Engineering Economics

Dr. Pradyot Ranjan Jena

School of Management

NATIONAL INSTITUTE OF TECHNOLOGY KARNATAKA, SURATHKAL

Course Outline

Course Title	Engineering Economics
Semester	V Semester B.Tech
Course Code	HU 300
Instructor	Dr. Pradyot Ranjan Jena

COURSE DESCRIPTION

The purpose of this course is to help students gain an understanding of the economic factors inherent in engineering design and decision-making. Any engineering project must be not only physically realizable but also economically feasible. The principal aim of this subject is to provide students with some basic techniques of economic analysis to understand the economic process.

Meeting Time

- You can email me at jpradyot@gmail.com

- If you have any queries and any doubts meet me in my office at School of Management.

Course Objectives

- Become acquainted with basic economic concepts such as demand and supply, price, competition, interest rate, profit, inflation, GDP, GNP etc.
- Develop a significant understanding of the **time value of money**.
- Develop the ability to apply various methods for economic analysis of **alternatives**.
- Increase student's knowledge of the impact that interest rate, taxes, inflation have on economic and engineering decisions.
- Develop the ability to estimate project **cash flows** for design alternatives including tax implications.
- Understand the fundamentals of profit and loss analysis and **benefit –cost** analysis.
- Develop the ability to make **replacement** decisions.
- Basic understanding of **depreciation** methods.

Course Content

Basic economic concepts and problems

- Introduction to Engineering Economics- Physical and economic efficiency

- Micro Economic Concepts:

  Demand and Supply, Elasticity and applications,

  Value and Utility, Law of Diminishing Marginal Utility, Indifference Curves, Cost Concepts, Market Equilibrium, Demand forecasting

- Macro Economic Concepts:

  Macroeconomic Aggregates, Growth and Development , Environment and Development, Human Development Index, Growth and Science &Technology,

Course Content

Methods of economic analysis in Engineering

- Time value of money, Interest rate calculations.
- Present worth, Annual equivalent, Future worth, Internal rate of return, Capitalized equivalent, Capital recovery with return. Selection among alternatives, Break-even analysis.

Course Content

Evaluating replacement alternatives

- Replacement analysis, the economic life of an asset, Retirement or abandonment decisions.
- Evaluating public activities: The nature of public activities, Benefit-cost analysis, Cost-effectiveness analysis

Depreciation accounting

- Basic depreciation methods. Basic terminology for Income taxes, Depreciation and Income taxes.

Estimating economic elements

- Cost Estimation, Location Decisions

Texts / References:

- Leland Blank P.E. and Anthony Tarquin P.E., “Engineering Economy”, 7th ed., McGraw Hill, 2012.
- Sullivan W.G., Bontadelli J.A. and Wicks E.M., “Engineering Economy “, 14th ed., Pearson Education Asia, New Delhi,
- Thuesen G.J. and Fabrycky W.J., “Engineering Economy “, 9th ed., Prentice Hall of India, New Delhi, 2002.
- Newnan Donald G., Eschenbach Ted G., Lavelle Jerome P., “Engineering Economic Analysis”, Oxford University Press, 2004.
- N.Gregory Mankiw, “Principles of Economics”, Thomson, 2002.
- Karl E Case, Ray C Fair, and Sharon E Oster, “Principles of Micro Economics”, 11th Edition, Pearson Education, 2014.
- Research articles and case studies.

Weightage for Assessments

• Mid – Semester exams of 1 hour 30 minutes (50 Marks)	25%
• Continuous Assessment	25%
• End-Semester exams of 3 hours (100 Marks) (Full syllabus)	50%
	100%

Pedagogy

The instructional tools consists of lectures, reading concurrent articles, case studies, problem solving and group discussions.

Assessment

Surprise tests, quizzes, assignments, class participation and group interaction will be considered for continuous assessment.

Attendance

As per regulations in Under Graduate Programme Curriculum 2012.

Engineering and Engineering Economy

Introduction

- Engineering activities are not an end in themselves. They are a means for satisfying human wants.
- Engineers have two concerns: 1) Materials and Forces of Nature, and 2) Needs of People
- Resource constraints is responsible for closely associating Engineering with Economics.
- Engineering projects need to be not just physically feasible but economically also.

Engineering and Science

- Engineering is not a science but an application of science. It is an art of adopting skill and knowledge of science.
- Accreditation Board for Engineering and Technology defines Engineering as, “Engineering is a profession in which knowledge of the mathematical and natural sciences are gained by study, experience, and practice is applied with judgment to develop ways to utilize economically the materials and forces of nature for the benefit of mankind”.

Contd..

- Role of a scientist is to add to humankind's accumulated body of knowledge and discover universal laws of behavior.
- Role of engineer is to apply this to particular situations to produce products and services.
- Engineering activities rarely are carried out for the satisfaction that may be derived from them directly. Instead, their use is confined to satisfying human wants

Contd..

- Modern civilization depends to a large degree on engineering. For e.g. transportation, communication, national defense and other goods and services used to facilitate work.
- Science is the foundation upon which engineer builds.
- Engineering activity is responsible for improvement in general standard of living.

Bi-Environmental Nature of Engineering

- Engineers are confronted with two environments:
- 1) Physical Environment.
- 2) Economic Environment.
- The success of engineering is to create products and services with the knowledge of physical laws. However, the worth these products and services lies in their utility measured in economic terms.

Contd..

- Physical environment is governed by physical laws which are more exact and much is known with certainty.
- Economic environment is governed by economic laws which are influenced by human behavior. These laws are less exact compared to physical laws.
- Quantification is possible to a large extent in economic environment due to similar reaction of human beings over space and across time to similar events.

- Engineers may have a tendency to disregard economic environment. But role of an engineer goes much beyond physical environment to economic and managerial as well.
- There is also an argument that engineers must confine to physical factors and economic and humanistic factors should be handled by others.

- Engineers can readily extend their inherent ability of analysis to become proficient in the analysis of the economic aspects of engineering application.
- Engineers who will be eventually engaged in managerial activities will find such proficiency is necessary.
- It is the objective of engineering economy to prepare engineers to cope with bi- environmental nature of engineering.

Physical and Economic Efficiency

- There is limited resources and as a result it is necessary to produce greatest output with limited input.
- Opportunity cost:
- Engineering is concerned with physical efficiency: i.e. **output/input**.
- Physical efficiency is always less than 100%

Contd..

- At second level there is economic efficiency i.e. **worth/cost**.
- Economic efficiency must be over 100% to consider a project.
- In final evaluation of ventures, even though engineering plays a major role, economic efficiency must take precedence over physical efficiency.
- Economic efficiency concept brings to the fore all complexities of economic environment.

Engineering for Economic Competitiveness

- Producers strive for sustainable competitive advantage in the market place. (Bajaj Chetak)
- Through the life cycle approach to engineering, economic competitiveness can be enhanced.
- Identification of need, conceptual/preliminary design, detailed design and development, production/construction, utilization and finally phase out and disposal.

- Generally, engineers have focused mainly on the acquisition phase i.e. up to production/construction.
- However, recent experience shows that product competitiveness cannot be achieved through efforts applied largely after product comes in to market place.
- As a result, it is essential that engineers need to be sensitive in the early stages of life cycle.

Engineering Economics

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Demand, Supply, and Market Equilibrium

Firms and Households: The Basic Decision-Making Units

firm An organization that transforms resources (inputs) into products (outputs). Firms are the primary producing units in a market economy.

entrepreneur A person who organizes, manages, and assumes the risks of a firm, taking a new idea or a new product and turning it into a successful business.

households The consuming units in an economy.

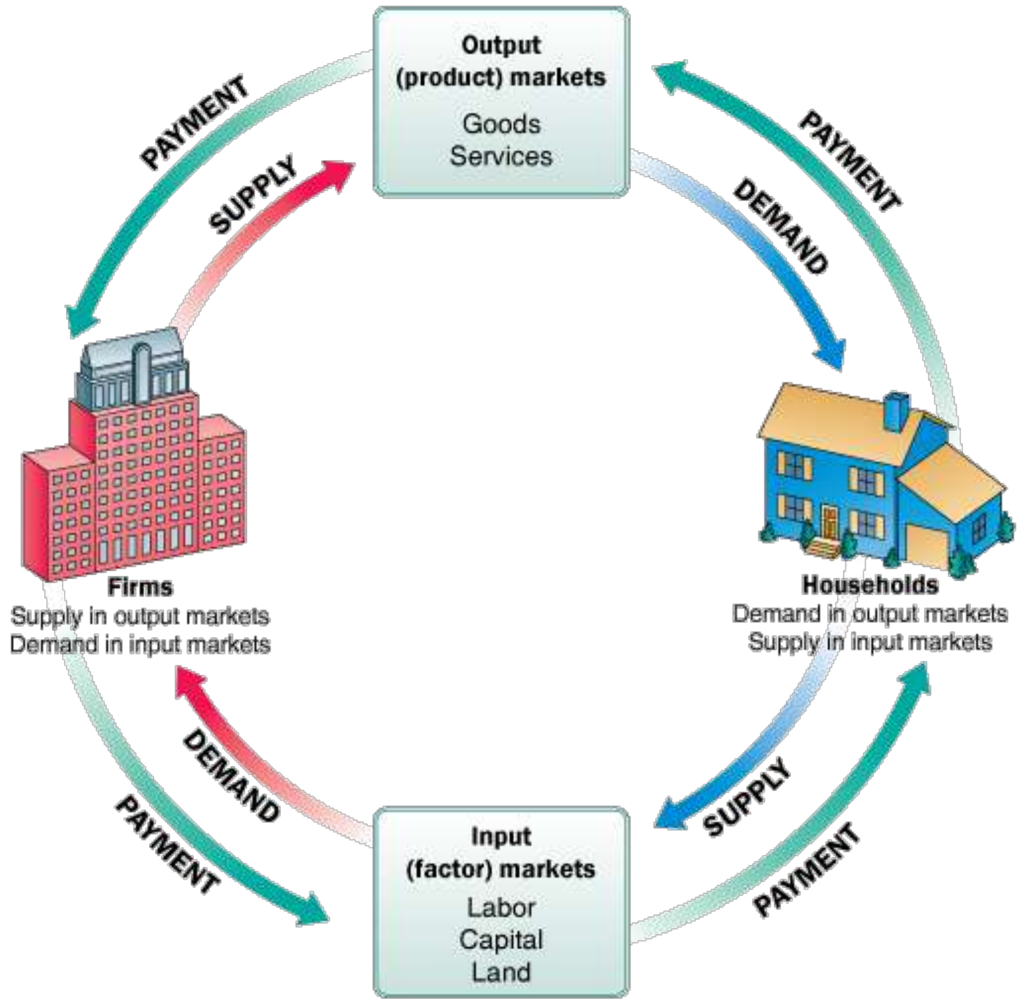
Input Markets and Output Markets: The Circular Flow

product *or* output markets The markets in which goods and services are exchanged.

input *or* factor markets The markets in which the resources used to produce goods and services are exchanged.

FIGURE 1 The Circular Flow of Economic Activity

Here goods and services flow clockwise: Labor services supplied by households flow to firms, and goods and services produced by firms flow to households. Payment (usually money) flows in the opposite (counterclockwise) direction: Payment for goods and services flows from households to firms, and payment for labor services flows from firms to households.



labor market The input/factor market in which households supply work for wages to firms that demand labor.

capital market The input/factor market in which households supply their savings, for interest or for claims to future profits, to firms that demand funds to buy capital goods.

land market The input/factor market in which households supply land or other real property in exchange for rent.

factors of production The inputs into the production process. Land, labor, and capital are the three key factors of production.

Input and output markets are connected through the behavior of both firms and households. Firms determine the quantities and character of outputs produced and the types and quantities of inputs demanded. Households determine the types and quantities of products demanded and the quantities and types of inputs supplied.

Demand in Product/Output Markets

A household's decision about what quantity of a particular output, or product, to demand depends on a number of factors, including:

- The *price of the product* in question.
- The *income available* to the household.
- The household's *amount of accumulated wealth*.
- The *prices of other products* available to the household.
- The household's *tastes and preferences*.
- The household's *expectations* about future income, wealth, and prices.

quantity demanded The amount (number of units) of a product that a household would buy in a given period if it could buy all it wanted at the current market price.

Changes in Quantity Demanded versus Changes in Demand

The most important relationship in individual markets is that between market price and quantity demanded.

Changes in the price of a product affect the *quantity demanded* per period. Changes in any other factor, such as income or preferences, affect *demand*. Thus, we say that an increase in the price of Coca-Cola is likely to cause a decrease in the *quantity of Coca-Cola demanded*. However, we say that an increase in income is likely to cause an increase in the *demand* for most goods.

Price and Quantity Demanded: The Law of Demand

demand schedule Shows how much of a given product a household would be willing to buy at different prices for a given time period.

demand curve A graph illustrating how much of a given product a household would be willing to buy at different prices.

TABLE 3.1 Alex's Demand Schedule for Gasoline	
Price (per Gallon)	**Quantity Demanded (Gallons per Week)**
$ 8.00	0
7.00	2
6.00	3
5.00	5
4.00	7
3.00	10
2.00	14
1.00	20
0.00	26

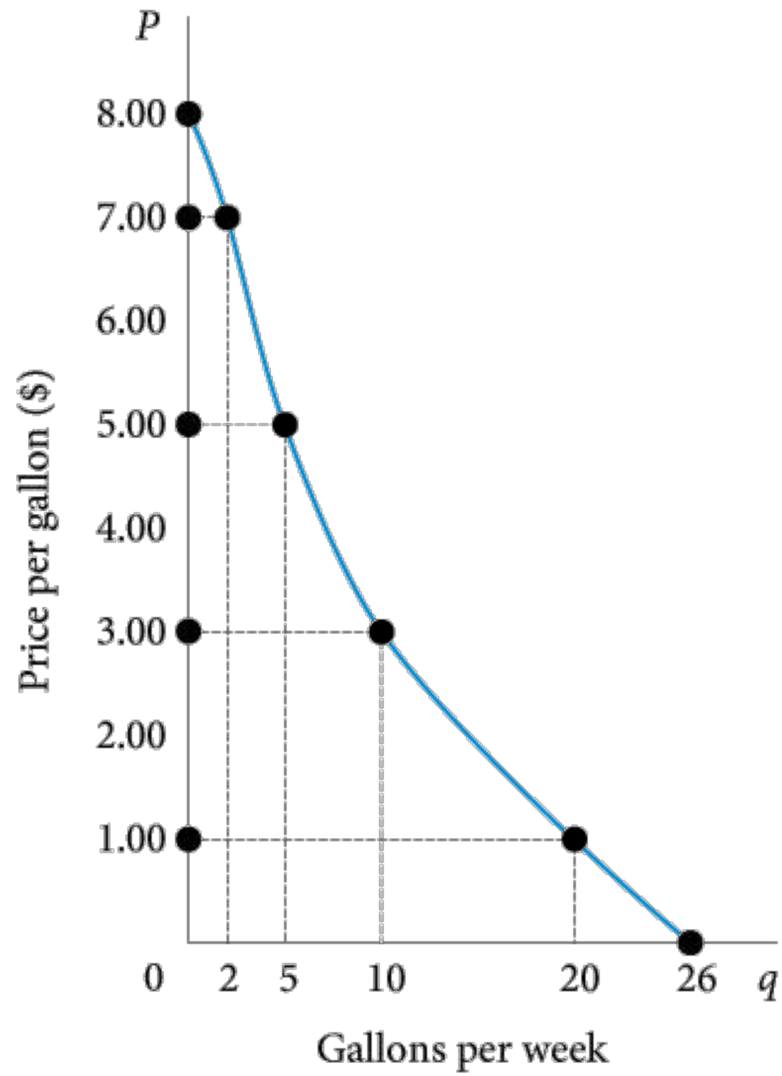


FIGURE 3.2 Alex's Demand Curve

The relationship between price (P) and quantity demanded (q) presented graphically is called a demand curve.

Demand curves have a negative slope, indicating that lower prices cause quantity demanded to increase.

Note that Alex's demand curve is blue; demand in product markets is determined by household choice.

Demand Curves Slope Downward

law of demand The negative relationship between price and quantity demanded: As price rises, quantity demanded decreases; as price falls, quantity demanded increases.

It is reasonable to expect quantity demanded to fall when price rises, *ceteris paribus*, and to expect quantity demanded to rise when price falls, *ceteris paribus*. Demand curves have a negative slope.

Other Properties of Demand Curves

1. They have a negative slope.

2. They intersect the quantity (*X*) axis a result of time limitations and diminishing marginal utility.

3. They intersect the price (*Y*) axis, a result of limited income and wealth.

The actual shape of an individual household demand curve—whether it is steep or flat, whether it is bowed in or bowed out—depends on the unique tastes and preferences of the household and other factors.

Other Determinants of Household Demand

Income and Wealth

income The sum of all a household's wages, salaries, profits, interest payments, rents, and other forms of earnings in a given period of time. It is a flow measure.

wealth *or* net worth The total value of what a household owns minus what it owes. It is a stock measure.

normal goods Goods for which demand goes up when income is higher and for which demand goes down when income is lower.

inferior goods Goods for which demand tends to fall when income rises.

Prices of Other Goods and Services

substitutes Goods that can serve as replacements for one another; when the price of one increases, demand for the other increases.

complements, complementary goods Goods that "go together"; a decrease in the price of one results in an increase in demand for the other and vice versa.

Tastes and Preferences

Income, wealth, and prices of goods available are the three factors that determine the combinations of goods and services that a household is *able* to buy.

Changes in preferences can and do manifest themselves in market behavior.

Within the constraints of prices and incomes, preference shapes the demand curve, but it is difficult to generalize about tastes and preferences. First, they are volatile. Second, tastes are idiosyncratic.

Expectations

What you decide to buy today certainly depends on today's prices and your current income and wealth.

There are many examples of the ways expectations affect demand.

Increasingly, economic theory has come to recognize the importance of expectations.

It is important to understand that demand depends on more than just *current* incomes, prices, and tastes.

Shift of Demand versus Movement Along a Demand Curve

TABLE 3.2 Shift of Alex's Demand Schedule Due to Increase in Income

	Schedule D_0	Schedule D_1
Price (per Gallon)	Quantity Demanded (Gallons per Week at an Income of $500 per Week)	Quantity Demanded (Gallons per Week at an Income of $700 per Week)
$ 8.00	0	3
7.00	2	5
6.00	3	7
5.00	5	10
4.00	7	12
3.00	10	15
2.00	14	19
1.00	20	24
0.00	26	30

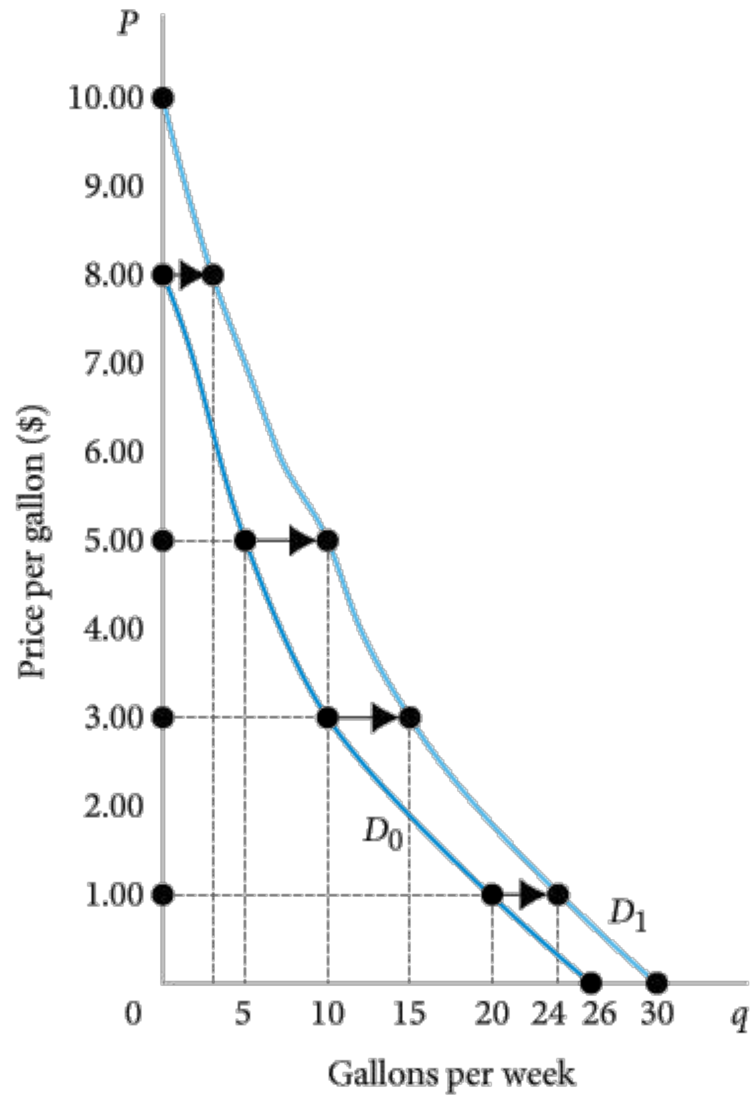


FIGURE 3.3 Shift of a Demand Curve following a Rise in Income

When the price of a good changes, we move *along* the demand curve for that good.

When any other factor that influences demand changes (income, tastes, and so on), the relationship between price and quantity is different; there is a *shift* of the demand curve, in this case from D_0 to D_1.

Gasoline is a normal good.

shift of a demand curve The change that takes place in a demand curve corresponding to a new relationship between quantity demanded of a good and price of that good. The shift is brought about by a change in the original conditions.

movement along a demand curve The change in quantity demanded brought about by a change in price.

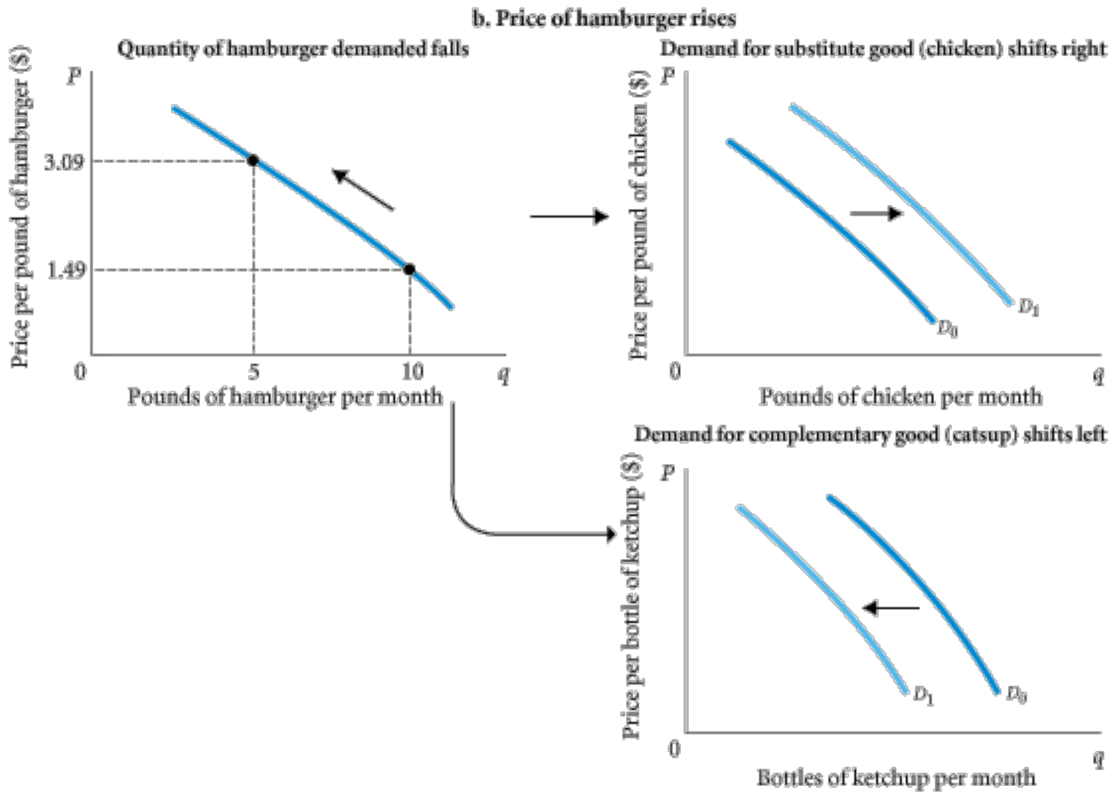
Change in price of a good or service leads to

→ Change in *quantity demanded* (movement along a demand curve).

→ Change in income, preferences, or prices of other goods or services leads to Change in *demand* (shift of a demand curve).

FIGURE 3.4 Shifts versus Movement Along a Demand Curve

a. When income increases, the demand for inferior goods *shifts to the left*
and the demand for normal goods *shifts to the right*.



b. If the price of hamburger rises, the quantity of hamburger demanded declines— this is a movement along the demand curve.
The same price rise for hamburger would shift the demand for chicken (a substitute for hamburger) to the right and the demand for ketchup (a complement to hamburger) to the left.

Supply in Product/Output Markets

Firms build factories, hire workers, and buy raw materials because they believe they can sell the products they make for more than it costs to produce them.

profit The difference between revenues and costs.

law of supply The positive relationship between price and quantity of a good supplied: An increase in market price will lead to an increase in quantity supplied, and a decrease in market price will lead to a decrease in quantity supplied.

supply curve A graph illustrating how much of a product a firm will sell at different prices.

TABLE 3.3 Clarence Brown's Supply Schedule for Soybeans

Price (per Bushel)	Quantity Supplied (Bushels per Year)
$1.50	0
1.75	10,000
2.25	20,000
3.00	30,000
4.00	45,000
5.00	45,000

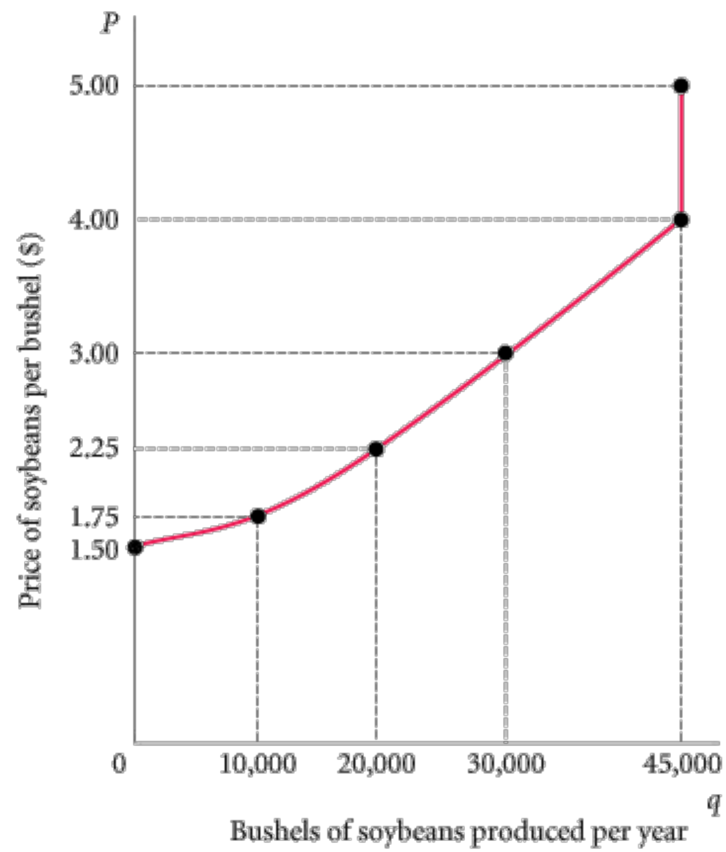


FIGURE 3.6 Clarence Brown's Individual Supply Curve

A producer will supply more when the price of output is higher. The slope of a supply curve is positive.

Other Determinants of Supply

The Cost of Production

For a firm to make a profit, its revenue must exceed its costs.

Cost of production depends on a number of factors, including the available technologies and the prices and quantities of the inputs needed by the firm (labor, land, capital, energy, and so on).

The Prices of Related Products

Assuming that its objective is to maximize profits, a firm's decision about what quantity of output, or product, to supply depends on:

1. The price of the good or service.

2. The cost of producing the product, which in turn depends on:
   - The price of required inputs (labor, capital, and land).
   - The technologies that can be used to produce the product.

3. The prices of related products.

As with demand, it is very important to distinguish between *movements along* supply curves (changes in quantity supplied) and *shifts in* supply curves (changes in supply):

Change in price of a good or service leads to
→ Change in *quantity supplied* (movement along a supply curve).

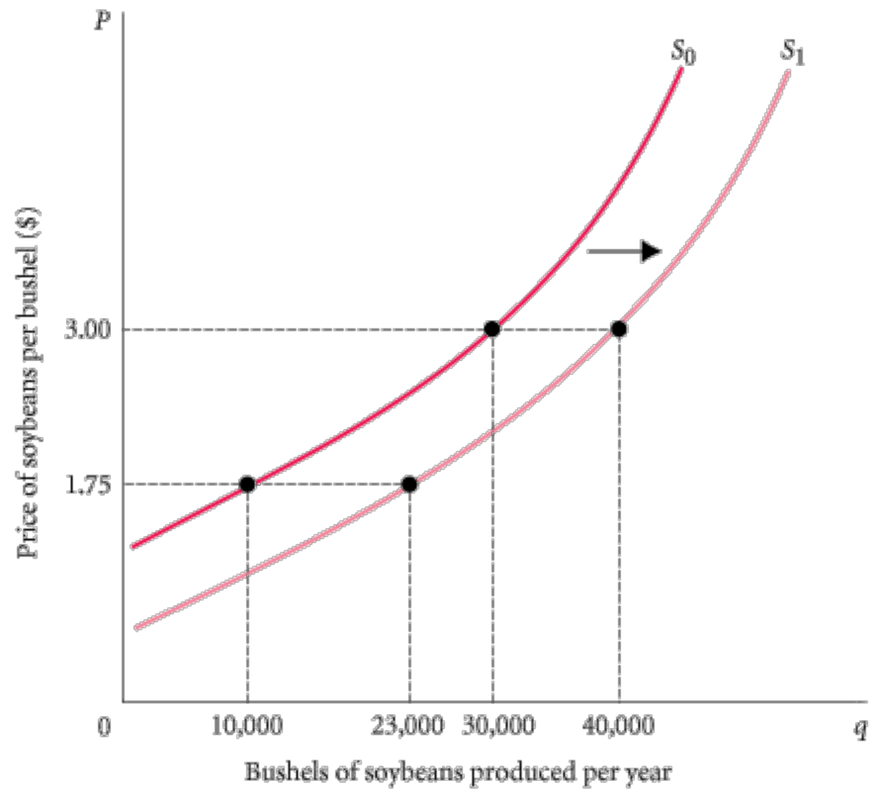
Change in costs, input prices, technology, or prices of related goods and services leads to
→ Change in *supply* (**shift of a supply curve**).

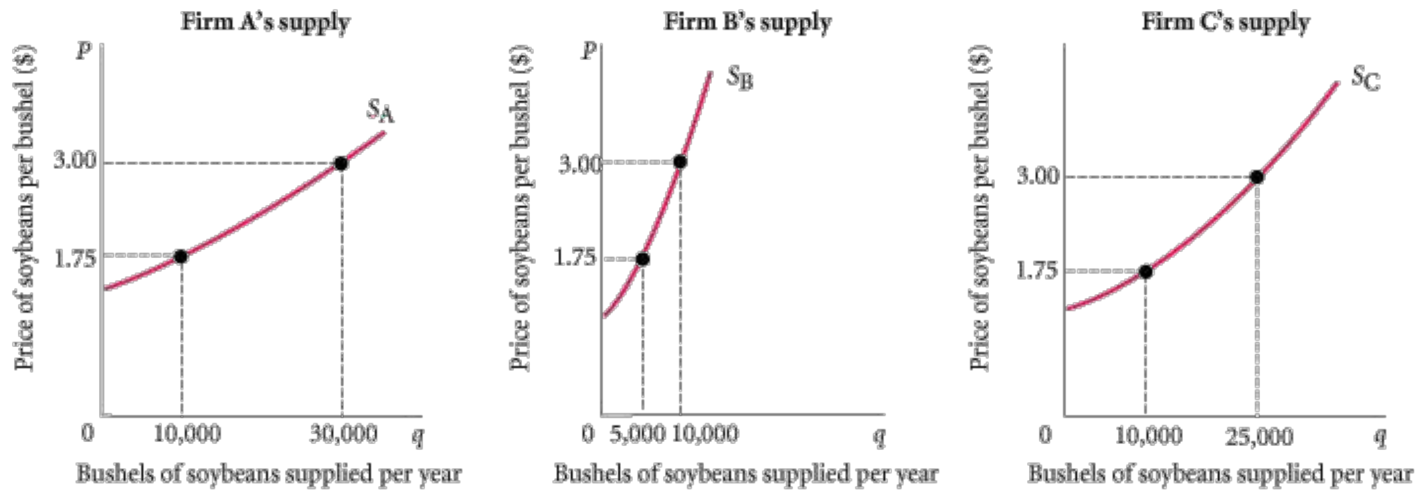
TABLE 3.4 Shift of Supply Schedule for Soybeans following Development of a New Disease-Resistant Seed Strain

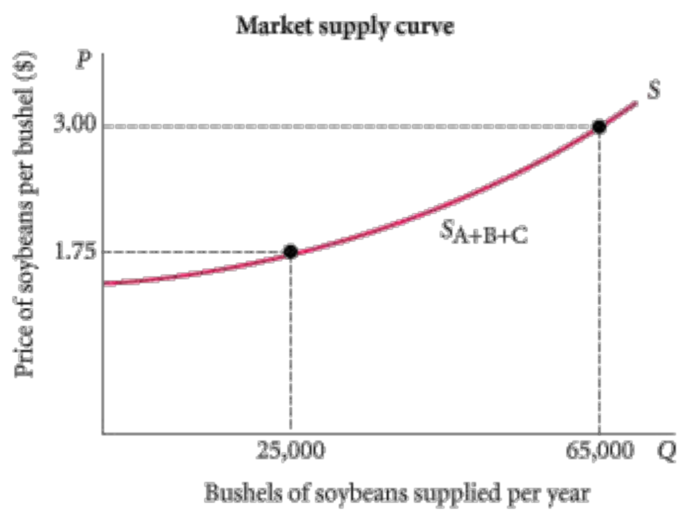
	Schedule S_0	Schedule S_1
Price (per Bushel)	Quantity Supplied (Bushels per Year Using Old Seed)	Quantity Supplied (Bushels per Year Using New Seed)
$1.50	0	5,000
1.75	10,000	23,000
2.25	20,000	33,000
3.00	30,000	40,000
4.00	45,000	54,000
5.00	45,000	54,000

FIGURE 3.7 Shift of the Supply Curve for Soybeans following Development of a New Seed Strain

When the price of a product changes, we move *along* the supply curve for that product; the quantity supplied rises or falls. When any other factor affecting supply changes, the supply curve *shifts*.







Price	Quantity (q) supplied by A	B	C	Total quantity supplied in the market (Q)
$3.00	30,000 +	10,000 +	25,000	= 65,000
1.75	10,000 +	5,000 +	10,000	= 25,000

FIGURE 3.8 Deriving Market Supply from Individual Firm Supply Curves

Total supply in the marketplace is the sum of all the amounts supplied by all the firms selling in the market. It is the sum of all the individual quantities supplied at each price.

