Computer Vision-IT416

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Image segmentation and clustering methods

- Image segmentation may be viewed as a clustering approach in which the pixels, that are satisfying a criterion, are grouped into a cluster while dissatisfying pixels are placed in different groups.

Broad Classification of Image Segmentation

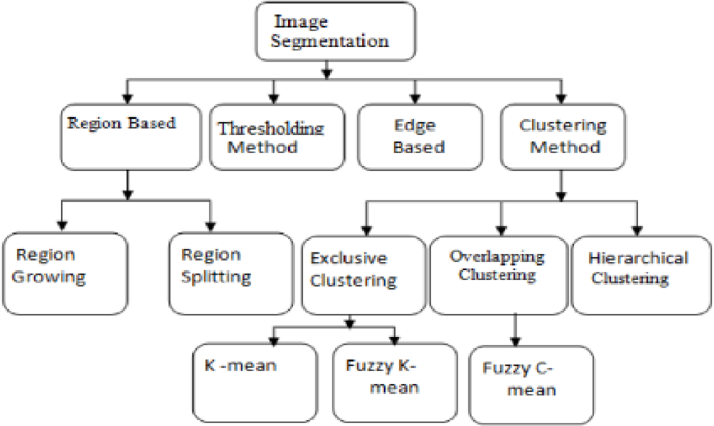

Image segmentation and clustering methods

- Image segmentation is an essential phase of computer vision in which useful information is extracted from an image that can range from finding objects while moving across a room to detect abnormalities in a medical image.
- As image pixels are generally unlabelled, the commonly used approach for the same is clustering.
- Two main clustering methods have been surveyed, namely hierarchical and partitional based clustering methods.
- As partitional clustering is computationally better, further, the partitional based clustering methods into three categories, namely K-means based methods, histogram-based methods, and meta-heuristic based methods.

- "A picture is worth a thousand words" is a famous idiom which signifies that processing an image may relieve more information than processing the textual data.
- In computer vision, image segmentation is the prime research area which corresponds to partitioning of an image into its constituent objects or region of interests (ROI)
- Generally, it assembles the image pixels into similar regions. It is a pre-processing phase of many image-based applications like biometric identification, medical imaging, object detection and classification, and pattern recognition. Some of the prominent applications are as follows.
- Content-based image retrieval: It corresponds to the searching of query-relevant digital images from large databases. The retrieval results are obtained as per the contents of the query image. To extract the contents from an image, image segmentation is performed.

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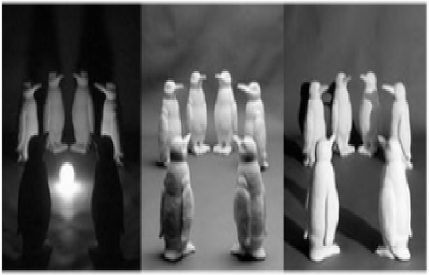
- Machine vision: It is the image-based technology for robotic inspection and analysis, especially at the industrial level. Here, segmentation extracts the information from the captured image related to a machine or processed material.
- Medical imaging: Today, image segmentation helps medical science in a number of ways from medical diagnosis to medical procedures. Some of the examples include segmentation of tumors for locating them, segmenting tissue to measure the corresponding volumes, and segmentation of cells for performing various digital pathological tasks like cell count, nuclei classification and many others.

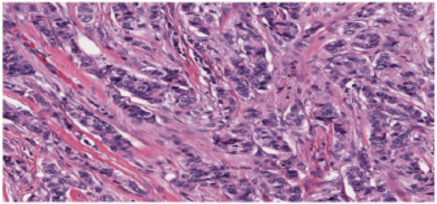
- Object recognition and detection: Object recognition and detection is an important application of computer vision. Here, an object may be referred to as a pedestrian or a face or some aerial objects like roads, forests, crops, etc. This application is indispensable to image segmentation as the extraction of the indented object from the image is priorly required.
- Video surveillance: In this, the video camera captures the movements of the region of interests and analysis them to perform an indented task such as identification of the action being performed in the captured video or controlling the traffic movement, counting the number of objects and many more. To perform the analysis, segmentation of the region of interest is foremost required.

- Though segmenting an image into the constituent ROI may end up as a trivial task for humans, it is relatively complex from the perspective of the computer vision. There are number of challenges which may affects the performance of an image segmentation method.
- Illumination variation: It is a fundamental problem in image segmentation and has severe effects on pixels. This variation occurs due to the different lighting conditions during the image capturing.



