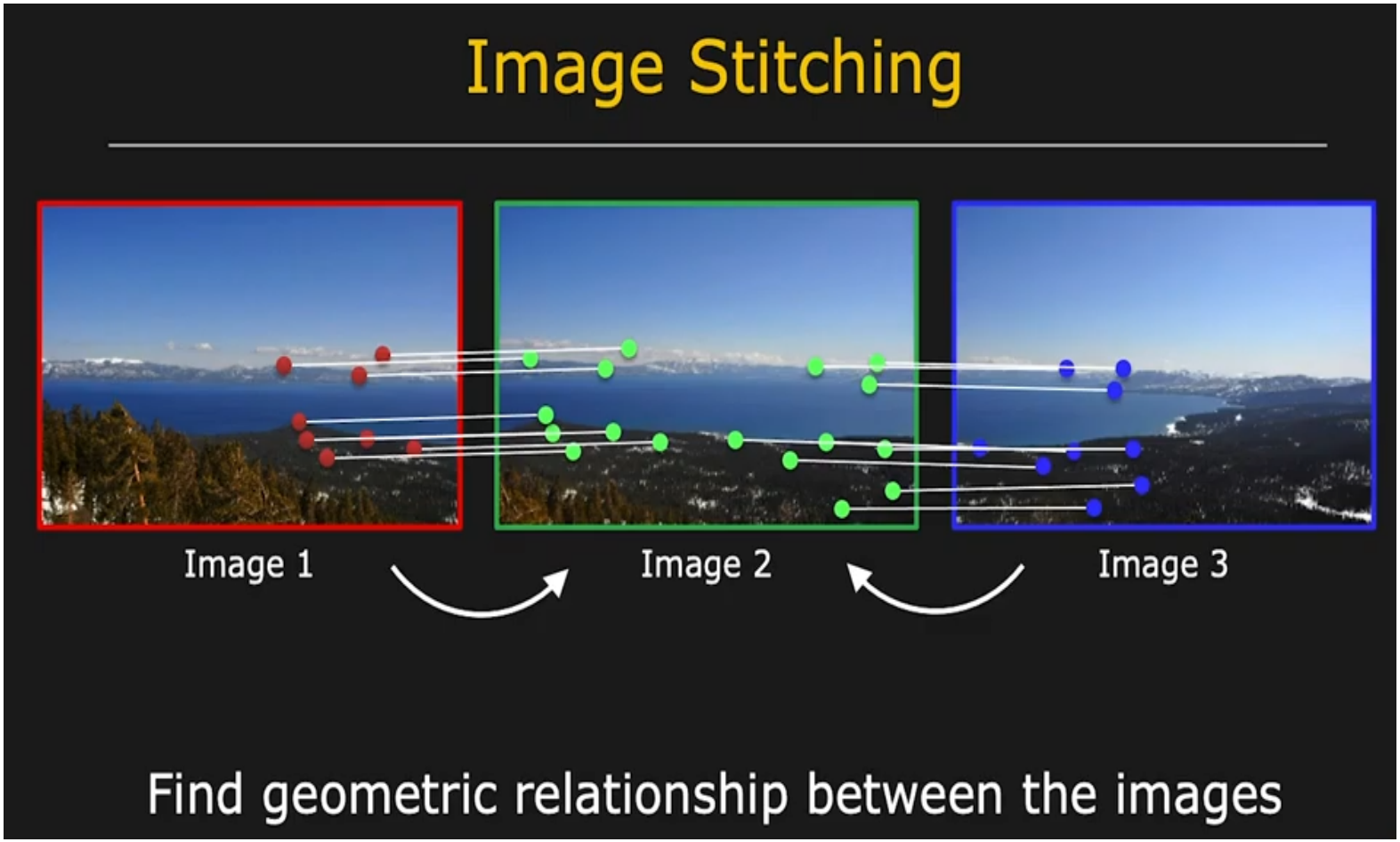


Image Stitching
Image 1
Image 2
Image 3
Find geometric relationship between the images