Market Equilibrium

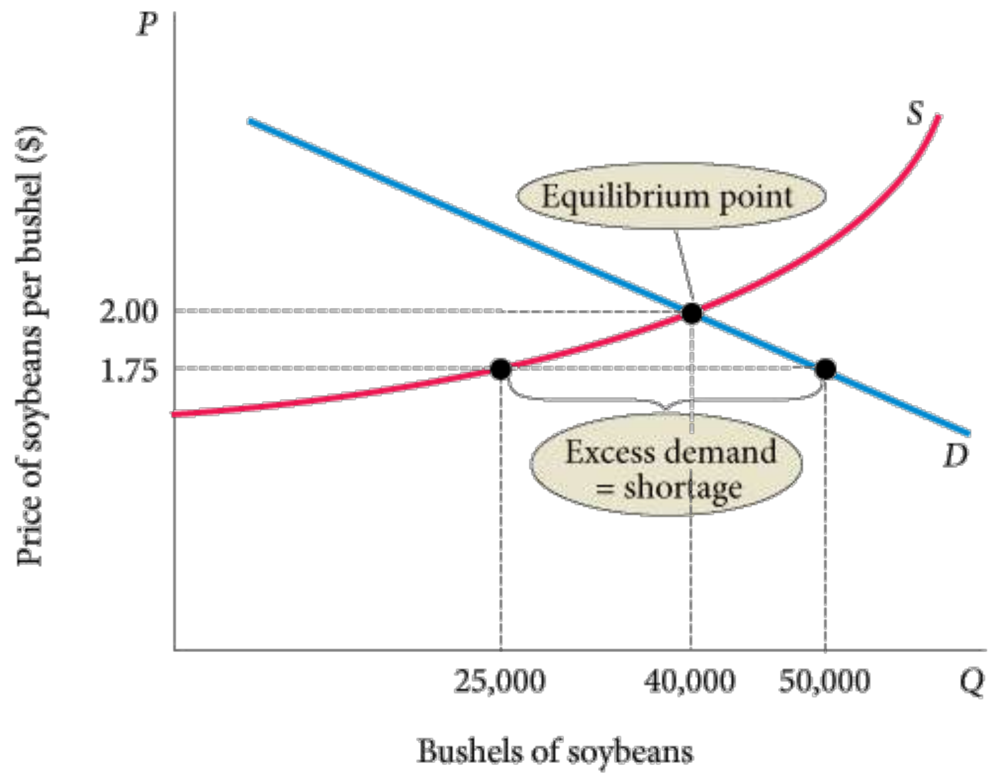
equilibrium The condition that exists when quantity supplied and quantity demanded are equal. At equilibrium, there is no tendency for price to change.

Excess Demand

excess demand *or* shortage The condition that exists when quantity demanded exceeds quantity supplied at the current price.

FIGURE 3.9 Excess Demand, or Shortage

At a price of $1.75 per bushel, quantity demanded exceeds quantity supplied.
When excess *demand* exists, there is a tendency for price to rise.
When quantity demanded equals quantity supplied, excess demand is eliminated and the market is in equilibrium. Here the equilibrium price is $2.00 and the equilibrium quantity is 40,000 bushels.



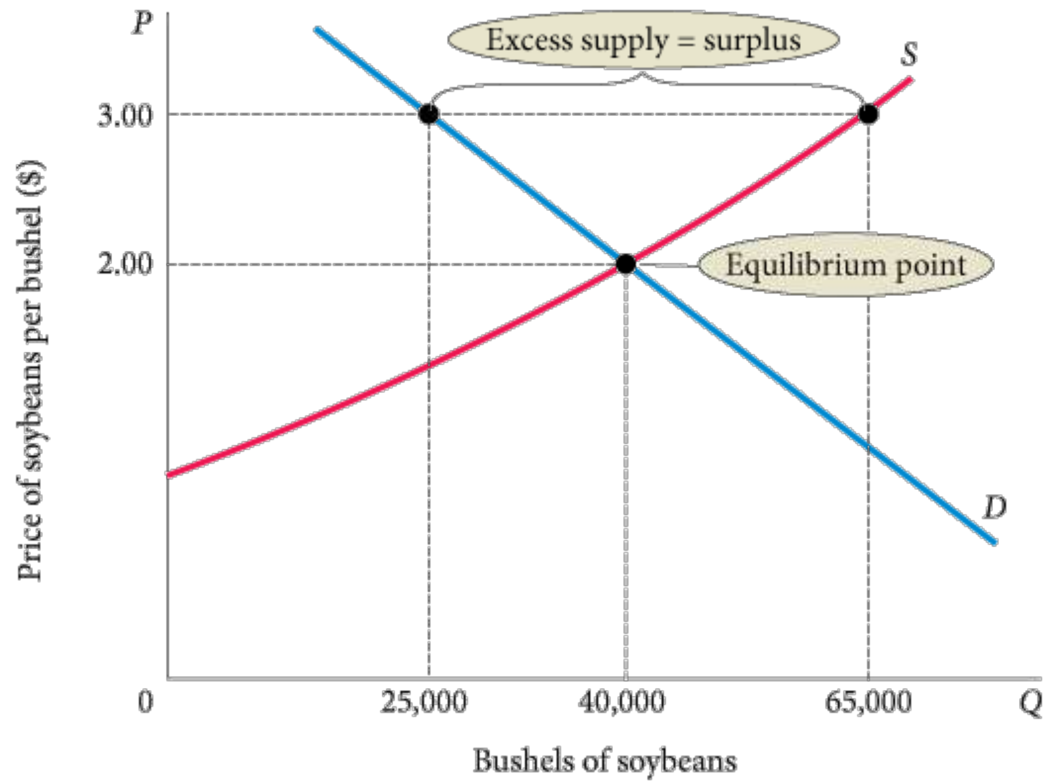
When quantity demanded exceeds quantity supplied, price tends to rise.
When the price in a market rises, quantity demanded falls and quantity supplied rises until an equilibrium is reached at which quantity demanded and quantity supplied are equal.

Excess Supply

excess supply *or* surplus The condition that exists when quantity supplied exceeds quantity demanded at the current price.

FIGURE 3.10 Excess Supply, or Surplus

At a price of $3.00, quantity supplied exceeds quantity demanded by 20,000 bushels. This excess supply will cause the price to fall.



When quantity supplied exceeds quantity demanded at the current price, the price tends to fall. When price falls, quantity supplied is likely to decrease and quantity demanded is likely to increase until an equilibrium price is reached where quantity supplied and quantity demanded are equal.

Changes In Equilibrium

When supply and demand curves shift, the equilibrium price and quantity change.

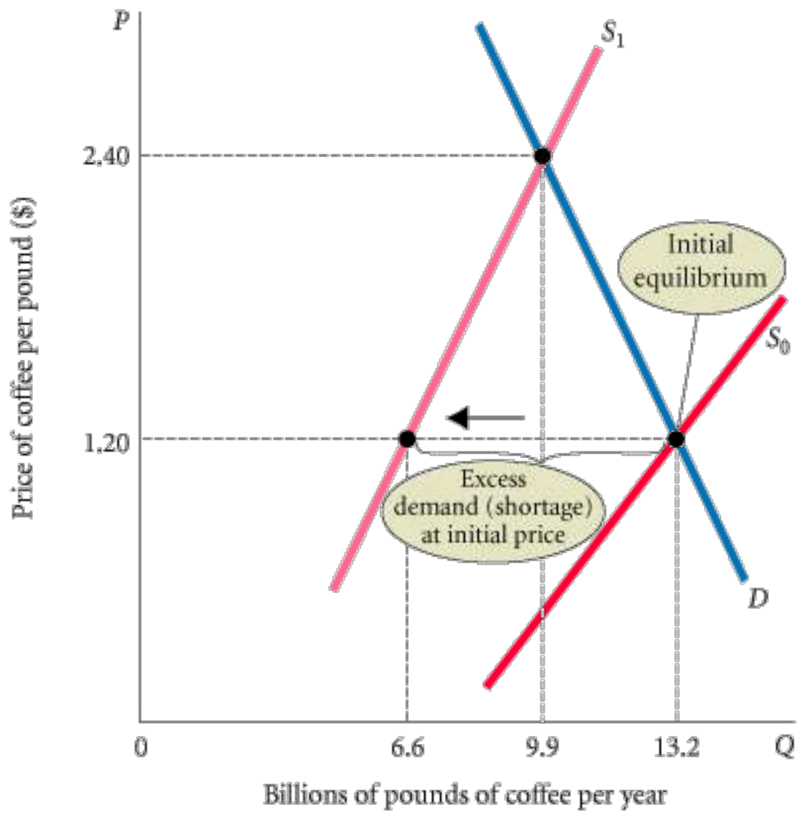


FIGURE 3.11 The Coffee Market: A Shift of Supply and Subsequent Price Adjustment

Before the freeze, the coffee market was in equilibrium at a price of $1.20 per pound.
At that price, quantity demanded equaled quantity supplied.
The freeze shifted the supply curve to the left (from S_0 to S_1), increasing the equilibrium price to $2.40.

ECONOMICS IN PRACTICE

Coffee or Tea?

China is rapidly changing, and tea-drinking habits are no exception. Chinese consumers have discovered coffee!

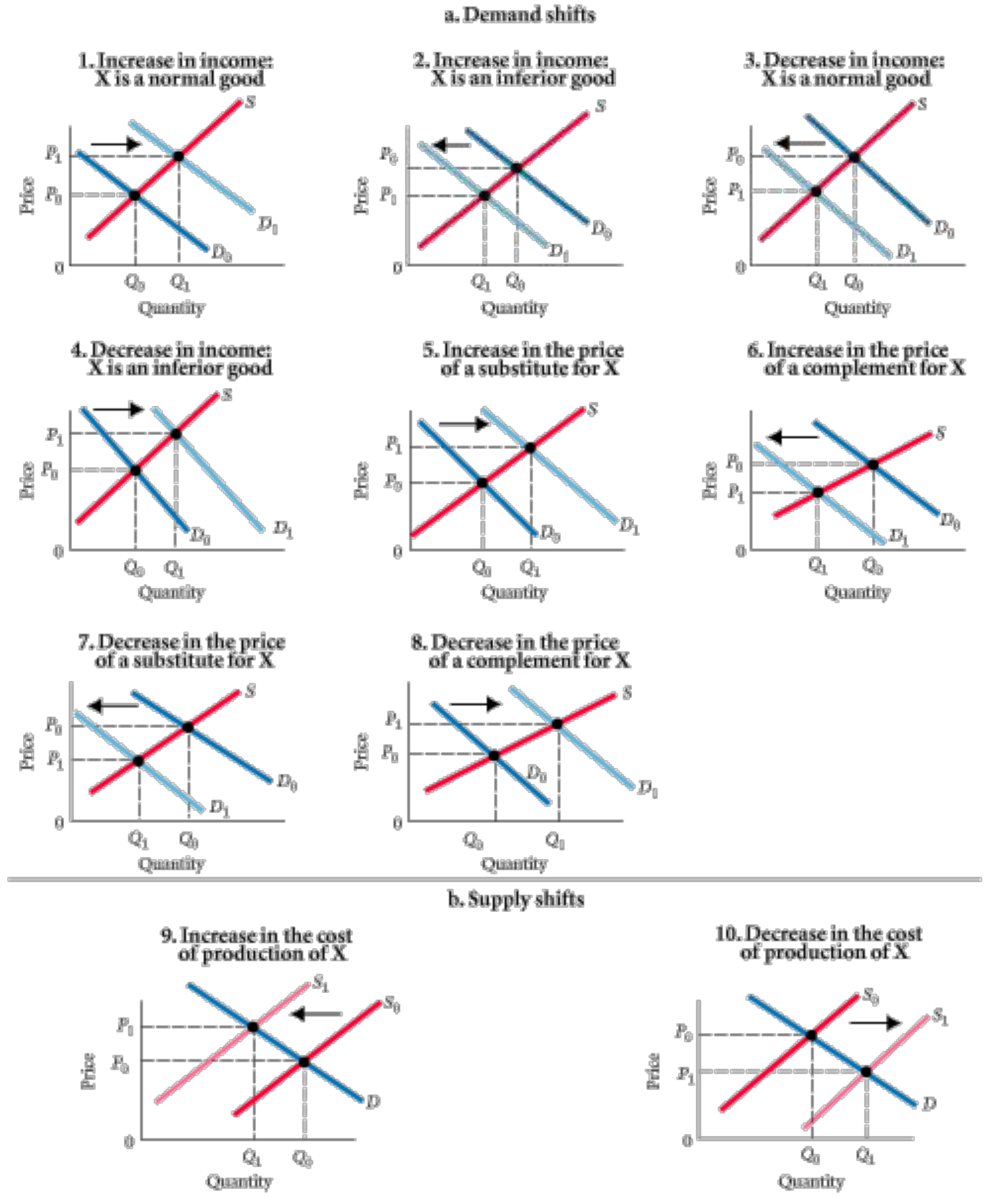
Some observers suggest that the fast pace of current day China is more compatible with coffee drinking than tea. Perhaps coffee drinking is a complement to economic growth?

With new and large populations now interested in coffee, the world demand for coffee shifts rightward. This is good news for coffee growers. As you already know from this chapter, however, how good that news really is from the point of view of coffee prices depends on the supply side as well!

THINKING PRACTICALLY

1. Show in a graph the effect that the growth in China's interest in coffee will likely have on coffee prices? What features of supply determine how big the price increase will be?

FIGURE 3.12 Examples of Supply and Demand Shifts for Product X



Demand and Supply in Product Markets: A Review

Here are some important points to remember about the mechanics of supply and demand in product markets:

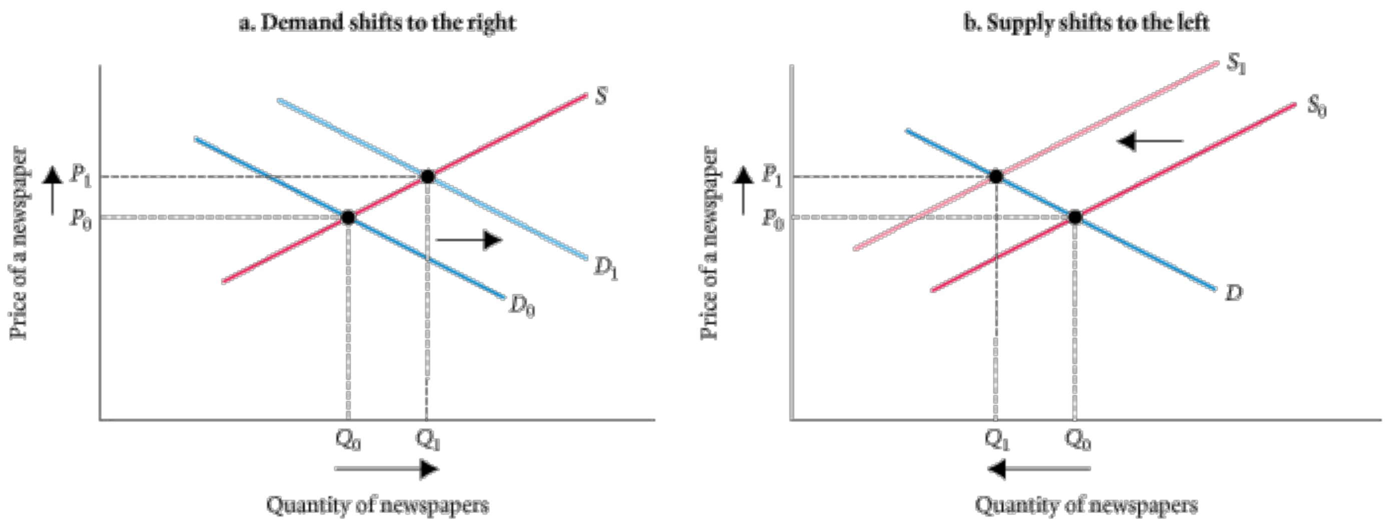
1. A demand curve shows how much of a product a household would buy if it could buy all it wanted at the given price. A supply curve shows how much of a product a firm would supply if it could sell all it wanted at the given price.
2. Quantity demanded and quantity supplied are always per time period—that is, per day, per month, or per year.
3. The demand for a good is determined by price, household income and wealth, prices of other goods and services, tastes and preferences, and expectations.

4. The supply of a good is determined by price, costs of production, and prices of related products. Costs of production are determined by available technologies of production and input prices.
5. Be careful to distinguish between movements along supply and demand curves and shifts of these curves. When the price of a good changes, the quantity of that good demanded or supplied changes—that is, a movement occurs along the curve. When any other factor changes, the curve shifts, or changes position.
6. Market equilibrium exists only when quantity supplied equals quantity demanded at the current price.

ECONOMICS IN PRACTICE

Why Do the Prices of Newspapers Rise?

In 2006, the average price for a daily edition of a Baltimore newspaper was $0.50. In 2007, the average price had risen to $0.75.



Looking Ahead: Markets and the Allocation of Resources

You can already begin to see how markets answer the basic economic questions of what is produced, how it is produced, and who gets what is produced.

- Demand curves reflect what people are willing and able to pay for products; demand curves are influenced by incomes, wealth, preferences, prices of other goods, and expectations.
- Firms in business to make a profit have a good reason to choose the best available technology—lower costs mean higher profits.
- When a good is in short supply, price rises. As it does, those who are willing and able to continue buying do so; others stop buying.

Assignment

Name	**Quantity**	**Maximum price** willing to pay
Mary	1	4
Bob	1	1
Jane	1	5
Ed	1	3
Alice	1	2

Use this data to construct a demand schedule and a demand curve.

Elasticity

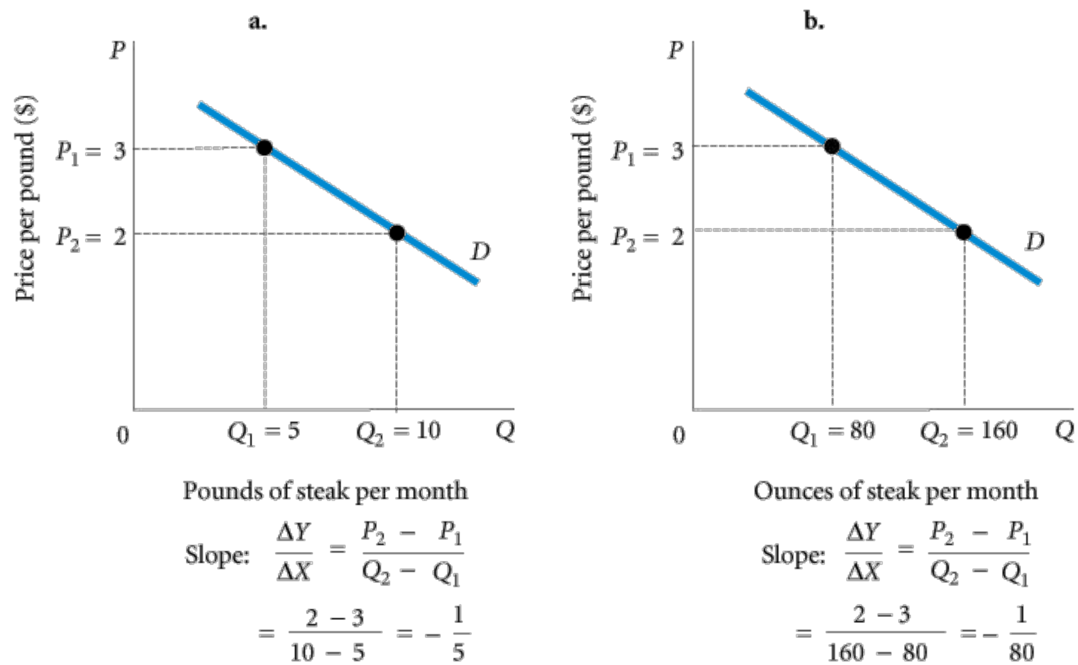
CHAPTER OUTLINE

elasticity A general concept used to quantify the response in one variable when another variable changes.

$$\text{elasticity of } A \text{ with respect to } B = \frac{\%\Delta A}{\%\Delta B}$$

Price Elasticity of Demand

Slope and Elasticity



▲ **FIGURE 5.1 Slope Is Not a Useful Measure of Responsiveness**

Changing the unit of measure from pounds to ounces changes the numerical value of the demand slope dramatically, but the behavior of buyers in the two diagrams is identical.

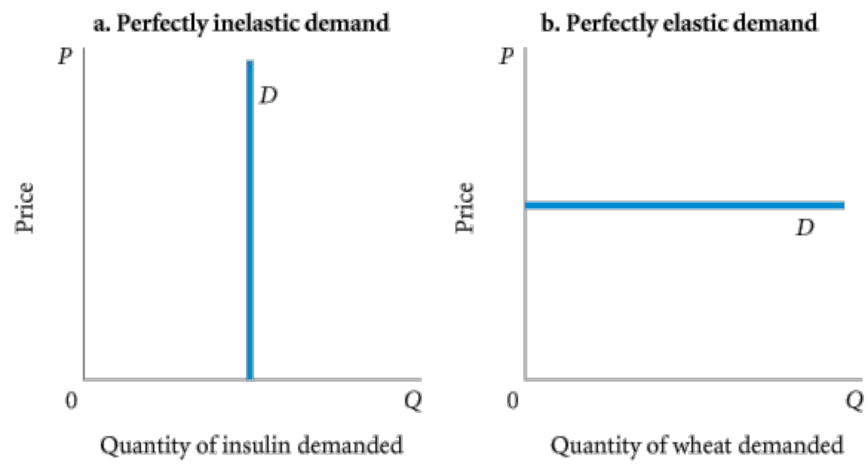
price elasticity of demand The ratio of the percentage of change in quantity demanded to the percentage of change in price; measures the responsiveness of quantity demanded to changes in price.

$$\text{price elasticity of demand} = \frac{\%\ \text{change in quantity demanded}}{\%\ \text{change in price}}$$

Types of Elasticity

perfectly inelastic demand Demand in which quantity demanded does not respond at all to a change in price.

perfectly elastic demand Demand in which quantity drops to zero at the slightest increase in price.



▲ **FIGURE 5.2 Perfectly Inelastic and Perfectly Elastic Demand Curves**

Figure 5.2(a) shows a perfectly inelastic demand curve for insulin.
Price elasticity of demand is zero.
Quantity demanded is fixed; it does not change at all when price changes.
Figure 5.2(b) shows a perfectly elastic demand curve facing a wheat farmer.
A tiny price increase drives the quantity demanded to zero.
In essence, perfectly elastic demand implies that individual producers can sell all they want at the going market price but cannot charge a higher price.

elastic demand A demand relationship in which the percentage change in quantity demanded is larger than the percentage change in price in absolute value (a demand elasticity with an absolute value greater than 1).

inelastic demand Demand that responds somewhat, but not a great deal, to changes in price. Inelastic demand always has a numerical value between zero and 1.

unitary elasticity A demand relationship in which the percentage change in quantity of a product demanded is the same as the percentage change in price in absolute value (a demand elasticity of 1).

A warning:

You must be very careful about signs. Because it is generally understood that demand elasticities are negative (demand curves have a negative slope), they are often reported and discussed without the negative sign.

Calculating Elasticities

Calculating Percentage Changes

To calculate percentage change in quantity demanded using the initial value as the base, the following formula is used:

$$\begin{aligned}\%\ \text{change in quantity demanded} &= \frac{\text{change in quantity demanded}}{Q_1} \times 100\% \\ &= \frac{Q_2 - Q_1}{Q_1} \times 100\%\end{aligned}$$

We can calculate the percentage change in price in a similar way. Once again, let us use the initial value of P—that is, P_1—as the base for calculating the percentage. By using P_1 as the base, the formula for calculating the percentage of change in P is

$$\begin{aligned}\%\text{ change in price} &= \frac{\text{change in price}}{P_1} \times 100\% \\ &= \frac{P_2 - P_1}{P_1} \times 100\%\end{aligned}$$

Elasticity Is a Ratio of Percentages

Once the changes in quantity demanded and price have been converted to percentages, calculating elasticity is a matter of simple division. Recall the formal definition of elasticity:

$$\text{price elasticity of demand} = \frac{\%\ \text{change in quantity demanded}}{\%\ \text{change in price}}$$

The Midpoint Formula

midpoint formula A more precise way of calculating percentages using the value halfway between P_1 and P_2 for the base in calculating the percentage change in price and the value halfway between Q_1 and Q_2 as the base for calculating the percentage change in quantity demanded.

$$\%\ \text{change in quantity demanded} = \frac{\text{change in quantity demanded}}{(Q_1 + Q_2)/2} \times 100\%$$

$$= \frac{Q_2 - Q_1}{(Q_1 + Q_2)/2} \times 100\%$$

Point Elasticity

point elasticity A measure of elasticity that uses the slope measurement.

We have defined elasticity as the percentage change in quantity demanded divided by the percentage change in price. We can write this as

$$\frac{\dfrac{\Delta Q}{Q_1}}{\dfrac{\Delta P}{P_1}}$$

Where Δ denotes a small change and Q_1 and P_1 refer to the original price and quantity demanded.
This can be rearranged and written as

$$\frac{\Delta Q}{\Delta P} \cdot \frac{P_1}{Q_1}$$

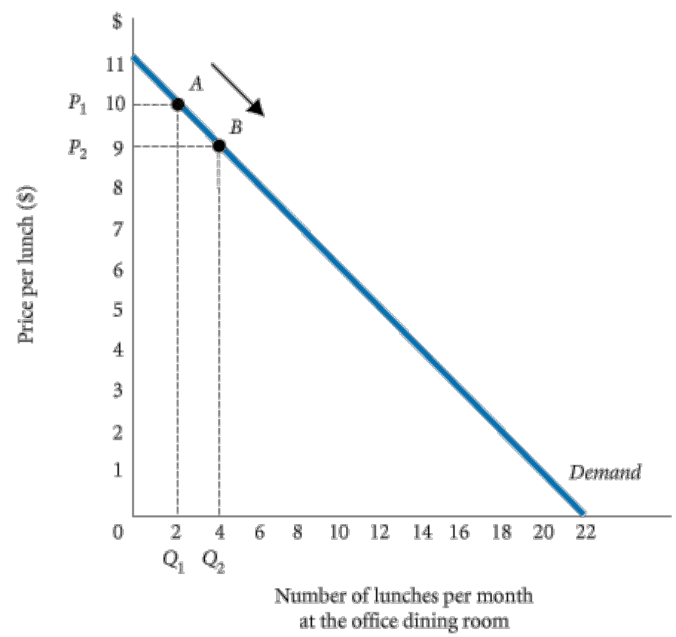
Notice that $\Delta Q/\Delta P$ is the reciprocal of the slope.

Elasticity Changes Along a Straight-Line Demand Curve

TABLE 5.1 Demand Schedule for Office Dining Room Lunches

Price (per Lunch)	Quantity Demanded (Lunches per Month)
$11	0
10	2
9	4
8	6
7	8
6	10
5	12
4	14
3	16
2	18
1	20
0	22

▼ **FIGURE 5.3 Demand Curve for Lunch at the Office Dining Room**



To calculate price elasticity of demand between points *A* and *B* on the demand curve, first calculate the percentage change in quantity demanded:

$$\% \text{ change in quantity demanded} = \frac{4-2}{(2+4)/2} \times 100\% = \frac{2}{3} \times 100\% = 66.7\%$$

Elasticity Changes Along a Straight-Line Demand Curve

TABLE 5.1 Demand Schedule for Office Dining Room Lunches

Price (per Lunch)	Quantity Demanded (Lunches per Month)
$11	0
10	2
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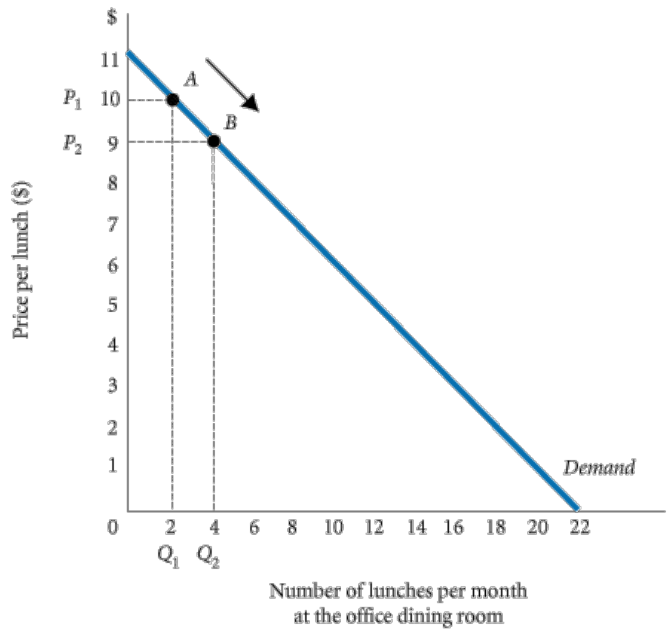
▼ **FIGURE 5.3 Demand Curve for Lunch at the Office Dining Room**



Next, calculate the percentage change in price:

$$\%\,\text{change in price} = \frac{9-10}{(10+9)/2} \times 100\% = \frac{-1}{9.5} \times 100\% = -10.5\%$$

Elasticity Changes Along a Straight-Line Demand Curve

TABLE 5.1 Demand Schedule for Office Dining Room Lunches

Price (per Lunch)	Quantity Demanded (Lunches per Month)
$11	0
10	2
9	4
8	6
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3	16
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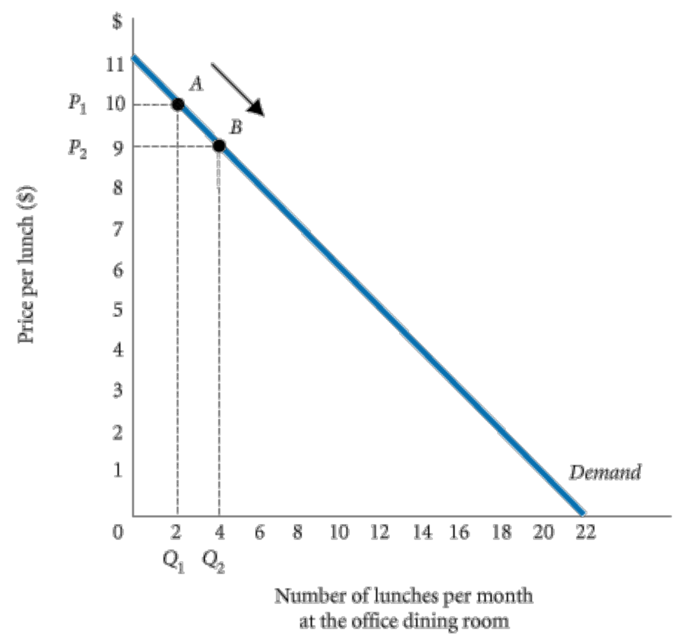
1	20
0	22

▼ FIGURE 5.3 Demand Curve for Lunch at the Office Dining Room



Finally, calculate elasticity:

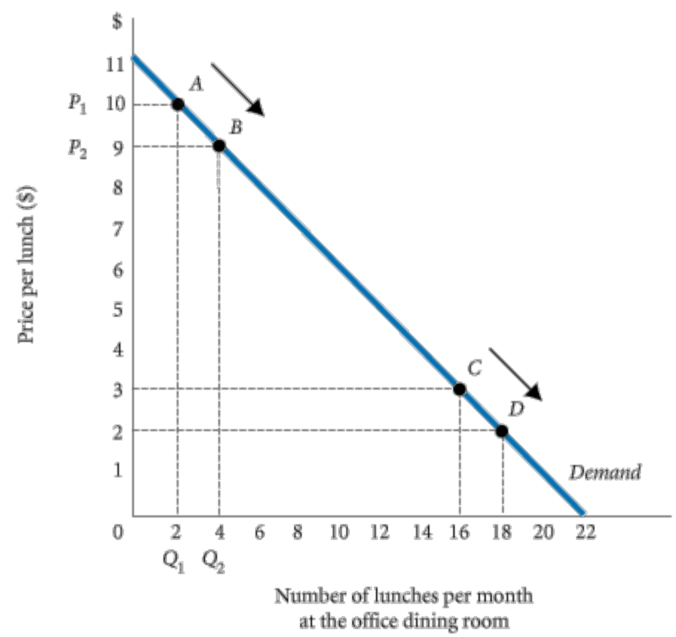
$$\text{elasticity of demand} = \frac{66.7\%}{-10.5\%} = -6.33$$

Elasticity Changes Along a Straight-Line Demand Curve

TABLE 5.1 Demand Schedule for Office Dining Room Lunches

Price (per Lunch)	Quantity Demanded (Lunches per Month)
$11	0
10	2
9	4
8	6
7	8
6	10
5	12
4	14
3	16
2	18
1	20
0	22

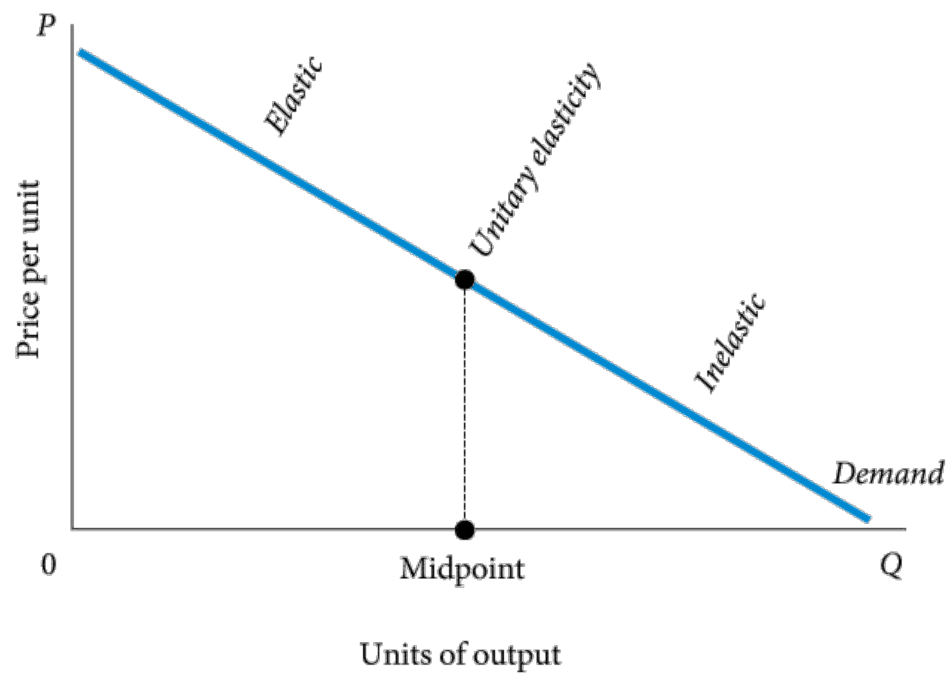
▼ FIGURE 5.3 Demand Curve for Lunch at the Office Dining Room



Between points *A* and *B*, demand is quite elastic at −6.33.

Between points *C* and *D*, demand is quite inelastic at −.294. (You can work this number out for yourself using the midpoint formula.)