- Intra-class variation: One of the major issues in this field is the existence of the region of interest in a number of different forms or appearances. Figure 1b depicts an example of chairs that are shown in different shapes, each having a different appearance. Such intra-class variation often makes the segmentation procedure difficult. Thus, a segmentation method should be invariant to such kind of variations.

- Background complexity: Image with a complex background is a major challenge. Segmenting an image as the region of interests may mingle with the complex environment and constraints. The dark blue color regions in the image represent the nuclei region which is generally defined as the region of interests in histopathological applications like nuclei count or cancer detection. It can be observed that the background is too complex due to which the nuclei regions do not have clearly defined boundaries. Therefore, such background complexities degrade the performance of segmentation methods.



Clustering Method

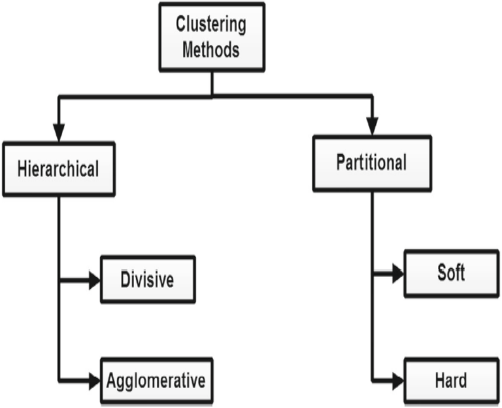
- To mitigate the challenges and learning from unlabelled data, the common approach is to group the data based on certain similarity or dissimilarity measures followed by labelling of each group. This approach of grouping the data is generally termed as clustering.
- The main objective of a clustering method is to classify the unlabelled pixels into homogeneous groups that have maximum resemblance, i.e. to achieve maximum similarity within the clusters and minimum dissimilarity among the clusters.

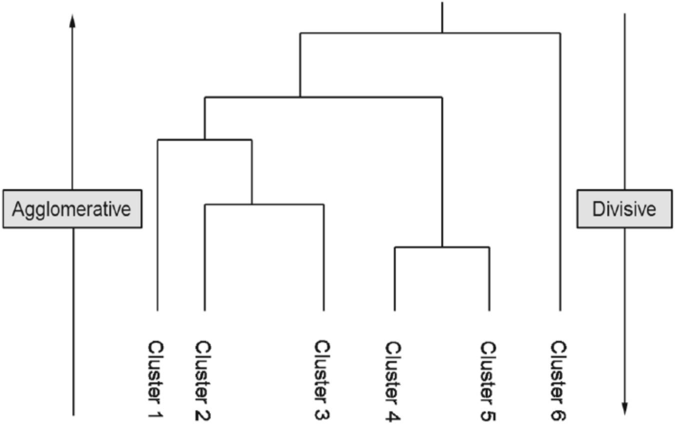
Mathematically, the clustering procedure on an image (X) of size ($m \times n$), defined over d-dimensions, generates K clusters $\{C_1, C_2, \cdots, C_K\}$ subject to the following conditions:

- $C_i \neq \emptyset$, for $i = 1, 2, \cdots, K$
- $C_i \cap C_j = \emptyset$, for i *and* $j = 1, 2, \cdots, K$ and $i \neq j$
- $\cup_{i=1}^{K} C_i = X$

- The first condition ensures that there will be at least one pixel in every formed cluster.
- The next condition implies that all the formed clusters will be mutually exclusive, i.e. a pixel will not be assigned to two clusters.
- The last condition states that the data values assigned to all the clusters will represent the complete image.

- In hierarchical clustering, the grouping of data is performed at different similarity levels which is schematized by a tree-like structure termed as a dendrogram




Agglomerative
Divisive
Cluster 1
Cluster 2
Cluster 3
Cluster 4
Cluster 5
Cluster 6

- In divisive clustering, recursive hierarchical data splitting is performed in a top-down fashion to generate the clusters.
- All the data items belong to a single cluster initially. This single cluster is further split into smaller clusters until a termination criteria is satisfied or until each data item forms its own cluster.
- On the other side, the agglomerative clustering is performed in the bottom-up fashion where data points are merged hierarchically to produce clusters.
- Initially, each data item defines itself as a cluster which is further merged into bigger clusters, until a termination criteria is meet or until a single cluster is formed consisting of all the data items.