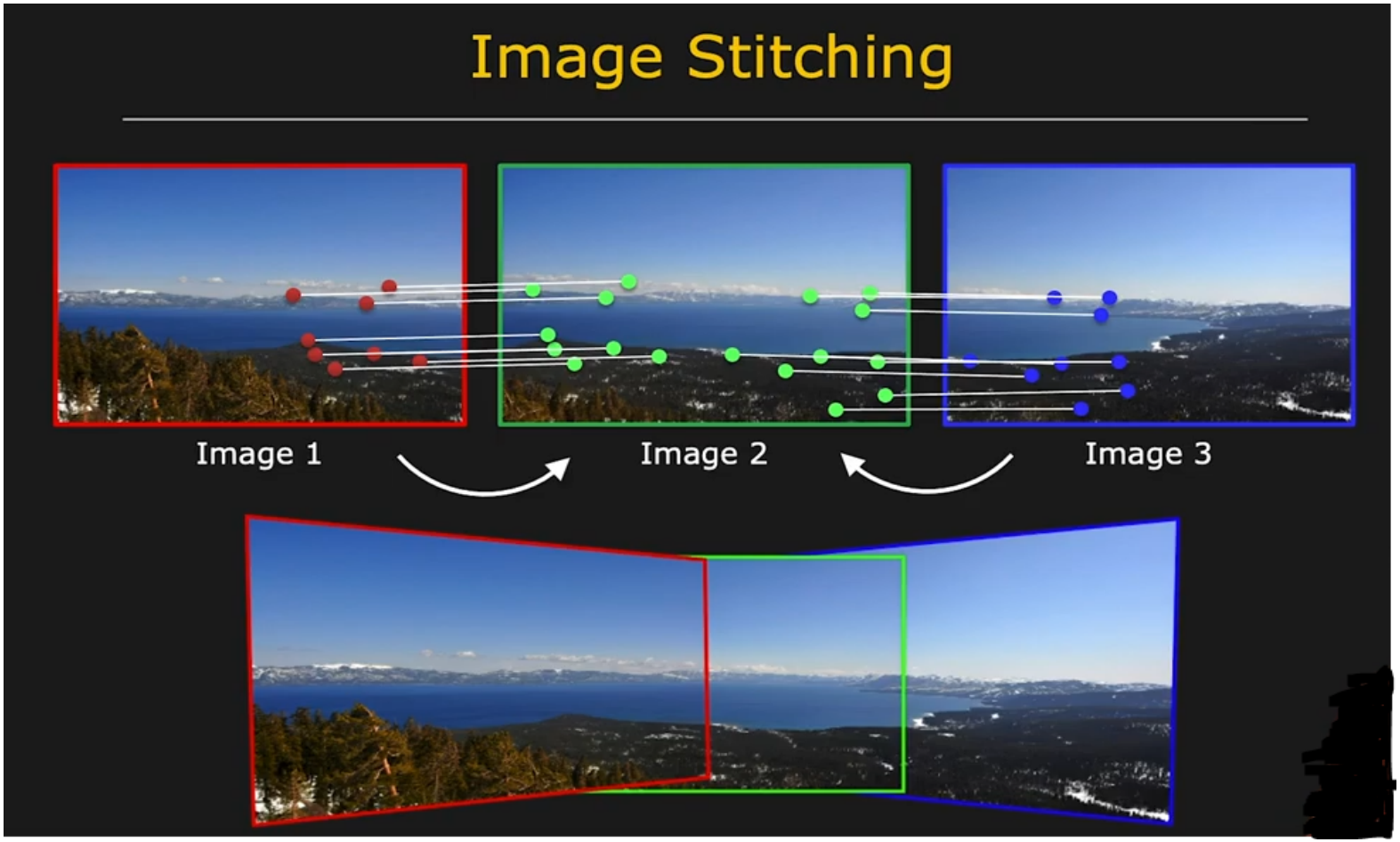


Image Stitching
Image 1
Image 2
Image 3

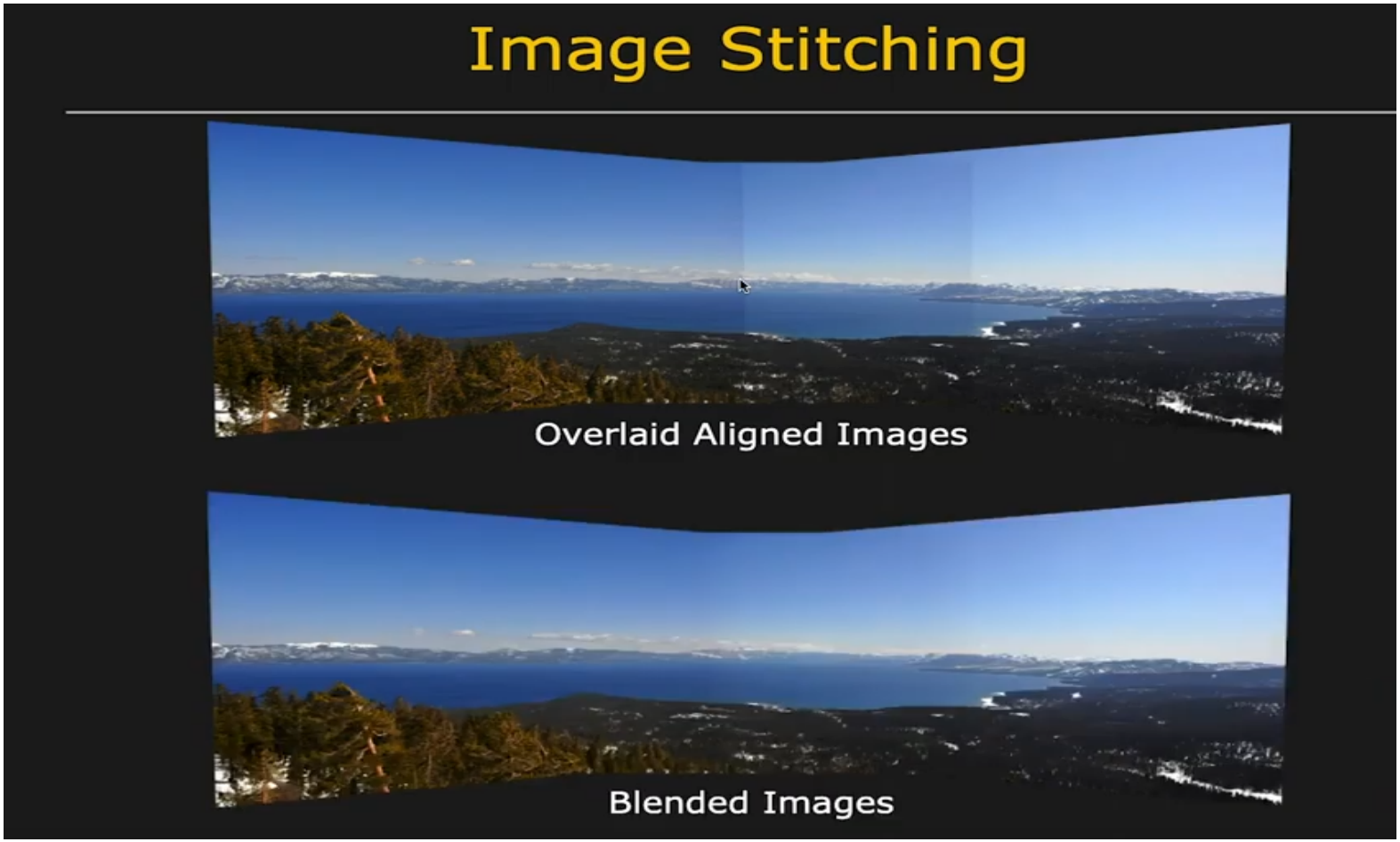


Image Stitching
Overlaid Aligned Images
Blended Images

Image Stitching

Combine multiple photos to create a larger photo

Topics:

(1) 2x2 Image Transformations

(2) 3x3 Image Transformations

(3) Computing Homography

(4) Dealing with Outliers: RANSAC

(5) Warping and Blending Images

Image Manipulation

Image Filtering: Change range (brightness)

$$g(x,y) = T_r(f(x,y))$$

Image Manipulation

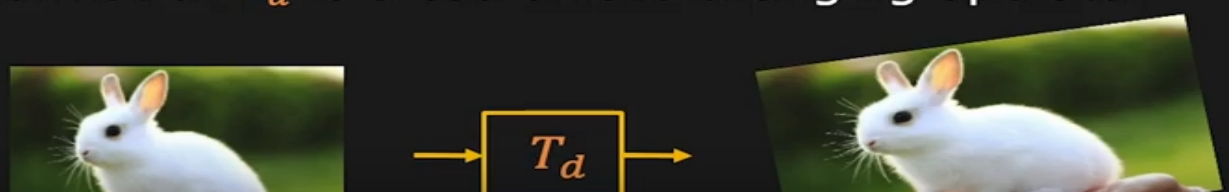
Image Filtering: Change range (brightness)

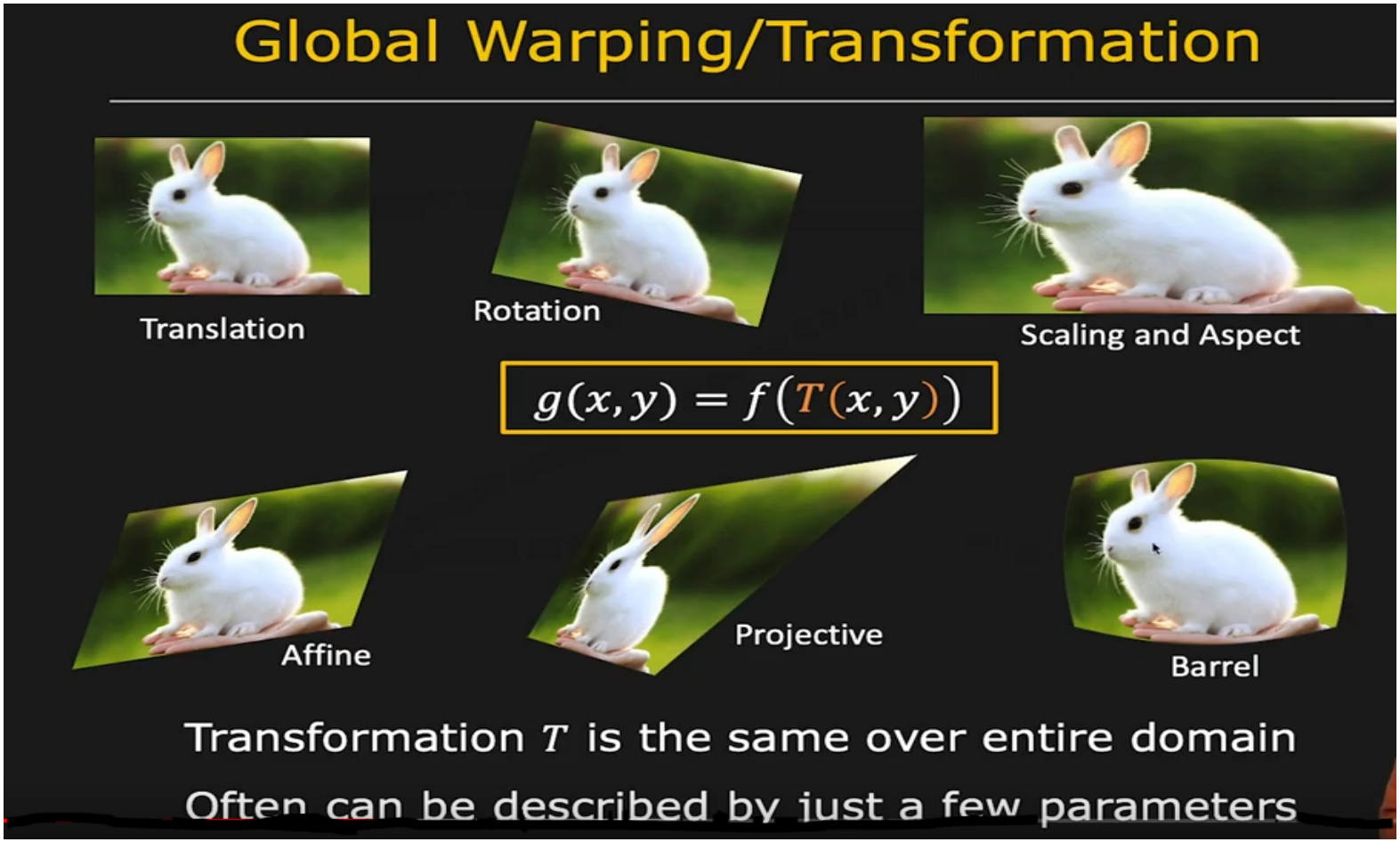
$$g(x,y) = T_r(f(x,y))$$

Image Warping: Change domain (location)

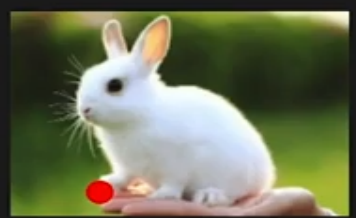
$$g(x,y) = f(T_d(x,y))$$

Transformation T_d is a coordinate changing operator




Global Warping/Transformation
Translation
Rotation
Scaling and Aspect
$g(x,y) = f(T(x,y))$
Affine
Projective
Barrel
Transformation T is the same over entire domain
Often can be described by just a few parameters

2x2 Linear Transformations



T

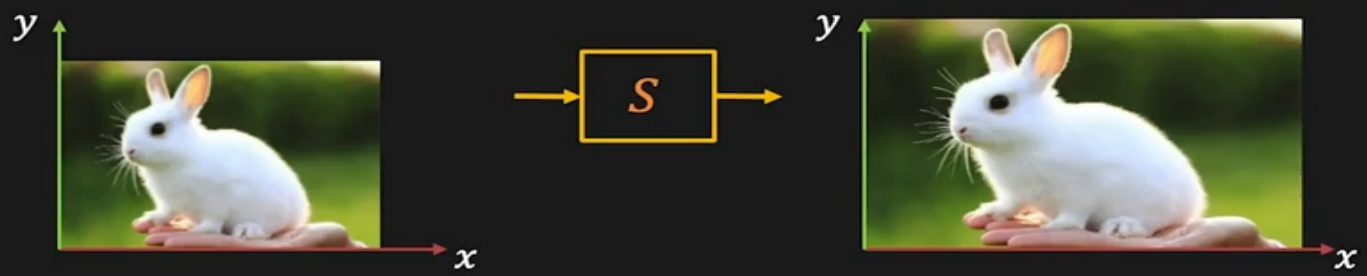
$\mathbf{p}_1 = (x_1, y_1)$

$\mathbf{p}_2 = (x_2, y_2)$

T can be represented by a matrix.

$$\mathbf{p}_2 = T\mathbf{p}_1 \qquad \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = T \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \qquad \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

Scaling (Stretching or Squishing)