▼ **FIGURE 5.4** **Point Elasticity Changes Along a Demand Curve**



Elasticity and Total Revenue

In any market, $P \times Q$ is total revenue (TR) received by producers:

$TR = P \times Q$
total revenue = price × quantity

When price (P) declines, quantity demanded (Q_D) increases. The two factors, P and Q_D, move in opposite directions:

effects of price changes on quantity demanded:

$P \uparrow \rightarrow Q_D \downarrow$
and
$P \downarrow \rightarrow Q_D \uparrow$

Because total revenue is the product of *P* and *Q*, whether *TR* rises or falls in response to a price increase depends on which is bigger: the percentage increase in price or the percentage decrease in quantity demanded.

effect of price increase on a product with inelastic demand:	$\uparrow P \times Q_D \downarrow = TR \uparrow$

If the percentage decline in quantity demanded following a price increase is larger than the percentage increase in price, total revenue will fall.

effect of price increase on a product with elastic demand:	$\uparrow P \times Q_D \downarrow = TR \downarrow$

The opposite is true for a price cut. When demand is elastic, a cut in price increases total revenues:

effect of price cut on a product with elastic demand:	$\downarrow P \times Q_{\mathrm{D}} \uparrow = TR \uparrow$

When demand is inelastic, a cut in price reduces total revenues:

effect of price cut on a product with inelastic demand:	$\downarrow P \times Q_{\mathrm{D}} \uparrow = TR \downarrow$

Other Important Elasticities

Income Elasticity of Demand

income elasticity of demand A measure of the responsiveness of demand to changes in income.

$$\text{income elasticity of demand} = \frac{\%\ \text{change in quantity demanded}}{\%\ \text{change in income}}$$

Cross-Price Elasticity of Demand

cross-price elasticity of demand A measure of the response of the quantity of one good demanded to a change in the price of another good.

$$\text{cross-price elasticity of demand} = \frac{\%\ \text{change in quantity of } Y \text{ demanded}}{\%\ \text{change in price of } X}$$

Elasticity of Supply

elasticity of supply A measure of the response of quantity of a good supplied to a change in price of that good. Likely to be positive in output markets.

$$\text{elasticity of supply} = \frac{\%\text{ change in quantity supplied}}{\%\text{ change in price}}$$

elasticity of labor supply A measure of the response of labor supplied to a change in the price of labor.

$$\text{elasticity of labor supply} = \frac{\%\text{ change in quantity of labor supplied}}{\%\text{ change in the wage rate}}$$

Utility, Indifference Curve and Budget Constraint

The Budget Constraint

budget constraint The limits imposed on household choices by income, wealth, and product prices.

TABLE 6.1 Possible Budget Choices of a Person Earning $1,000 per Month after Taxes

Option	Monthly Rent	Food	Other Expenses	Total	Available?
A	$ 400	$250	$350	$1,000	Yes
B	600	200	200	1,000	Yes
C	700	150	150	1,000	Yes
D	1,000	100	100	1,200	No

choice set *or* **opportunity set** The set of options that is defined and limited by a budget constraint.

Preferences, Tastes, Trade-Offs, and Opportunity Cost

Within the constraints imposed by limited incomes and fixed prices, households are free to choose what they will and will not buy.

Whenever a household makes a choice, it is weighing the good or service that it chooses against all the other things that the same money could buy.

As long as a household faces a limited budget—and all households ultimately do—the real cost of any good or service is the value of the other goods and services that could have been purchased with the same amount of money.

The Equation of the Budget Constraint

In general, the budget constraint can be written

$$P_X X + P_Y Y = I,$$

where P_X = the price of X, X = the quantity of X consumed, P_Y = the price of Y, Y = the quantity of Y consumed, and I = household income.

The Basis of Choice: Utility

utility The satisfaction a product yields.

Diminishing Marginal Utility

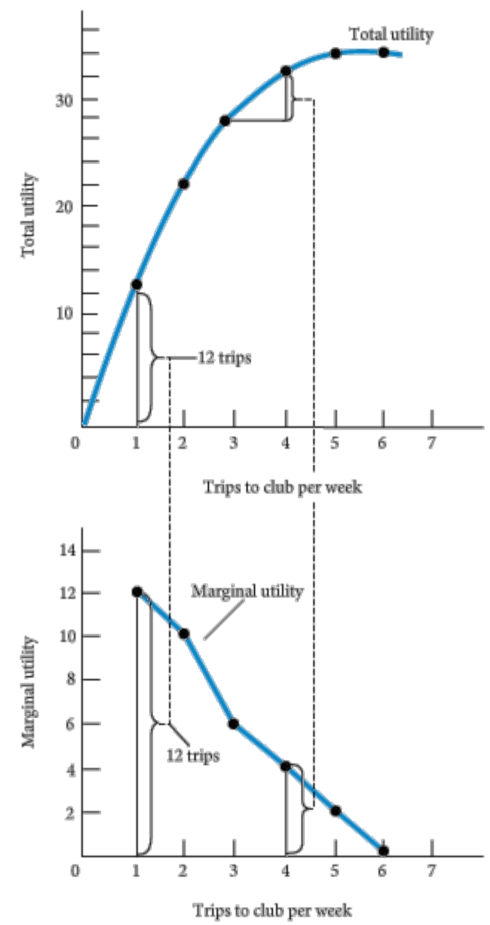
marginal utility (*MU*) The additional satisfaction gained by the consumption or use of *one more* unit of a good or service.

total utility The total amount of satisfaction obtained from consumption of a good or service.

law of diminishing marginal utility The more of any one good consumed in a given period, the less satisfaction (utility) generated by consuming each additional (marginal) unit of the same good.

TABLE 6.2 Total Utility and Marginal Utility of Trips to the Club per Week

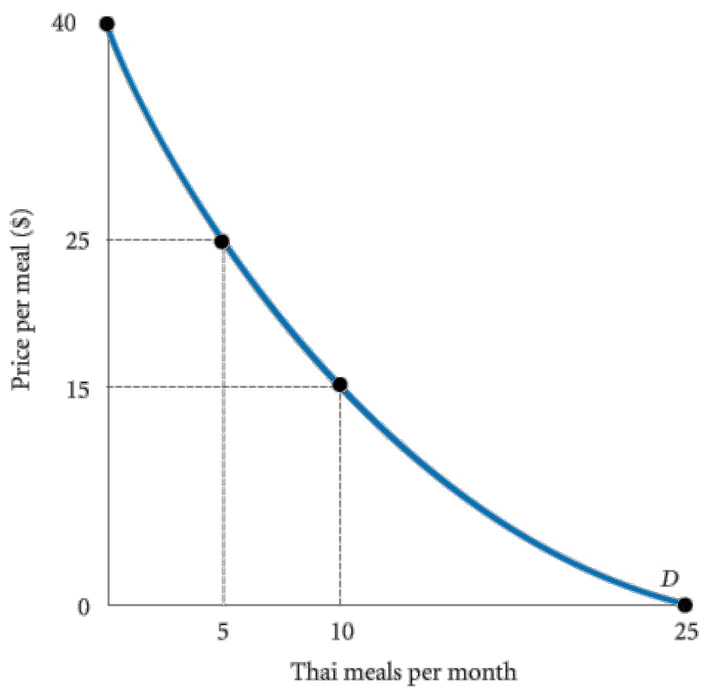
Trips to Club	Total Utility	Marginal Utility
1	12	12
2	22	10
3	28	6
4	32	4
5	34	2
6	34	0



▶ **FIGURE 6.3 Graphs of Frank's Total and Marginal Utility**

Marginal utility is the additional utility gained by consuming one additional unit of a commodity. When marginal utility is zero, total utility stops rising.

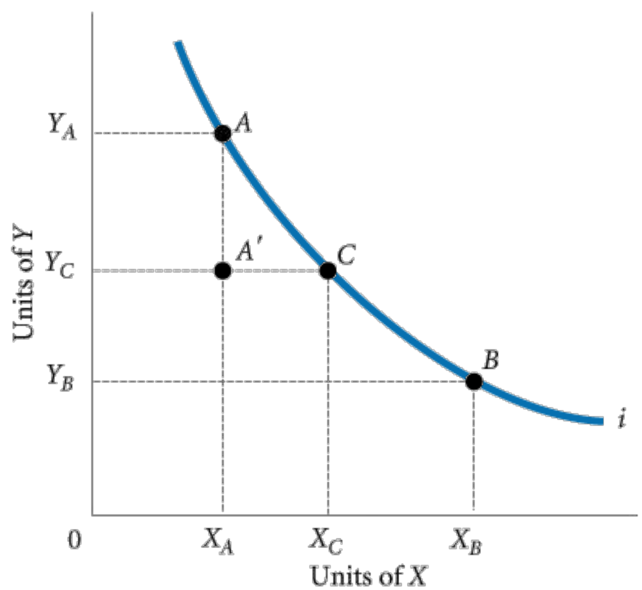
Diminishing Marginal Utility and Downward-Sloping Demand



◄ **FIGURE 6.4 Diminishing Marginal Utility and Downward-Sloping Demand**

At a price of $40, the utility gained from even the first Thai meal is not worth the price.
However, a lower price of $25 lures Ann and Tom into the Thai restaurant 5 times a month. (The utility from the sixth meal is not worth $25.)
If the price is $15, Ann and Tom will eat Thai meals 10 times a month—until the marginal utility of a Thai meal drops below the utility they could gain from spending $15 on other goods.
At 25 meals a month, they cannot tolerate the thought of another Thai meal even if it is free.

Deriving Indifference Curves



◄ **FIGURE 6A.1 An Indifference Curve**

An indifference curve is a set of points, each representing a combination of some amount of good *X* and some amount of good *Y*, that all yield the same amount of total utility.

The consumer depicted here is indifferent between bundles *A* and *B*, *B* and *C*, and *A* and *C*. Because "more is better," our consumer is unequivocally worse off at *A′* than at *A*.

Properties of Indifference Curves

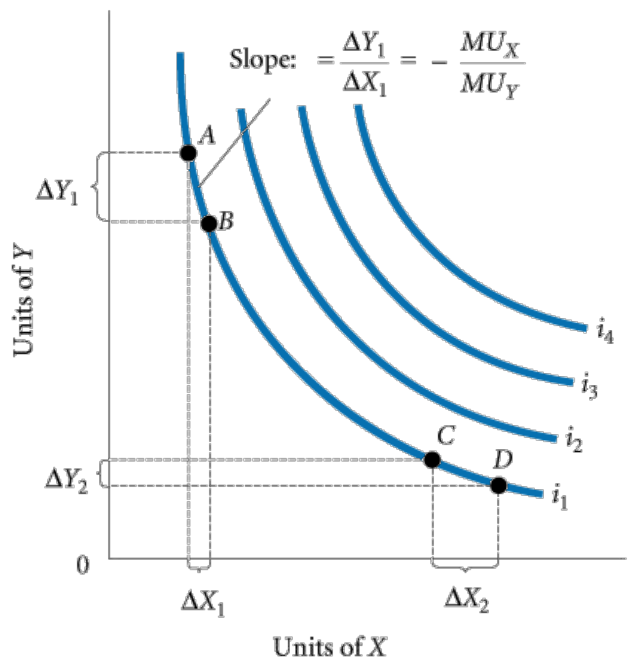


FIGURE 6A.2 A Preference Map: A Family of Indifference Curves
Each consumer has a unique family of indifference curves called a preference map. Higher indifference curves represent higher levels of total utility.

$$MU_X \cdot \Delta X = -(MU_Y \cdot \Delta Y)$$

When we divide both sides by MU_Y and by ΔX, we obtain

$$\frac{\Delta Y}{\Delta X} = -\left(\frac{MU_X}{MU_Y}\right)$$

The slope of an indifference curve is the ratio of the marginal utility of *X* to the marginal utility of *Y*, and it is negative.

Consumer Choice

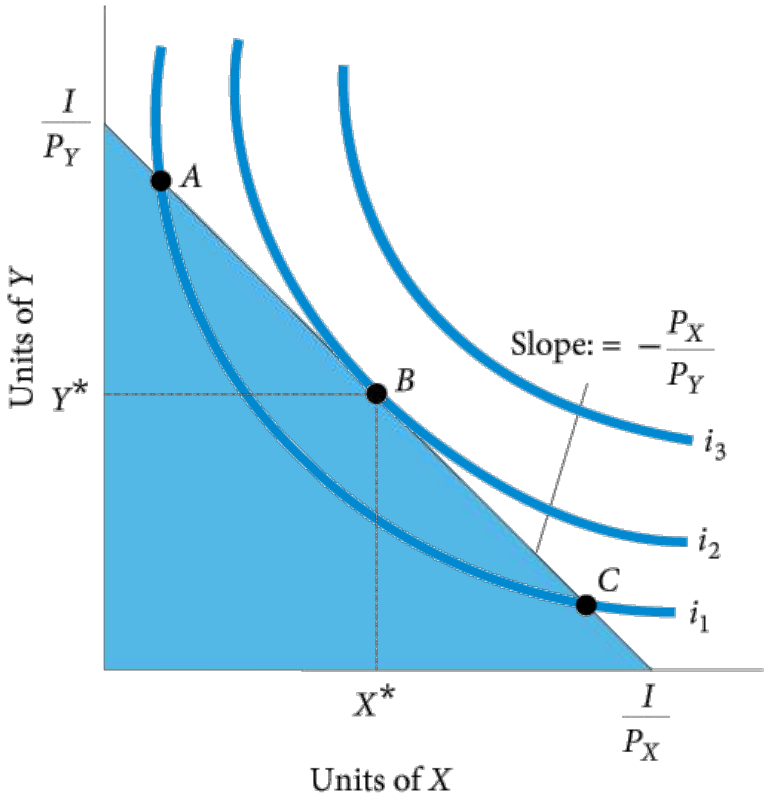


FIGURE 6A.3 Consumer Utility-Maximizing Equilibrium

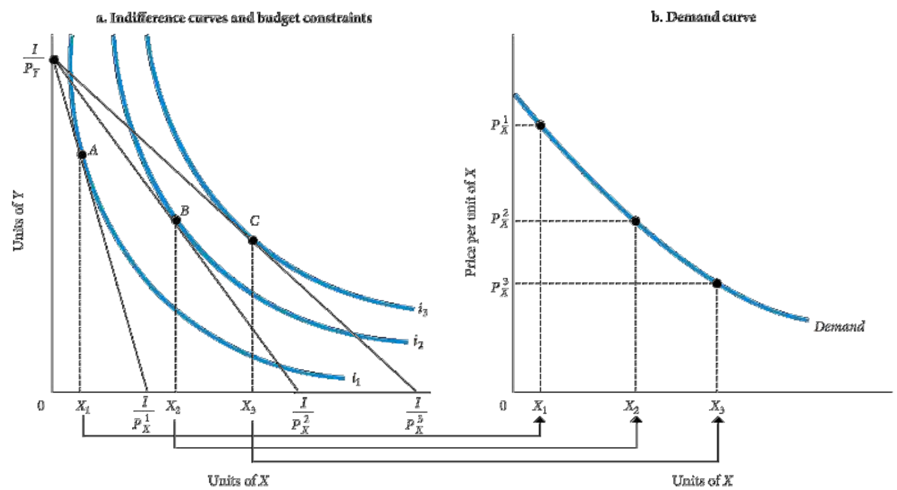
Consumers will choose the combination of *X* and *Y* that maximizes total utility. Graphically, the consumer will move along the budget constraint until the highest possible indifference curve is reached. At that point, the budget constraint and the indifference curve are tangent. This point of tangency occurs at *X** and *Y** (point *B*).

$$\underbrace{-\frac{MU_X}{MU_Y}}_{\text{slope of indifference curve}} = \underbrace{-\frac{P_X}{P_Y}}_{\text{slope of budget constraint}}$$

By multiplying both sides of this equation by MU_Y and dividing both sides by P_X, we can rewrite this utility-maximizing rule as

$$\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y}$$

Deriving a Demand Curve from Indifference Curves and Budget Constraints



▲ **FIGURE 6A.4 Deriving a Demand Curve from Indifference Curves and Budget Constraint**

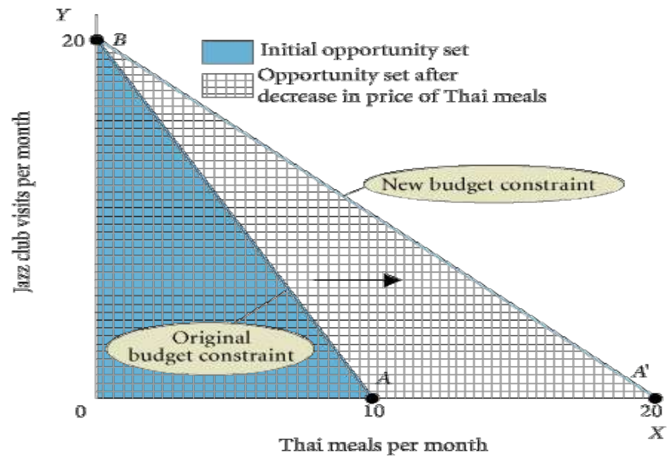
Indifference curves are labeled i_1, i_2, and i_3; budget constraints are shown by the three diagonal lines from I/P_Y to I/P^1_X, I/P^2_X and I/P^3_X. Lowering the price of *X* from P^1_X to P^2_X and then to P^3_X swivels the budget constraint to the right. At each price, there is a different utility-maximizing combination of *X* and *Y*. Utility is maximized at point *A* on i_1, point *B* on i_2, and point *C* on i_3.
Plotting the three prices against the quantities of *X* chosen results in a standard downward-sloping demand curve.

A Question

- Suppose the price of good X, Px = 50 and price of Y, Py = 150, the total income is 1000 which you have planned to spend on X and Y.
- You have already bought 8 units of X. How many units of Y can you buy?

Budget Constraints Change When Prices Rise or Fall

When the price of a good decreases, the budget constraint swivels to the right, increasing the opportunities available and expanding choice.



The Utility-Maximizing Rule

In general, utility-maximizing consumers spread out their expenditures until the following condition holds:

$$\text{utility-maximizing rule: } \frac{MU_X}{P_X} = \frac{MU_Y}{P_Y} \text{ for all goods,}$$

where MU_X is the marginal utility derived from the last unit of X consumed, MU_Y is the marginal utility derived from the last unit of Y consumed, P_X is the price per unit of X, and P_Y is the price per unit of Y.

utility-maximizing rule Equating the ratio of the marginal utility of a good to its price for all goods.

diamond/water paradox A paradox stating that (1) the things with the greatest value in use frequently have little or no value in exchange and (2) the things with the greatest value in exchange frequently have little or no value in use.

Price Change: Income and Substitution Effects

THE IMPACT OF A PRICE CHANGE

- **Economists often separate the impact of a price change into two components:**
  - **the substitution effect; and**
  - **the income effect.**

THE IMPACT OF A PRICE CHANGE

- The substitution effect involves the substitution of good x_1 for good x_2 or vice-versa due to a change in relative prices of the two goods.
- The income effect results from an increase or decrease in the consumer's real income or purchasing power as a result of the price change.
- The sum of these two effects is called the price effect.

THE IMPACT OF A PRICE CHANGE

- **The decomposition of the price effect into the income and substitution effect can be done in several ways**
- **There are two main methods:**

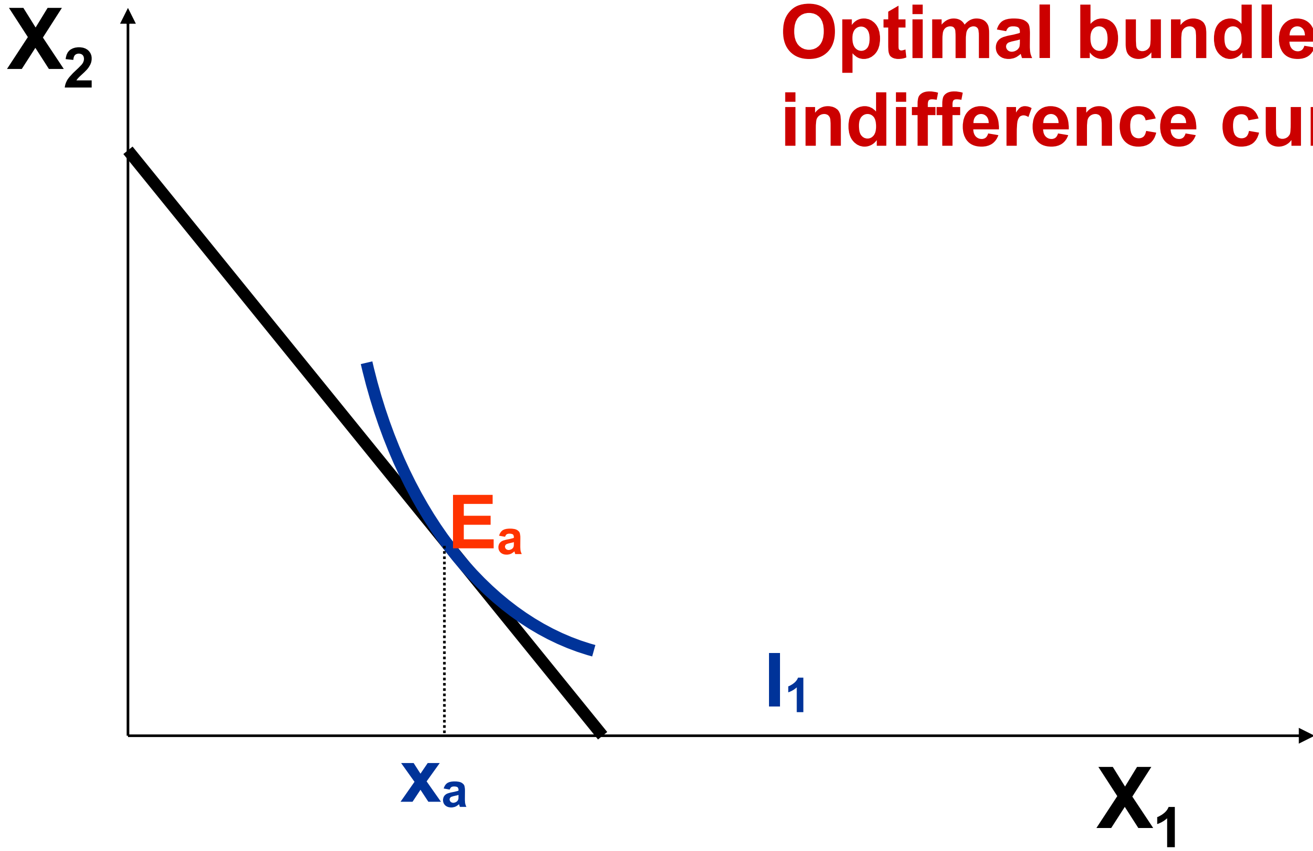
(i) The Hicksian method; and

(ii) The Slutsky method

THE HICKSIAN METHOD

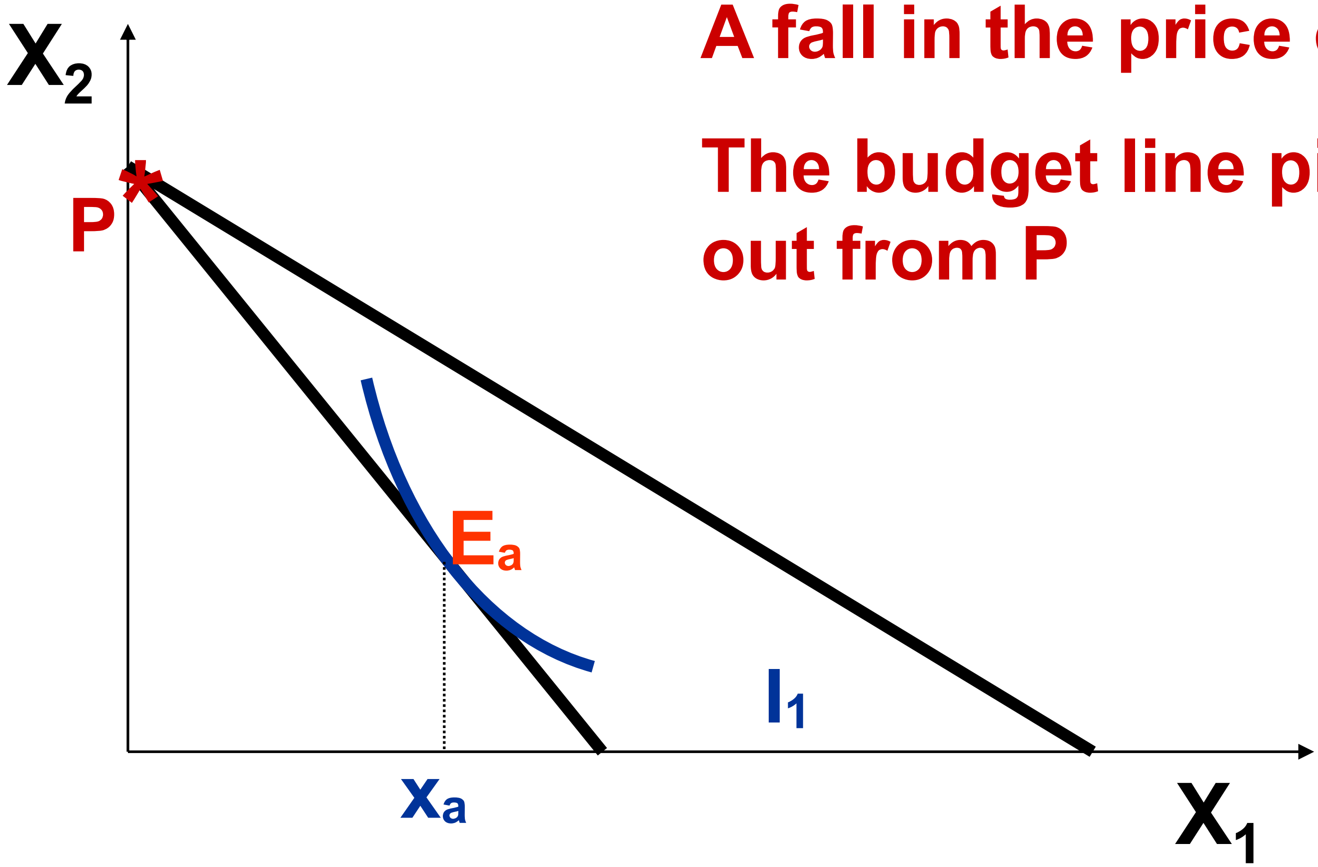
- Sir John R.Hicks (1904-1989)
- Awarded the Nobel Laureate in Economics (with Kenneth J. Arrrow) in 1972 for work on general equilibrium theory and welfare economics.

THE HICKSIAN METHOD



Optimal bundle is E_a, on indifference curve I_1.

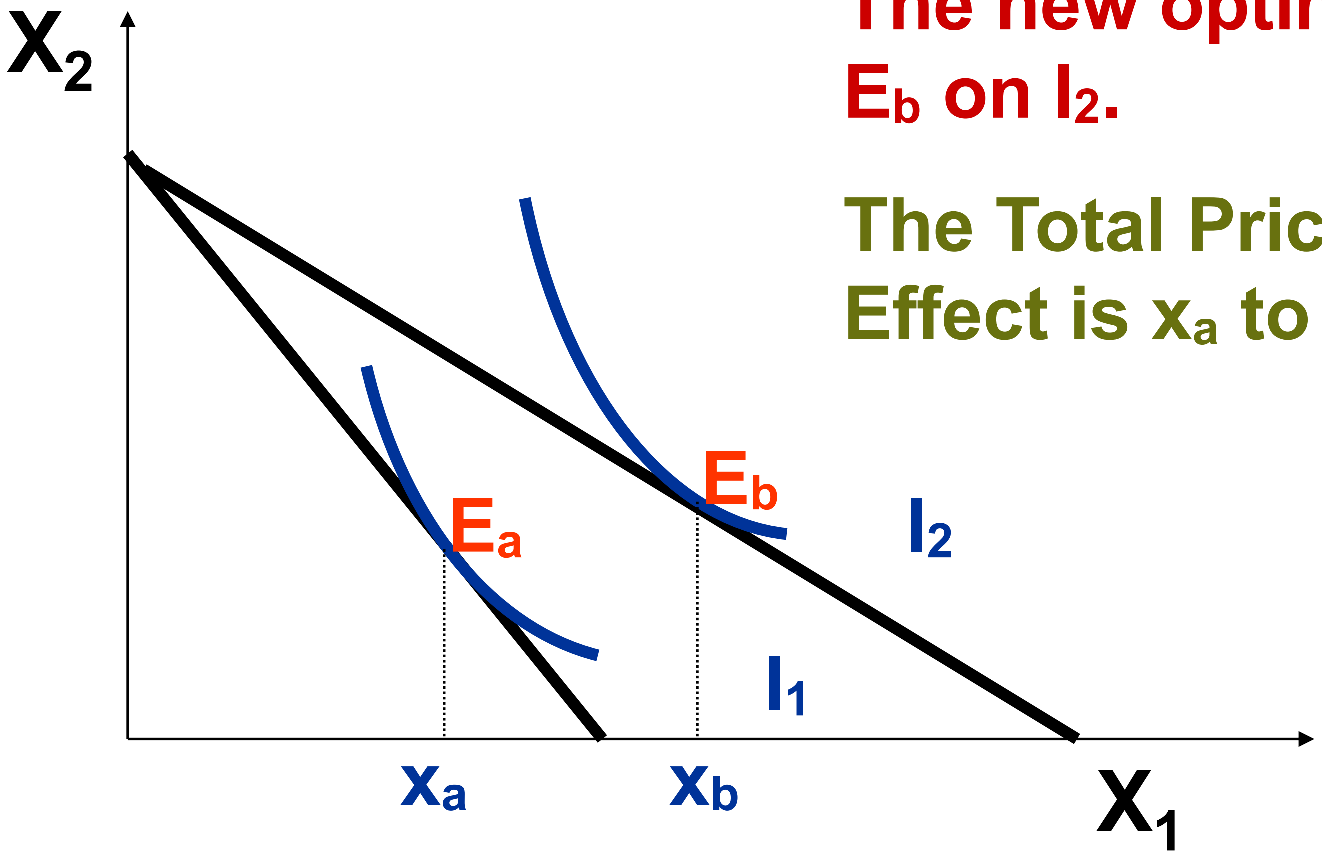
THE HICKSIAN METHOD



A fall in the price of X_1

The budget line pivots out from P

THE HICKSIAN METHOD



The new optimum is E_b on I_2.

The Total Price Effect is x_a to x_b

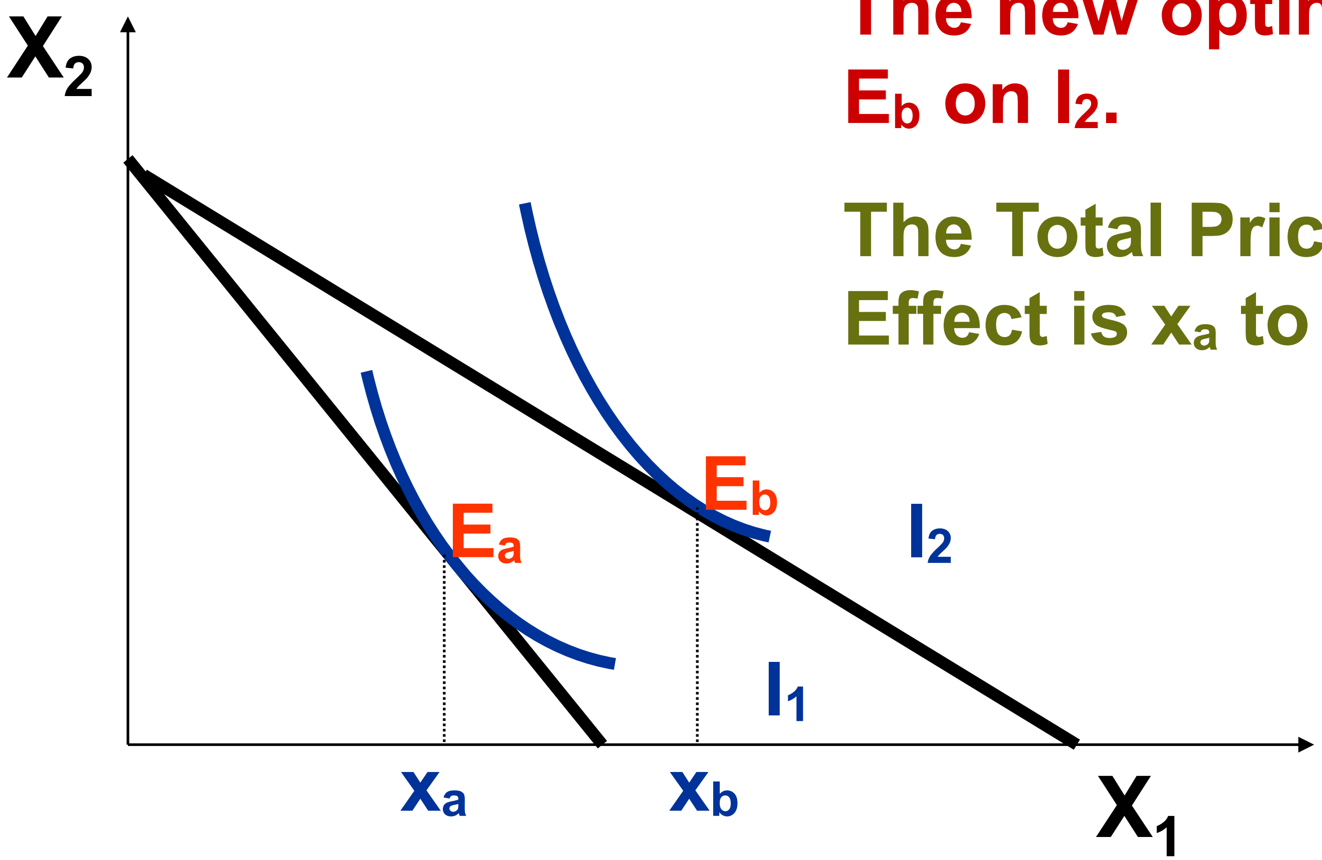
THE HICKSIAN METHOD

- **To isolate the substitution effect we ask….**

  “what would the consumer’s optimal bundle be if s/he faced the new lower price for X_1 but experienced no change in real income?”