- In general, divisive clustering is more complex than the agglomerative approach, as it partitions the data until each cluster contains a single data item.
- The divisive clustering will be computationally efficient if the each cluster is not partitioned to individual data leaves.
- The time complexity of a naive agglomerative clustering is $O(n^3)$ which can be reduced to $O(n^2)$ using optimization algorithms.
- On the contrary, the time complexity of divisive clustering is $\Omega(n^2)$
- Moreover, divisive clustering is also more accurate since it considers the global distribution of data while partitioning data in top-level partitioning.

- The pseudocode of the divisive and agglomerative clustering approaches are presented in Algorithms 1 and 2 respectively.

Algorithm 1 Agglomerative clustering.

Input: Initialize the number of clusters to be formed.
Output: Clustered data.
Consider each data point as a cluster;
while clustering condition is not satisfied **do**
    Perform merging of two clusters having minimum inter-cluster distance;
end while

Algorithm 2 Divisive clustering.

Input: Initialize the number of clusters to be formed.
Output: Clustered data.
Consider all data points as a single cluster;
while clustering condition is not satisfied **do**
    Divide the cluster into two clusters resulting in the largest inter-cluster distance;
end while

Partitional clustering

- The partitional clustering is relatively popular and preferred over the hierarchical cluster- ing, especially for a large dataset, due to its computational efficiency .
- In this clustering approach, the notion of similarity is used as the measurement parameter.
- Generally, partitional clustering groups the data items into clusters according to some objective function such that data items in a cluster are more similar than the data items in the other clusters.
- To achieve this, the similarity of each data item is measured with every cluster. Moreover, in partitional clustering, the general notion of the objective function is the minimization of the within-cluster similarity criteria which is usually computed by using Euclidean distance.
- The objective function expresses the goodness of each formed cluster and returns the best representation from the generated clusters.

Partitional clustering methods for image segmentation

Sub-categories	Methods	Remarks
Soft	FCM, FCS,	Grouping based on objective function;
	FLAME	Preferred for large datasets.
Hard	Kmeans-based	Low time complexity;
	histogram-based	Number of clusters need to be known priorly;
	Metaheuristic-based	Dependance over the initial clusters.

- Soft clustering methods assign each data to either two or more clusters with a degree of belongingness (or membership) iteratively.
- The degree of belongingness illustrates the level of association among data more reasonably. The belongingness of a data item with a cluster is a continuous value in the interval [0, 1] and depends upon the objective function.

- Particularly, FCM is the most widely used and popular method of this approach. It returns a set of K fuzzy clusters by minimizing the objective function defined in (1)

$$\sum_{i=1}^{N}\sum_{k=1}^{K}\mu_{ik}^{m} \parallel x_i - v_k \parallel^2, m \geq 1. \tag{1}$$

where, $\mu_{ik} \in [0, 1]$ and corresponds to membership degree for i^{th} pixel with k^{th} cluster. Equation (1) is optimized iteratively by updating μ_{ik} and v_k according to (2) and (3) respectively.

$$\mu_{ik} = \frac{1}{\sum_{j=1}^{K}\left(\frac{\|x_i - v_k\|}{\|x_i - v_j\|}\right)^{\frac{2}{m-1}}} \tag{2}$$

$$v_k = \frac{\sum_{i=1}^{N}\mu_{ik}^{m}x_i}{\sum_{i=1}^{N}\mu_{ik}^{m}} \tag{3}$$

- Normally, the exponent of the fuzzy partition matrix (m) is kept as $m \geq 1$. This regulates the number of pixels that can have membership with more than one cluster.

The pseudo-code of FCM is presented in Algorithm 3

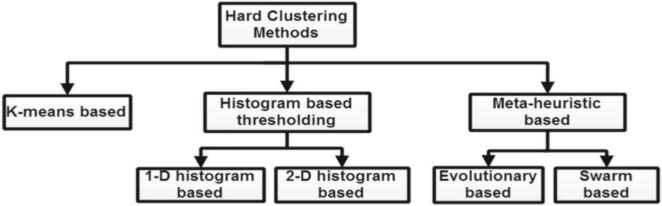
Algorithm 3 Fuzzy C-Means (FCM).

Input: Initialize the number of clusters to be formed.
Output: Clustered data.
Randomly initialize the cluster centroids;
while clustering condition is not satisfied or centroids do not change **do**
    Use (2) to compute the fuzzy partition matrix (U);
    Compute the objective function;
    Update the cluster centroids by (3);
end while

- The inclusion of degree of belongingness benefits soft clustering methods in a number of ways like, relatively high clustering accuracy, faster in generating approximate solutions,and efficient in handling incomplete or heterogeneous data.