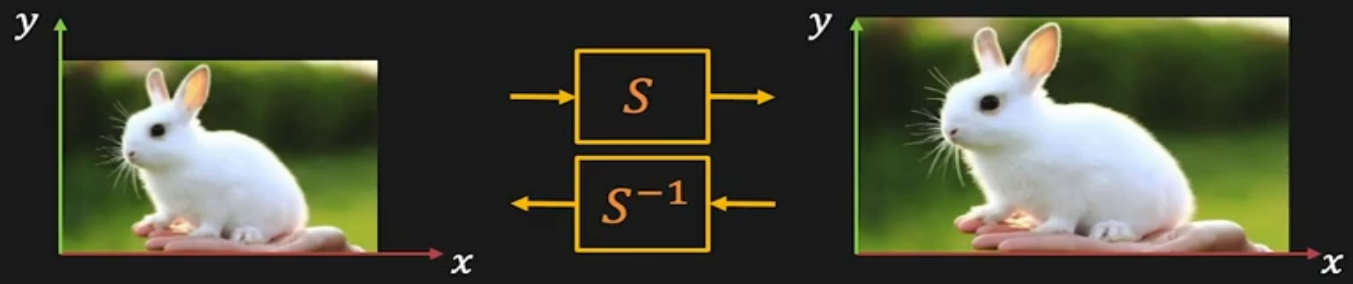


Forward:

$$x_2 = ax_1 \qquad y_2 = by_1$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = S \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

Scaling (Stretching or Squishing)



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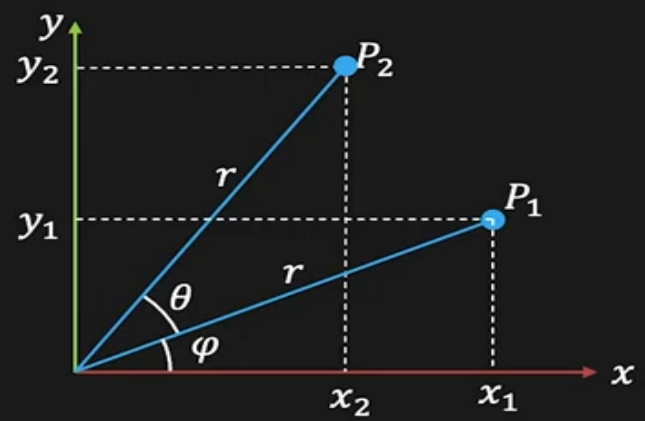
$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = S \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

Inverse:

$$x_1 = \frac{1}{a} x_2 \qquad y_1 = \frac{1}{b} y_2$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = S^{-1} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1/a & 0 \\ 0 & 1/b \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

2D Rotation

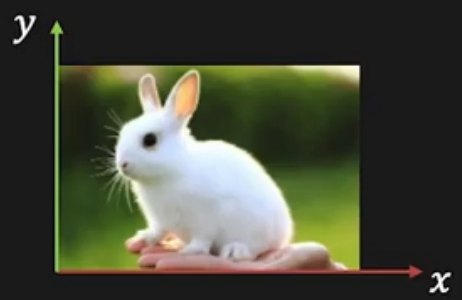


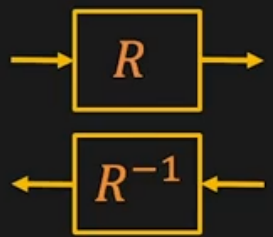
$$x_1 = r\cos(\varphi)$$
$$y_1 = r\sin(\varphi)$$

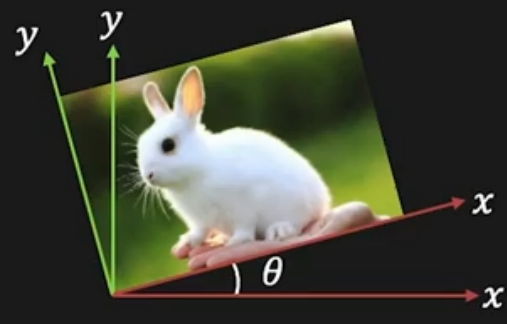
$$x_2 = r\cos(\varphi+\theta)$$
$$x_2 = r\cos\varphi\cos\theta - r\sin\varphi\sin\theta$$
$$x_2 = x_1\cos\theta - y_1\sin\theta$$

$$y_2 = r\sin(\varphi+\theta)$$
$$y_2 = r\cos\varphi\sin\theta + r\sin\varphi\cos\theta$$
$$y_2 = x_1\sin\theta + y_1\cos\theta$$

Rotation







Forward:

$$x_2 = x_1 cos\theta - y_1 sin\theta$$

$$y_2 = x_1 sin\theta + y_1 cos\theta$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = R \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

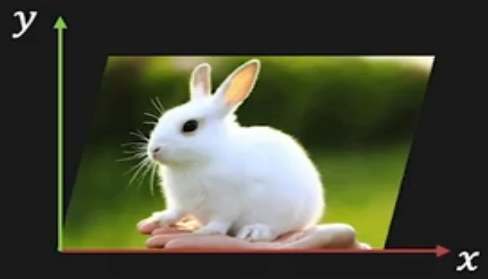
Inverse:

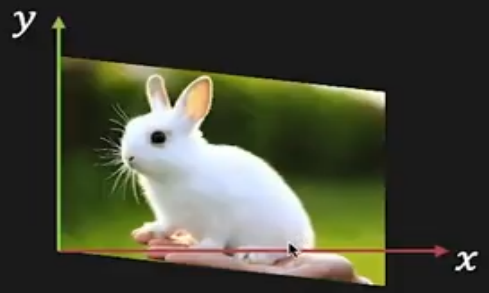
$$x_1 = x_2 cos\theta + y_2 sin\theta$$

$$y_1 = -x_2 sin\theta + y_2 cos\theta$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = R^{-1} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

Skew





Horizontal Skew:

$$x_2 = x_1 + m_x y_1$$

$$y_2 = y_1$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = S_x \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 1 & m_x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

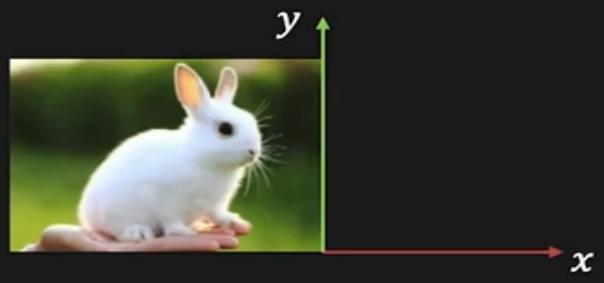
Vertical Skew:

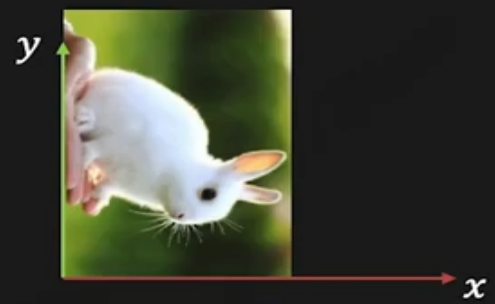
$$x_2 = x_1$$

$$y_2 = m_y x_1 + y_1$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = S_x \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ m_y & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

Mirror





Mirror about Y-axis:

$$x_2 = -x_1$$
$$y_2 = y_1$$

$$M_y = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

Mirror about line $y = x$:

$$x_2 = y_1$$
$$y_2 = x_1$$

$$M_{xy} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

2x2 Matrix Transformations

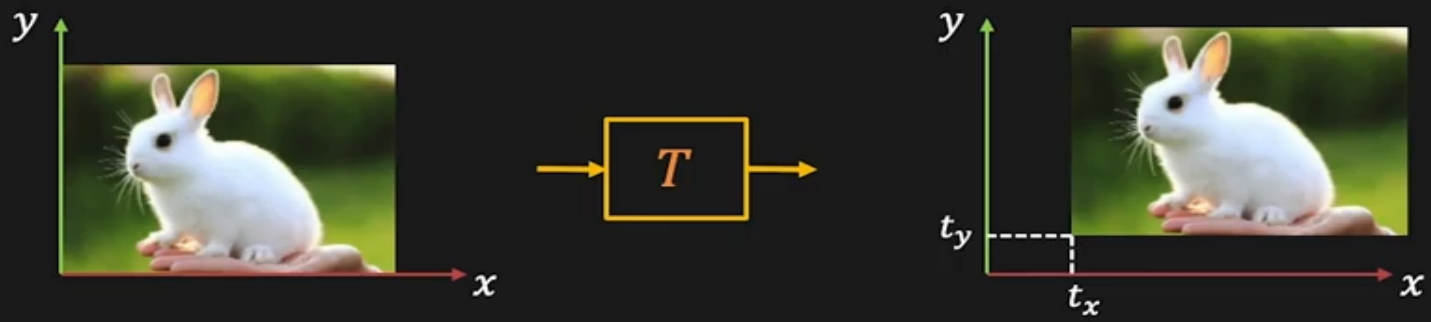
Any transformation of the form:

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

- Origin maps to the origin
- Lines map to lines
- Parallel lines remain parallel
- Closed under composition

$$\left.\begin{aligned} \mathbf{p}_2 &= T_{21}\mathbf{p}_1 \\ \mathbf{p}_3 &= T_{32}\mathbf{p}_2 \\ \mathbf{p}_3 &= T_{31}\mathbf{p}_1 \end{aligned}\right\} \quad \mathbf{p}_3 = T_{32}\mathbf{p}_2 = T_{32}T_{21}\mathbf{p}_1 \Rightarrow T_{31} = T_{32}T_{21}$$

Translation



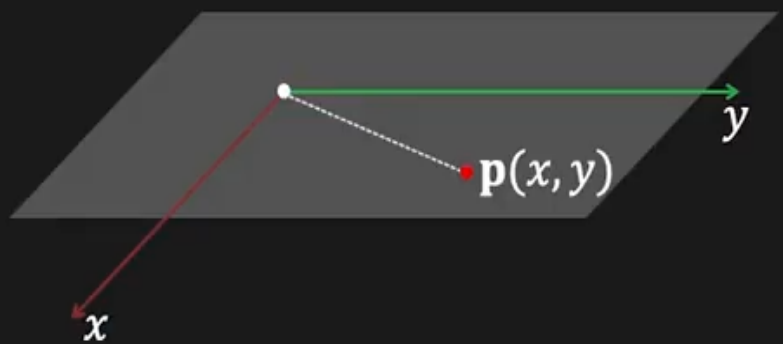
$$x_2 = x_1 + t_x \qquad y_2 = y_1 + t_y$$

Can translation be expressed as a 2x2 matrix? No.