- **This amounts to returning the consumer to the original indifference curve (I_1)**

THE HICKSIAN METHOD

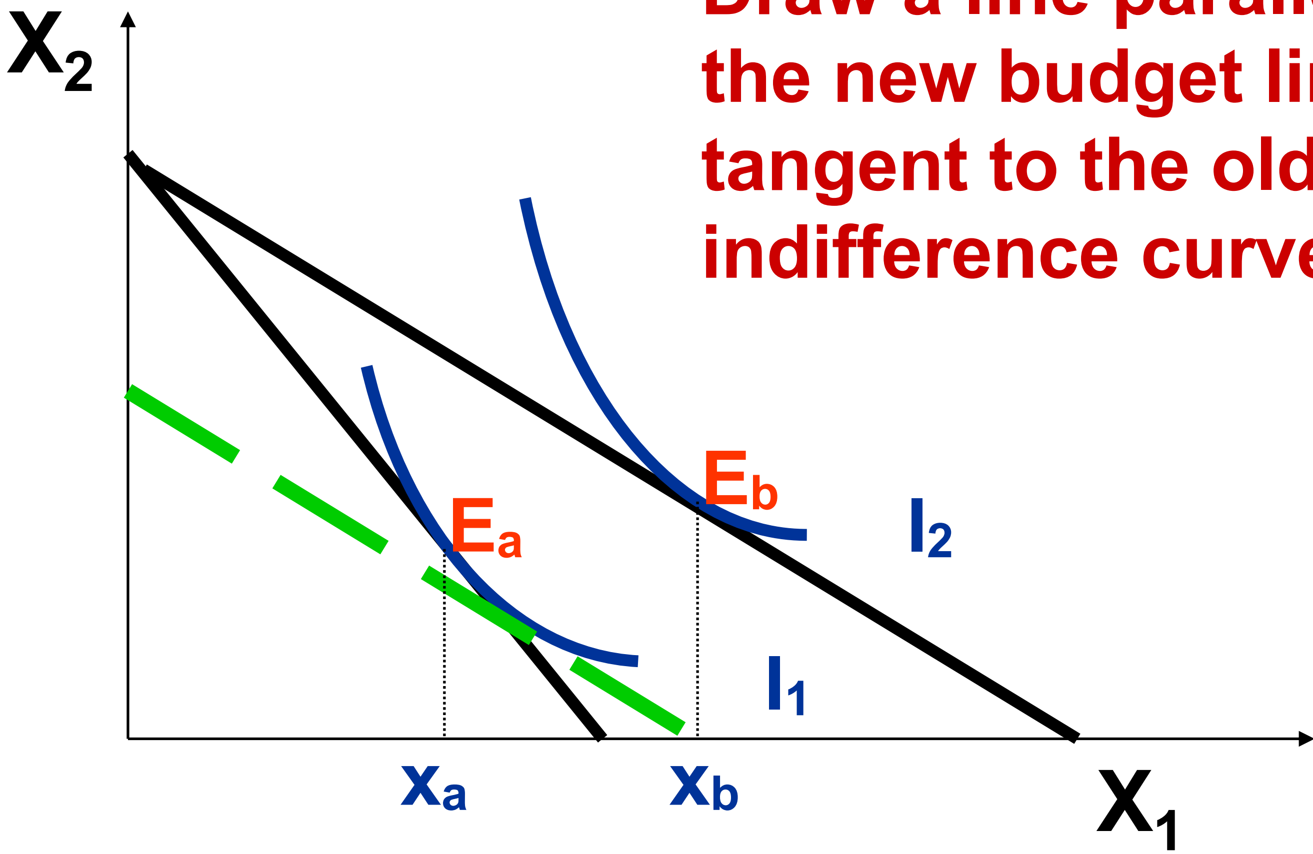


The new optimum is E_b on I_2.

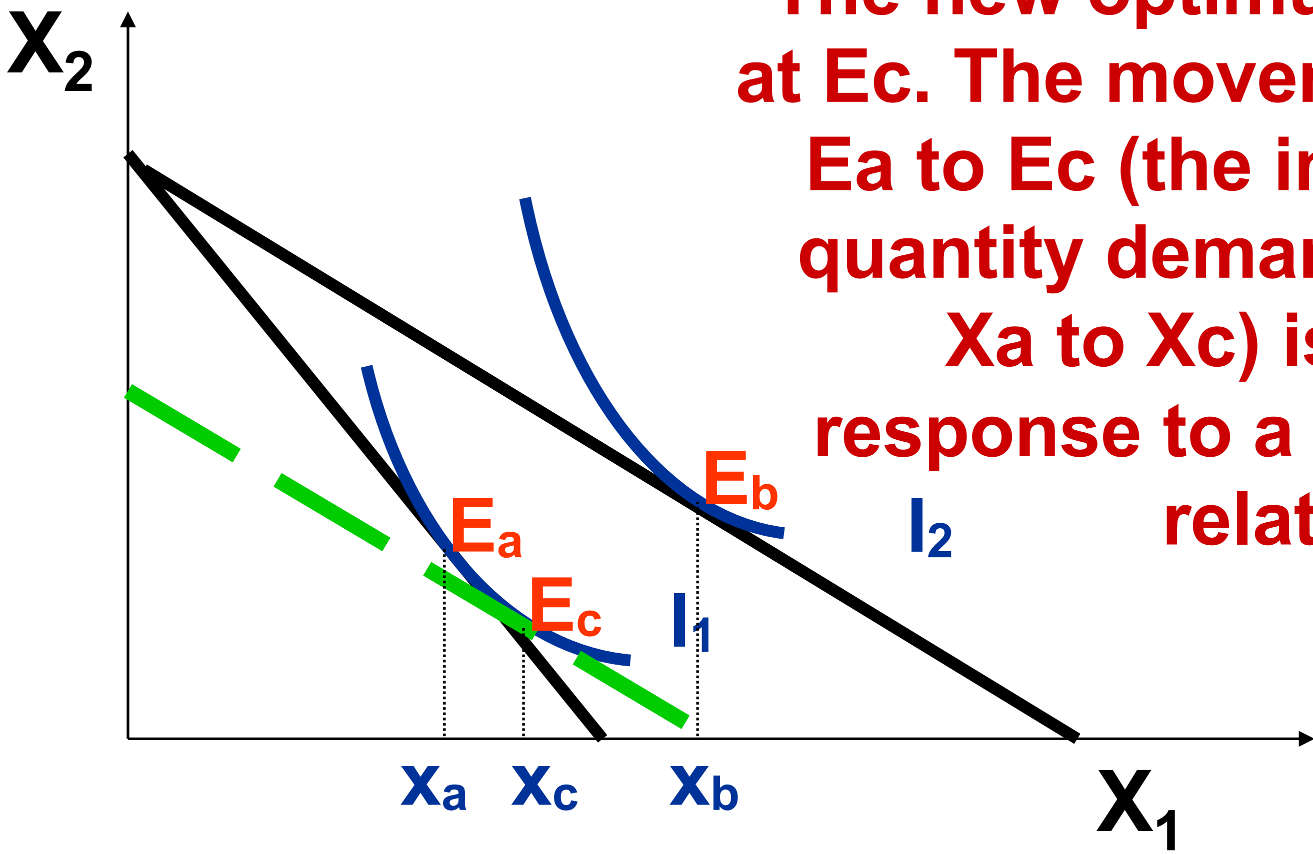
The Total Price Effect is x_a to x_b

THE HICKSIAN METHOD

Draw a line parallel to the new budget line and tangent to the old indifference curve

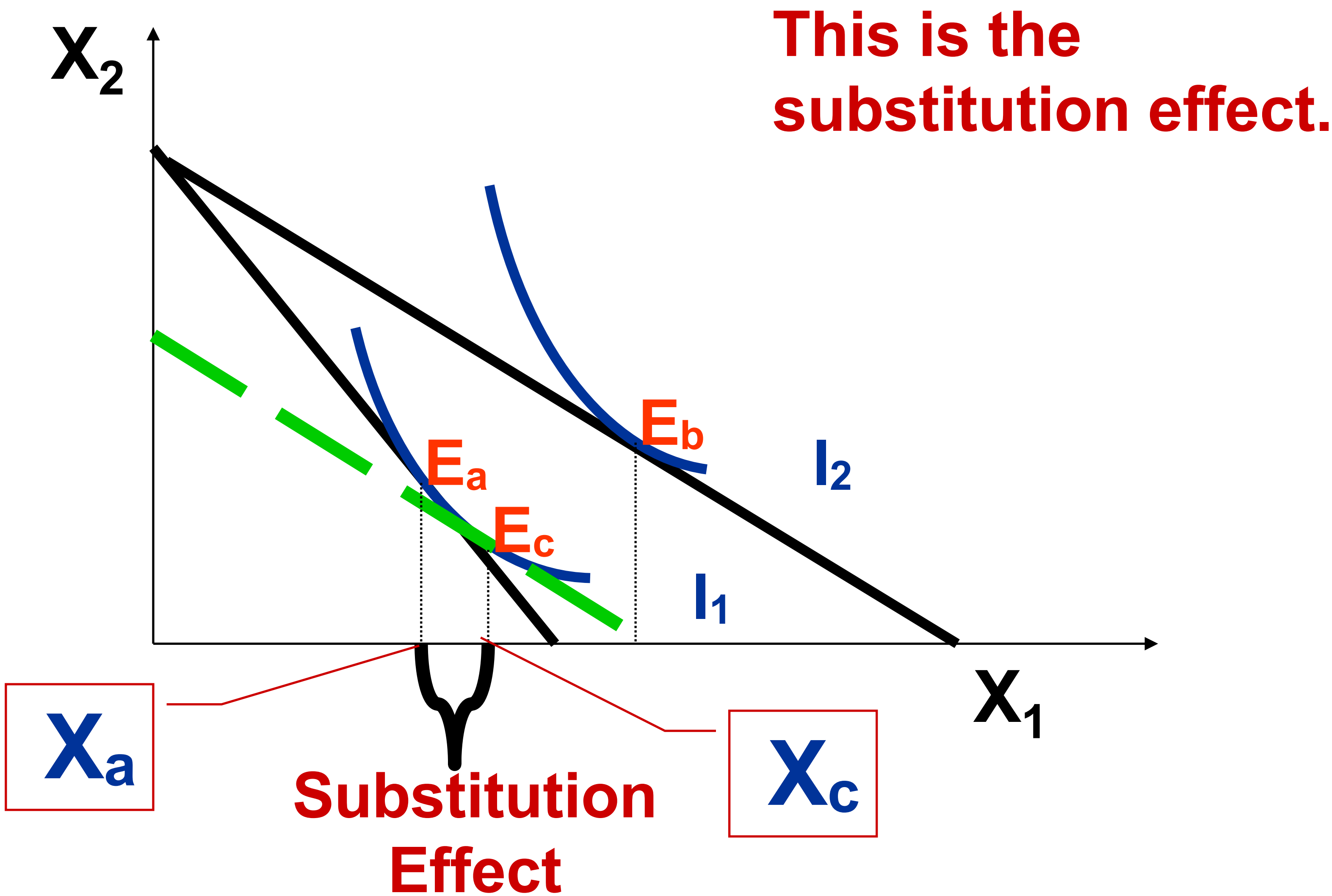


THE HICKSIAN METHOD



The new optimum on I_1 is at Ec. The movement from Ea to Ec (the increase in quantity demanded from Xa to Xc) is solely in response to a change in relative prices

THE HICKSIAN METHOD

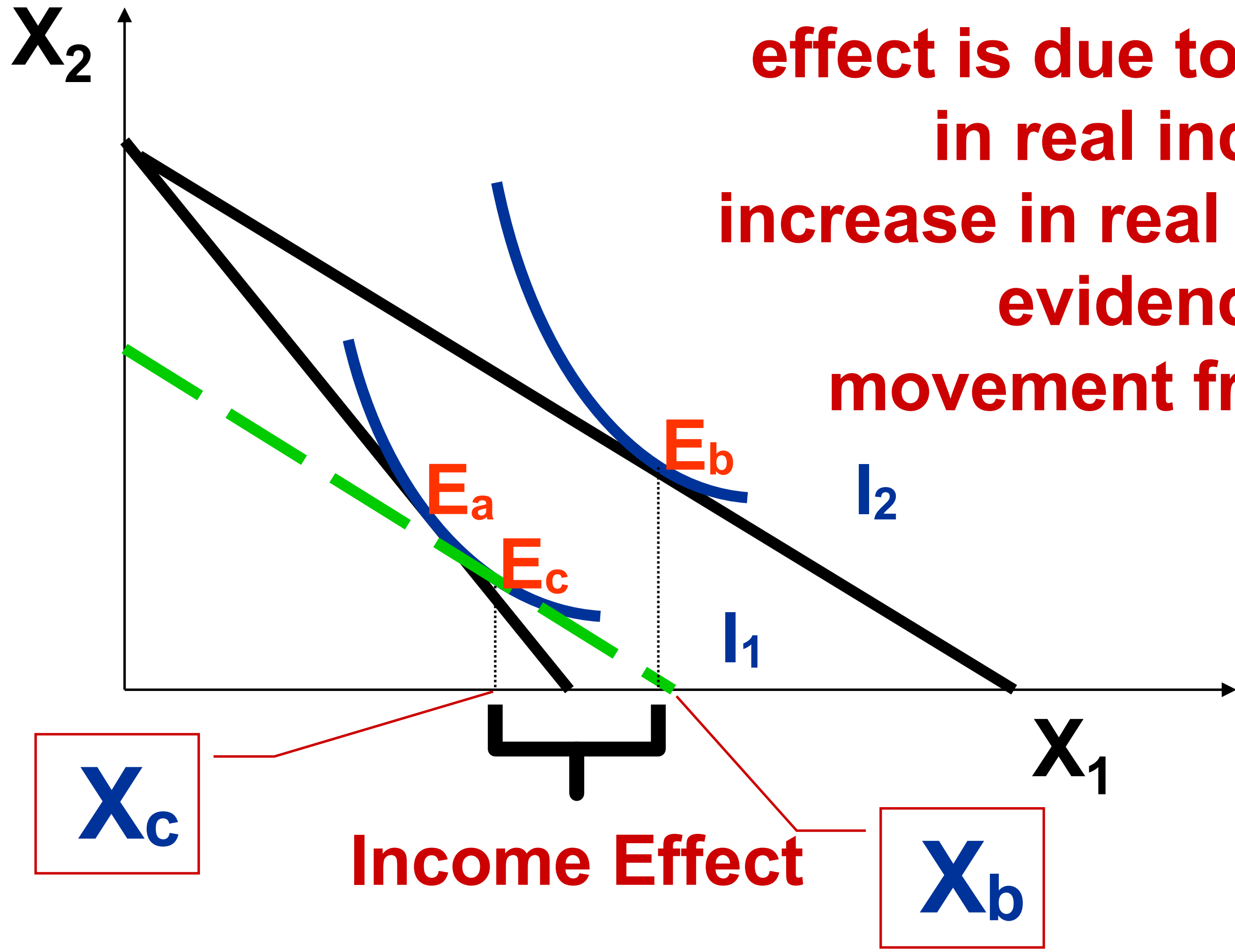


THE HICKSIAN METHOD

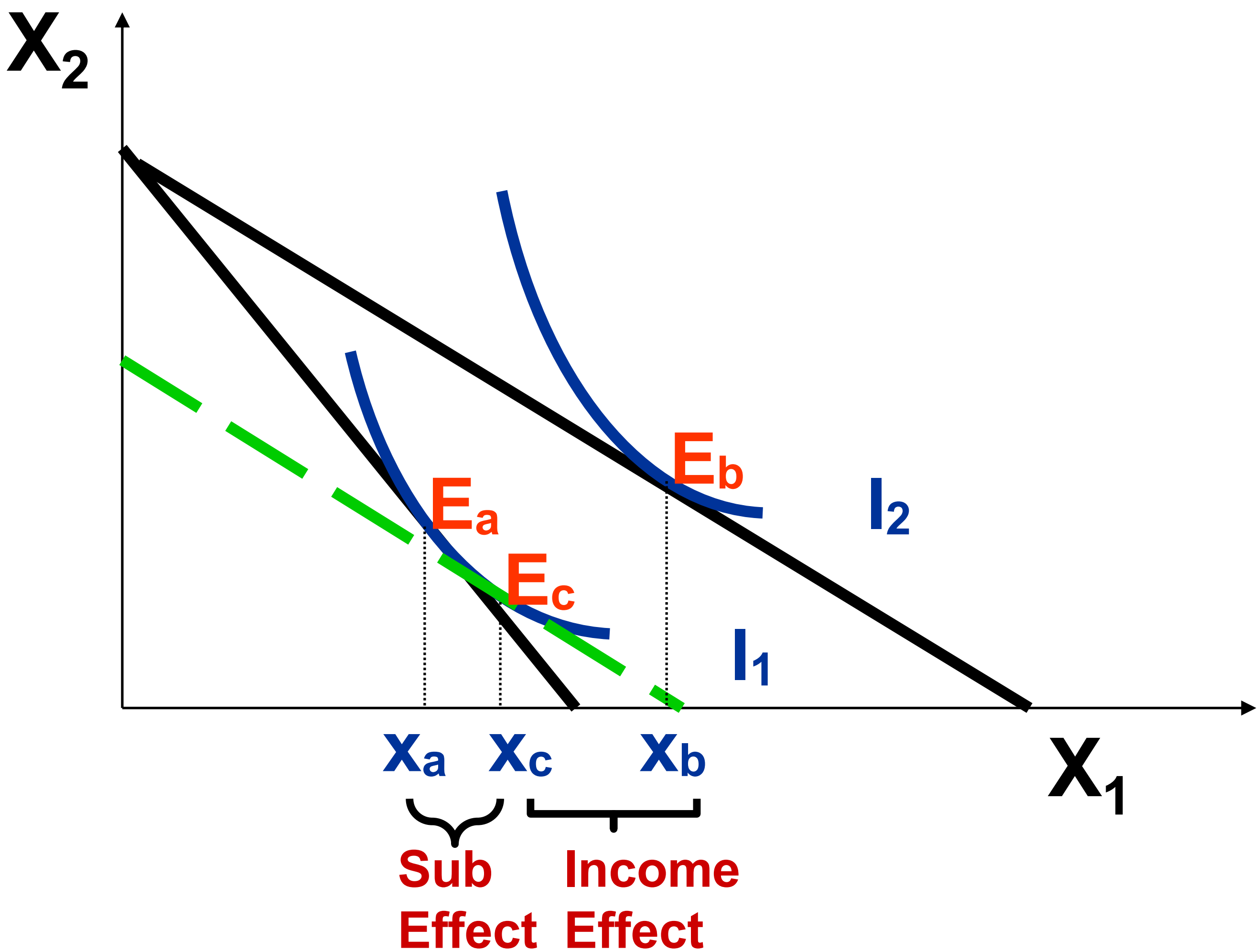
- To isolate the income effect …
- Look at the remainder of the total price effect
- This is due to a change in real income.

THE HICKSIAN METHOD

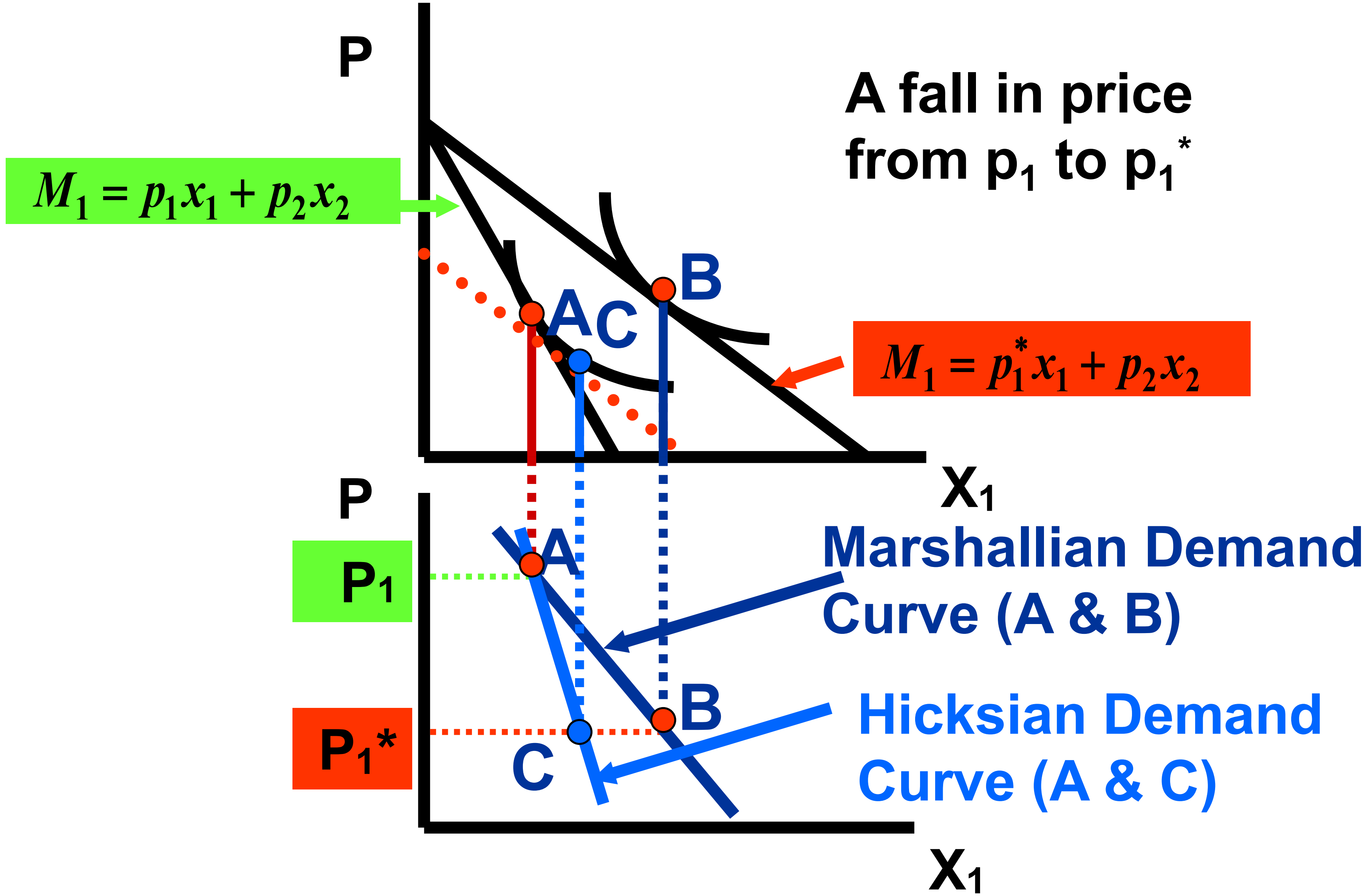
The remainder of the total effect is due to a change in real income. The increase in real income is evidenced by the movement from I_1 to I_2



THE HICKSIAN METHOD



HICKSIAN ANALYSIS and DEMAND CURVES



HICKSIAN ANALYSIS and DEMAND CURVES

Hicksian (compensated) demand curves cannot be upward-sloping (i.e. substitution effect cannot be positive)

NORMAL GOODS

- Since both the substitution and income effects increase demand when own-price falls, a normal good's ordinary demand curve slopes downwards.
- The "Law" of Downward-Sloping Demand therefore always applies to normal goods.

INFERIOR GOODS

- Some goods are (sometimes) inferior (i.e. demand is reduced by higher income).
- The substitution and income effects “oppose” each other when an inferior good’s own price changes.

Cost Curves

Short-Run versus Long-Run Decisions

short run The period of time for which two conditions hold: The firm is operating under a fixed scale (fixed factor) of production, and firms can neither enter nor exit an industry.

long run That period of time for which there are no fixed factors of production: Firms can increase or decrease the scale of operation, and new firms can enter and existing firms can exit the industry.

Costs in the Short Run

fixed cost Any cost that does not depend on the firms' level of output. These costs are incurred even if the firm is producing nothing. There are no fixed costs in the long run.

variable cost A cost that depends on the level of production chosen.

total cost (*TC*) Total fixed costs plus total variable costs.

$$TC = TFC + TVC$$

Fixed Costs

Total Fixed Cost (*TFC*)

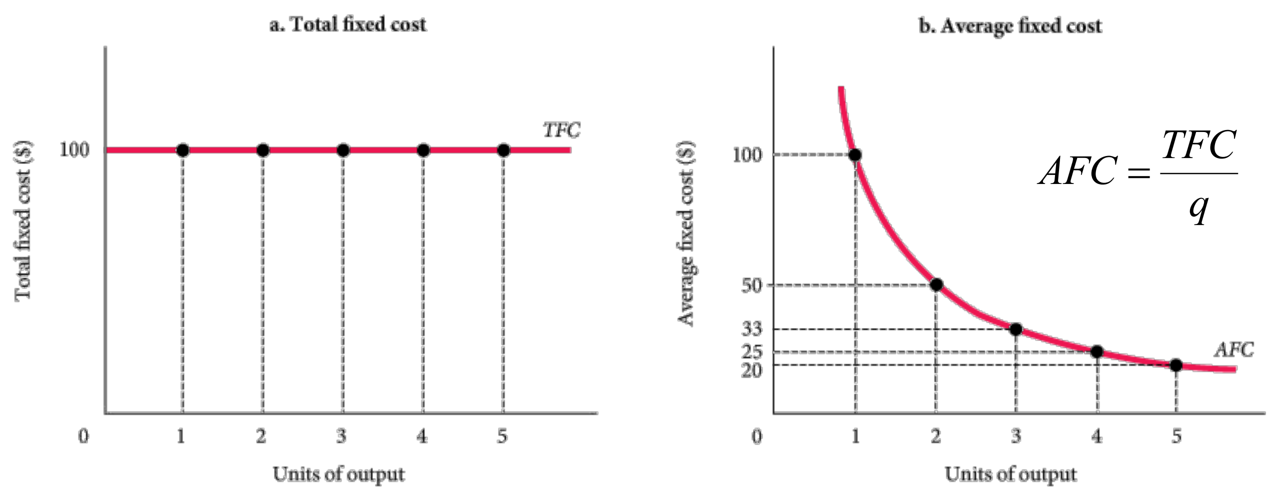
total fixed costs (*TFC*) *or* overhead The total of all costs that do not change with output even if output is zero.

TABLE 8.1 Short-Run Fixed Cost (Total and Average) of a Hypothetical Firm

(1) *q*	(2) *TFC*	(3) *AFC (TFC/q)*
0	$100	$ –
1	100	100
2	100	50
3	100	33
4	100	25
5	100	20

Average Fixed Cost (*AFC*)

average fixed cost (*AFC*) Total fixed cost divided by the number of units of output; a per-unit measure of fixed costs.



▲ **FIGURE 8.2 Short-Run Fixed Cost (Total and Average) of a Hypothetical Firm**

Average fixed cost is simply total fixed cost divided by the quantity of output. As output increases, average fixed cost declines because we are dividing a fixed number ($1,000) by a larger and larger quantity.

spreading overhead The process of dividing total fixed costs by more units of output. Average fixed cost declines as quantity rises.

Variable Costs

Total Variable Cost (*TVC*)

total variable cost (*TVC*) The total of all costs that vary with output in the short run.

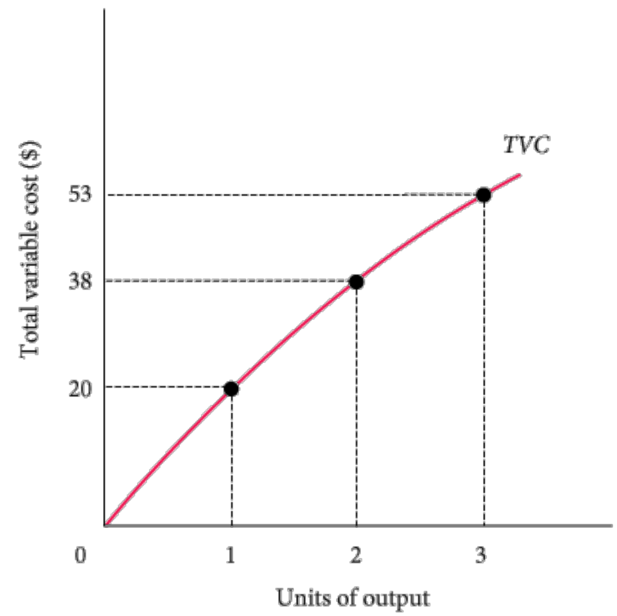
TABLE 8.2 Derivation of Total Variable Cost Schedule from Technology and Factor Prices

Produce	Using Technique	Units of Input Required (Production Function) *K*	*L*	Total Variable Cost Assuming P_K = \$2, P_L = \$1 $TVC = (K \times P_K) + (L \times P_L)$
1 unit of output	*A*	10	7	(10 x \$2) + (7 x \$1) = \$27
	B	6	8	(6 x \$2) + (8 x \$1) = \$20
2 units of output	*A*	16	8	(16 x \$2) + (8 x \$1) = \$40
	B	11	16	(11 x \$2) + (16 x \$1) = \$38
3 units of output	*A*	19	15	(19 x \$2) + (15 x \$1) = \$38
	B	18	22	(18 x \$2) + (22 x \$1) = \$58

total variable cost curve A graph that shows the relationship between total variable cost and the level of a firm's output.

▶ **FIGURE 8.3 Total Variable Cost Curve**

In Table 8.2, total variable cost is derived from production requirements and input prices.
A total variable cost curve expresses the relationship between *TVC* and total output.



Marginal Cost (*MC*)

marginal cost (*MC*) The increase in total cost that results from producing 1 more unit of output. Marginal costs reflect changes in variable costs.

TABLE 8.3 Derivation of Marginal Cost from Total Variable Cost

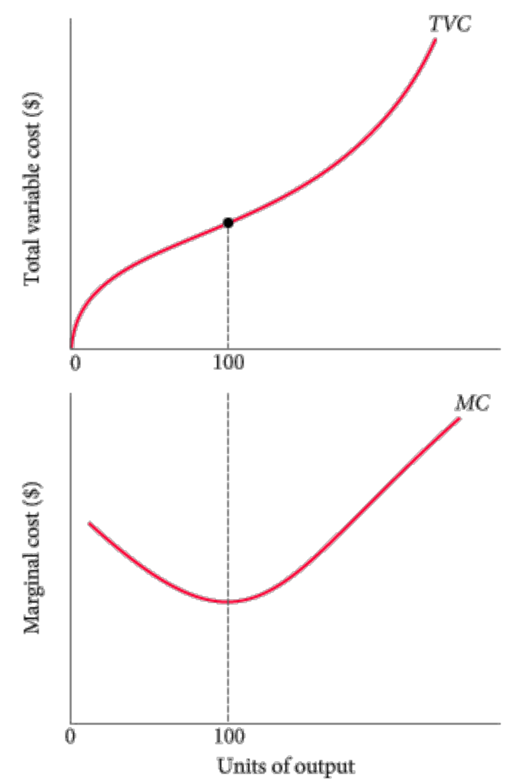
Units of Output	Total Variable Costs ($)	Marginal Costs ($)
0	0	
1	20	20
2	38	18
3	53	15

Graphing Total Variable Costs and Marginal Costs

▶ **FIGURE 8.5 Total Variable Cost and Marginal Cost for a Typical Firm**

Total variable costs always increase with output.
Marginal cost is the cost of producing each additional unit.
Thus, the marginal cost curve shows how total variable cost changes with single-unit increases in total output.

$$\text{slope of } TVC = \frac{\Delta TVC}{\Delta q} = \frac{\Delta TVC}{1} = \Delta TVC = MC$$



Average Variable Cost (*AVC*)

average variable cost (*AVC*) Total variable cost divided by the number of units of output.

$$AVC = \frac{TVC}{q}$$

TABLE 8.4 Short-Run Costs of a Hypothetical Firm

(1) *q*	(2) *TVC*	(3) *MC* (Δ *TVC*)	(4) *AVC* (*TVC/q*)	(5) *TFC*	(6) *TC* (*TVC* + *TFC*)	(7) *AFC* (*TFC/q*)	(8) *ATC* (*TC/q* or *AFC* + *AVC*)
0	$ 0.00	$ -	$ -	$ 100.00	$ 100.00	$ -	$ -
1	20.00	20.00	20.00	100.00	120.00	100.00	120.00
2	38.00	18.00	19.00	100.00	138.00	50.00	69.00
3	53.00	15.00	17.66	100.00	153.00	33.33	51.00
4	65.00	12.00	16.25	100.00	165.00	25.00	41.25
5	75.00	10.00	15.00	100.00	175.00	20.00	35.00
6	83.00	8.00	13.83	100.00	183.50	16.67	30.50
7	94.50	11.50	13.50	100.00	194.50	14.28	27.78
8	108.00	13.50	13.50	100.00	208.00	12.50	26.00
9	128.50	20.50	14.28	100.00	228.50	11.11	25.39
10	168.50	40.00	16.85	100.00	268.50	10.00	26.85

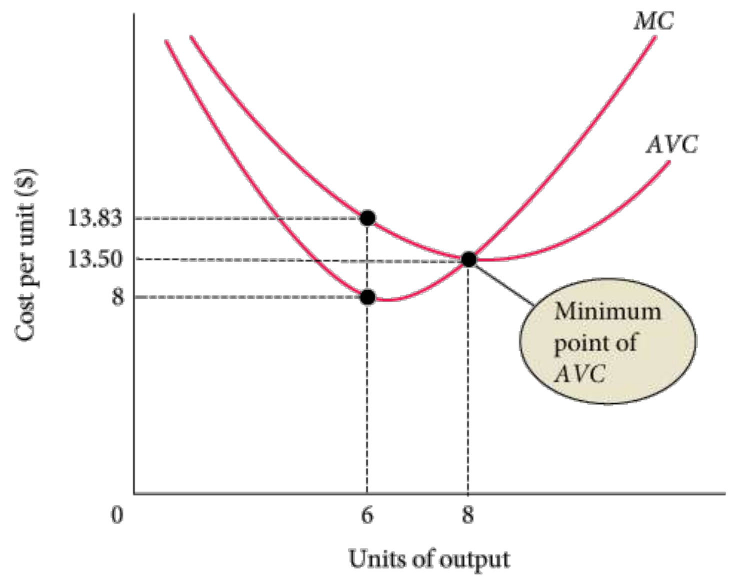
Graphing Average Variable Costs and Marginal Costs

▶ **FIGURE 8.6 More Short-Run Costs**

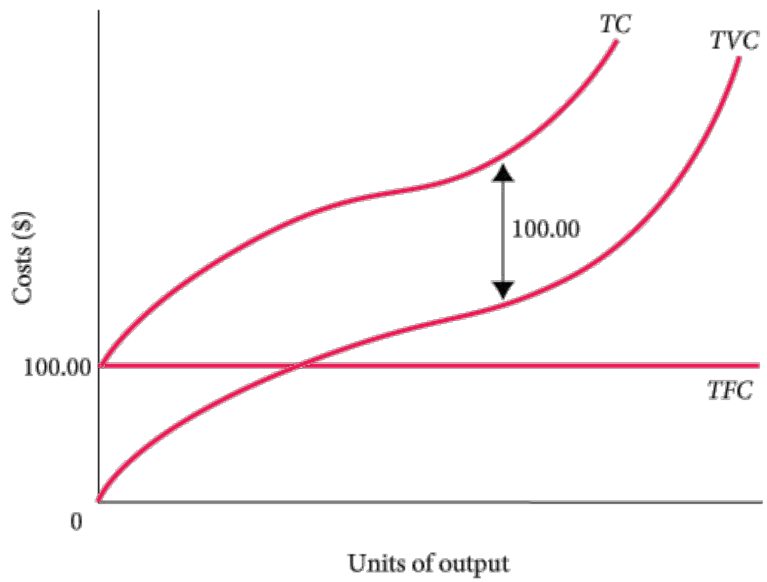
When marginal cost is *below* average cost, average cost is declining.

When marginal cost is *above* average cost, average cost is increasing.

Rising marginal cost intersects average variable cost at the minimum point of *AVC*.



Total Costs



▲ **FIGURE 8.7 Total Cost = Total Fixed Cost + Total Variable Cost**

Adding *TFC* to *TVC* means adding the same amount of total fixed cost to every level of total variable cost.

Thus, the total cost curve has the same shape as the total variable cost curve; it is simply higher by an amount equal to *TFC*.