Hard clustering methods

- Hard clustering methods iteratively partition the data into disjoint clusters according to the objective function. Generally, the objective function is the sum of squared Euclidean distance between data and associated centroid which is to be minimized.
- Usually, the centre of the clustered data is considered as the centroid of the clusters in these methods. Moreover, in contrast to soft clustering, hard clustering assigns data to a single cluster only i.e., each data will have the degree of belongingness as either 0 or 1.
- Classification of hard clustering methods



Kmeans-based methods

- In Kmeans-based methods, the cluster centroid is updated by taking the mean of all the data items assigned to the corresponding cluster.
- This is iteratively continued until some defined convergence criterion is met.
- Although methods of this category have merits like relatively low time complexity, simple in nature, and guaranteed convergence, there are number of limitations which need to be handled.
- These limitations include the number of clusters to be formed needs to be known priorly, solution quality depends on the initial clusters and number of formed clusters, not appropriate on data having non-convex distribution, follow hill-climbing strategy and hence usually traps into local optima, and relatively sensitive to outliers, noise, and initialization phase.

K-means: K-means [48] partitions a set of data points, $X = \{x_1, \cdots, x_n\}$, into a number of clusters (k). It performs partition based on the similarity criteria which is usually the sum of squared error defined in (4).

$$J = \sum_{i=1}^{k} sum_{x_j \in X} ||x_j - m_i||^2 \tag{4}$$

where, m_i is the centroid of cluster i which is collectively represented as $M = \{m_1, \cdots, m_k\}$ for corresponding clusters, $C = \{c_1, \cdots, c_k\}$. This method iteratively minimizes the criterion function J. Further, the formed clusters C and corresponding centroids M are updated as given by (5) and (6) respectively.

$$x_i \in c_l, \ if \ l = argmin_{l=1}^{k} ||x_j - m_i||^2 \tag{5}$$

$$m_i = \frac{\sum_{x_i \in c_l} x_i}{|c_l|} \tag{6}$$

for $1 \leq i \leq N$ and $1 \leq l \leq k$.

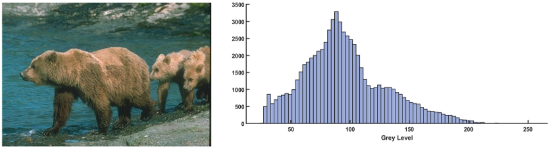
- K-means method has the time complexity of O(nkt) where, n, k, and t correspond to the number of data items, number of clusters to be formed, and maximum iterations respectively. However, this method is biased towards initial cluster centroids and usually traps into local minima. Moreover, solutions vary with the number of clusters.
- The pseudo-code of the K-means method is presented in Algorithm 4

Algorithm 4 K-means.

```
Input: Initialize the number of clusters to be formed.
Output: Clustered data.
Randomly select distinct data points as initial cluster centroids;
while clustering condition is not satisfied or centroids do not change do
    Compute the objective function, defined in (4);
    Assign each data point to the cluster which is closest;
    Update the cluster centroids;
end while
```

Histogram-based methods

- This category includes methods which perform segmentation by constructing histogram according to the frequency of the intensity values in an image.
- These methods identify a set of optimal threshold values which partitions the histogram. This partitioning results in grouping of intensity values in different clustering. Therefore, histogram-based segmentation methods can be considered as clustering approach.
- Generally, a single feature of an image is considered, usually the intensity, to define a 1-dimensional (1D) histogram.



a b

Fig. 6 1D view of grey level histogram of an image taken from BSDS300 [49] a Image b 1D histogram

1D histogram-based segmentation method

To understand the mathematical formulation of 1D histogram-based segmentation method, consider an RGB image of N number of pixels each having intensity from $\{0, 1, 2, \cdots, L-1\}$ in each plane. Suppose, n_i corresponds to the number of pixels for i^{th} intensity level. Therefore, the probability distribution (p_i) of i^{th} intensity level in the image is defined as the probability of occurrence of the image pixels with i^{th} intensity level which is formulated as (7).

$$p_i = \frac{n_i}{N},\ 0 \leq i \leq L-1 \tag{7}$$

The mean intensity of an image plane is determined by (8).