Homogenous Coordinates

The homogenous representation of a 2D point $\mathbf{p} = (x, y)$ is a 3D point $\tilde{\mathbf{p}} = (\tilde{x}, \tilde{y}, \tilde{z})$. The third coordinate $\tilde{z} \neq 0$ is fictitious such that:

$$x = \frac{\tilde{x}}{\tilde{z}} \qquad y = \frac{\tilde{y}}{\tilde{z}}$$

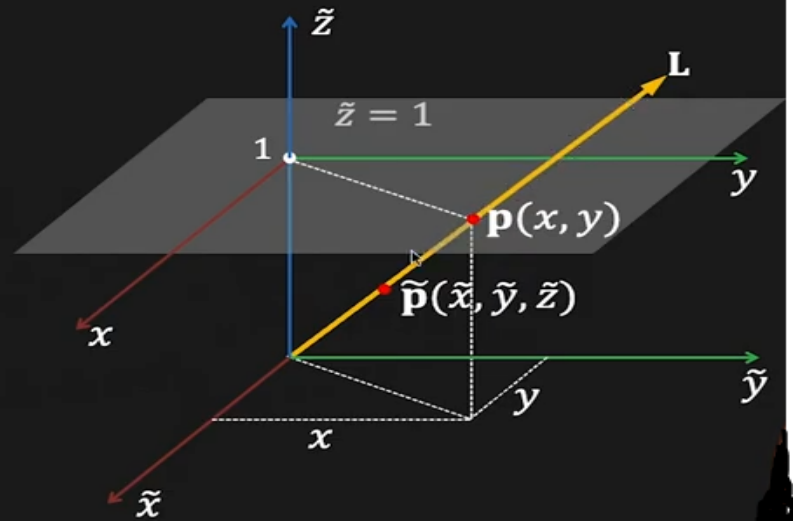


Homogenous Coordinates

The homogenous representation of a 2D point $\mathbf{p} = (x, y)$ is a 3D point $\tilde{\mathbf{p}} = (\tilde{x}, \tilde{y}, \tilde{z})$. The third coordinate $\tilde{z} \neq 0$ is fictitious such that:

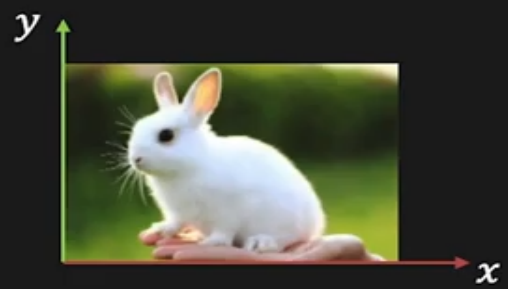
$$x = \frac{\tilde{x}}{\tilde{z}} \qquad y = \frac{\tilde{y}}{\tilde{z}}$$

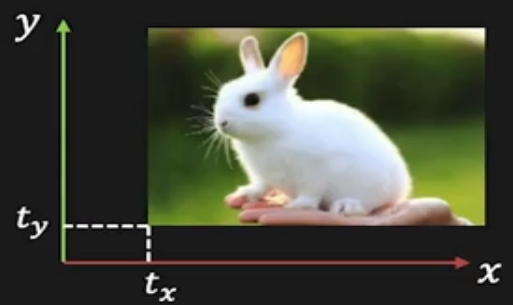
$$\mathbf{p} \equiv \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{z}x \\ \tilde{z}y \\ \tilde{z} \end{bmatrix} \equiv \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{bmatrix} = \tilde{\mathbf{p}}$$



Every point on line L (except origin) represen
the homogenous coordinate of $\mathbf{p}(x, y)$

Translation





$$x_2 = x_1 + t_x \qquad y_2 = y_1 + t_y$$

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

Scaling, Rotation, Skew, Translation

$$\begin{bmatrix}\tilde{x}_2\\ \tilde{y}_2\\ \tilde{z}_2\end{bmatrix} = \begin{bmatrix}s_x & 0 & 0\\ 0 & s_y & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ y_1\\ 1\end{bmatrix}$$

Scaling

$$\begin{bmatrix}\tilde{x}_2\\ \tilde{y}_2\\ \tilde{z}_2\end{bmatrix} = \begin{bmatrix}1 & m_x & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ y_1\\ 1\end{bmatrix}$$

Skew

$$\begin{bmatrix}\tilde{x}_2\\ \tilde{y}_2\\ \tilde{z}_2\end{bmatrix} = \begin{bmatrix}1 & 0 & t_x\\ 0 & 1 & t_y\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ y_1\\ 1\end{bmatrix}$$

Translation

$$\begin{bmatrix}\tilde{x}_2\\ \tilde{y}_2\\ \tilde{z}_2\end{bmatrix} = \begin{bmatrix}cos\theta & -sin\theta & 0\\ sin\theta & cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ y_1\\ 1\end{bmatrix}$$

Rotation

Affine Transformation

Any transformation of the form:

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix}$$

Affine Transformation

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$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix}$$

- Origin does not necessarily map to the origin
- Lines map to lines
- Parallel lines remain parallel
- Closed under composition

Projective Transformation

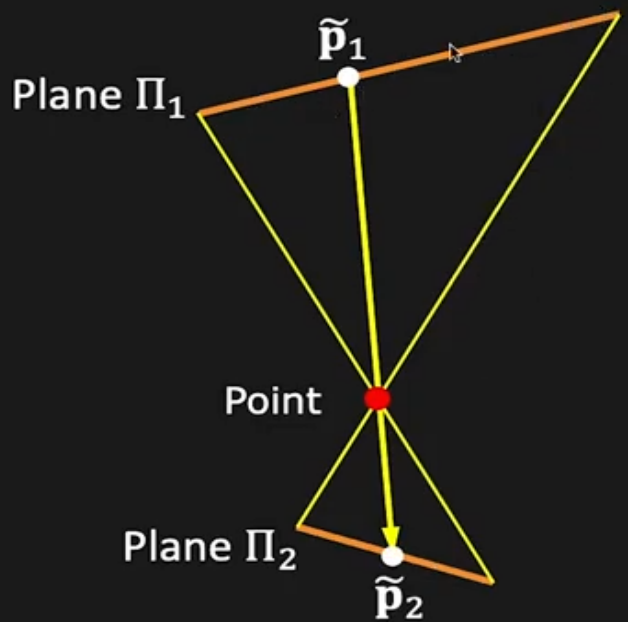
Any transformation of the form:

$$\begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix} \qquad \tilde{\mathbf{p}}_2 = H\tilde{\mathbf{p}}_1$$

Also called Homography

Projective Transformation

Mapping of one plane to another through a point



$$\tilde{\mathbf{p}}_2 = H\tilde{\mathbf{p}}_1$$

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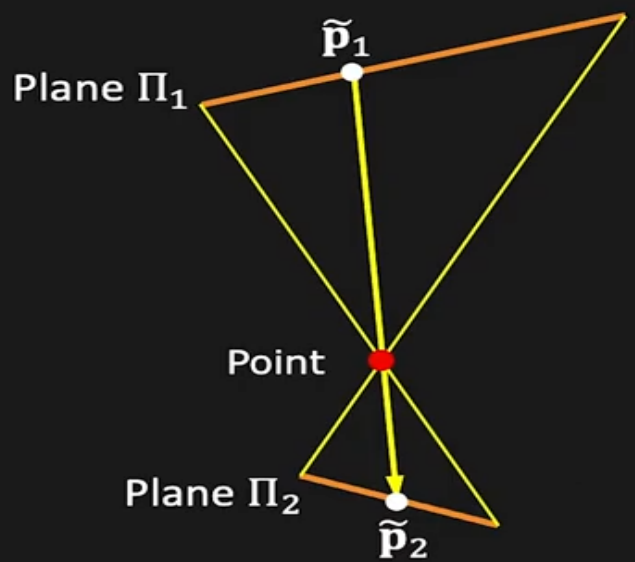
Projective Transformation

Mapping of one plane to another through a point



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Same as imaging a plane through a pinhole

Projective Transformation

Homography can only be defined up to a scale.

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} \equiv k \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix}$$

Projective Transformation

Homography can only be defined up to a scale.