Average Total Cost (*ATC*)

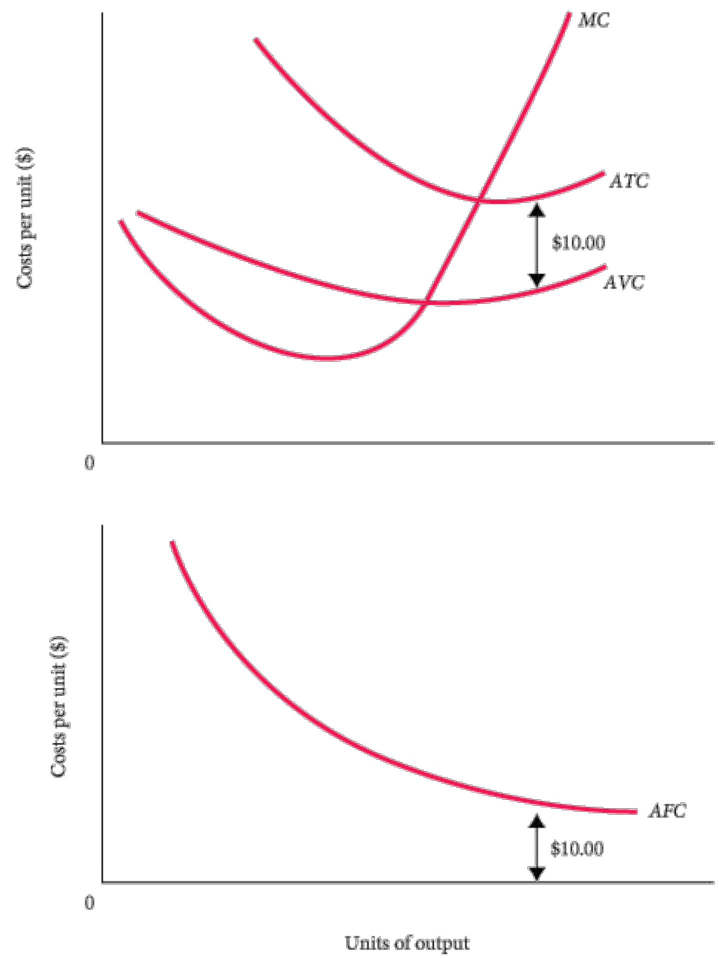
average total cost (*ATC*) Total cost divided by the number of units of output.

$$ATC = \frac{TC}{q}$$

$$ATC = AFC + AVC$$

Average Total Cost = Average Variable Cost + Average Fixed Cost

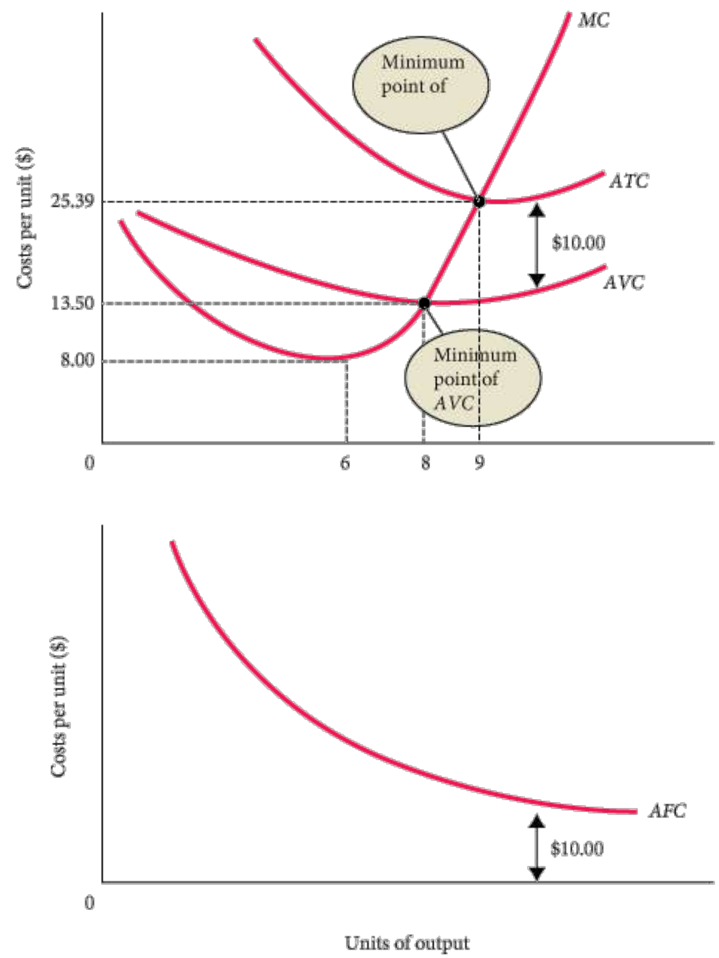
To get average total cost, we add average fixed and average variable costs at all levels of output. Because average fixed cost falls with output, an ever-declining amount is added to *AVC*. Thus, *AVC* and *ATC* get closer together as output increases, but the two lines never meet.



The Relationship Between Average Total Cost and Marginal Cost

The relationship between average *total* cost and marginal cost is exactly the same as the relationship between average *variable* cost and marginal cost.

If marginal cost is *below* average total cost, average total cost will *decline* toward marginal cost. If marginal cost is *above* average total cost, average total cost will *increase*. As a result, marginal cost intersects average *total* cost at *ATC*'s minimum point for the same reason that it intersects the average *variable* cost curve at its minimum point.



Short-Run Costs: A Review

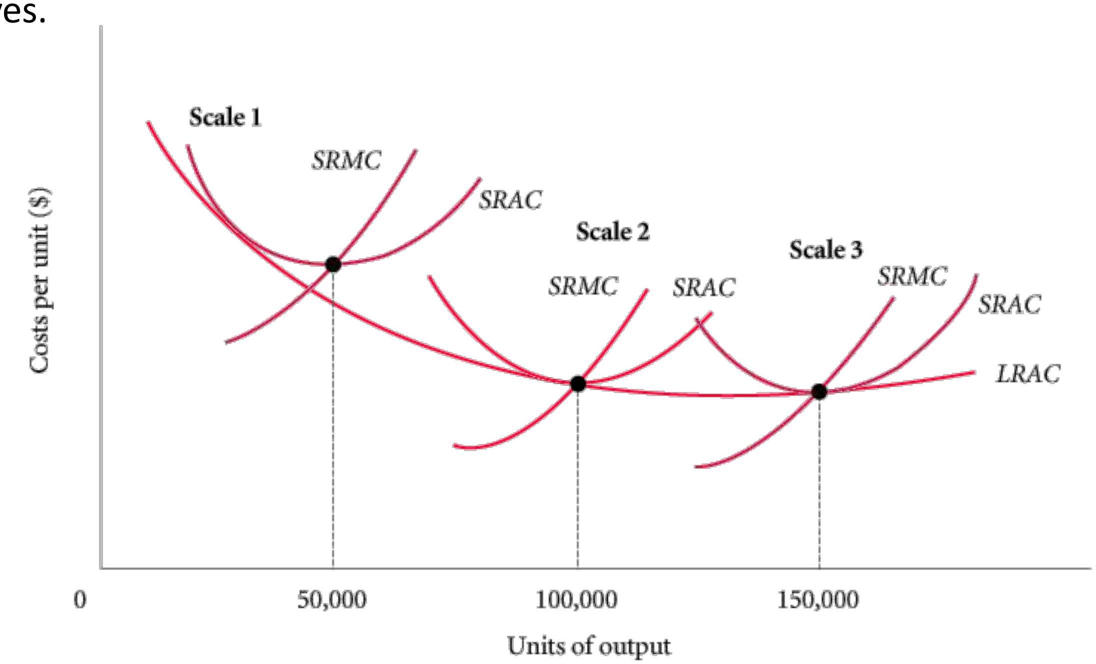
TABLE 8.5 A Summary of Cost Concepts

Term	Definition	Equation
Accounting costs	Out-of-pocket costs or costs as an accountant would define them. Sometimes referred to as *explicit costs*.	–
Economic costs	Costs that include the full opportunity costs of all inputs. These include what are often called *implicit costs*.	–
Total fixed costs (*TFC*)	Costs that do not depend on the quantity of output produced. These must be paid even if output is zero.	–
Total variable costs (*TVC*)	Costs that vary with the level of output.	–
Total cost (*TC*)	The total economic cost of all the inputs used by a firm in production.	$TC = TFC + TVC$
Average fixed costs (*AFC*)	Fixed costs per unit of output.	$AFC = TFC/q$
Average variable costs (*AVC*)	Variable costs per unit of output.	$AVC = TVC/q$
Average total costs (*ATC*)	Total costs per unit of output.	$ATC = TC/q \quad ATC = AFC + AVC$
Marginal costs (*MC*)	The increase in total cost that results from producing 1 additional unit of output.	$MC = \Delta TC/\Delta q$

Why short run average cost curves are U-shaped?

Long-run Average cost curve

long-run average cost curve (*LRAC*) The “envelope” of a series of short-run cost curves.



minimum efficient scale (MES) The smallest size at which the long-run average cost curve is at its minimum.

Profit Maximization in Perfectly Competitive Market

Output Decisions: Revenues, Costs, and Profit Maximization

Perfect Competition

perfect competition An industry structure in which there are many firms, each small relative to the industry, producing identical products and in which no firm is large enough to have any control over prices. In perfectly competitive industries, new competitors can freely enter and exit the market.

homogeneous products Undifferentiated products; products that are identical to, or indistinguishable from, one another.

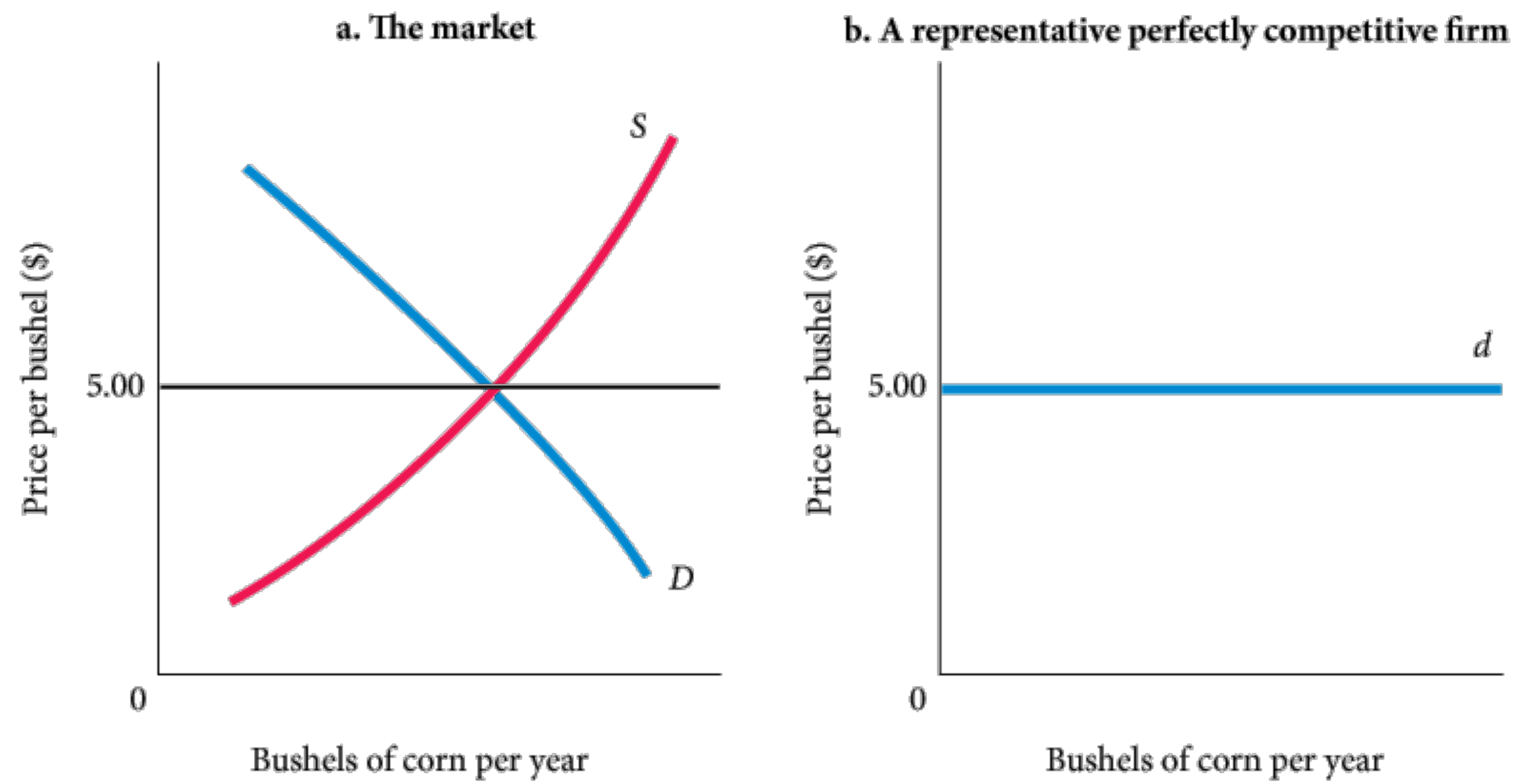


FIGURE 8.9 Demand Facing a Single Firm in a Perfectly Competitive Market

If a representative firm in a perfectly competitive market raises the price of its output above $5.00, the quantity demanded of *that firm's* output will drop to zero.

Each firm faces a perfectly elastic demand curve, *d*.

Total Revenue and Marginal Revenue

total revenue (*TR*) The total amount that a firm takes in from the sale of its product: the price per unit times the quantity of output the firm decides to produce (*P* x *q*).

$$\text{total revenue} = \text{price} \times \text{quantity}$$

$$TR = P \times q$$

marginal revenue (*MR*) The additional revenue that a firm takes in when it increases output by one additional unit. In perfect competition, *P* = *MR*.

The *marginal revenue curve and the demand curve facing a competitive firm are identical*. The horizontal line in Figure 8.9(b) can be thought of as both the demand curve facing the firm and its marginal revenue curve:

$$P^* = d = MR$$

The Profit-Maximizing Level of Output

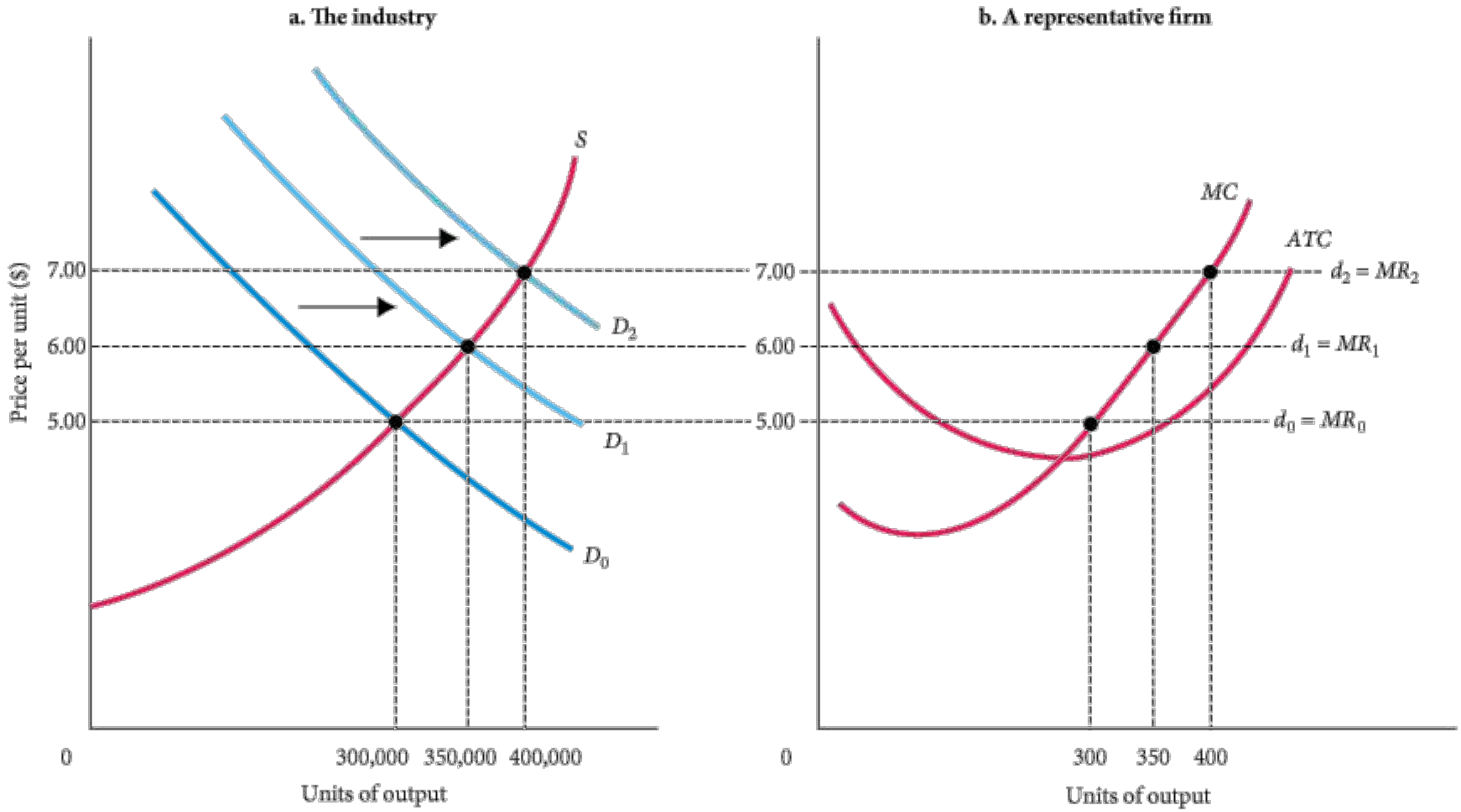
As long as marginal revenue is greater than marginal cost, even though the difference between the two is getting smaller, added output means added profit. Whenever marginal revenue exceeds marginal cost, the revenue gained by increasing output by 1 unit per period exceeds the cost incurred by doing so.

The profit-maximizing perfectly competitive firm will produce up to the point where the price of its output is just equal to short-run marginal cost—the level of output at which $P^* = MC$.

The profit-maximizing output level for *all* firms is the output level where $MR = MC$.
In perfect competition, however, MR = P, as shown earlier. Hence, for perfectly competitive firms, we can rewrite our profit-maximizing condition as $P = MC$.

Important note: The key idea here is that firms will produce as long as marginal revenue exceeds marginal cost.

The Short-Run Supply Curve



At any market price, the marginal cost curve shows the output level that maximizes profit.

Thus, the marginal cost curve of a perfectly competitive profit-maximizing firm is the firm's short-run supply curve.

Minimizing Losses

- If total revenue exceeds total variable cost, the excess revenue can be used to offset fixed costs and reduce losses, and it will pay the firm to keep operating.
- If total revenue is smaller than total variable cost, the firm that operates will suffer losses in excess of fixed costs. In this case, the firm can minimize its losses by shutting down.

Producing at a Loss to Offset Fixed Costs

shutdown point The lowest point on the average variable cost curve. When price falls below the minimum point on *AVC*, total revenue is insufficient to cover variable costs and the firm will shut down and bear losses equal to fixed costs.

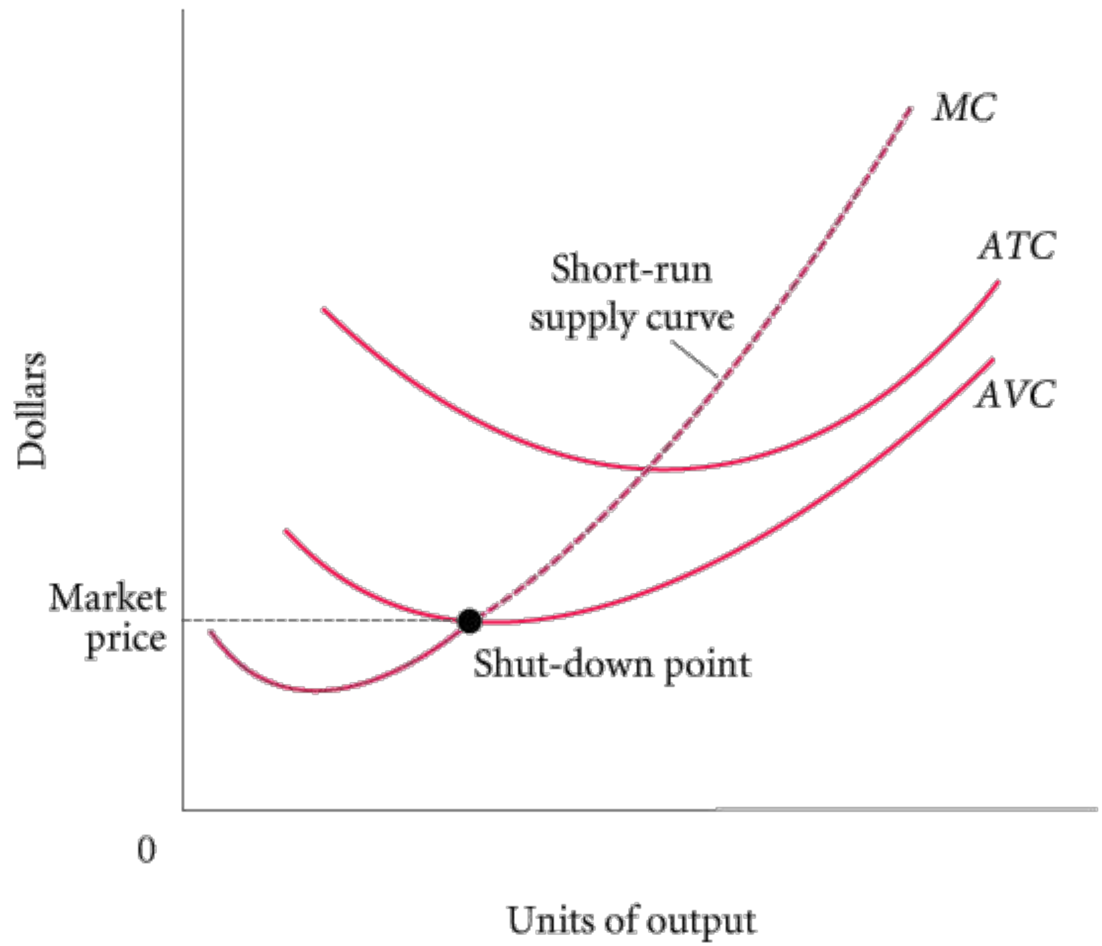


 FIGURE 9.2 Short-Run Supply Curve of a Perfectly Competitive Firm

At prices below average variable cost, it pays a firm to shut down rather than continue operating. Thus, the short-run supply curve of a competitive firm is the part of its marginal cost curve that lies *above* its average variable cost curve.

Long-Run Directions: A Review

TABLE 9.2 Profits, Losses, and Perfectly Competitive Firm Decisions in the Long and Short Run

	Short-Run Condition	Short-Run Decision	Long-Run Decision
Profits	$TR > TC$	$P = MC$: operate	Expand: new firms enter
Losses	1. $TR \geq TVC$	$P = MC$: operate (loss < total fixed cost)	Contract: firms exit
	2. $TR < TVC$	Shut down: loss = total fixed cost	Contract: firms exit

U-Shaped Long-Run Average Costs

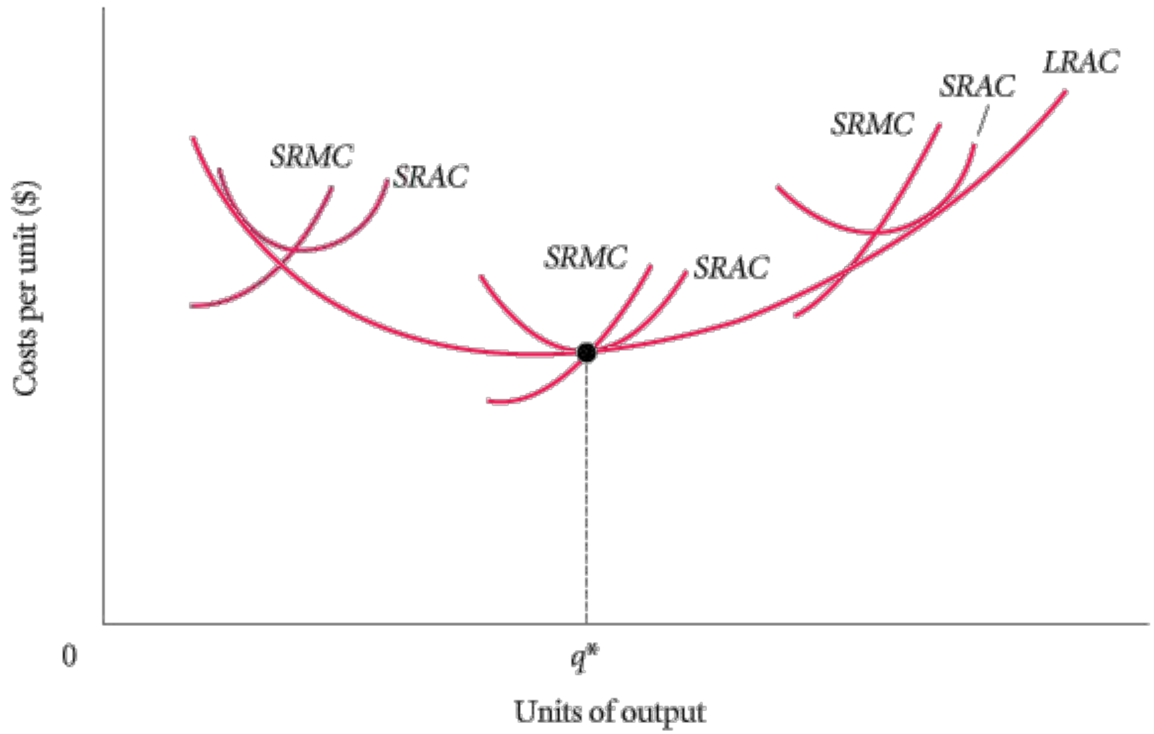


FIGURE 9.5 A Firm Exhibiting Economies and Diseconomies of Scale

Economies of scale push this firm's average costs down to *q**.
Beyond *q**, the firm experiences diseconomies of scale;
*q** is the level of production at lowest average cost, using optimal scale.

optimal scale of plant The scale of plant that minimizes average cost.

Long-Run Adjustments to Short-Run Conditions

Short-Run Profits: Moves In and Out of Equilibrium

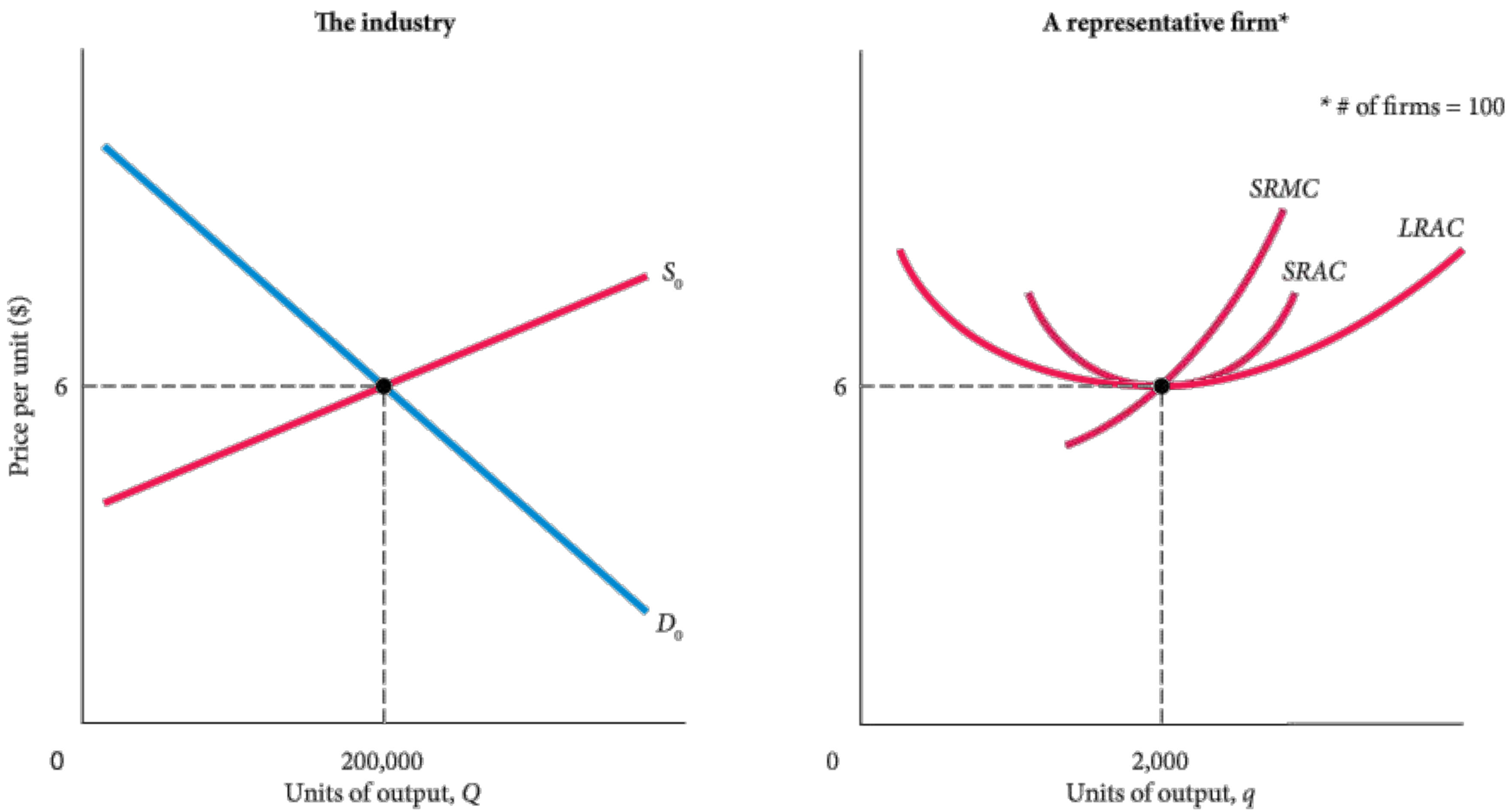


FIGURE 9.6 Equilibrium for an Industry with U-shaped Cost Curves

In equilibrium, each firm has

$$SRMC = SRAC = LRAC$$

Firms make no excess profits so that

$$P = SRMC = SRAC = LRAC$$

and there are enough firms so that supply equals demand.

FIGURE 15.1 Characteristics of Different Market Organizations

	Number of firms	Products differentiated or homogeneous	Price a decision variable	Easy entry	Distinguished by	Examples
Perfect competition	Many	Homogeneous	No	Yes	Market sets price	Wheat farmer Textile firm
Monopoly	One	One version or many versions of a product	Yes	No	Still constrained by market demand	Public utility Patented drug
Monopolistic competition	Many	Differentiated	Yes, but limited	Yes	Price and quality competition	Restaurants Hand soap
Oligopoly	Few	Either	Yes	Limited	Strategic behavior	Automobiles Aluminum

Imperfect Competition and Market Power: Core Concepts

imperfectly competitive industry An industry in which individual firms have some control over the price of their output.

market power An imperfectly competitive firm's ability to raise price without losing all of the quantity demanded for its product.

Forms of Imperfect Competition and Market Boundaries

A *monopoly* is an industry with a single firm in which the entry of new firms is blocked.

An *oligopoly* is an industry in which there is a small number of firms, each large enough so that its presence affects prices.

Firms that differentiate their products in industries with many producers and free entry are called *monopolistic competitors*.

pure monopoly An industry with a single firm that produces a product for which there are no close substitutes and in which significant barriers to entry prevent other firms from entering the industry to compete for profits.

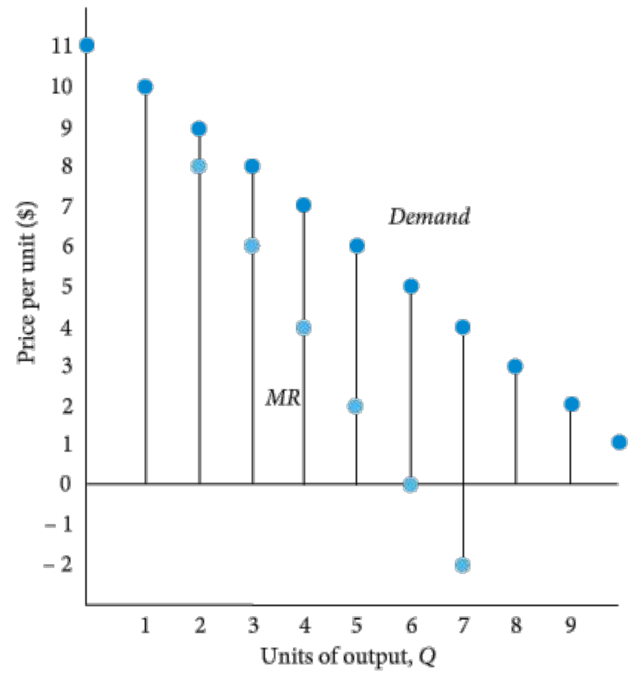
Price and Output Decisions in Pure Monopoly Markets

Demand in Monopoly Markets

Marginal Revenue and Market Demand

TABLE 13.1 Marginal Revenue Facing a Monopolist

(1) Quantity	(2) Price	(3) Total Revenue	(4) Marginal Revenue
0	$11	0	–
1	10	$10	$10
2	9	18	8
3	8	24	6
4	7	28	4
5	6	30	2
6	5	30	0
7	4	28	−2
8	3	24	−4
9	2	18	−6
10	1	10	−8

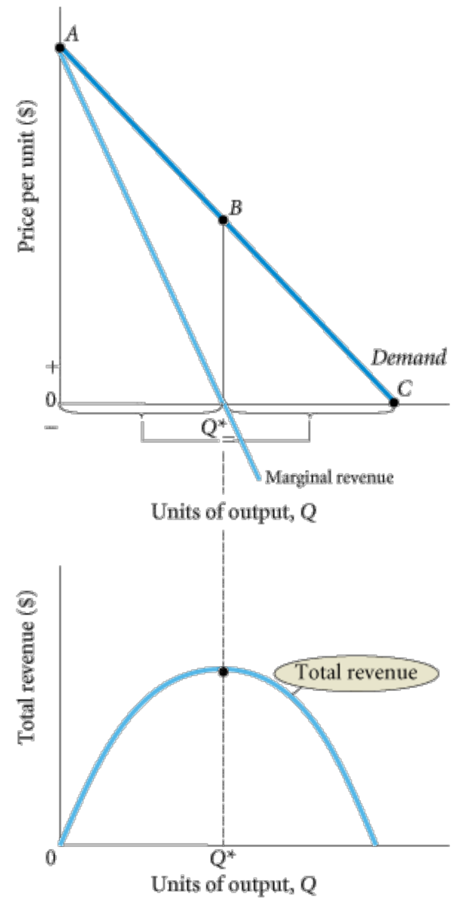


▲ **FIGURE 13.2 Marginal Revenue Curve Facing a Monopolist**

At every level of output except 1 unit, a monopolist's marginal revenue (*MR*) is below price.
This is so because (1) we assume that the monopolist must sell all its product at a single price (no price discrimination) and (2) to raise output and sell it, the firm must lower the price it charges.
Selling the additional output will raise revenue, but this increase is offset somewhat by the lower price charged for all units sold. Therefore, the increase in revenue from increasing output by 1 (the marginal revenue) is less than the price.

▶ **FIGURE 13.3 Marginal Revenue and Total Revenue**

A monopoly's marginal revenue curve bisects the quantity axis between the origin and the point where the demand curve hits the quantity axis.
A monopoly's *MR* curve shows the change in total revenue that results as a firm moves along the segment of the demand curve that lies exactly above it.