$$\mu = \sum_{i=1}^{L} i p_i \tag{8}$$

To group the image pixels into n clusters $\{C_1, C_2, \cdots, C_n\}$, $n-1$ thresholds, $(t_1, t_2, \cdots, t_{n-1})$, are required which are represented as (9).

$$v(x, y) = \begin{cases} 0, & v(x, y) \leq t_1 \\ \frac{t_1+t_2}{2}, & t_1 < v(x, y) \leq t_2 \\ & . \\ & . \\ & . \\ \frac{t_{n-2}+t_{n-1}}{2}, & t_{n-2} < v(x, y) \leq t_{n-1} \\ L-1, & v(x, y) > t_{n-1} \end{cases} \tag{9}$$

where $v(x, y)$ corresponds to pixel intensity at (x, y) location of a $M \times N$ image. The partition $C_j, 1 \leq j \leq n$ consists of pixels with intensity value more than t_{j-1} and less than equals to t_j. The frequency of cluster C_j for each plane is computed as (10).

$$w_j = \sum_{i=t_{j-1}+1}^{t_j} p_i \tag{10}$$

The mean for the cluster C_j can be calculated by (11) and the inter-class variance is formulated in (12).

$$\mu_j = \sum_{i=t_{j-1}+1}^{t_j} i p_i / w_j \tag{11}$$

$$\sigma^2 = \sum_{j=1}^{n} w_j (\mu_j - \mu)^2 \tag{12}$$

To group the pixels into clusters based on intensity, the maximization of inter-class variance [(12)] is considered. Therefore, the objective function tries to maximize the fitness function, defined in (13):

$$\phi = \max_{1<t_1<\cdots<t_{n-1}<L} \{\sigma^2(t)\} \tag{13}$$

- The performance of 1D histogram-based methods is generally unsatisfactory as they consider only single feature like intensity level information of an image and do not deal with spatial correlation among the pixels which is an important parameter for segmentation.
- To achieve the same, a new histogram has been introduced for image segmentation, termed as 2D-histogram, which has been proven to be better.
- A 2D-histogram considers two features of an image at a time. In literature, these features are selected from a set of three image features, namely pixel intensity, average pixel intensity, and pixel gradient.
- Generally, the segmentation methods based on the histogram are efficient as they require only a single pass through the pixels to construct the histogram.

Metaheuristic-based methods

- This category involves the use of metaheuristic-based approaches to obtain optimal clusters by updating random solutions according to mathematical formulation and optimality criteria (or objective function).
- Generally, a meta-heuristic method solves an optimization problem whose objective function is to perform either maximization or minimization of a cost function (f (x)) with a given set of constraints.
- The mathematically formulation of a maximization optimization problem is presented in (14).

$$Maximize_{\{x \in \mathbb{R}^d\}} f(x) \tag{14}$$

$$such\ that: \ a_i(x) \geq 0, \tag{15}$$

$$b_j(x) = 0, \tag{16}$$

$$c_k(x) \leq 0 \tag{17}$$

where $x = (x_1, x_2, x_3, \cdots, x_d)^T$ is the set of decision variables which is defined over $d-$ dimensions and $\mathbb{R}^d$ is the search space of the problem. $a_i(x)$, $b_j(x)$, and $c_k(x)$ correspond to the different constraints applicable to an optimization problem. Actual constraints depend on the considered optimization problem.

Metaheuristic-based methods

- Each meta-heuristic algorithm mimics a particular natural phenomena which may belong to evolutionary, physical, or biological.
- In literature, two common aspects that are often found in these algorithms are exploration and exploitation.
- Exploration represents the diversification in the search space wherein the existing solutions are updated with the intention of exploring the search space. This helps in exploring the new solutions, prevents the stagnation problem, and responsible for achieving the global solution.
- The exploitation, which corresponds to the intensification of the current solution, performs the local search around the currently generated solutions. In this, the goal is to exploit the search space and responsible for convergence to the optimal solution.
- Generally, meta-heuristic algorithms may broadly be classified into two categories, namely evolutionary and swarm algorithms.