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} \equiv k \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix}$$

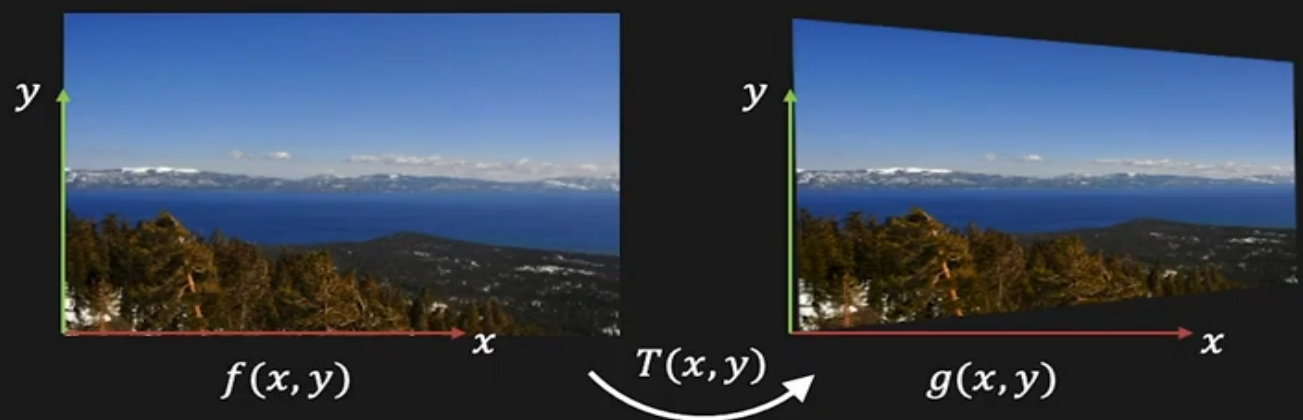
If we fix scale such that $\sqrt{\Sigma(h_{ij})^2} = 1$ then 8 free parameters

- Origin does not necessarily map to the origin
- Lines map to lines
- Parallel lines do not necessarily remain parallel
- Closed under composition

Warping Images

Given a transformation T and a image $f(x,y)$, compute the transformed image $g(x,y)$

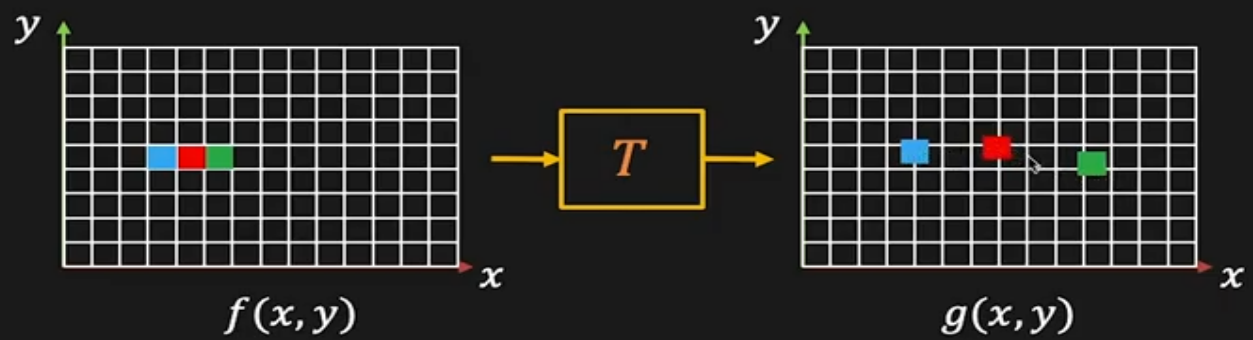
$$g(x,y) = f(T(x,y))$$



Forward Warping

Send each pixel (x, y) in $f(x, y)$ to its corresponding location $T(x, y)$ in $g(x, y)$

$$g(x, y) = f(T(x, y))$$



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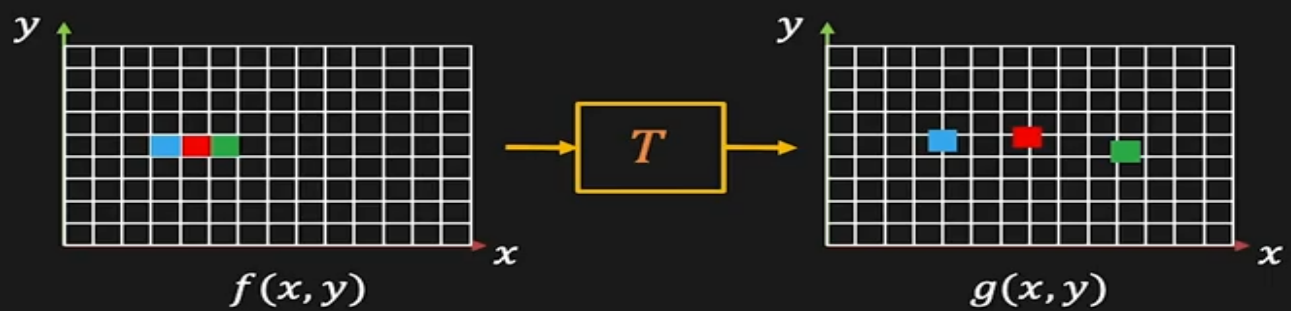
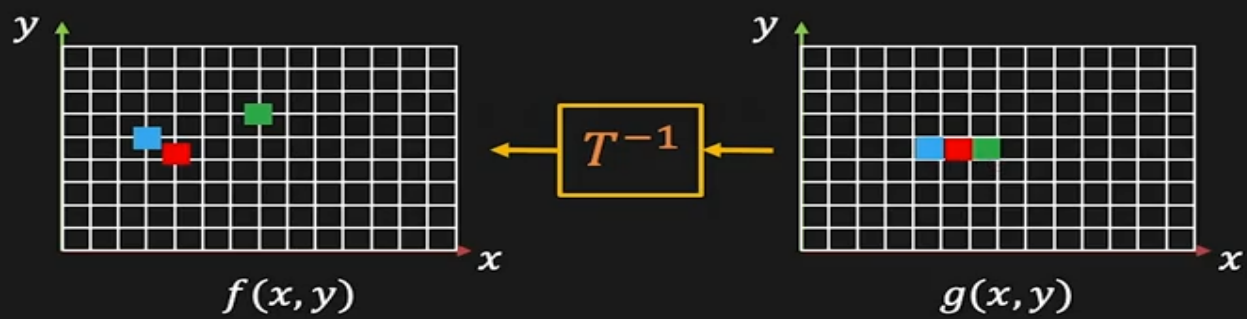


What if pixel lands in between pixels?
What if not all pixels in $g(x, y)$ are filled?

Backward Warping

Get each pixel (x, y) in $g(x, y)$ from its corresponding location $T^{-1}(x, y)$ in $f(x, y)$

$$g(x, y) = f(T(x, y))$$



What if pixel lands between pixels?
Use Nearest Neighbor or Interpolate

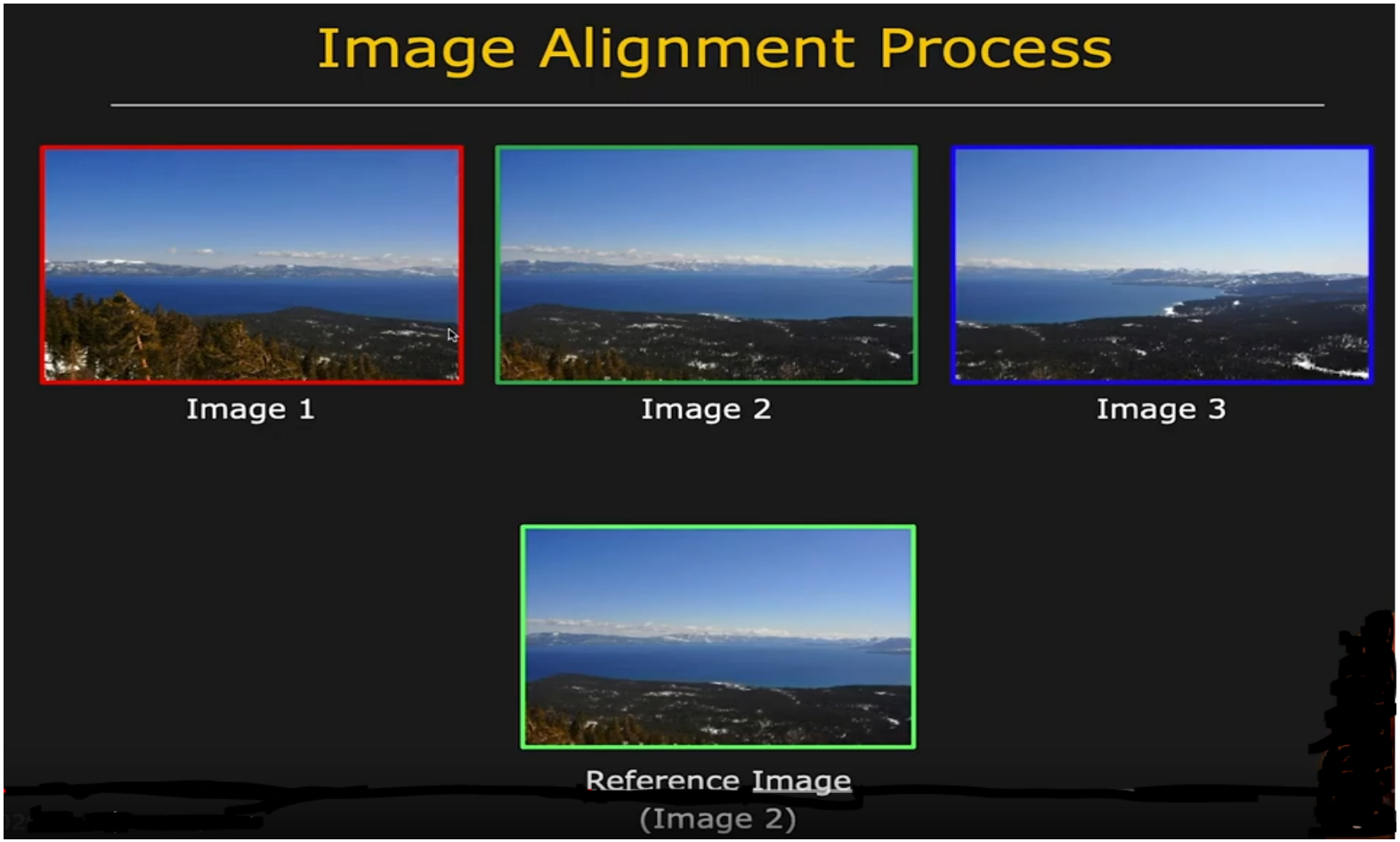


Image Alignment Process
Image 1
Image 2
Image 3
Reference Image
(Image 2)