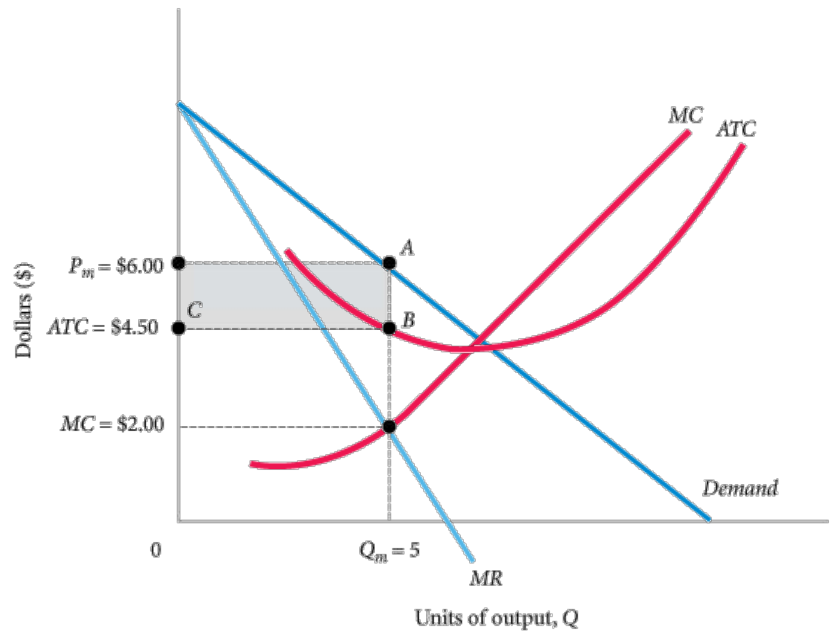
The Monopolist's Profit-Maximizing Price and Output

▶ **FIGURE 13.4 Price and Output Choice for a Profit-Maximizing Monopolist**

A profit-maximizing monopolist will raise output as long as marginal revenue exceeds marginal cost. Maximum profit is at an output of 5 units per period and a price of $6.

Above 5 units of output, marginal cost is greater than marginal revenue; increasing output beyond 5 units would reduce profit.

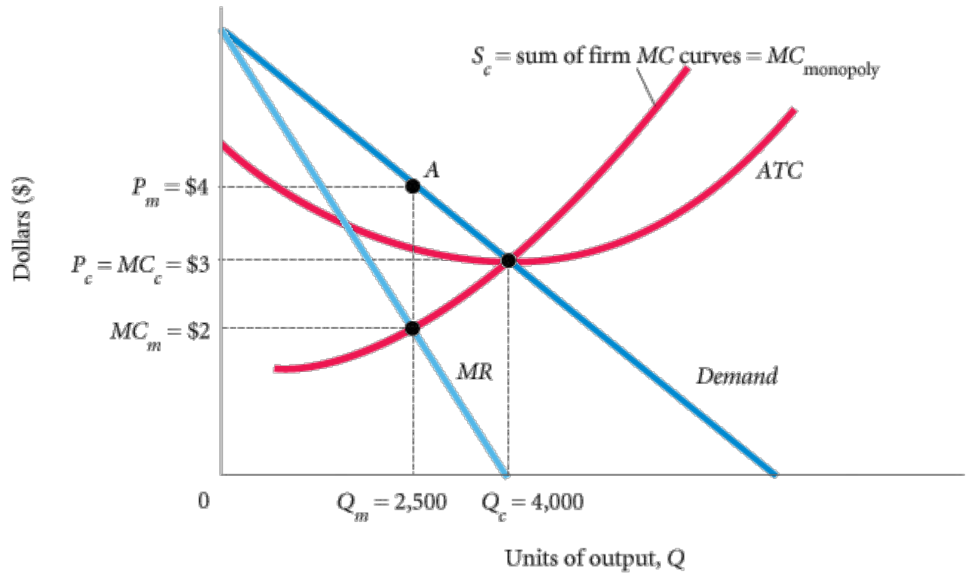
At 5 units, $TR = P_mAQ_m0$, $TC = CBQ_m0$, and profit = P_mABC.



The Absence of a Supply Curve in Monopoly

A monopoly firm has no supply curve that is independent of the demand curve for its product.

A monopolist sets both price and quantity, and the amount of output that it supplies depends on its marginal cost curve and the demand curve that it faces.



▲ **FIGURE 13.6 Comparison of Monopoly and Perfectly Competitive Outcomes for a Firm with Constant Returns to Scale**

In the newly organized monopoly, the marginal cost curve is the same as the supply curve that represented the behavior of all the independent firms when the industry was organized competitively.

Quantity produced by the monopoly will be less than the perfectly competitive level of output, and the monopoly price will be higher than the price under perfect competition.

Under monopoly, $P = P_m$ = \$4 and $Q = Q_m$ = 2,500.

Under perfect competition, $P = P_c$ = \$3 and $Q = Q_c$ = 4,000.

Monopoly in the Long Run: Barriers to Entry

barriers to entry Factors that prevent new firms from entering and competing in imperfectly competitive industries.

Economies of Scale

natural monopoly An industry that realizes such large economies of scale that single-firm production of that good or service is most efficient.

Industry Characteristics

monopolistic competition A common form of industry (market) structure characterized by a large number of firms, no barriers to entry, and product differentiation.

TABLE 15.1 Percentage of Value of Shipments Accounted for by the Largest Firms in Selected Industries, 2002

Industry Designation	Four Largest Firms	Eight Largest Firms	Twenty Largest Firms	Number of Firms
Travel trailers and campers	38	45	58	733
Games, toys	39	48	63	732
Wood office furniture	34	43	56	546
Book printing	33	54	68	560
Curtains and draperies	17	25	38	1,778
Fresh or frozen seafood	14	24	48	529
Women's dresses	18	23	48	528
Miscellaneous plastic products	6	10	18	6,775

Product Differentiation and Advertising

product differentiation A strategy that firms use to achieve market power. Accomplished by producing goods that differ from others in the market.

How Many Varieties?

In well-working markets, the level of product variety reflects the underlying heterogeneity of consumers' tastes in that market, the gains if any from coordination, and cost economies from standardization.

In industries that are monopolistically competitive, differences in consumer tastes, lack of need for coordination, and modest or no scale economies from standardization give rise to a large number of firms, each of which has a different product.

How Do Firms Differentiate Products?

horizontal differentiation Products differ in ways that make them better for some people and worse for others.

vertical differentiation A product difference that, from everyone's perspective, makes a product better than rival products.

behavioral economics A branch of economics that uses the insights of psychology and economics to investigate decision making.

oligopoly A form of industry (market) structure characterized by a few dominant firms. Products may be homogenous or differentiated.

Oligopolists compete with one another not only in price but also in developing new products, marketing and advertising those products, and developing complements to use with the products.

Macro Economic Aggregates

Measuring National Output and National Income

national income and product accounts Data collected and published by the government describing the various components of national income and output in the economy.

Gross Domestic Product

gross domestic product (GDP) The total market value of all final goods and services produced within a given period by factors of production located within a country.

GDP is the total market value of a country's output. It is the market value of all final goods and services produced within a given period of time by factors of production located within a country.

Final Goods and Services

final goods and services Goods and services produced for final use.

intermediate goods Goods that are produced by one firm for use in further processing by another firm.

value added The difference between the value of goods as they leave a stage of production and the cost of the goods as they entered that stage.

In calculating GDP, we can sum up the value added at each stage of production or we can take the value of final sales. We do not use the value of total sales in an economy to measure how much output has been produced.

Macro Economic Aggregates

Measuring National Output and National Income

Gross Domestic Product

Final Goods and Services
Exclusion of Used Goods and Paper Transactions
Exclusion of Output Produced Abroad by Domestically Owned Factors of Production

Calculating GDP

The Expenditure Approach
The Income Approach

Nominal versus Real GDP

Calculating Real GDP
Calculating the GDP Deflator
The Problems of Fixed Weights

Limitations of the GDP Concept

GDP and Social Welfare
The Informal Economy
Gross National Income per Capita

national income and product accounts Data collected and published by the government describing the various components of national income and output in the economy.

Gross Domestic Product

gross domestic product (GDP) The total market value of all final goods and services produced within a given period by factors of production located within a country.

GDP is the total market value of a country's output. It is the market value of all final goods and services produced within a given period of time by factors of production located within a country.

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In calculating GDP, we can sum up the value added at each stage of production or we can take the value of final sales. We do not use the value of total sales in an economy to measure how much output has been produced.

Exclusion of Used Goods and Paper Transactions

GDP is concerned only with new, or current, production. Old output is not counted in current GDP because it was already counted when it was produced.

GDP does not count transactions in which money or goods changes hands but in which no new goods and services are produced.

Exclusion of Output Produced Abroad by Domestically Owned Factors of Production

GDP is the value of output produced by factors of production *located within a country*.

gross national product (GNP) The total market value of all final goods and services produced within a given period by factors of production owned by a country's citizens, regardless of where the output is produced.

Calculating GDP

expenditure approach A method of computing GDP that measures the total amount spent on all final goods and services during a given period.

income approach A method of computing GDP that measures the income—wages, rents, interest, and profits—received by all factors of production in producing final goods and services.

The Expenditure Approach

There are four main categories of expenditure:

- Personal consumption expenditures (C): household spending on consumer goods
- Gross private domestic investment (I): spending by firms and households on new capital, that is, plant, equipment, inventory, and new residential structures
- Government consumption and gross investment (G)
- Net exports ($EX - IM$): net spending by the rest of the world, or exports (EX) minus imports (IM)

$$\text{GDP} = C + I + G + (EX - IM)$$

TABLE 6.2 Components of GDP: The Expenditure Approach

	Billions of Dollars		Percentage of GDP	
Personal consumption expenditures (C)	11,119.5		70.9	
Durable goods		1,218.8		7.8
Nondurable goods		2,563.0		16.3
Services		7,337.7		46.8
Gross private domestic investment (I)	2,059.5		13.1	
Nonresidential		1,616.6		10.3
Residential		382.4		2.4
Change in business inventories		60.6		0.4
Government consumption and gross investment (G)	3,063.6		19.5	
Federal		1,214.2		7.7
State and local		1,849.4		11.8
Net exports *(EX – IM)*	−566.7		−3.6	
Exports (*EX*)		2,179.7		13.9
Imports (*IM*)		2,746.3		17.5
Gross domestic product	15,676.0		100.0	

Note: Numbers may not add exactly because of rounding.

Personal Consumption Expenditures (*C*)

personal consumption expenditures (*C*) Expenditures by consumers on goods and services.

durable goods Goods that last a relatively long time, such as cars and household appliances.

nondurable goods Goods that are used up fairly quickly, such as food and clothing.

services The things we buy that do not involve the production of physical things, such as legal and medical services and education.

Gross Private Domestic Investment (*I*)

gross private domestic investment (*I*) Total investment in capital—that is, the purchase of new housing, plants, equipment, and inventory by the private (or nongovernment) sector.

nonresidential investment Expenditures by firms for machines, tools, plants, and so on.

residential investment Expenditures by households and firms on new houses and apartment buildings.

change in business inventories The amount by which firms' inventories change during a period. Inventories are the goods that firms produce now but intend to sell later.

Change in Business Inventories

GDP = Final sales + Change in business inventories

Gross Investment versus Net Investment

depreciation The amount by which an asset's value falls in a given period.

gross investment The total value of all newly produced capital goods (plant, equipment, housing, and inventory) produced in a given period.

net investment Gross investment minus depreciation.

$$\text{capital}_{\text{end of period}} = \text{capital}_{\text{beginning of period}} + \text{net investment}$$

Government Consumption and Gross Investment (*G*)

government consumption and gross investment (*G*) Expenditures by federal, state, and local governments for final goods and services.

Net Exports (*EX* − *IM*)

net exports (*EX* − *IM*) The difference between exports (sales to foreigners of U.S.-produced goods and services) and imports (U.S. purchases of goods and services from abroad). The figure can be positive or negative.

net national product (NNP) Gross national product minus depreciation; a nation's total product minus what is required to maintain the value of its capital stock.

TABLE 6.4 GDP, GNP, NNP, and National Income, 2012

	Dollars (Billions)
GDP	15,676.0
Plus: Receipts of factor income from the rest of the world	+774.1
Less: Payments of factor income to the rest of the world	−537.0
Equals: **GNP**	15,913.1
Less: Depreciation	−2,011.4
Equals: **Net national product (NNP)**	13,901.7
Less: Indirect business tax	−68.5
Equals: **National income**	13,833.2

The Income Approach

national income The total income earned by the factors of production owned by a country's citizens.

compensation of employees Includes wages, salaries, and various supplements—employer contributions to social insurance and pension funds, for example—paid to households by firms and by the government.

proprietors' income The income of unincorporated businesses.

rental income The income received by property owners in the form of rent.

corporate profits The income of corporations.

net interest The interest paid by business.

TABLE 6.5 National Income, Personal Income, Disposable Personal Income, and Personal Saving

	Dollars (Billions)
National income	13,833.2
Less: Amount of national income not going to households	−430.8
Equals: **Personal income**	13,402.4
Less: Personal income taxes	−1,471.9
Equals: **Disposable personal income**	11,930.6
Less: Personal consumption expenditures	−11,119.5
Personal interest payments	−172.3
Transfer payments made by households	−168.1
Equals: **Personal saving**	470.8
Personal saving as a percentage of disposable personal income:	3.9%

disposable personal income ***or*** **after-tax income** Personal income minus personal income taxes. The amount that households have to spend or save.

personal saving The amount of disposable income that is left after total personal spending in a given period.

personal saving rate The percentage of disposable personal income that is saved. If the personal saving rate is low, households are spending a large amount relative to their incomes; if it is high, households are spending cautiously.

Nominal versus Real GDP

current prices The current prices that we pay for goods and services.

nominal GDP Gross domestic product measured in current prices.

weight The importance attached to an item within a group of items.

Calculating Real GDP

TABLE 6.6 A Three-Good Economy

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Production		**Price per Unit**		**GDP in Year 1 in Year 1 Prices**	**GDP in Year 2 in Year 1 Prices**	**GDP in Year 1 in Year 2 Prices**	**GDP in Year 2 in Year 2 Prices**
	Year 1 Q_1	**Year 2** Q_2	**Year 1** P_1	**Year 2** P_2	$P_1 \times Q_1$	$P_1 \times Q_2$	$P_2 \times Q_1$	$P_2 \times Q_2$
Good *A*	6	11	$0.50	$0.40	$3.00	$5.50	$2.40	$4.40
Good *B*	7	4	0.30	1.00	2.10	1.20	7.00	4.00
Good *C*	10	12	0.70	0.90	7.00	8.40	9.00	10.80
Total					$12.10	$15.10	$18.40	$19.20
					Nominal GDP in year 1			Nominal GDP in year 2

base year The year chosen for the weights in a fixed-weight procedure.

fixed-weight procedure A procedure that uses weights from a given base year.

Calculating the GDP Deflator

Policy makers not only need good measures of how real output is changing but also good measures of how the overall price level is changing.

The GDP deflator is one measure of the overall price level.

The Problems of Fixed Weights

The use of fixed-price weights does not account for the responses in the economy to supply shifts.

The fixed-weight procedure ignores the substitution away from goods whose prices are increasing and toward goods whose prices are decreasing or increasing less rapidly.

Limitations of the GDP Concept

GDP and Social Welfare

If crime levels went down, society would be better off, but a decrease in crime is not an increase in output and is not reflected in GDP.

An increase in leisure is also an increase in social welfare, sometimes associated with a *decrease* in GDP.

Most nonmarket and domestic activities, such as housework and child care, are not counted in GDP even though they amount to real production.

GDP also has nothing to say about the distribution of output among individuals in a society.

ECONOMICS IN PRACTICE

Green Accounting

Recently many economists and policy makers have become concerned about the exclusion of one particularly large and important nonmarket activity from the national income accounts: the environment.

The market goods that many industries produce go into the national income and product accounts, but the environmental costs of air pollution are not subtracted.

Recent work by Nick Muller, Robert Mendelsohn, and Bill Nordhaus estimates that including properly valued air pollution in the national income and product accounts as an offset to the value of the marketed goods produced by some industries would make their contribution to our nation's GDP negative!

The Informal Economy

informal economy The part of the economy in which transactions take place and in which income is generated that is unreported and therefore not counted in GDP.

Human Development Index (HDI)

Understanding Indexes

- **What is an index?**
- An index is a composite of indicators that produces a single calculation which can then be ranked.

The Human Development Index (HDI)

…is the best known composite index of social and economic well-being…

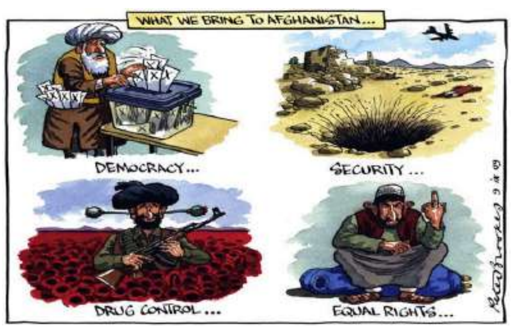
The Concept of Human Development

"The basic purpose of development is to enlarge people's choices. In principle, these choices can be infinite and can change over time. People often value achievements that do not show up at all, or not immediately, in income or growth figures: greater access to knowledge, better nutrition and health services, more secure livelihoods, security against crime and physical violence, satisfying leisure hours, political and cultural freedoms and sense of participation in community activities. The objective of development is to create an enabling environment for people to enjoy long, healthy and creative lives."

Mahbub ul Haq -- Founder of the Human Development Report

Calculating HDI: New Method

- Indicator that measures the overall development of a nation; Life expectancy at birth, longevity, Education, Income
- It is the statistic used to rank countries by level of standard of living and quality of life.
- It goes from 0 to 1 (1 –most ; 0- worst)



Components of HDI

- **Life expectancy at birth:** Number of years a newborn infant could expect to live if prevailing patterns of age-specific mortality rates at the time of birth stay the same throughout the infant's life.
- **Mean years of schooling**: Average number of years of education received by people aged 25 and older, converted from education attainment levels using official durations of each level.
- **Expected years of schooling**: Number of years of schooling that a child of school entrance age can expect to receive if prevailing patterns of age-specific enrolment rates persist throughout the child's life.

Contd..

- **Gross national income (GNI) per capita**: Aggregate income of an economy generated by its production and its ownership of factors of production, less the incomes paid for the use of factors of production owned by the rest of the world, converted to international dollars using purchasing power parity (PPP) rates, divided by midyear population.

Contd..

- **Human Development Index (HDI)**: A composite index measuring average achievement in three basic dimensions of human development—a long and healthy life, knowledge and a decent standard of living.

Calculating Human Development Index

- The Human Development Index (HDI) is a summary measure of human development.
- It measures the average achievements in a country in three basic dimensions of human development: a long and healthy life, access to knowledge and a decent standard of living.
- The HDI is the geometric mean of normalized indices measuring achievements in each dimension.

Steps to estimate the Human Development Index

- **Step 1. Creating the dimension indices**
- Minimum and maximum values (goalposts) are set in order to transform the indicators into indices between 0 and 1.
- These goalposts act as the 'natural zeroes' and 'aspirational goals', respectively, from which component indicators are standardized

Goal Posts

Dimension	Indicator	Minimum	Maximum
Health	Life expectancy (years)	20	85
Education	Expected years of schooling	0	18
	Mean years of schooling	0	15
Standard of living	Gross national income per capita (PPP 2011 $)	100	75,000

Goal Posts Contd..

- The justification for placing the natural zero for life expectancy at 20 years is based on historical evidence.
- Societies can subsist without formal education, justifying the education minimum of 0 years.
- The maximum for mean years of schooling, 15, is the projected maximum of this indicator for 2025.
- The maximum for expected years of schooling, 18, is equivalent to achieving a master’s degree in most countries.

Goal Posts Contd..

- The low minimum value for gross national income (GNI) per capita, $100, is justified by the considerable amount of unmeasured subsistence and nonmarket production in economies close to the minimum, which is not captured in the official data.
- The maximum is set at $75,000 per capita.
- Kahneman and Deaton (2010) have shown that there is a virtually no gain in human development and well-being from annual income beyond $75,000.

- the sub indices are calculated as follows:

$$\text{Dimension index} = \frac{\text{actual value} - \text{minimum value}}{\text{maximum value} - \text{minimum value}}. \quad (1)$$

How is the HDI calculated?

- LEI = Life Expectancy Index
- EI = Education Index
- II = Income Index

$$\mathrm{HDI} = \sqrt[3]{\mathrm{LEI} \cdot \mathrm{EI} \cdot \mathrm{II}}.$$

Formula for HDI calculation

- Life Expectancy Index (LEI) = $\frac{LE-20}{85-20}$
- Education Index (EI) = (MYSI + EYSI)/2

  MYSI = MYS/15

  EYSI = EYS/18

- Income Index (II) = $\frac{\ln(GNIpc)-\ln(100)}{\ln(75000)-\ln(100)}$
- Finally, the HDI is the geometric mean of the previous three normalized indices:

$$HDI = \sqrt[3]{LEI.EI.II}$$

Methodology used to express income

- The World Bank's 2014 World Development Indicators database contains estimates of GNI per capita in 2011 purchasing power parity (PPP) terms for many countries.
- For countries missing this indicator, the Human Development Report Office calculates it by converting GNI from current to constant terms using two steps.
- First, the value of nominal GNI per capita is converted into PPP terms for the base year (2011).
- Second, a time series of GNI per capita in 2011 PPP terms is constructed by applying the real growth rates to the GNI per capita in PPP terms for the base year.
- The real growth rate is implied by the ratio of the nominal growth of current GNI per capita in local currency terms to the GDP deflator.

Purchasing Power Parity (PPP) $

- Official PPP conversion rates are produced by the International Comparison Program, whose surveys periodically collect thousands of prices of matched goods and services in many countries.
- The last round of this exercise refers to 2011 and covered 180 countries.

Is the HDI Enough to Measure a Country’s Level of Development?

- According to the UNDP, the answer is:
- “Not at all.”
- “The concept of human development is much broader than what can be captured in the HDI, or any other composite indices…”
- “The HDI and the other composite indices can only offer a broad proxy on some of the key issues of human development…”
- “A fuller picture of a country's level of human development requires analysis of other human development indicators and information.”

Interest Rates, Cash Flow and Equivalence

Time Value of Money (TVM)

Description: TVM explains the change in the amount of money over time for funds owed by or owned by a corporation (or individual)

- Corporate investments are expected to earn a return
- Investment involves money
- Money has a 'time value'

The time value of money is the most important concept in engineering economy

Interest and Interest Rate

- Interest – the manifestation of the time value of money
  - Fee that one pays to use someone else's money
  - Difference between an ending amount of money and a beginning amount of money

    ➢ Interest = amount owed now – principal

- Interest rate – Interest paid over a time period expressed as a percentage of principal

    ➢ $$\text{Interest rate (\%)} = \frac{\text{interest accrued per time unit}}{\text{principal}} \times 100\%$$

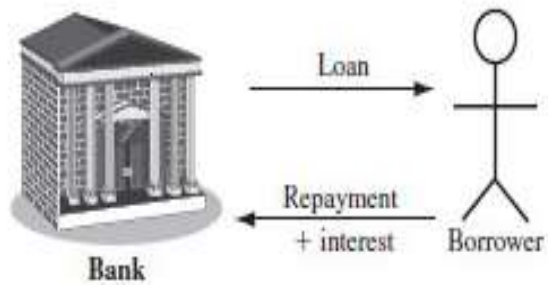
Rate of Return

- Interest earned over a period of time is expressed as a percentage of the original amount (principal)

$$\text{Rate of return (\%)} = \frac{\text{interest accrued per time unit}}{\text{original amount}} \text{ x } 100\%$$

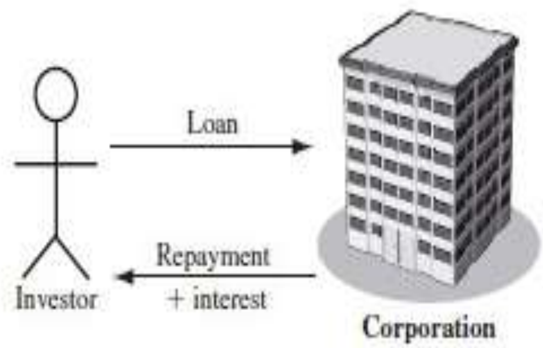
- Borrower’s perspective – interest rate paid
- Lender’s or investor’s perspective – rate of return earned

Interest paid



Interest rate

Interest earned



Rate of return

Commonly used Symbols

t = time, usually in periods such as years or months

P = value or amount of money at a time *t* designated as present or time 0

F = value or amount of money at some future time, such as at *t* = *n* periods in the future

A = series of consecutive, equal, end-of-period amounts of money

n = number of interest periods; years, months

i = interest rate or rate of return per time period; percent per year or month

Cash Flows: Terms

- Cash Inflows – Revenues (R), receipts, incomes, savings generated by projects and activities that flow in. Plus sign used
- Cash Outflows – Disbursements (D), costs, expenses, taxes caused by projects and activities that flow out. Minus sign used

- Net Cash Flow (NCF) for each time period:

  NCF = cash inflows – cash outflows = R – D

- End-of-period assumption:

  Funds flow at the end of a given interest period

Cash Flows: Estimating

- ✓ Point estimate – A single-value estimate of a cash flow element of an alternative

  Cash inflow: Income = $150,000 per month

- ✓ Range estimate – Min and max values that estimate the cash flow

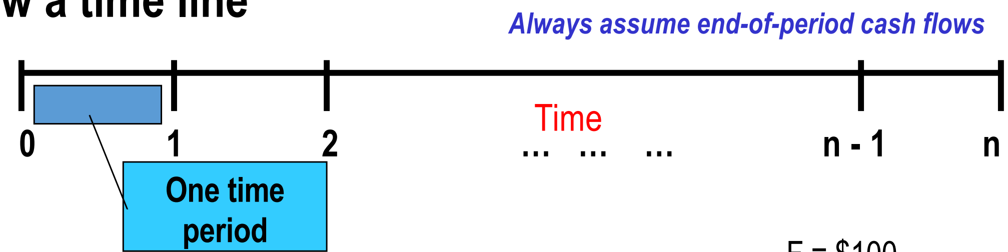
  Cash outflow: Cost is between $2.5 M and $3.2 M

Point estimates are commonly used; however, range estimates with probabilities attached provide a better understanding of variability of economic parameters used to make decisions

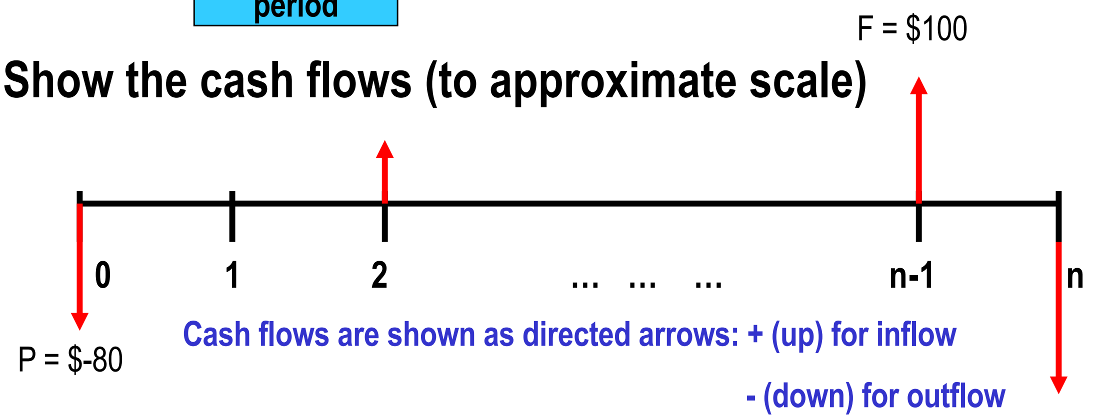
Cash Flow Diagrams

What a typical cash flow diagram might look like

Draw a time line



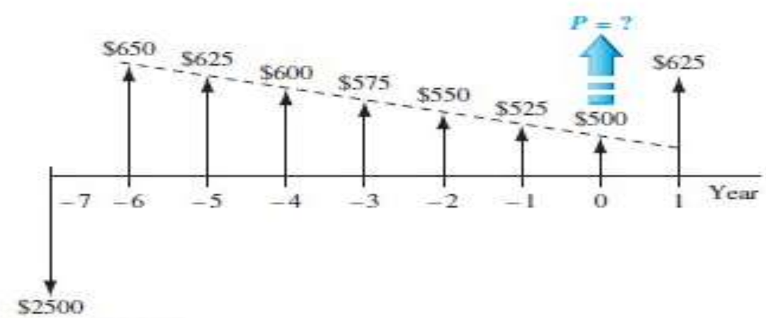
Show the cash flows (to approximate scale)



Cash Flow Diagram Example

Plot observed cash flows over last 8 years and estimated sale next year for $150. Show present worth (P) arrow at present time, t = 0

End of Year	Income	Cost	Net Cash Flow
−7	$ 0	$2500	$−2500
−6	750	100	650
−5	750	125	625
−4	750	150	600
−3	750	175	575
−2	750	200	550
−1	750	225	525
0	750	250	500
1	750 + 150	275	625



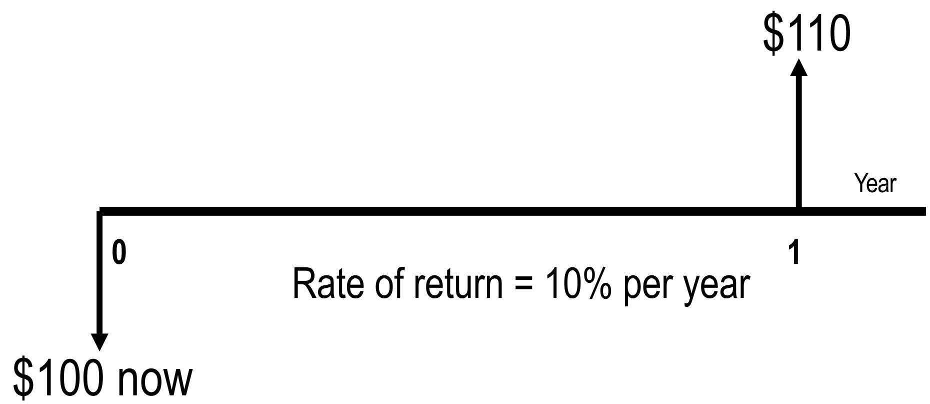
Economic Equivalence

Definition: Combination of interest rate (rate of return) and time value of money to determine different amounts of money at different points in time that are economically equivalent

How it works: Use rate *i* and time *t* in upcoming relations to move money (values of P, F and A) between time points *t* = 0, 1, …, n to make them equivalent (not equal) at the rate *i*

Example of Equivalence

Different sums of money at different times may be equal in economic value at a given rate



$100 now is economically equivalent to $110 one year from now, if the $100 is invested at a rate of 10% per year.

Simple and Compound Interest

- Simple Interest

Interest is calculated using principal only

Interest = (principal)(number of periods)(interest rate)

I = Pni

Example: $100,000 lent for 3 years at simple *i* = 10% per year. What is repayment after 3 years?

Interest = 100,000(3)(0.10) = $30,000

Total due = 100,000 + 30,000 = $130,000

Simple and Compound Interest

- Compound Interest

Interest is based on principal plus all accrued interest

That is, interest compounds over time

Interest = (principal + all accrued interest) (interest rate)

Interest for time period *t* is

$$I_t = \left(P + \sum_{j=1}^{j=t-1} I_j \right)(i)$$

Compound Interest Example

Example: \$100,000 lent for 3 years at i = 10% per year compounded. What is repayment after 3 years?

Interest, year 1:	$I_1 = 100{,}000(0.10) = \$10{,}000$
Total due, year 1:	$T_1 = 100{,}000 + 10{,}000 = \$110{,}000$
Interest, year 2:	$I_2 = 110{,}000(0.10) = \$11{,}000$
Total due, year 2:	$T_2 = 110{,}000 + 11{,}000 = \$121{,}000$
Interest, year 3:	$I_3 = 121{,}000(0.10) = \$12{,}100$
Total due, year 3:	$T_3 = 121{,}000 + 12{,}100 = \$133{,}100$

Compounded: \$133,100 Simple: \$130,000

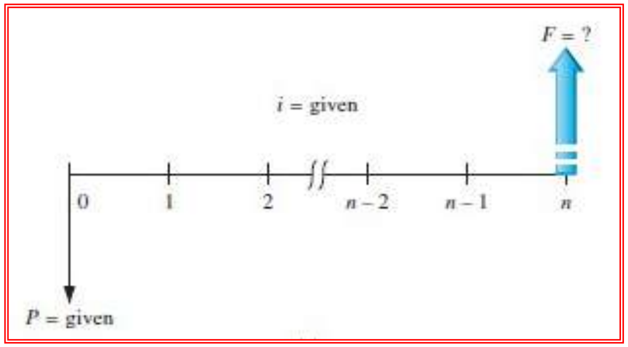
Factors: How Time and Interest Affect Money

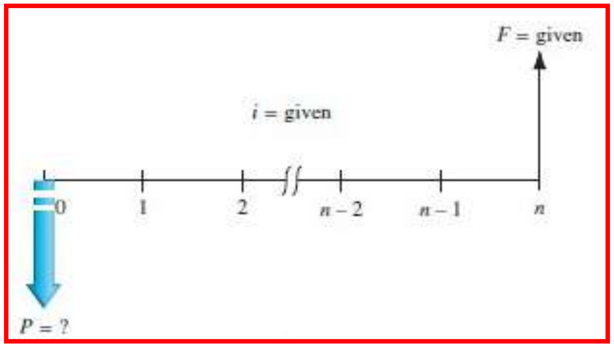
LEARNING OUTCOMES

1. **F/P and P/F Factors**
2. **P/A and A/P Factors**
3. **F/A and A/F Factors**
4. **Factor Values**
5. **Arithmetic Gradient**
6. **Geometric Gradient**
7. **Find i or n**

Single Payment Factors (F/P and P/F)

Single payment factors involve only P and F. **Cash flow diagrams are as follows:**





Formulas are as follows:

$$F = P(1 + i)^n$$

$$P = F[1 / (1 + i)^n]$$

Terms in parentheses or brackets are called *factors*. Values are in tables for i and n values

Factors are represented in *standard factor notation such as (F/P,i,n)*,

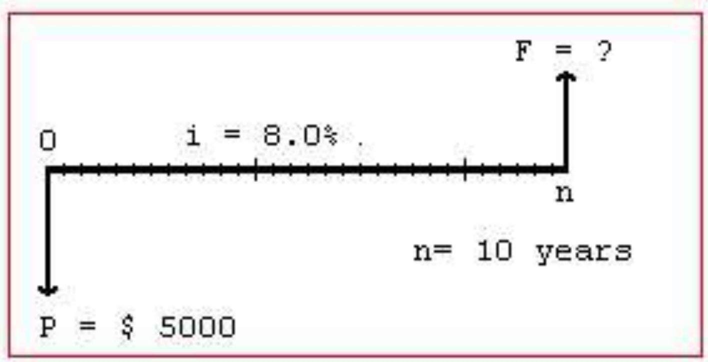
where letter to left of slash is what is sought; letter to right represents what is given

Example: Finding Future Value

A person deposits $5000 into an account which pays interest at a rate of 8% per year. The amount in the account after 10 years is closest to:

(A) $2,792 (B) $9,000 (C) $10,795 (D) $12,165

The cash flow diagram is:



Solution:

F = P(F/P,i,n)

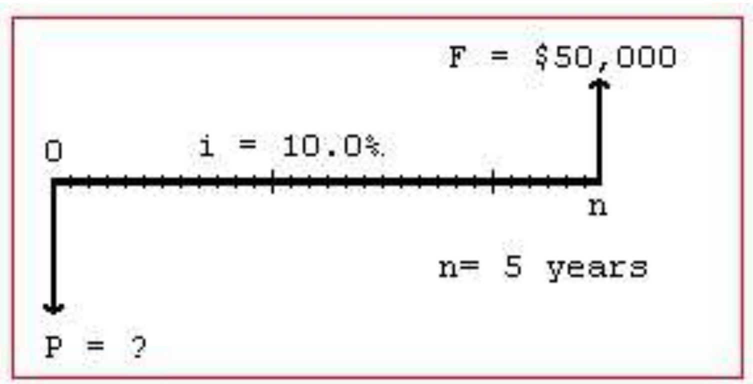
Answer is ?

