Computer Vision-IT416

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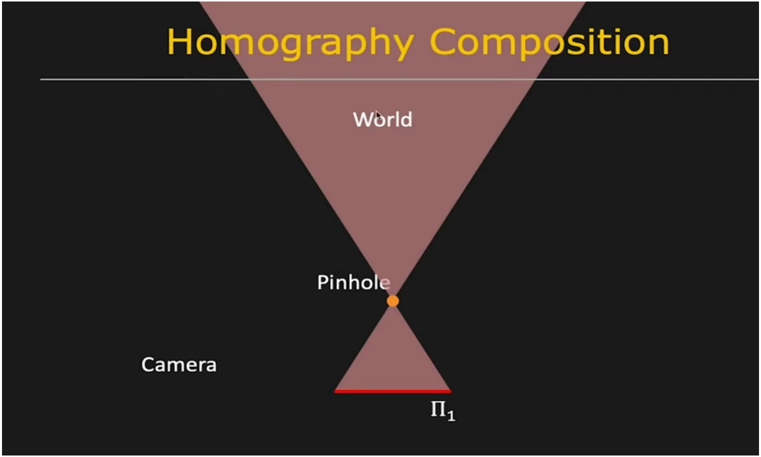
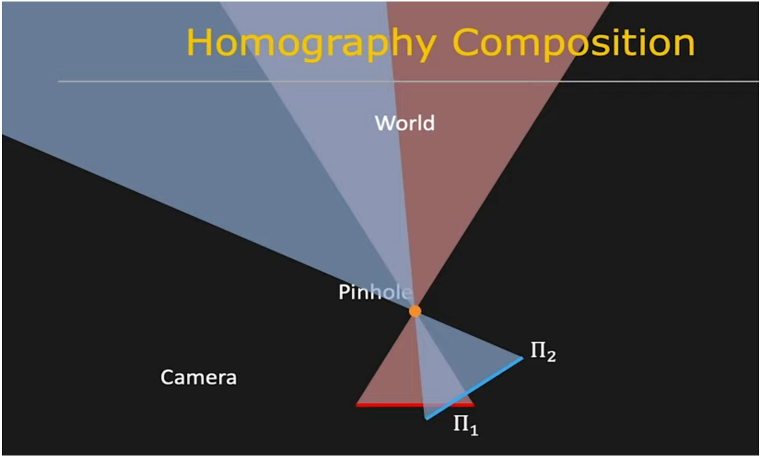
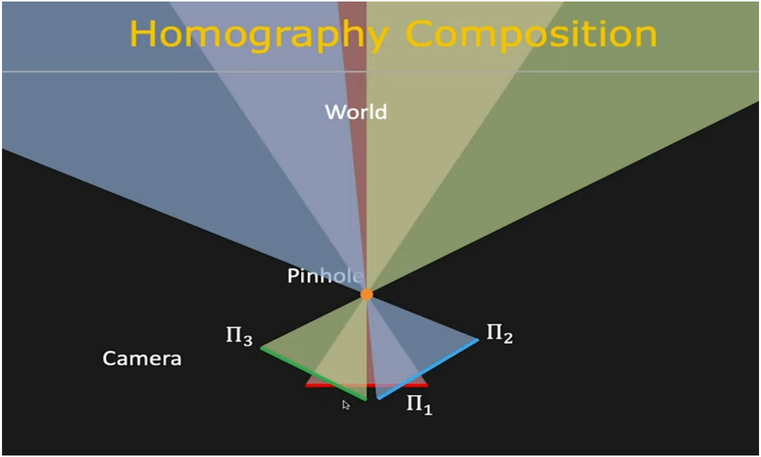
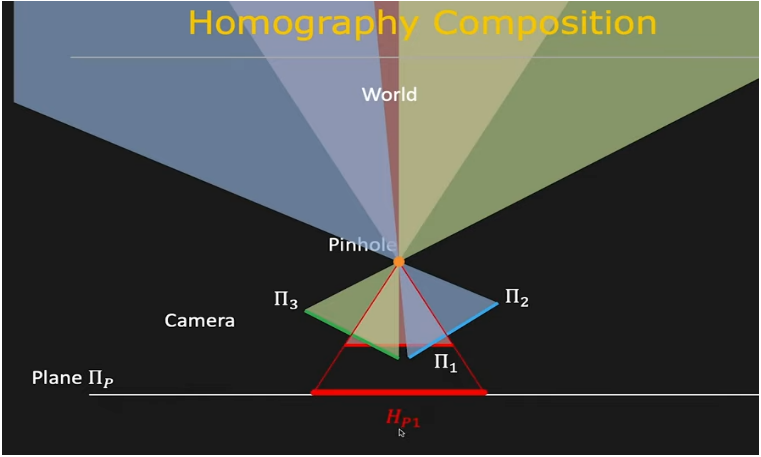
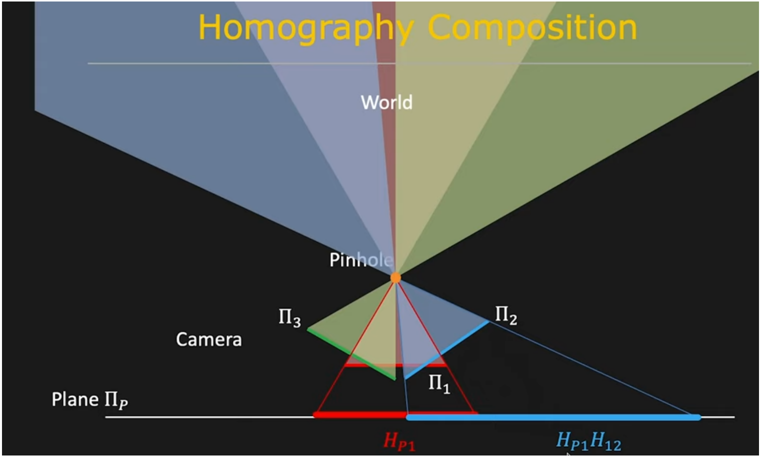