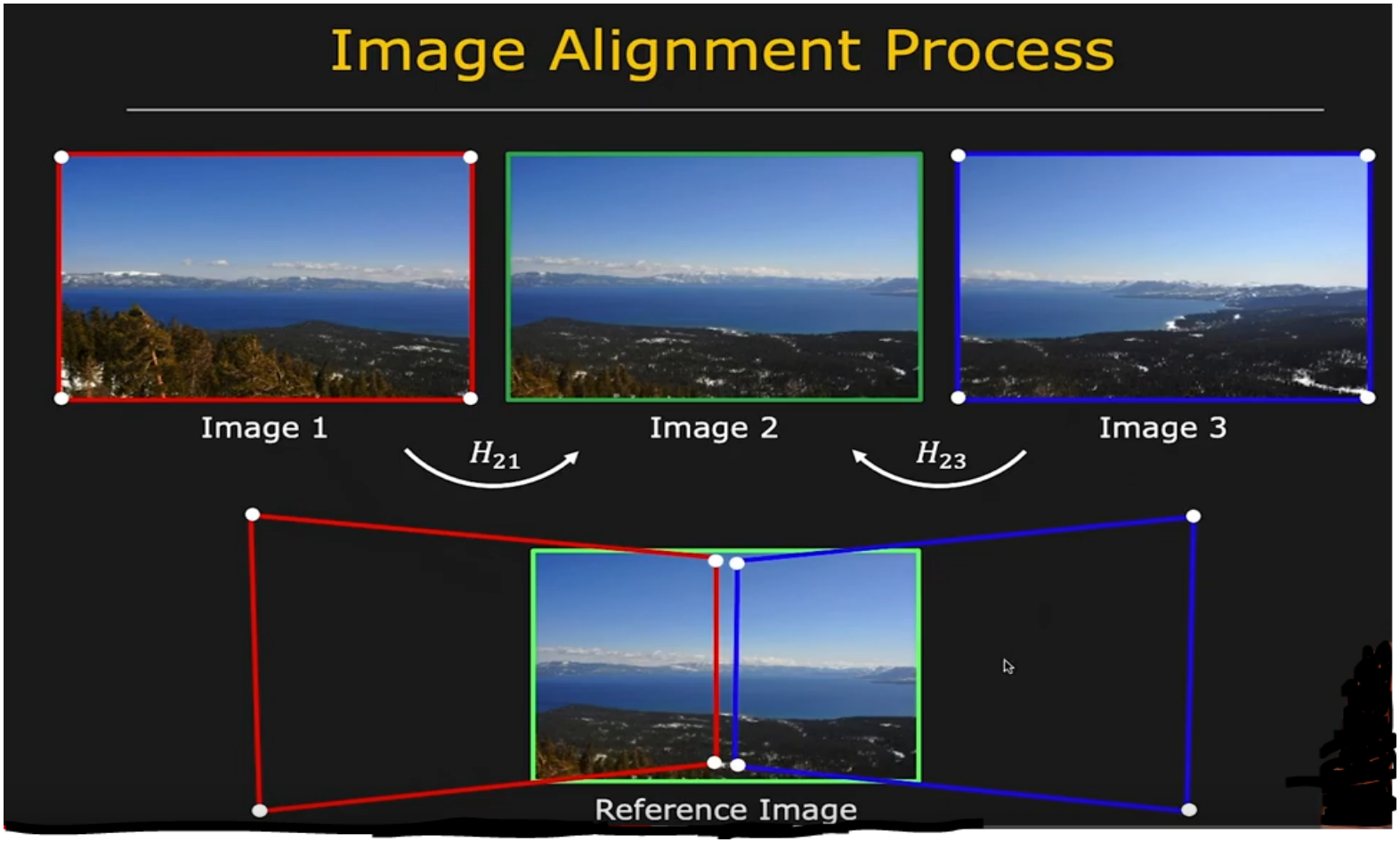


Image Alignment Process
Image 1
Image 2
Image 3
H_{21}
H_{23}
Reference Image

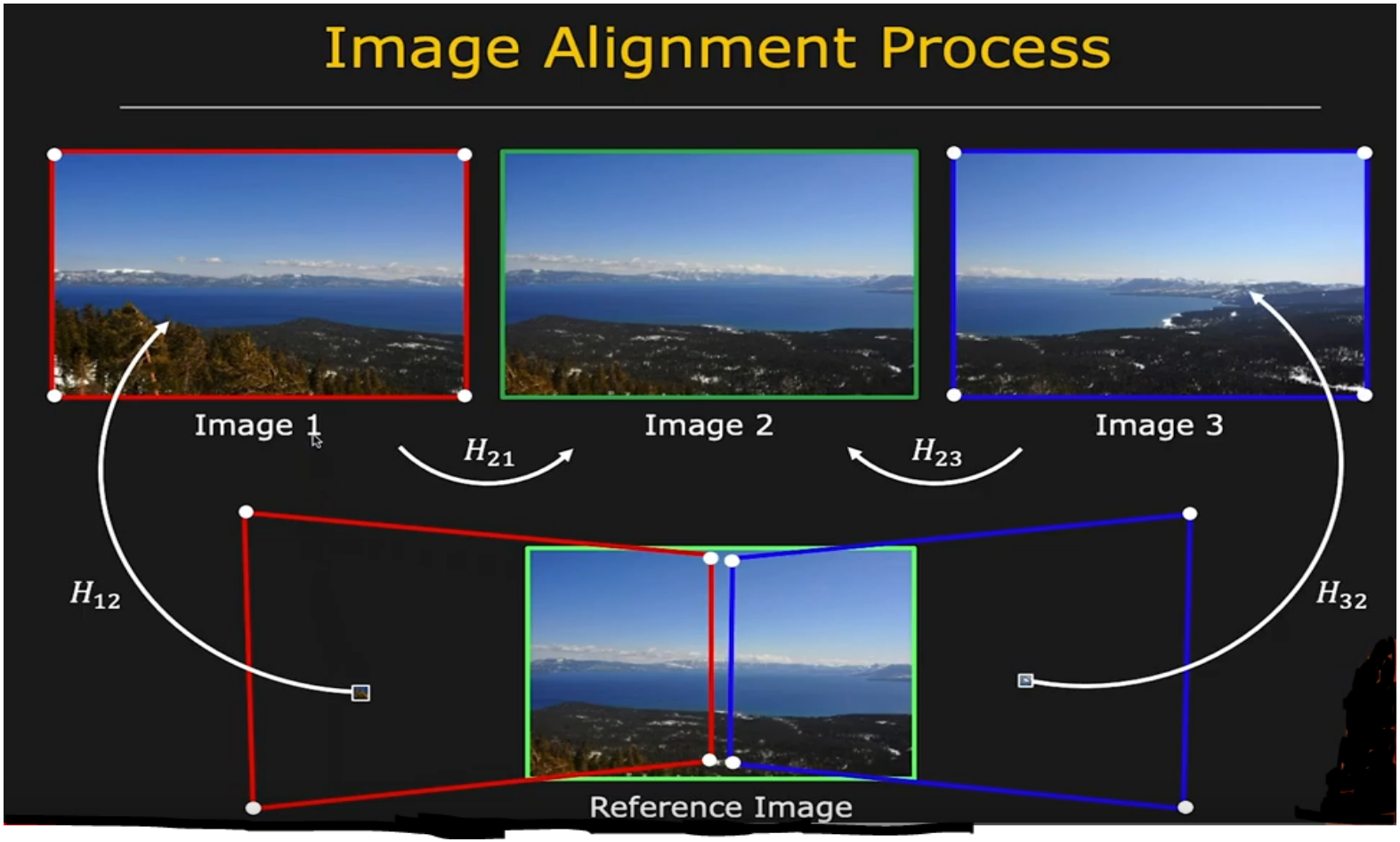


Image Alignment Process
Image 1
Image 2
Image 3
H_{21}
H_{23}
H_{12}
H_{32}
Reference Image