Example: Finding Present Value

A small company wants to make a single deposit now so it will have enough money to purchase a backhoe costing $50,000 five years from now. If the account will earn interest of 10% per year, the amount that must be deposited now is nearest to:

(A) $10,000 (B) $ 31,050 (C) $ 33,250 (D) $319,160

The cash flow diagram is:



Solution:

P = F(P/F,i,n)

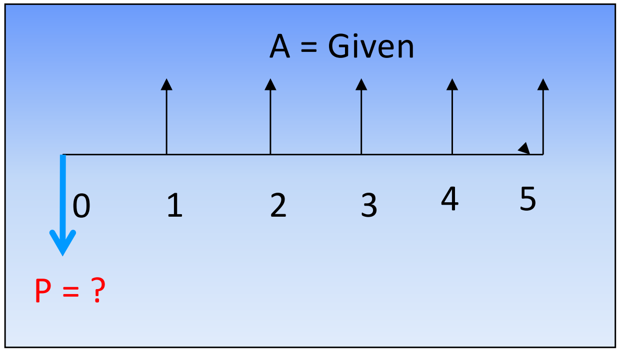
Answer is ?

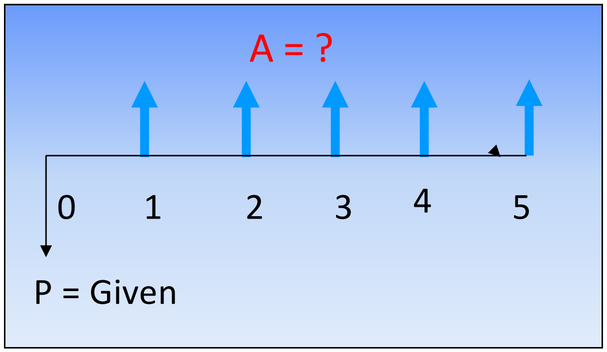
Uniform Series Involving P/A and A/P

The uniform series factors that involve **P and A** are derived as follows:

(1) Cash flow occurs in ***consecutive*** interest periods
(2) Cash flow amount is ***same*** in each interest period

The cash flow diagrams are:





P = A(P/A,i,n) ⟸ Standard Factor Notation ⟹ A = P(A/P,i,n)

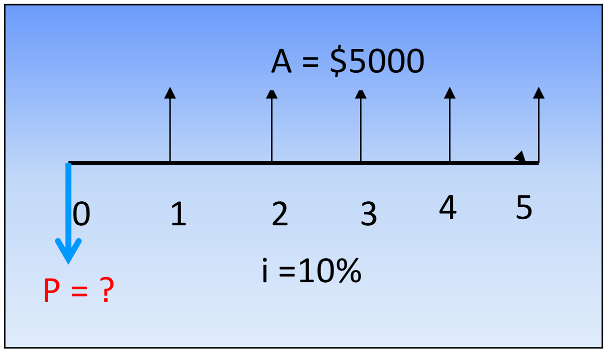
Note: P is one period *Ahead* of first A value

Example: Uniform Series Involving P/A

A chemical engineer believes that by modifying the structure of a certain water treatment polymer, his company would earn an extra $5000 per year. At an interest rate of 10% per year, how much could the company afford to spend now to just break even over a 5 year project period?

(A) $11,170 **(B) 13,640** **(C) $15,300** **(D) $18,950**

The cash flow diagram is as follows:



Solution:

P = 5000(P/A,10%,5)

Answer is ?

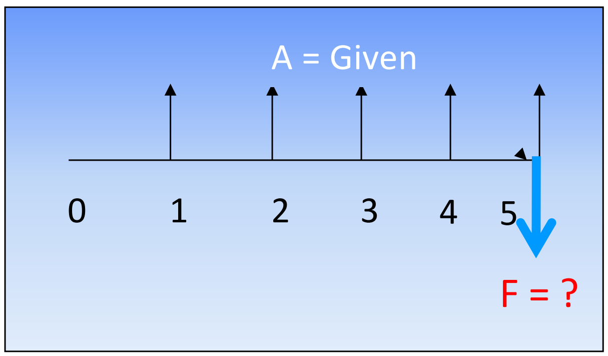
Uniform Series Involving F/A and A/F

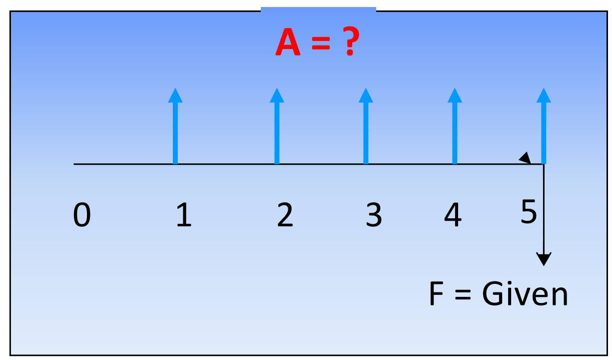
The uniform series factors that involve **F and A** are derived as follows:

(1) Cash flow occurs in ***consecutive*** interest periods

(2) Last cash flow occurs in ***same*** period as F

Cash flow diagrams are:





$F = A(F/A,i,n)$ Standard Factor Notation $A = F(A/F,i,n)$

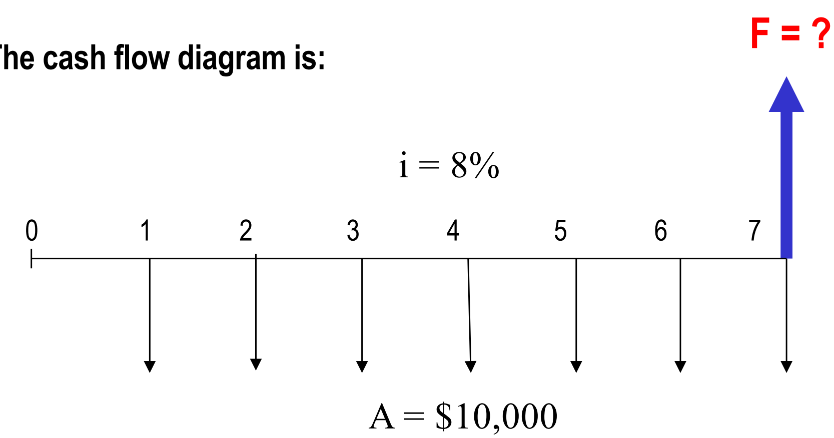
Note: F takes place in the *same* period as last A

Example: Uniform Series Involving F/A

An industrial engineer made a modification to a chip manufacturing process that will save her company $10,000 per year. At an interest rate of 8% per year, how much will the savings amount to in 7 years?

(A) $45,300 (B) $68,500 (C) $89,228 (D) $151,500

The cash flow diagram is:



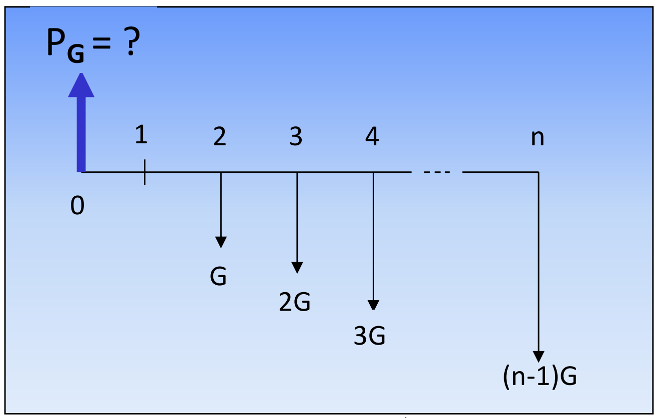
Solution:

$F = 10{,}000(F/A, 8\%, 7)$

Arithmetic Gradients

Arithmetic gradients *change* by the *same amount* each period

The cash flow diagram for the P_G of an arithmetic gradient is:



Standard factor notation is

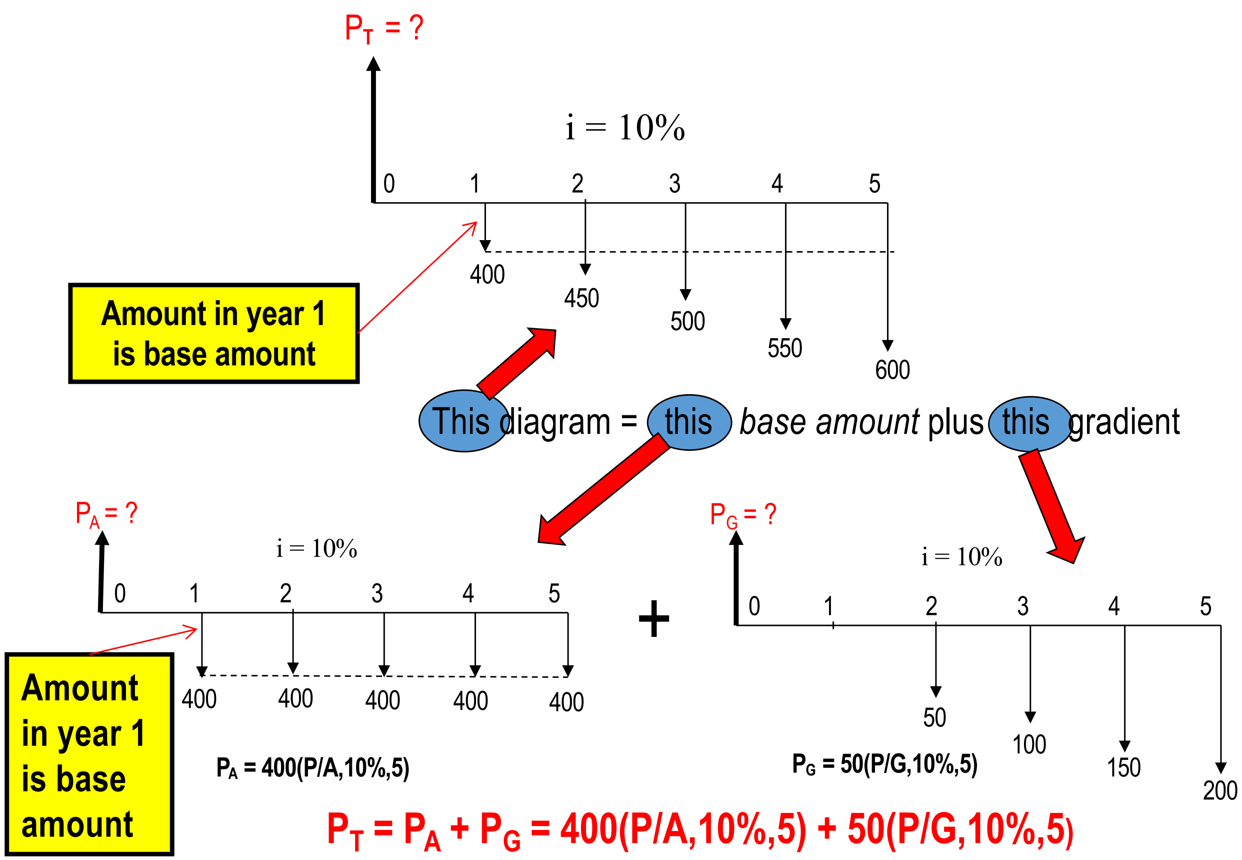
$P_G = G(P/G,i,n)$

G starts between periods 1 and 2 (not between 0 and 1)

This is because cash flow in year 1 is usually not equal to G and is handled separately as a *base amount* (shown on next slide)

Note that P_G is located Two Periods Ahead of the first change that is equal to G

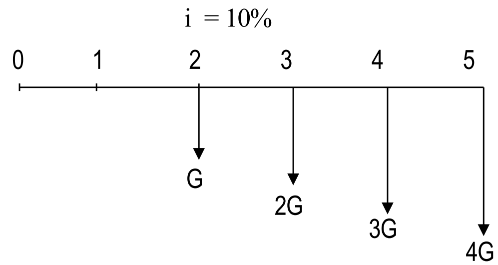
Typical Arithmetic Gradient Cash Flow

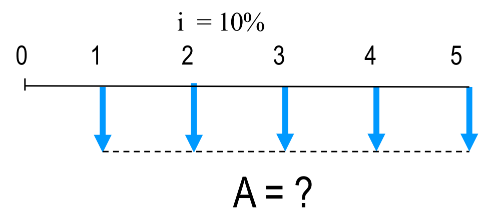


$$P_T = P_A + P_G = 400(P/A,10\%,5) + 50(P/G,10\%,5)$$

Converting Arithmetic Gradient to A

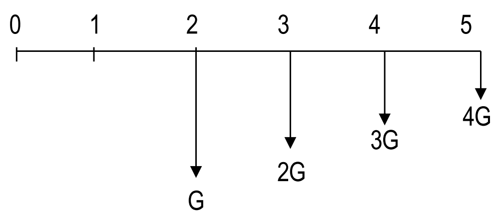
Arithmetic gradient can be converted into equivalent A value using G(A/G,i,n)





General equation when base amount is involved is

A = base amount + G(A/G,i,n)



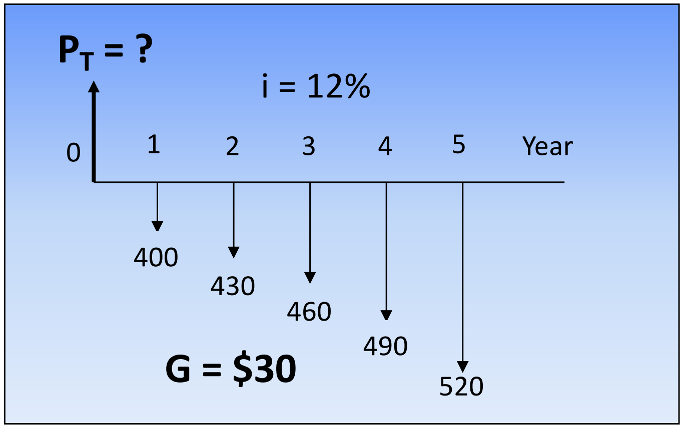
For decreasing gradients, change plus sign to minus

A = base amount - G(A/G,i,n)

Example: Arithmetic Gradient

The present worth of \$400 in year 1 and amounts increasing by \$30 per year through year 5 at an interest rate of 12% per year is closest to:

(A) \$1532 (B) \$1,634 (C) \$1,744 (D) \$1,829



Solution:

$$P_T = 400(P/A,12\%,5) + 30(P/G,12\%,5)$$
$$= 400(3.6048) + 30(6.3970)$$
$$= \$1,633.83$$

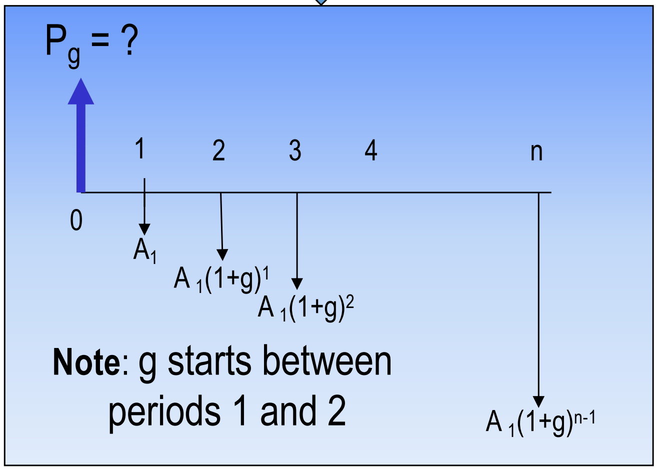
Answer is (B)

The cash flow could also be converted into an **A** value as follows:

Geometric Gradients

Geometric gradients **change by the *same percentage* each period**

Cash flow diagram for present worth of geometric gradient



There are ***no tables*** for geometric factors

Use following equation for $g \neq i$:

$$P_g = A_1\{1- [(1+g)/(1+i)]^n\}/(i-g)$$

where: A_1 = cash flow in period 1

g = rate of increase

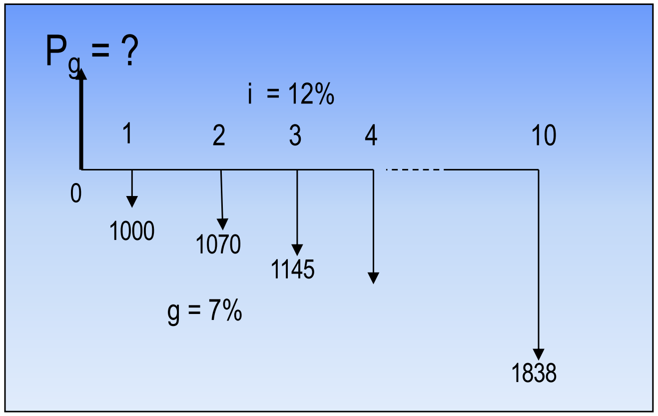
If $g = i$, $P_g = A_1 n/(1+i)$

Note: If g is negative, change signs in front of both g values

Example: Geometric Gradient

Find the present worth of $1,000 in year 1 and amounts increasing by 7% per year through year 10. Use an interest rate of 12% per year.

(a) $5,670 (b) $7,333 (c) $12,670 (d) $13,550



Solution:

$P_g = 1000[1-(1+0.07/1+0.12)^{10}]/(0.12-0.07)$

$= \$7,333$

Answer is ?

To find A, multiply P_g by (A/P,12%,10)

Unknown Interest Rate i

Unknown interest rate problems involve solving for i,
given n and 2 other values (P, F, or A)

(Usually requires a trial and error solution or interpolation in interest tables)

Procedure: Set up equation with all symbols involved and solve for i

A contractor purchased equipment for \$60,000 which provided income of \$16,000 per year for 10 years. The annual rate of return of the investment was closest to:

(a) 15% (b) 18% (c) 20% (d) 23%

Solution: Can use either the P/A or A/P factor. Using A/P:

$$60{,}000(A/P,i\%,10) = 16{,}000$$
$$(A/P,i\%,10) = 0.26667$$

From A/P column at n = 10 in the interest tables, i is between 22% and 24% Answer is (d)

Unknown Recovery Period n

Unknown recovery period problems involve solving for n, given i and 2 other values (P, F, or A)

(Like interest rate problems, they usually require a trial & error solution or interpolation in interest tables)

Procedure: Set up equation with all symbols involved and solve for n

A contractor purchased equipment for $60,000 that provided income of $8,000 per year. At an interest rate of 10% per year, the length of time required to recover the investment was closest to:

(a) 10 years **(b) 12 years** **(c) 15 years** **(d) 18 years**

Solution: Can use either the P/A or A/P factor. Using A/P:

$$60{,}000(A/P,10\%,n) = 8{,}000$$

$$(A/P,10\%,n) = 0.13333$$

From A/P column in i = 10% interest tables, n is between 14 and 15 years **Answer is (c)**

Summary of Important Points

- In P/A and A/P factors, P is *one period ahead* of first A
- In F/A and A/F factors, F is in *same period as last* A
- To find untabulated factor values, best way is to use *formula or spreadsheet*
- For arithmetic gradients, gradient G starts between *periods 1 and 2*
- Arithmetic gradients have 2 parts, *base amount* (year 1) and *gradient amount*
- For geometric gradients, gradient g starts been *periods 1 and 2*
- In geometric gradient formula, A_1 is amount in *period 1*
- To find unknown i or n, *set up equation involving all terms* and solve for i or n

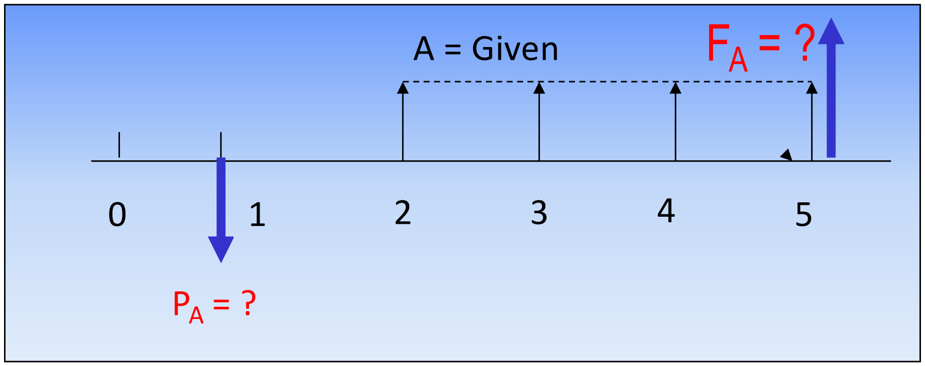
Shifted Series

Shifted Uniform Series

A shifted uniform series starts at a time *other than period 1*

The cash flow diagram below is an example of a shifted series

Series starts in period 2, not period 1



Shifted series usually require the use of *multiple factors*

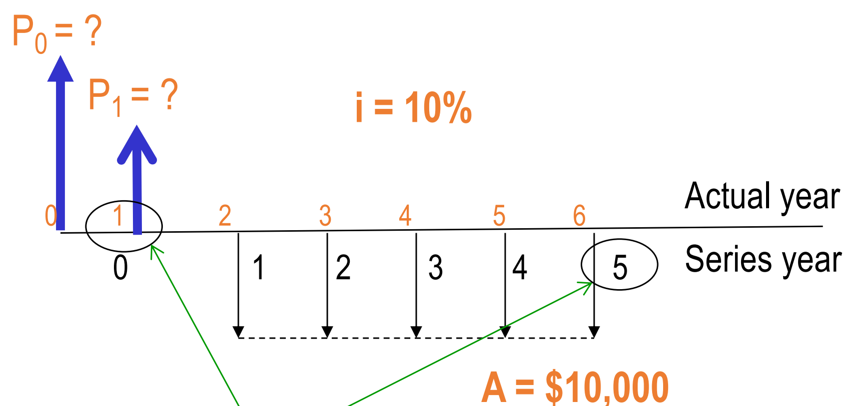
Remember: **When using P/A or A/P factor, P_A is always *one year ahead* of first A**

When using F/A or A/F factor, F_A is in *same year as last A*

Example Using P/A Factor: Shifted Uniform Series

The present worth of the cash flow shown below at i = 10% is:

(a) \$25,304 (b) \$29,562 (c) \$34,462 (d) \$37,908



Solution: (1) Use P/A factor with n = 5 (for 5 arrows) to get P_1 **in year 1**

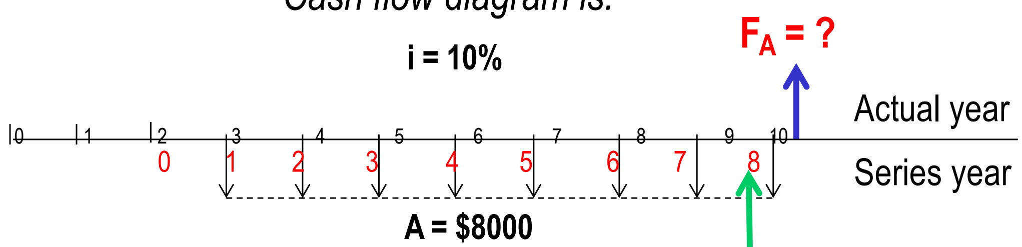
(2) Use P/F factor with n = 1 to move P_1 back for P_0 **in year 0**

P_0 = P_1(P/F,10%,1) = A(P/A,10%,5)(P/F,10%,1) = 10,000(3.7908)(0.9091) = \$34,462

Example Using F/A Factor: Shifted Uniform Series

How much money would be available in year 10 if \$8000 is deposited each year in years 3 through 10 at an interest rate of 10% per year?

Cash flow diagram is:



Solution: Re-number diagram to determine n = 8 (number of arrows)

$$F_A = 8000(F/A,10\%,8)$$
$$= 8000(11.4359)$$
$$= \mathbf{\$91,487}$$

Shifted Series and Random Single Amounts

For cash flows that include *uniform series* **and** *randomly placed single amounts*:

→ *Uniform series procedures* are applied to the *series amounts*

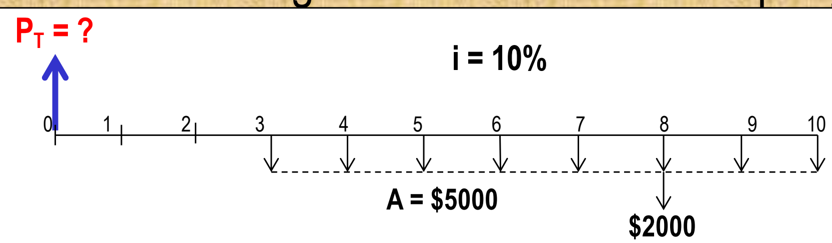
→ *Single amount formulas* are applied to the *one-time cash flows*

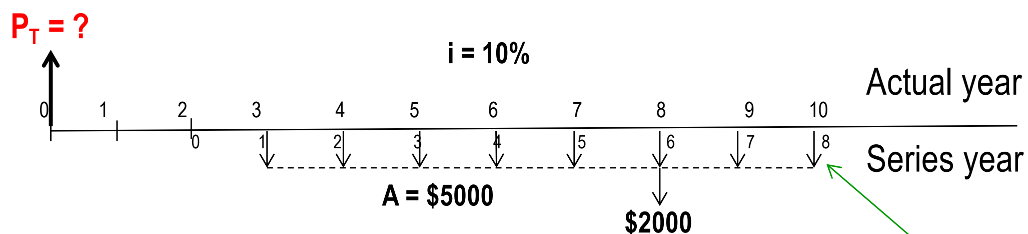
The resulting values are then *combined* per the problem statement

The following slides illustrate the procedure

Example: Series and Random Single Amounts

Find the present worth in year 0 for the cash flows shown using an interest rate of 10% per year.

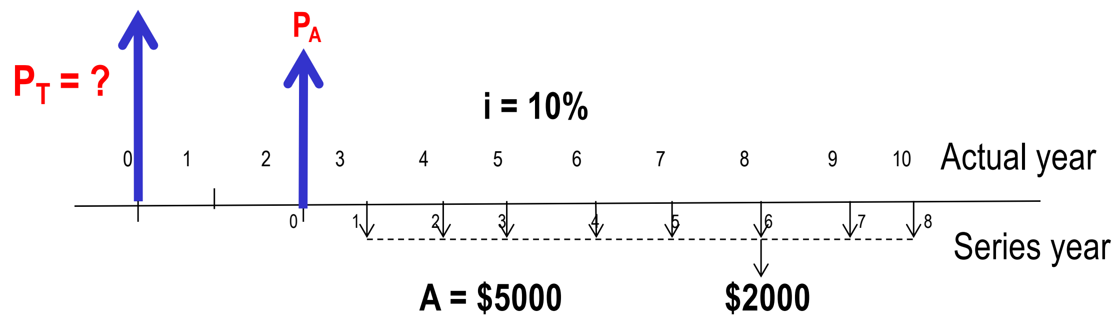




Solution:

First, re-number cash flow diagram to get n for uniform series: n = 8

Example: Series and Random Single Amounts



Use P/A to get P_A in year 2: $P_A = 5000(P/A,10\%,8) = 5000(5.3349) = \$26,675$

Move P_A back to year 0 using P/F: $P_0 = 26,675(P/F,10\%,2) = 26,675(0.8264) = \$22,044$

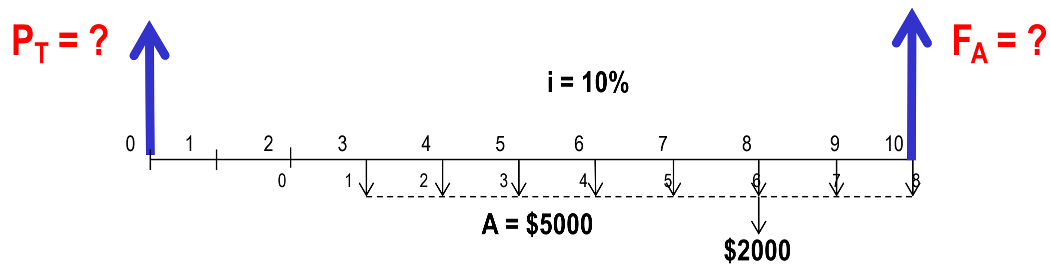
Move $2000 single amount back to year 0: $P_{2000} = 2000(P/F,10\%,8) = 2000(0.4665) = \$933$

Now, add P_0 and P_{2000} to get P_T: $P_T = 22,044 + 933 = \$22,977$

Example Worked a Different Way

(Using F/A instead of P/A for uniform series)

The same re-numbered diagram from the previous slide is used



Solution: Use F/A to get F_A in actual year 10: $F_A = 5000(F/A,10\%,8) = 5000(11.4359) = \$57,180$

Move F_A back to year 0 using P/F: $P_0 = 57,180(P/F,10\%,10) = 57,180(0.3855) = \$22,043$

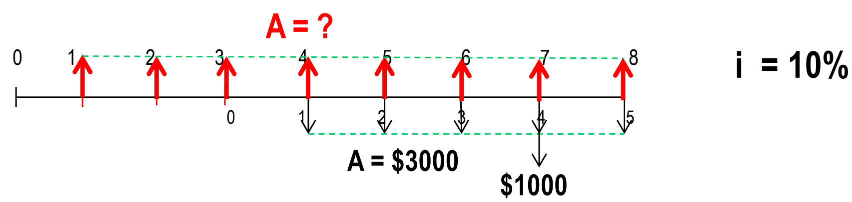
Move $2000 single amount back to year 0: $P_{2000} = 2000(P/F,10\%,8) = 2000(0.4665) = \$933$

Now, add two P values to get P_T: $P_T = 22,043 + 933 = \$22,976$ **Same as before**

As shown, there are usually multiple ways to work equivalency problems

Example: Series and Random Amounts

Convert the cash flows shown below (black arrows) into an equivalent annual worth A in years 1 through 8 (red arrows) at i = 10% per year.



Approaches:

1. Convert all cash flows into P in year 0 and use A/P with n = 8

2. Find F in year 8 and use A/F with n = 8

Solution:

Solve for F: **F = 3000(F/A,10%,5) + 1000(F/P,10%,1)**
= 3000(6.1051) + 1000(1.1000)
= $19,415

Find A: **A = 19,415(A/F,10%,8)**
= 19,415(0.08744)
= $1698

Shifted Arithmetic Gradients

Shifted gradient begins at a time other than between periods 1 and 2

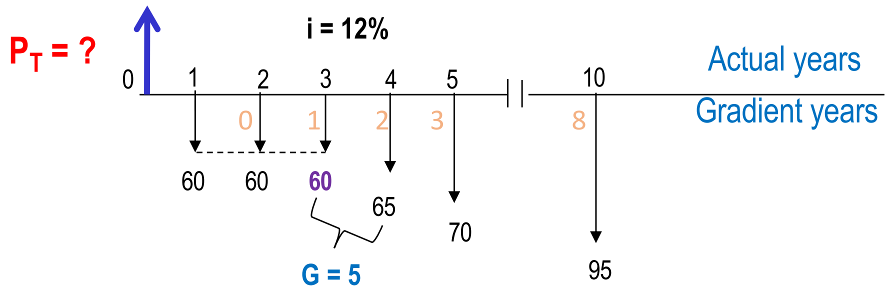
Present worth P_G is located 2 periods before gradient starts

Must use multiple factors to find P_T in actual year 0

To find equivalent A series, find P_T at actual time 0 and apply (A/P,i,n)

Example: Shifted Arithmetic Gradient

John Deere expects the cost of a tractor part to increase by \$5 per year beginning 4 years from now. If the cost in years 1-3 is \$60, determine the *present worth in year 0* of the cost through year 10 at an interest rate of 12% per year.



Solution: First find P_2 for G = \$5 *and* base amount (\$60) in actual year 2

$P_2 = 60(P/A,12\%,8) + 5(P/G,12\%,8) = \$370.41$

Next, move P_2 back to year 0

$P_0 = P_2(P/F,12\%,2) = \$295.29$

Next, find P_A for the \$60 amounts of years 1 and 2

$P_A = 60(P/A,12\%,2) = \$101.41$

Finally, add P_0 and P_A to get P_T in year 0

$P_T = P_0 + P_A =$ **\$396.70**

Shifted Geometric Gradients

- Shifted gradient begins at a time other than between periods 1 and 2

- Equation yields P_g for *all* cash flows (base amount A_1 is included)

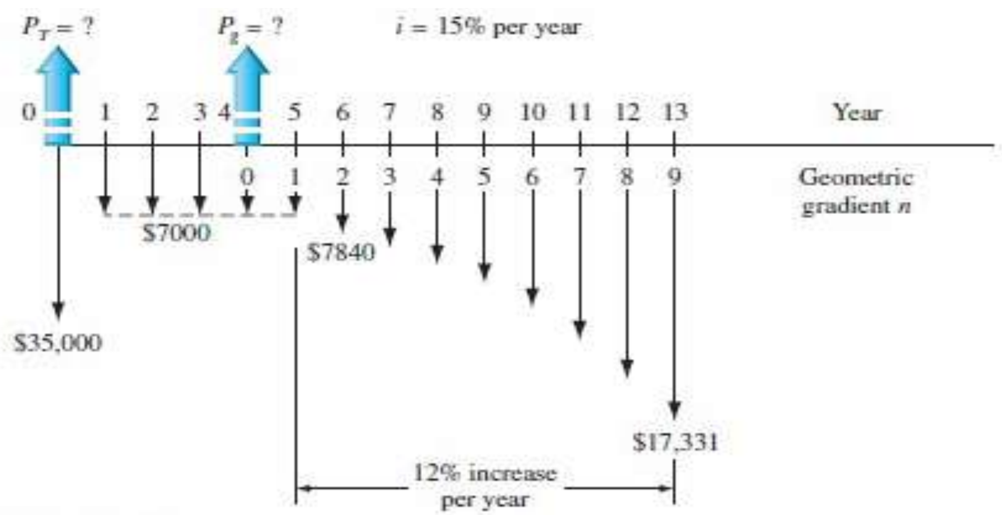
Equation ($i \neq g$): P

- For negative gradient, change signs on both g values

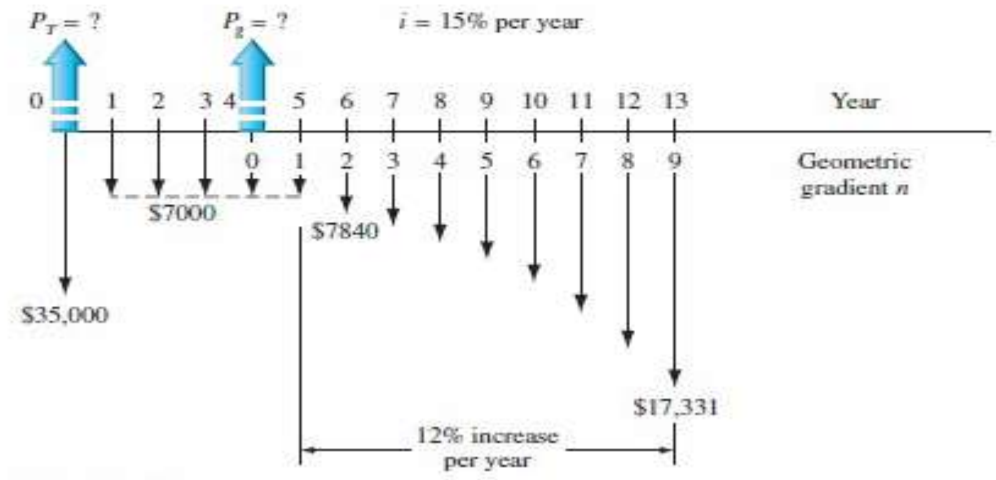
There are no tables for geometric gradient factors

Example: Shifted Geometric Gradient

Weirton Steel signed a 5-year contract to purchase water treatment chemicals from a local distributor for $7000 per year. When the contract ends, the cost of the chemicals is expected to increase by 12% per year for the next 8 years. If an initial investment in storage tanks is $35,000, determine the equivalent present worth in year 0 of all of the cash flows at i = 15% per year.