Homography Composition
World
Pinhole
Camera
Π_1


Homography Composition
World
Pinhole
Camera
Π_2
Π_1


Homography Composition
World
Pinhole
Camera
Π_3
Π_2
Π_1


Homography Composition
World
Pinhole
Π_3
Π_2
Camera
Π_1
Plane Π_P
H_{P1}



Homography Composition
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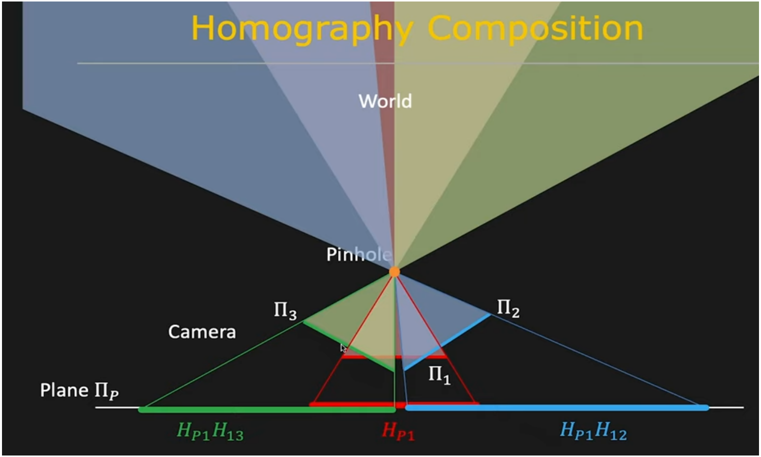
Pinhole
Π_3
Π_2
Camera
Π_1
Plane Π_P
H_{P1}
$H_{P1}H_{12}$