Metaheuristic-based methods

- Evolutionary-based algorithms are based on evolution theories such as Darwin's evolutionary theory. The evolutionary algorithms work on the principle of generating better individuals with the course of generation by combining the best individuals of the current generation.
- Genetic algorithm (GA), evolutionary strategy (ES), differential evolution (DE) , biogeography-based optimization (BBO), and probability-based incremental learning (PBIL).
- On the other side, swarm-based algorithms behave like the swarm of agents, such as fishes or birds, to achieve optimal results.
- Some algorithms of this category are particle swarm optimization (PSO) , ant colony optimization (ACO) , gravitational search algorithm (GSA) ,spider monkey optimization (SMO), grey-wolf optimizer (GWO) , cuckoo search (CS) , and military dog based optimizer (MDO).

The pseudo-code for a metaheuristic-based clustering method is presented in Algorithm 5

- Generally, these clustering-based methods are better than other clustering methods in terms of independence from the initial parameter settings and return global optimal solution.

Algorithm 5 Metaheuristic-based clustering method.

Input: Initialize the number of clusters to be formed.
Output: Clustered data.
Choose a set of random population where each individual of the population corresponds to a cluster centroid.
while clustering condition is not satisfied or centroids do not change **do**
    Compute the fitness value of each solution through the considered objective function;
    Update the solution according to the meta-heuristic algorithm;
    Assign each data point to the nearest cluster centroid;
end while

Performance evaluation parameters

- The performance evaluation of a method is necessary to assure the validity of a method. This section lists out various performance measures that researchers have found useful for the quantitive evaluation of an image segmentation method.
- Confusion matrix (CM): It is a widely used representation for the assessing the effi- cacy of a classification method. However, it can be used to analyze the results of a clustering method too.
- In context of clustering, the number of correctly clustered patterns (true positive (TP), true negative (TN)) and wrongly clustered patterns (false positive (FP), false negative (FN)) can be easily identified from the confusion matrix.
- The confusion matrix (CM) of size NxN represents that there are N classes (or clusters).

- Based on CM, precision, recall and accuracy can also be computed using (18) – (20).

$$Precision = \frac{TP}{TP + FP} \tag{18}$$

$$Recall = \frac{TP}{TP + FN} \tag{19}$$

$$Accuracy = \frac{TP + TN}{TP + TN + FP + FN} \tag{20}$$

- Intersection of Union (IoU): Intersection of Union is the ratio of the number of com- mon pixels between X and Y to the total number of pixels in X and Y. Here, X and Y correspond to the segmented image and ground truth respectively. The formulation of IoU is depicted in (21).

$$IoU = \frac{|X \cap Y|}{|X| + |Y|} \tag{21}$$

Dice-Coefficient (DC): Dice-Coefficient is defined as twice the number of common pixels divided by total number of pixels in X and Y, where X corresponds to the segmented image and Y is the ground truth. DC is mathematically defined as (22).

$$DC = \frac{2|X \cap Y|}{|X| + |Y|} \tag{22}$$

Boundary Displacement Error (BDE): This parameter computes the average boundary pixels displacement error between two segmented images as depicted in (23). The error of one boundary pixel is defined as it's distance from the closest pixel in the other boundary image.

$$\mu_{LA}(u, v) = \begin{cases} \frac{u-v}{L-1} & 0 < u - v \\ 0 & u - v < 0 \end{cases} \tag{23}$$

Probability Rand Index (PRI): Probability Rand Index finds labelling consistency between the segmented image and its ground truth. It counts such fraction of pairs of pixels and average the result across all ground truths of a given image as shown in (24).

$$R = \frac{a+b}{a+b+c+d} = \frac{a+b}{n} \tag{24}$$

Variation of Information (VOI): Variation of Information as shown in (25) computes the randomness in one segmentation in terms of the distance from given segmentation.

$$VOI(X;Y) = H(X) + H(Y) - 2I(X,Y) \tag{25}$$

Given a set of n elements $S = \{o_1, \ldots, o_n\}$ and two partitions of S to compare, $X = \{X_1, \ldots, X_r\}$, a partition of S into r subsets, and $Y = \{Y_1, \ldots, Y_s\}$, a partition of S into s subsets, define the following:

- a, the number of pairs of elements in S that are in the **same** subset in X and in the **same** subset in Y
- b, the number of pairs of elements in S that are in **different** subsets in X and in **different** subsets in Y
- c, the number of pairs of elements in S that are in the **same** subset in X and in **different** subsets in Y
- d, the number of pairs of elements in S that are in **different** subsets in X and in the **same** subset in Y

The Rand index, R, is:[1][2]

$$R = \frac{a+b}{a+b+c+d} = \frac{a+b}{\binom{n}{2}}$$

Intuitively, $a+b$ can be considered as the number of agreements between X and Y and $c+d$ as the number of disagreements between X and Y.