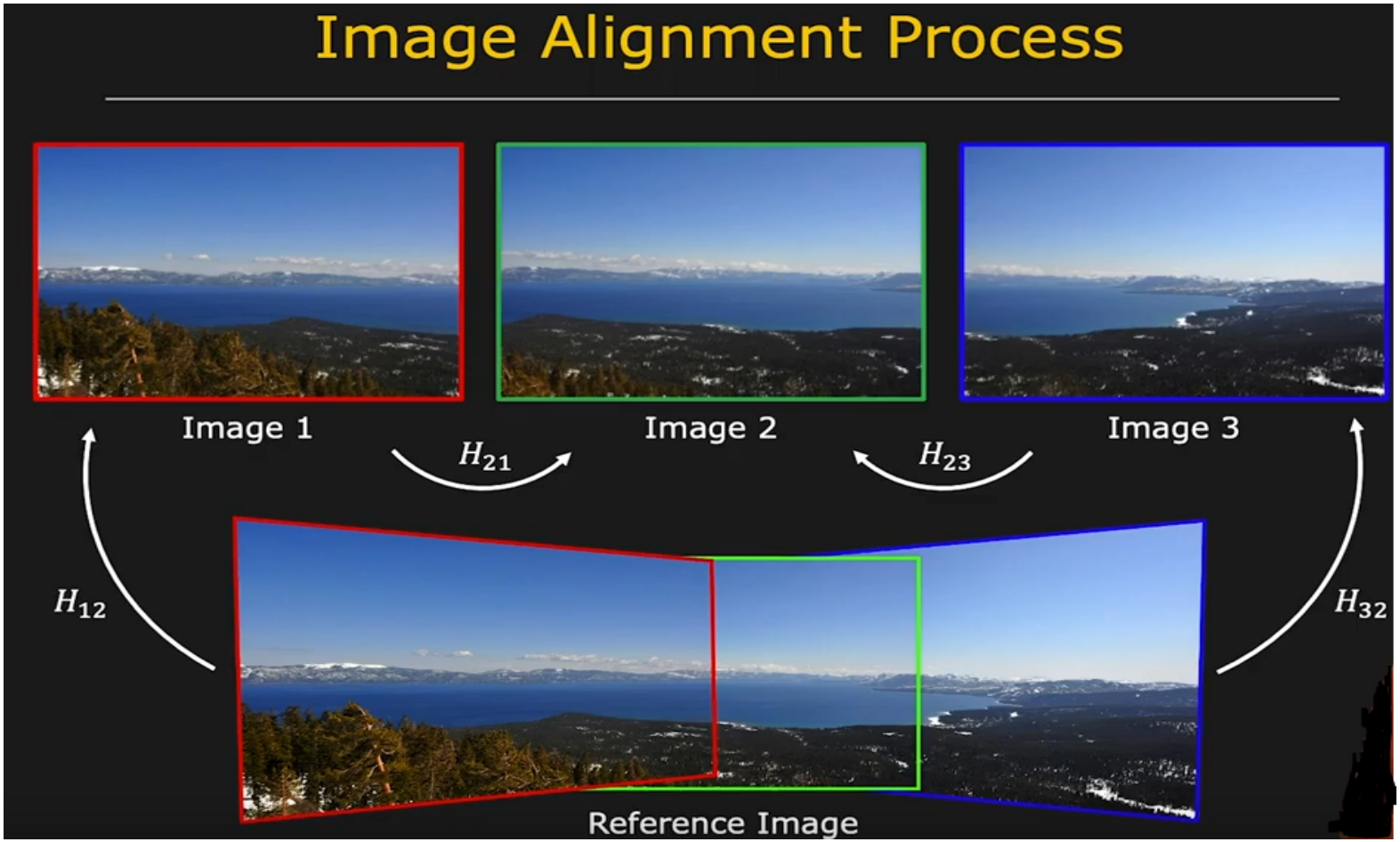
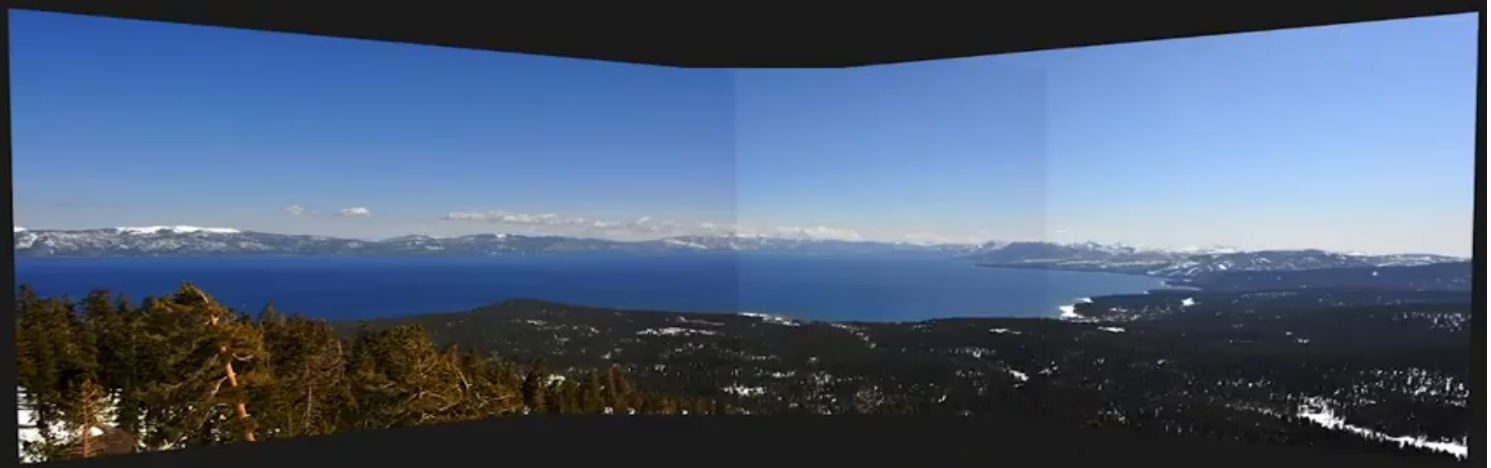


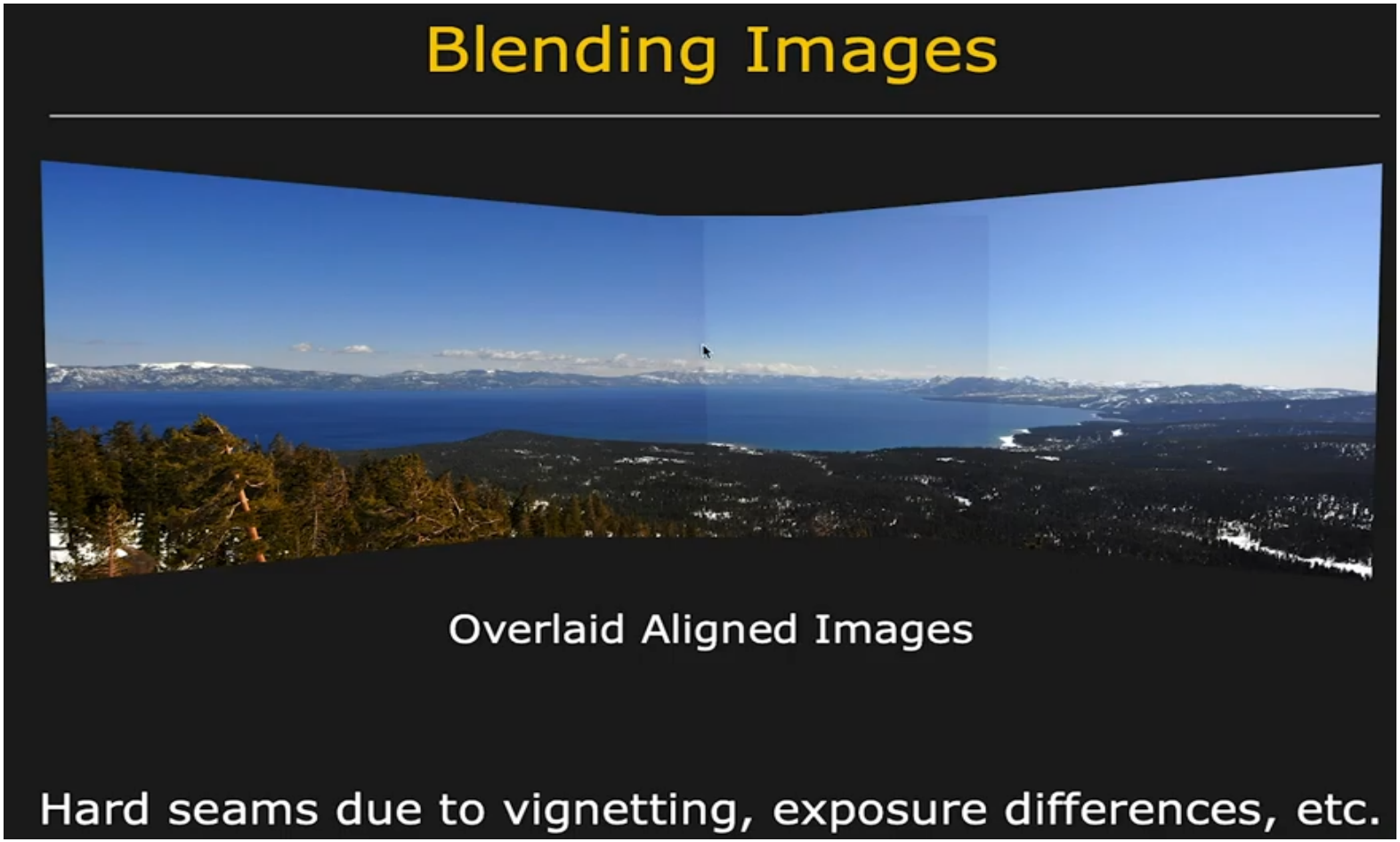
Image Alignment Process
Image 1
Image 2
Image 3
H_{21}
H_{23}
H_{12}
H_{32}
Reference Image