Homography Composition
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Π_2
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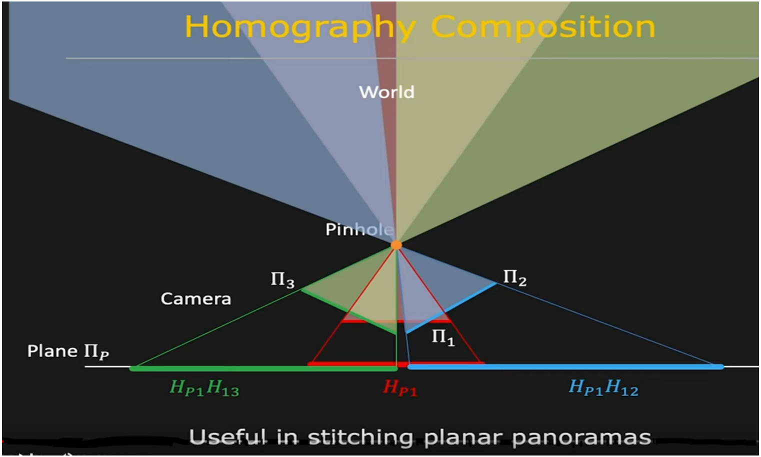
$H_{P1}H_{13}$
H_{P1}
$H_{P1}H_{12}$


Homography Composition
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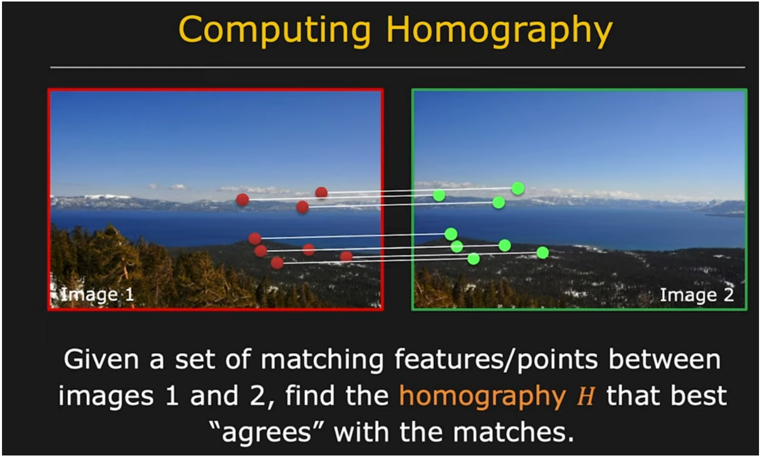
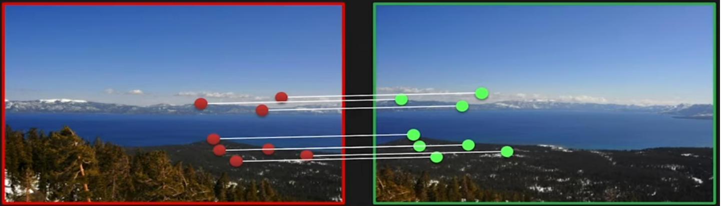
$H_{P1}H_{13}$
H_{P1}
$H_{P1}H_{12}$
Useful in stitching planar panoramas


Computing Homography
Image 1
Image 2
Given a set of matching features/points between images 1 and 2, find the homography H that best "agrees" with the matches.

Computing Homography



Source Image

Destination Image

$$\begin{bmatrix} x_d \\ y_d \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{x}_d \\ \tilde{y}_d \\ \tilde{z}_d \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x_s \\ y_s \\ 1 \end{bmatrix}$$

How many unknowns?

Computing Homography

For a given pair i of corresponding points:

$$x_d{}^{(i)} = \frac{\tilde{x}_d{}^{(i)}}{\tilde{z}_d{}^{(i)}} = \frac{h_{11}x_s^{(i)} + h_{12}y_s^{(i)} + h_{13}}{h_{31}x_s^{(i)} + h_{32}y_s^{(i)} + h_{33}}$$

$$y_d{}^{(i)} = \frac{\tilde{y}_d{}^{(i)}}{\tilde{z}_d{}^{(i)}} = \frac{h_{21}x_s^{(i)} + h_{22}y_s^{(i)} + h_{23}}{h_{31}x_s^{(i)} + h_{32}y_s^{(i)} + h_{33}}$$