Since the denominator is the total number of pairs, the Rand index represents the *frequency of occurrence* of agreements over the total pairs, or the probability that X and Y will agree on a randomly chosen pair.

$\binom{n}{2}$ is calculated as $n(n-1)/2$.

Similarly, one can also view the Rand index as a measure of the percentage of correct decisions made by the algorithm. It can be computed using the following formula:

$$RI = \frac{TP+TN}{TP+FP+FN+TN}$$

where TP is the number of true positives, TN is the number of true negatives, FP is the number of false positives, and FN is the number of false negatives.

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Global Consistency Error (GCE): A refinement in one segmentation over the other is depicted by the value of GCE as given in (26). If two segmentations are related this way then they are considered as consistent, i.e. both can represent the same natural image segmentation at different scales.

$$GCE = \frac{1}{n}\left\{\sum_i E(s_1, s_2, p_i), \sum_i E(s_2, s_1, p_i)\right\} \tag{26}$$

Structural Similarity Index (SSIM): Structural Similarity Index measures the similarity between two images by taking initial uncompressed or distortion-free image as the reference and computed as (27). It incorporates important perceptual phenomena such as luminance masking and contrast masking.

$$SSIM = \frac{(2 \times \bar{x} \times \bar{y} + c_1)(2 \times \sigma_{xy} + c_2)}{(\sigma_x^2 + \sigma_{y)}^2) \times ((\overline{x})^2 + (\overline{y})^2 + c_1)} \tag{27}$$

Feature Similarity Index (FSIM): Feature Similarity Index is a quality score that uses the phase congruency (PC), which is a dimensionless measure and shows the significance of a local structure. It is calculated by (28).

$$FSIM = \frac{\sum_{x \in \Omega} S_L(x).PC_m(x)}{\sum_{x \in \Omega} PC_m(x)} \tag{28}$$

Root Mean squared error (RMSE): The root-mean-squared error computes the difference between sample value predicted by a model or an estimator and actual value. The formulation is shown in (29).

$$\mathrm{RMSE}(\hat{\theta}) = \sqrt{\mathrm{MSE}(\hat{\theta})} = \sqrt{\mathrm{E}((\hat{\theta} - \theta)^2)} \tag{29}$$

Peak Signal to Noise Ratio (PSNR in dB): Peak signal-to-noise ratio is defined as the ratio between the maximum possible power of a signal and the power of corrupting noise and is calculated using (30). In general, a higher value of PSNR represents high quality reconstruction and is defined via MSE.

$$PSNR = 10log_{10} \frac{(2^n - 1)^2}{\sqrt{MSE}} \tag{30}$$

Normalized Cross-Correlation (NCC): Normalized cross-correlation is used for template matching where images are first normalized due to lighting and exposure conditions. It is calculated by subtracting the mean of original and segmented images from the corresponding images, and divided by their standard deviations. Let $t(x, y)$ is the segmented image of $f(x, y)$, then NCC is calculated by (31).

$$\frac{1}{n}\sum_{x,y}\frac{1}{\sigma_f\sigma_t}\left(f(x, y)-\overline{f}\right)\left(t(x, y)-\overline{t}\right) \tag{31}$$

Average Difference (AD): This parameter represents the average difference between the pixel values and is computed by (32).

$$AD=\frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}(x(i, j)-y(i, j)) \tag{32}$$

Maximum Difference (MD): This parameter finds the maximum of error signal by taking the difference between original image and segmented image and defined in (33).

$$MD = max\mid x(i,j) - y(i,j)\mid \tag{33}$$

Normalized Absolute Error (NAE): The normalized absolute difference between the original and corresponding segmented image gives NAE and is calculated by (34).

$$NAE = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N}\mid x(i,j) - y(i,j)\mid}{\sum_{i=1}^{M}\sum_{j=1}^{N}\mid x(i,j)\mid} \tag{34}$$

- In the above mentioned parameters, IoU, DC, SSIM, FSIM, PRI, PSNR, and NCC show better segmentation on high values. A high value indicates that the segmented image is more close to the ground truth.
- To measure the same, these parameters compute values by considering the region of intersection between the segmented image and the ground truth which corresponds to matching the number of similar pixels.
- On the contrary, other indices prefer lower values for better segmentation as these measures compute error between the segmented image and the ground truth and aim at reducing the difference between them.