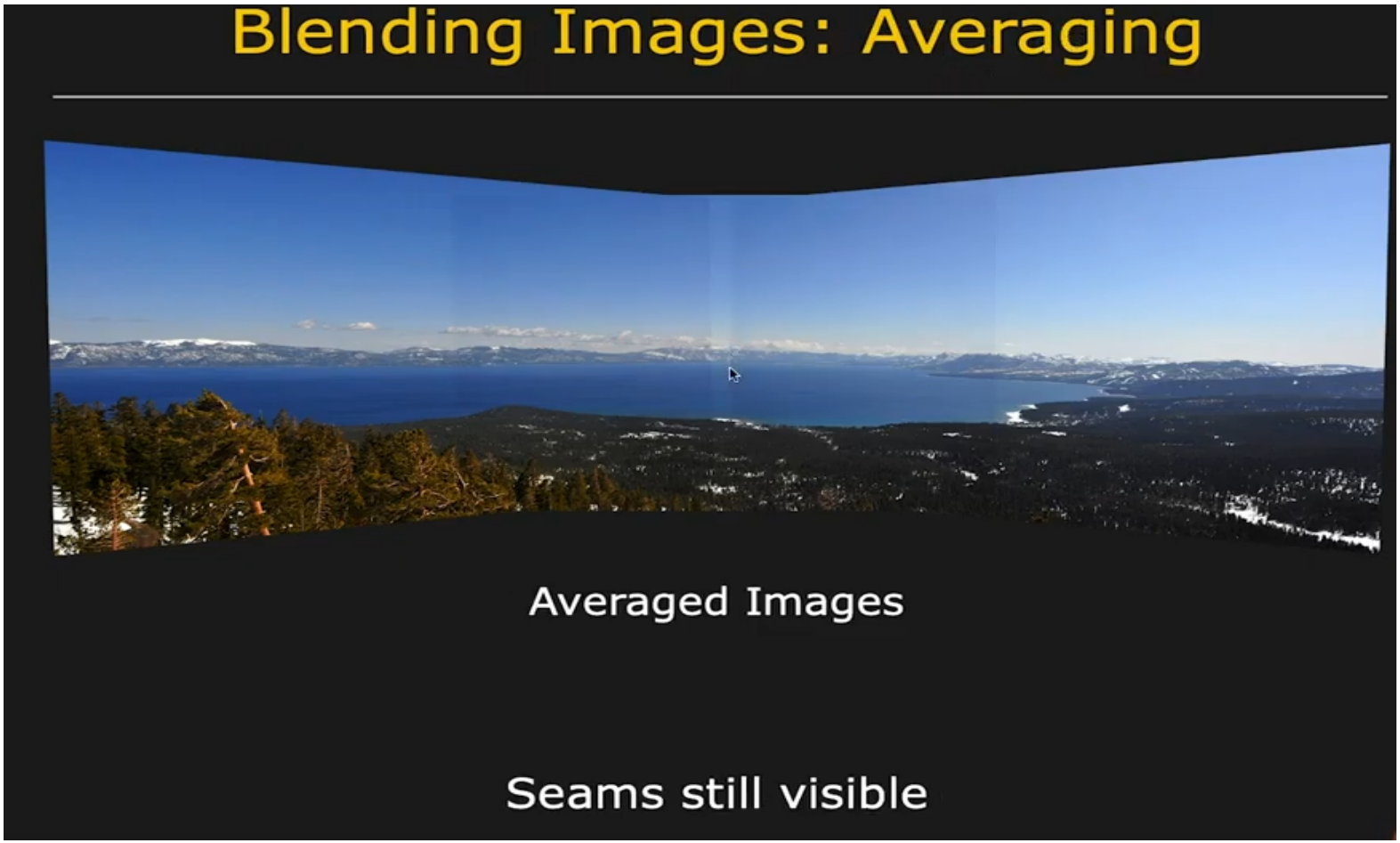
Blending Images



Overlaid Aligned Images

Hard seams due to vignetting, exposure differences, etc.


Blending Images
Overlaid Aligned Images
Hard seams due to vignetting, exposure differences, etc.


Blending Images: Averaging
Averaged Images
Seams still visible

Blending Images

Say we want to blend images I_1 and I_2 at the center

Image I_1

Image I_2

Blending Images

Say we want to blend images I_1 and I_2 at the center

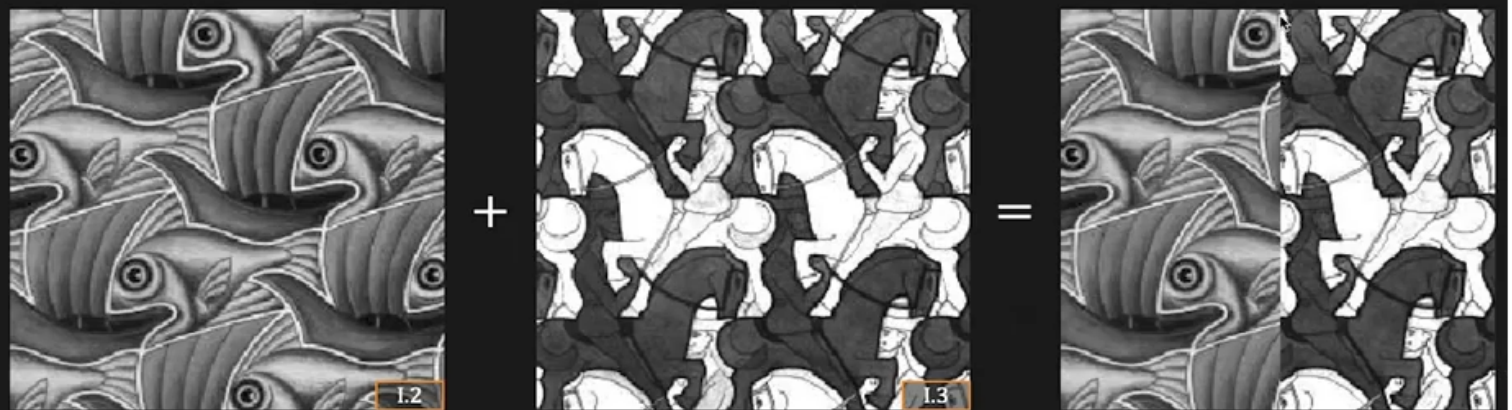


Image I_1 | Image I_2 | Hard overlay

Weight w_1 | Weight w_2

Blending Images

Say we want to blend images I_1 and I_2 at the center

Image I_1 + Image I_2

Weight w_1

Weight w_2

$$I_{blend} = \frac{w_1 I_1 + w_2 I_2}{w_1 + w_2}$$

Blending Images

Say we want to blend images I_1 and I_2 at the center

Image I_1 + Image I_2 = Blended Image I_{blend}

Weight w_1

Weight w_2

$$I_{blend} = \frac{w_1 I_1 + w_2 I_2}{w_1 + w_2}$$

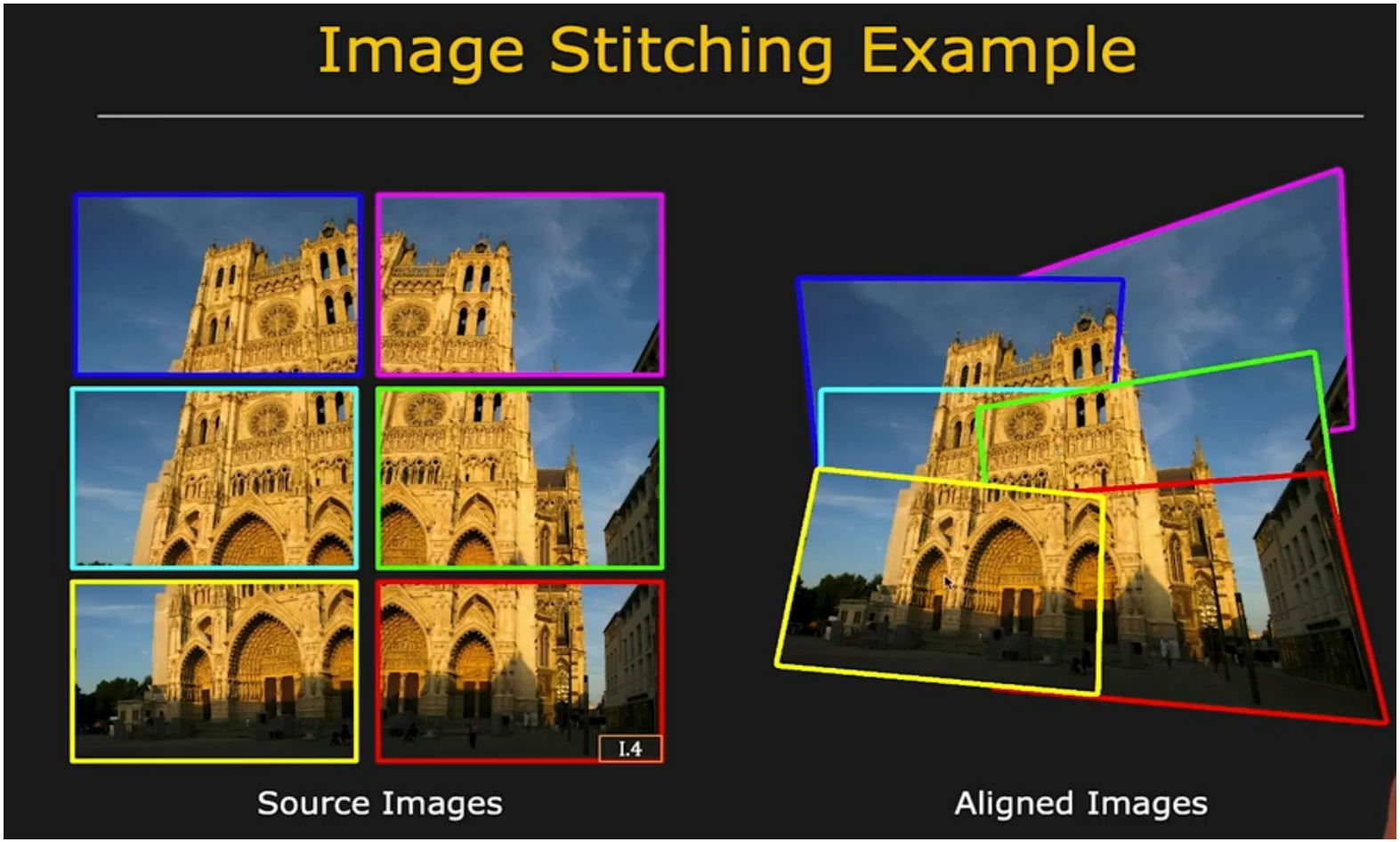


Image Stitching Example
I.4
Source Images
Aligned Images