Rearranging the terms:

$$x_d{}^{(i)}\left(h_{31}x_s^{(i)} + h_{32}y_s^{(i)} + h_{33}\right) = h_{11}x_s^{(i)} + h_{12}y_s^{(i)} + h_{13}$$

$$y_d{}^{(i)}\left(h_{31}x_s^{(i)} + h_{32}y_s^{(i)} + h_{33}\right) = h_{21}x_s^{(i)} + h_{22}y_s^{(i)} + h_{23}$$

Computing Homography

$$x_d^{(i)}\left(h_{31}x_s^{(i)} + h_{32}y_s^{(i)} + h_{33}\right) = h_{11}x_s^{(i)} + h_{12}y_s^{(i)} + h_{13}$$

$$y_d^{(i)}\left(h_{31}x_s^{(i)} + h_{32}y_s^{(i)} + h_{33}\right) = h_{21}x_s^{(i)} + h_{22}y_s^{(i)} + h_{23}$$

Rearranging the terms and writing as linear equation:

$$\begin{bmatrix} x_s^{(i)} & y_s^{(i)} & 1 & 0 & 0 & 0 & -x_d^{(i)}x_s^{(i)} & -x_d^{(i)}y_s^{(i)} & -x_d^{(i)} \\ 0 & 0 & 0 & x_s^{(i)} & y_s^{(i)} & 1 & -y_d^{(i)}x_s^{(i)} & -y_d^{(i)}y_s^{(i)} & -y_d^{(i)} \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Computing Homography

$$x_d^{(i)}\left(h_{31}x_s^{(i)} + h_{32}y_s^{(i)} + h_{33}\right) = h_{11}x_s^{(i)} + h_{12}y_s^{(i)} + h_{13}$$

$$y_d^{(i)}\left(h_{31}x_s^{(i)} + h_{32}y_s^{(i)} + h_{33}\right) = h_{21}x_s^{(i)} + h_{22}y_s^{(i)} + h_{23}$$

Rearranging the terms and writing as linear equation:

$$\underbrace{\boxed{\begin{bmatrix} x_s^{(i)} & y_s^{(i)} & 1 & 0 & 0 & 0 & -x_d^{(i)}x_s^{(i)} & -x_d^{(i)}y_s^{(i)} & -x_d^{(i)} \\ 0 & 0 & 0 & x_s^{(i)} & y_s^{(i)} & 1 & -y_d^{(i)}x_s^{(i)} & -y_d^{(i)}y_s^{(i)} & -y_d^{(i)} \end{bmatrix}}}_{\text{(Known)}} \underbrace{\boxed{\begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix}}}_{\substack{\mathbf{h} \\ \text{(Unknown)}}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

[redacted]

Computing Homography

Combining the equations for all corresponding points:

$$
\underbrace{\begin{bmatrix}
x_s^{(1)} & y_s^{(1)} & 1 & 0 & 0 & 0 & -x_d^{(1)}x_s^{(1)} & -x_d^{(1)}y_s^{(1)} & -x_d^{(1)} \\
0 & 0 & 0 & x_s^{(1)} & y_s^{(1)} & 1 & -y_d^{(1)}x_s^{(1)} & -y_d^{(1)}y_s^{(1)} & -y_d^{(1)} \\
 & & & & \vdots & & & & \\
x_s^{(i)} & y_s^{(i)} & 1 & 0 & 0 & 0 & -x_d^{(i)}x_s^{(i)} & -x_d^{(i)}y_s^{(i)} & -x_d^{(i)} \\
0 & 0 & 0 & x_s^{(i)} & y_s^{(i)} & 1 & -y_d^{(i)}x_s^{(i)} & -y_d^{(i)}y_s^{(i)} & -y_d^{(i)} \\
 & & & & \vdots & & & & \\
x_s^{(n)} & y_s^{(n)} & 1 & 0 & 0 & 0 & -x_d^{(n)}x_s^{(n)} & -x_d^{(n)}y_s^{(n)} & -x_d^{(n)} \\
0 & 0 & 0 & x_s^{(n)} & y_s^{(n)} & 1 & -y_d^{(n)}x_s^{(n)} & -y_d^{(n)}y_s^{(n)} & -y_d^{(n)}
\end{bmatrix}}_{\substack{A \\ \text{(Known)}}}
\underbrace{\begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix}}_{\substack{\mathbf{h} \\ \text{(Unknown)}}}
=
\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}
$$

Computing Homography

Combining the equations for all corresponding points:

$$\underbrace{\begin{bmatrix} x_s^{(1)} & y_s^{(1)} & 1 & 0 & 0 & 0 & -x_d^{(1)}x_s^{(1)} & -x_d^{(1)}y_s^{(1)} & -x_d^{(1)} \\ 0 & 0 & 0 & x_s^{(1)} & y_s^{(1)} & 1 & -y_d^{(1)}x_s^{(1)} & -y_d^{(1)}y_s^{(1)} & -y_d^{(1)} \\ & & & & & \vdots & & & \\ x_s^{(i)} & y_s^{(i)} & 1 & 0 & 0 & 0 & -x_d^{(i)}x_s^{(i)} & -x_d^{(i)}y_s^{(i)} & -x_d^{(i)} \\ 0 & 0 & 0 & x_s^{(i)} & y_s^{(i)} & 1 & -y_d^{(i)}x_s^{(i)} & -y_d^{(i)}y_s^{(i)} & -y_d^{(i)} \\ & & & & & \vdots & & & \\ x_s^{(n)} & y_s^{(n)} & 1 & 0 & 0 & 0 & -x_d^{(n)}x_s^{(n)} & -x_d^{(n)}y_s^{(n)} & -x_d^{(n)} \\ 0 & 0 & 0 & x_s^{(n)} & y_s^{(n)} & 1 & -y_d^{(n)}x_s^{(n)} & -y_d^{(n)}y_s^{(n)} & -y_d^{(n)} \end{bmatrix}}_{\substack{A \\ \text{(Known)}}} \underbrace{\begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix}}_{\substack{\mathbf{h} \\ \text{(Unknown)}}} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

Solve for $\mathbf{h}$: $\boxed{A\,\mathbf{h} = \mathbf{0}}$ such that $\|\mathbf{h}\|^2 = 1$

Constrained Least Squares

Solve for $\mathbf{h}$: $\boxed{A\,\mathbf{h} = \mathbf{0}}$ such that $||\mathbf{h}||^2 = 1$

Define least squares problem:

$$\min_{\mathbf{h}} \; ||A\mathbf{h}||^2 \quad \text{such that } ||\mathbf{h}||^2 = 1$$

We know that:

$$||A\mathbf{h}||^2 = (A\mathbf{h})^T(A\mathbf{h}) = \mathbf{h}^T A^T A\mathbf{h} \quad \text{and} \quad ||\mathbf{h}||^2 = \mathbf{h}^T\mathbf{h} = 1$$

$$\boxed{\min_{\mathbf{h}}(\mathbf{h}^T A^T A\mathbf{h}) \;\text{ such that }\; \mathbf{h}^T\mathbf{h} = 1}$$

Constrained Least Squares

$$\min_{\mathbf{h}}(\mathbf{h}^T A^T A\mathbf{h}) \text{ such that } \mathbf{h}^T\mathbf{h} = 1$$

Define Loss function $L(\mathbf{h},\lambda)$:

$$L(\mathbf{h},\lambda) = \mathbf{h}^T A^T A\mathbf{h} - \lambda(\mathbf{h}^T\mathbf{h} - 1)$$

Taking derivatives of $L(\mathbf{h},\lambda)$ w.r.t $\mathbf{h}$: $2A^T A\mathbf{h} - 2\lambda\mathbf{h} = \mathbf{0}$

$$\boxed{A^T A\mathbf{h} = \lambda\mathbf{h}}$$ Eigenvalue Problem

Eigenvector $\mathbf{h}$ with smallest eigenvalue λ of matrix $A^T A$ minimizes the loss function $L(\mathbf{h})$.