Example: Shifted Geometric Gradient



Gradient starts between actual years 5 and 6; these are gradient years 1 and 2. P_g is located in gradient year 0, which is actual year 4

$P_g = 7000\{1-[(1+0.12)/(1+0.15)]^9/(0.15-0.12)\} = \$49,401$

Move P_g and other cash flows to year 0 to calculate P_T

$P_T = 35,000 + 7000(P/A,15\%,4) + 49,401(P/F,15\%,4) = \$83,232$

Negative Shifted Gradients

For negative arithmetic gradients, change sign on G term from + to -

General equation for determining P: **P = present worth of base amount - P_G**

Changed from + to -

For negative geometric gradients, change signs on both g values

Changed from + to -

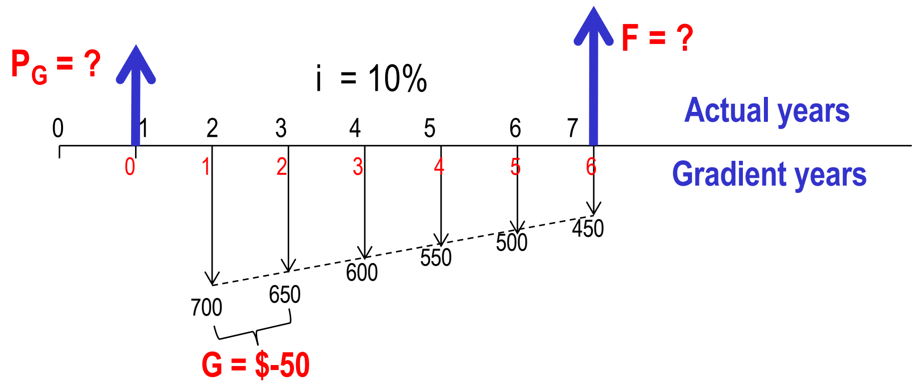
$$P_g = A_1\{1-[(1-g)/(1+i)]^n/(i+g)\}$$

Changed from - to +

All other procedures are the same as for positive gradients

Example: Negative Shifted Arithmetic Gradient

For the cash flows shown, find the future worth in year 7 at i = 10% per year



Solution: Gradient G first occurs between actual years 2 and 3; these are gradient years 1 and 2

P_G is located in gradient year 0 (actual year 1); base amount of $700 is in gradient years 1-6

$$P_G = 700(P/A,10\%,6) – 50(P/G,10\%,6) = 700(4.3553) – 50(9.6842) = \$2565$$

$$F = P_G(F/P,10\%,6) = 2565(1.7716) = \$4544$$

Summary of Important Points

P for shifted uniform series is ***one period ahead*** of first A;
n is equal to number of A values

F for shifted uniform series is in ***same period*** as last A;
n is equal to number of A values

For gradients, ***first change*** equal to G or g occurs
between **gradient years 1 and 2**

For **negative arithmetic** gradients, change sign on G from + to -

For **negative geometric** gradients, change sign on g from + to -

Nominal and Effective Interest Rates

LEARNING OUTCOMES

1. Understand interest rate statements
2. Use formula for effective interest rates
3. Determine interest rate for any time period
4. Determine payment period (PP) and compounding period (CP) for equivalence calculations
5. Make calculations for single cash flows
6. Make calculations for series and gradient cash flows with PP $\geq$ CP
7. Perform equivalence calculations when PP $<$ CP
8. Use interest rate formula for continuous compounding
9. Make calculations for varying interest rates

Interest Rate Statements

The terms ‘nominal’ and ‘effective’ enter into consideration when the interest period is *less than one year*.

New time-based definitions to understand and remember

Interest period (t) – period of time over which interest is expressed. For example, 1% *per month.*

Compounding period (CP) – Shortest time unit over which interest is charged or earned. For example,10% per year *compounded monthly.*

Compounding frequency (m) – Number of times compounding occurs within the interest period t. For example, at i = 10% per year, compounded monthly, interest would be *compounded 12 times* during the one year interest period.

Understanding Interest Rate Terminology

- A *nominal interest rate (r)* is obtained by multiplying an interest rate that is expressed over a short time period by the number of compounding periods in a longer time period: That is:

r = interest rate per period x number of compounding periods

Example: If i = 1% per month, nominal rate per year is
r = (1)(12) = 12% per year)

- *Effective interest rates (i)* take compounding into account (effective rates can be obtained from nominal rates via a formula to be discussed later).

IMPORTANT: Nominal interest rates are essentially simple interest rates. Therefore, they can *never* be used in interest formulas.

Effective rates must *always* be used hereafter in all interest formulas.

More About Interest Rate Terminology

There are 3 general ways to express interest rates as shown below

Sample Interest Rate Statements		**Comment**
(1)	i = 2% per month i = 12% per year	When no compounding period is given, rate is *effective*
(2)	i = 10% per year, comp'd semiannually i = 3% per quarter, comp'd monthly	When compounding period is given and it is *not the same* as interest period, it is *nominal*
(3)	i = effective 9.4%/year, comp'd semiannually i = effective 4% per quarter, comp'd monthly	When compounding period is given and rate is *specified as effective*, rate *is effective* over stated period

Effective Annual Interest Rates

Nominal rates are converted into effective annual rates via the equation:

$$i_a = (1 + i)^m - 1$$

where i_a = effective annual interest rate

i = effective rate for one compounding period

m = number times interest is compounded per year

Example: For a nominal interest rate of 12% per year, determine the nominal and effective rates per year for (a) quarterly, and (b) monthly compounding

Solution:

(a) Nominal r / year = 12% per year

Nominal r / quarter = 12/4 = 3.0% per quarter

Effective i / year = $(1 + 0.03)^4 - 1$ = 12.55% per year

(b) Nominal r /month = 12/12 = 1.0% per year

Effective i / year = $(1 + 0.01)^{12} - 1$ = 12.68% per year

Effective Interest Rates

Nominal rates can be converted into effective rates for any time period via the following equation:

$$i = (1 + r / m)^m - 1$$

where i = effective interest rate for any time period
r = nominal rate for same time period as i
m = no. times interest is comp'd in period specified for i

Spreadsheet function is = EFFECT(r%,m) where r = nominal rate per period specified for i

Example: For an interest rate of 1.2% per month, determine the nominal and effective rates (a) per quarter, and (b) per year

Solution:

(a) Nominal r / quarter = (1.2)(3) = 3.6% per quarter

Effective i / quarter = $(1 + 0.036/3)^3 - 1$ = 3.64% per quarter

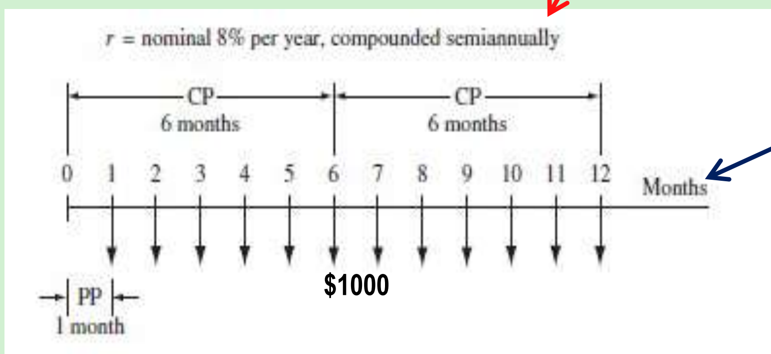
(b) Nominal i /year = (1.2)(12) = 14.4% per year

Effective i / year = $(1 + 0.144 / 12)^{12} - 1$ = 15.39% per year

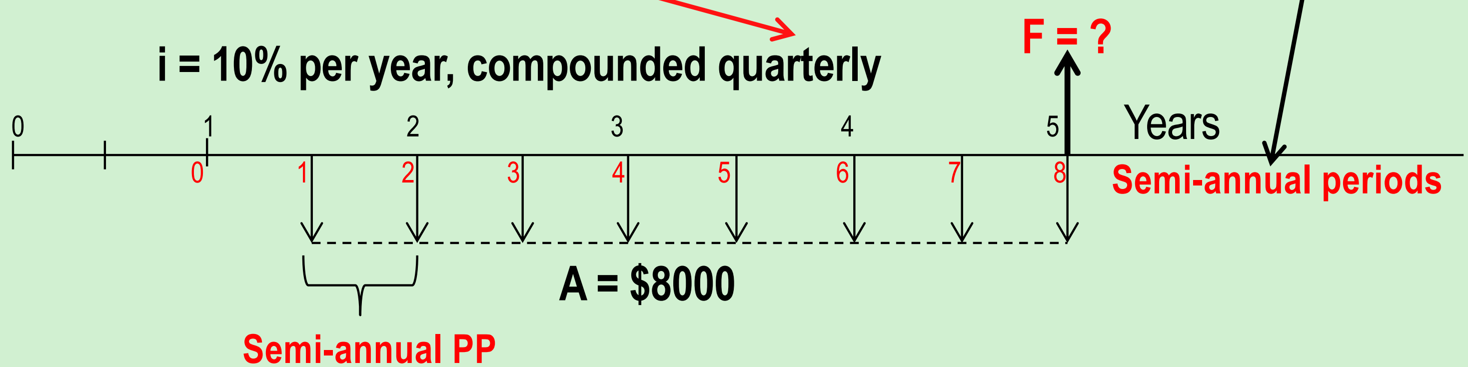
Equivalence Relations: PP and CP

New definition: **Payment Period (PP)** – Length of time between cash flows

In the diagram below, the compounding period (CP) is semiannual and the payment period (PP) is monthly



Similarly, for the diagram below, the CP is quarterly and the payment period (PP) is semiannual



Single Amounts with PP > CP

For problems involving single amounts, the payment period (PP) is usually longer than the compounding period (CP). For these problems, there are an infinite number of i and n combinations that can be used, with only two restrictions:

(1) The i must be an *effective* interest rate, and

(2) The time units on n must be *the same* as those of i

(i.e., if i is a rate per quarter, then n is the number of quarters between P and F)

There are two equally correct ways to determine i and n

Method 1: Determine effective interest rate over the compounding period CP, and set n equal to the number of compounding periods between P and F

Method 2: Determine the effective interest rate for any time period t, and set n equal to the total number of those **same time periods**.

Example: Single Amounts with PP ≥ CP

How much money will be in an account in 5 years if \$10,000 is deposited now at an interest rate of 1% per month? Use three different interest rates: (a) monthly, (b) quarterly , and (c) yearly.

(a) For monthly rate, 1% is effective [n = (5 years)×(12 CP per year = 60]

$$F = 10{,}000(F/P,1\%,60) = \$18{,}167$$

months
effective i per month

i and n must *always* have same time units

(b) For a quarterly rate, effective i/quarter = $(1 + 0.03/3)^3 - 1 = 3.03\%$

$$F = 10{,}000(F/P,3.03\%,20) = \$18{,}167$$

quarters
effective i per quarter

i and n must *always* have same time units

(c) For an annual rate, effective i/year = $(1 + 0.12/12)^{12} - 1 = 12.683\%$

$$F = 10{,}000(F/P,12.683\%,5) = \$18{,}167$$

years
effective i per year

i and n must *always* have same time units

Series with PP ≥ CP

For series cash flows, *first step* is to determine *relationship* between PP and CP

Determine if PP ≥ CP, or if PP < CP

When PP ≥ CP, the *only* procedure (2 steps) that can be used is as follows:

(1) First, find effective i per PP

Example: if PP is in quarters, *must* find effective *i/quarter*

(2) Second, determine n, the number of A values involved

Example: quarterly payments for 6 years yields $n = 4 \times 6 = 24$

Note: Procedure when PP < CP is discussed later

Example: Series with PP ≥ CP

How much money will be accumulated in 10 years from a deposit of \$500 every 6 months if the interest rate is 1% per month?

Solution: First, find relationship between PP and CP
PP = ***six months***, CP = ***one month***; Therefore, **PP > CP**

Since PP > CP, find effective i per PP of six months

Step 1. i /6 months = $(1 + 0.06/6)^6 - 1 = 6.15\%$

Next, determine n (number of 6-month periods)

Step 2: n = 10(2) = 20 six month periods

Finally, set up equation and solve for F

F = 500(F/A,6.15%,20) = \$18,692 (by factor or spreadsheet)

Series with PP < CP

Two policies: (1) interperiod cash flows earn *no interest* (most common)
(2) interperiod cash flows earn *compound interest*

For policy (1), *positive cash flows* are moved *to beginning of the interest period* in which they occur and *negative cash flows are moved to the end of the interest period*

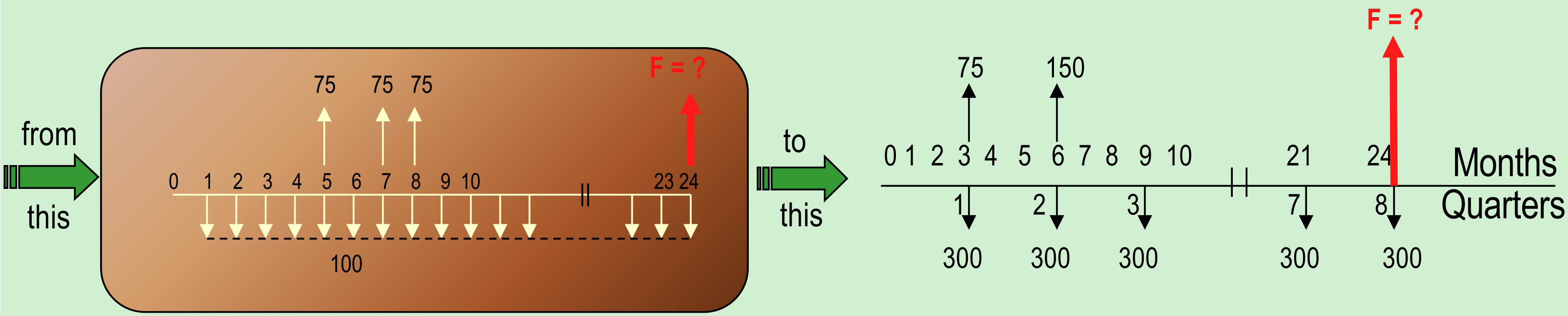
Note: The condition of PP < CP with no interperiod interest is the *only situation in which* the actual cash flow diagram is changed

For policy (2), cash flows are *not moved* and equivalent P, F, and A values are determined using the *effective interest rate per payment period*

Example: Series with PP < CP

A person deposits $100 per month into a savings account for 2 years. If $75 is withdrawn in months 5, 7 and 8 (in addition to the deposits), construct the cash flow diagram to determine how much will be in the account after 2 years at i = 6% per year, compounded quarterly. Assume there is no interperiod interest.

Solution: Since PP < CP with no interperiod interest, the cash flow diagram must be *changed using quarters as the time periods*



Continuous Compounding

When the interest period is infinitely small, interest is *compounded continuously.* Therefore, PP > CP and m increases.

Take limit as m → ∞ to find the effective interest rate equation

$$i = e^r - 1$$

Example: If a person deposits \$500 into an account every 3 months at an interest rate of 6% per year, compounded continuously, how much will be in the account at the end of 5 years?

Solution:

Payment Period: PP = 3 months

Nominal rate per *three months*: $r = 6\%/4 = 1.50\%$

Effective rate per 3 months: $i = e^{0.015} - 1 = 1.51\%$

F = 500(F/A,1.51%,20) = \$11,573

Varying Rates

When interest rates vary over time, use the interest rates associated with their respective time periods to find P

Example: Find the present worth of $2500 deposits in years 1 through 8 if the interest rate is 7% per year for the first five years and 10% per year thereafter.

Solution: P = 2,500(P/A,7%,5) + 2,500(P/A,10%,3)(P/F,7%,5)

= **$14,683**

An equivalent annual worth value can be obtained by replacing each cash flow amount with 'A' and setting the equation equal to the calculated P value

14,683 = A(P/A,7%,5) + A(P/A,10%,3)(P/F,7%,5)

A = **$2500 per year**

Summary of Important Points

Must understand: interest period, compounding period, compounding frequency, and payment period

Always use *effective rates* in interest formulas

$$i = (1 + r / m)^m - 1$$

Interest rates are stated different ways; must know how to get effective rates

For single amounts, make sure units on i and n are the same

Important Points (cont'd)

For uniform series with PP ≥ CP, find effective i over PP

For uniform series with PP < CP and no interperiod interest, move cash flows to match compounding period

For continuous compounding, use $i = e^r - 1$ to get effective rate

For varying rates, use stated i values for respective time periods

Present Worth Analysis

Present Worth Analysis

PURPOSE

Identify types of alternatives; and compare alternatives using a present worth basis

TOPICS

- Formulating alternatives
- Single and equal-life alternatives
- Different-life alternatives
- Capitalized cost alternative evaluation
- Independent alternatives

Formulating Alternatives

Types of alternatives

- ✓ **Mutually exclusive (ME)** - only one viable project can be accepted. Do-nothing (DN) alternative is selected if none are justified economically
- ✓ **Independent** - more than one project can be selected. DN is one of the projects
- ✓ **Do-nothing** – maintain status quo/current approach

Types of cash flow estimates for an alternative

- **Revenue** – estimates include costs, revenues and (possibly) savings
- **Cost** – only cost estimates included; revenues assumed equal for all alternatives

Formulating Alternatives

Much of the emphasis in professional engineering practice is on ME, cost alternatives. However, all tools in Eng Econ can be used to evaluate ME and independent alternatives that are revenue- or cost-based. Examples of both are included later.

Notes: P value of cash flows is now called PW, or present worth

P now represents first cost of an alternative

PW of a Single Alternative

Single project analysis

- **Calculate PW at stated MARR**
- **Criterion: If PW ≥ 0, project is economically justified**

Example: MARR = 10%
First cost, P = \$-2500
Annual revenue, R = \$2000
Annual cost, AOC = \$-900
Salvage value, S = \$200
Life, n = 5 years

$$
\begin{aligned}
PW &= P + S(P/F,10\%,5) \\
&\quad + (R - AOC)(P/A,10\%,5) \\
&= -2500 + 200(P/F,10\%,5) \\
&\quad + (2000-900)(P/A,10\%,5) \\
&= \$1794
\end{aligned}
$$

PW > 0; project is economically justified

Equal-life ME Alternatives

- Calculate PW of each alternative at MARR
- Equal-service of alternatives is assumed
- **Selection criterion: Select alternative with most favorable PW value, that is,**

numerically largest PW value

PW_1	PW_2	Select
$-1,500	$-500	2
-2,500	500	2
2,500	1,500	1

Note : Not the absolute value

Equal-life ME Alternatives

Example: Two ME cost alternatives for traffic analysis. Revenues are equal. MARR is 10% per year. Select one. (cont→)

Estimate	Electric-powered	Solar-powered
P, $/unit	-2,500	-6,000
AOC, $/year	-900	-50
S, $	200	100
n, years	5	5

Equal-life ME Alternatives

Determine PW_E and PW_S; select larger PW

$$PW_E = -2500-900(P/A,10\%,5)+200(P/F,10\%,5)$$
$$= \$-5788$$

$$PW_S = -6000-50(P/A,10\%,5)+100(P/F,10\%,5)$$
$$= \$-6127$$

Conclusion: $PW_E > PW_S$; select electric-powered

Different-life Alternatives

- PW evaluation always requires *equal-service* between all alternatives
- Two methods available:
  - Study period (same period for all alternatives)
  - Least common multiple (LCM) of lives for alternatives
- Study period method is recommended
- Evaluation approach: Determine each PW at stated MARR; select alternative with numerically largest PW

Different-life Alternatives

Study Period of length n years (periods)

- n is same for each alternative
- If life > n, use market value estimate in year n for salvage value
- If life < n, estimate costs for remaining years

Estimates outside time frame
of the study period are ignored

Different-life Alternatives

LCM Method

- Assumptions (may be unrealistic at times)
  - ✓ Same service needed for LCM years (e.g., LCM of 5 and 9 is 45 years!)
  - ✓ Alternatives available for multiple life cycles
  - ✓ Estimates are correct over all life cycles (true only if cash flow estimate changes match inflation/deflation rate)

Evaluation approach: obtain LCM, repeat purchase and life cycle for LCM years; calculate PW over LCM; select alternative with most favorable PW

Different-life Analysis - Example

	Location A	Location B
First cost, $	−15,000	−18,000
Annual lease cost, $ per year	−3,500	−3,100
Deposit return, $	1,000	2,000
Lease term, years	6	9

Use PW to select lower-cost alternative:

- For 5-year study period
- Using LCM of alternatives' lives

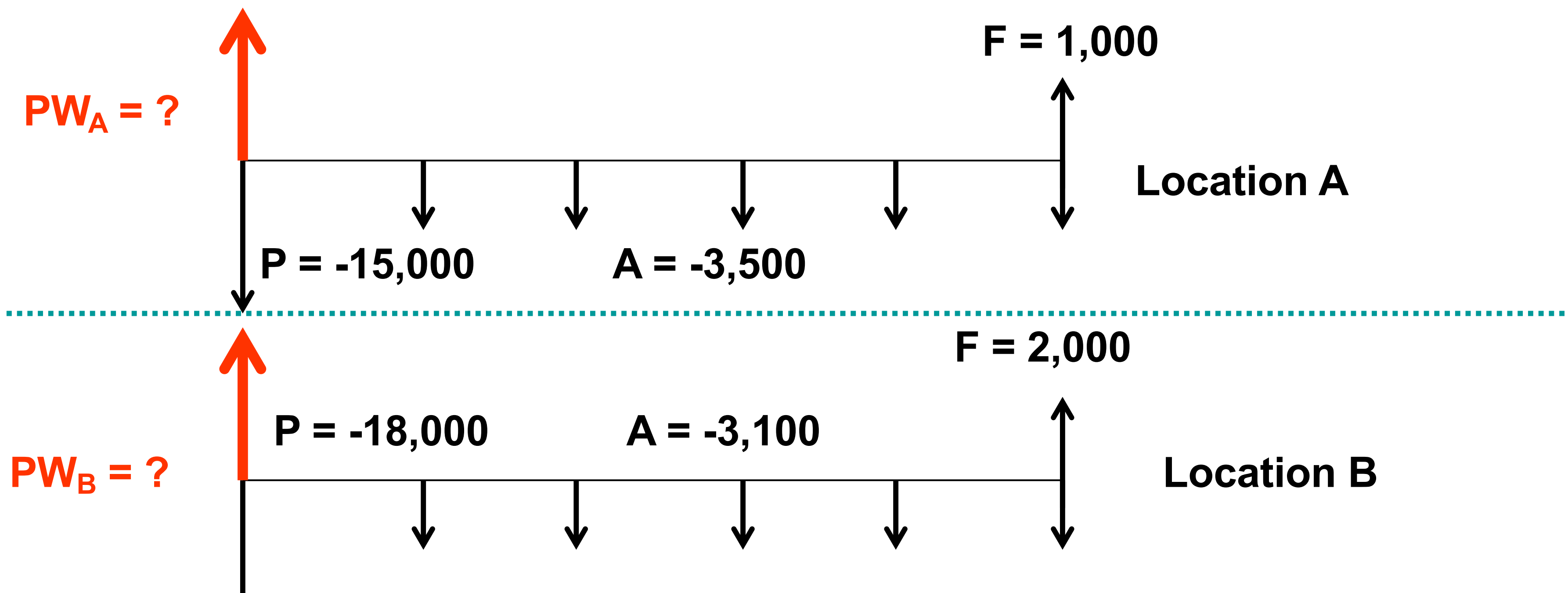
Assume MARR = 15% per year

(cont →)

Different-life Analysis - Example

Study period of 5 years

Assume deposit returns are good estimates after 5 years



For 5 years at i = 15%: PW_A = \$-26,236 and PW_B = \$-27,397

Select Location A with lower PW of costs

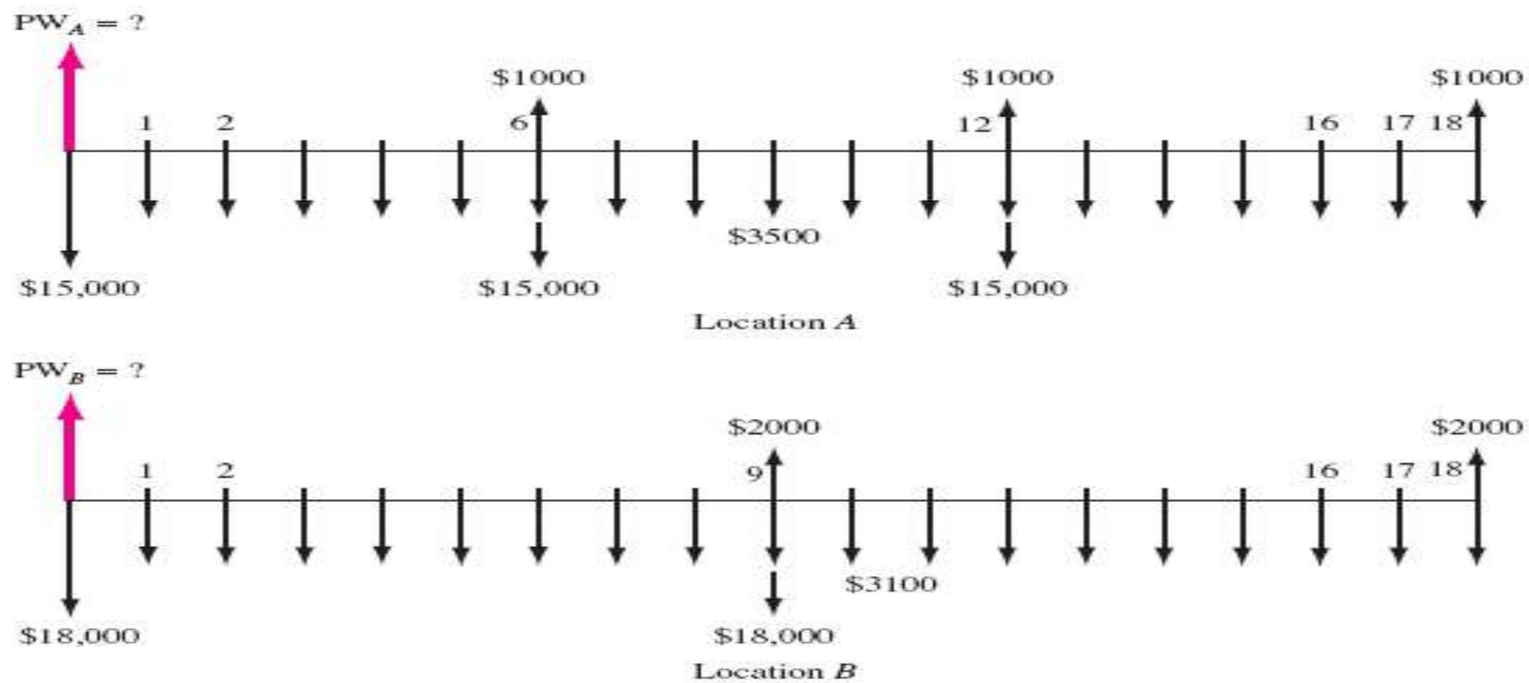
Different-life Analysis - Example

LCM evaluation

- LCM is 18 years
- Repurchase A twice (years 6 and 12)
- Repurchase B once (year 9)
- Assume all cash flow estimates (including first cost end-of-lease ‘deposit return’) are correct for repeated life cycles to total 18 years

(cont →)

Different-life Analysis - Example



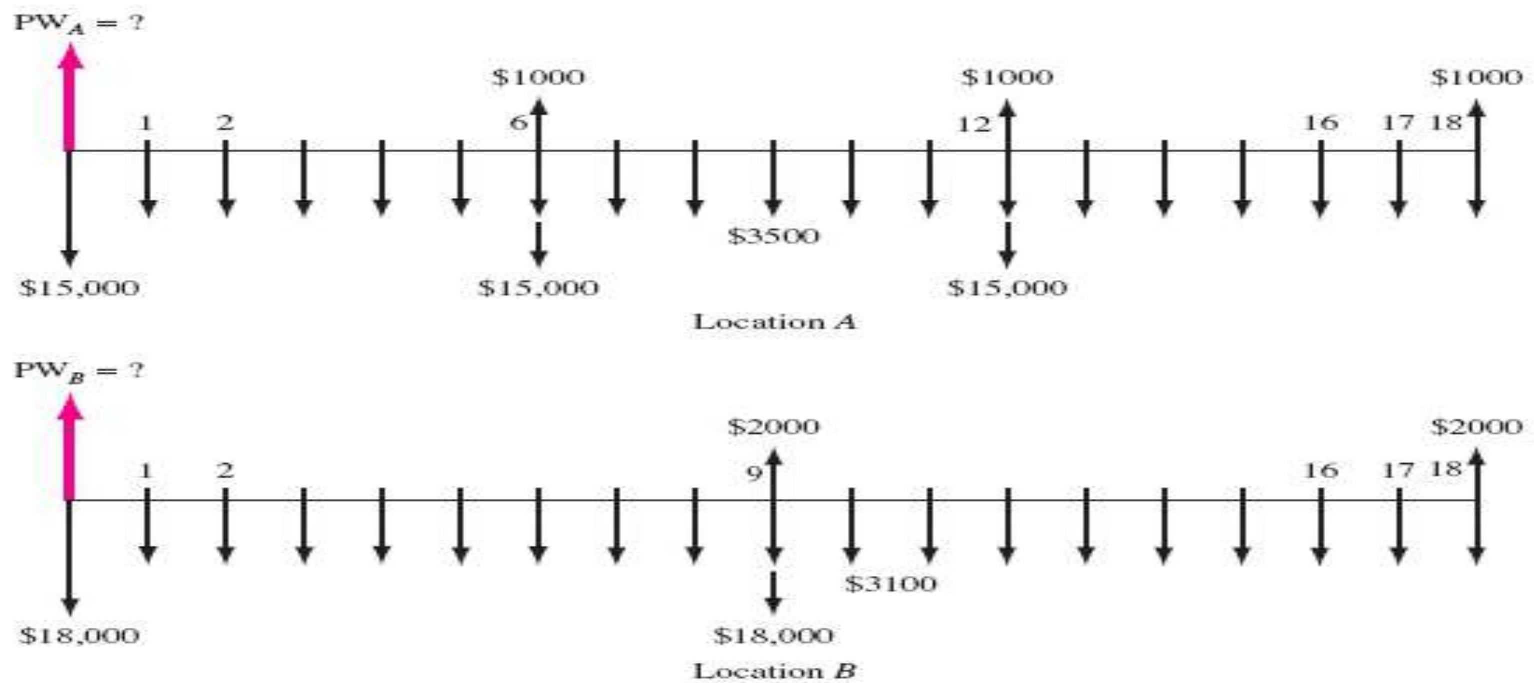
For 18 years at MARR = 15%: PW_A = \$-45,036

For 18 years at MARR = 15%: PW_B = \$-41,384

Select location B

Note: Selection changed from 5-year study period

Different-life Analysis - Example



For 18 years at MARR = 15%: PW_A = $-45,036

For 18 years at MARR = 15%: PW_B = $-41,384

Select location B

Note: Selection changed from 5-year study period

Future Worth Evaluation

- FW evaluation of alternatives is especially applicable for LARGE capital investment situations when maximizing the future worth of a corporation is important
- e.g., buildings, power generation, acquisitions

- Evaluation approach: Determine FW value from cash flows or PW with an *n* value in F/P factor
  - equal to study period, or
  - equal to LCM of alternatives' lives

Capitalized Cost (CC)

- PW of alternative that will last 'forever'
- Especially applicable to public project evaluation (dams, bridges, irrigation, hospitals, police, etc.)
- CC relation is derived using the limit as $n \rightarrow \infty$ for the P/A factor

$$\mathbf{PW} = A(P/A, i\%, n) = A\left[\frac{1 - \frac{1}{(1+i)^n}}{i}\right]$$

$$\text{PW} = A[1/i\,]$$

Capitalized Cost

- Refer to PW as CC when n is large (can be considered infinite). Then

$$CC = \frac{A}{i} = \frac{AW}{i}$$

and $AW = CC \times i$

Example: If $10,000 earns 10% per year, $1,000 is interest earned annually for eternity. Principal remains in tact

- Cash flows for CC computations are of two types -- recurring and nonrecurring

Capitalized Cost

Procedure to find CC

1. Draw diagram for 2 cycles of recurring cash flows and any nonrecurring amounts
2. Calculate PW (CC) for all nonrecurring amounts
3. Find AW for 1 cycle of recurring amounts; then add these to all A series applicable for all years 1 to ∞ (or long life)
4. Find CC for amount above using CC = AW/i
5. Add all CC values (steps 2 and 4)

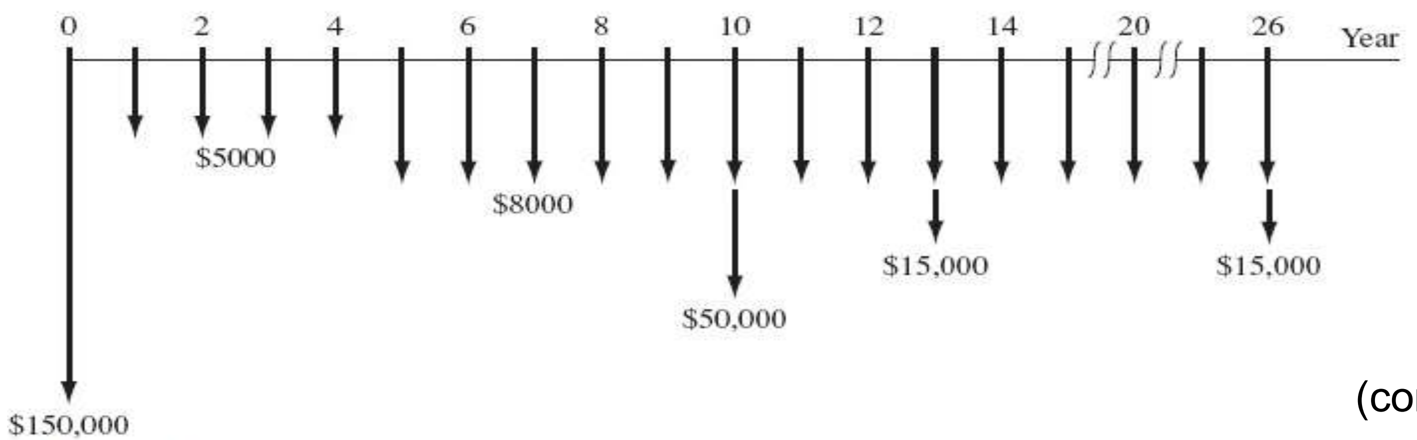
CC Computation - Example

Find CC and A values at i = 5% of long-term public project with cash flows below. Cycle time is 13 years.

Nonrecurring costs: first $150,000; one-time of $50,000 in year 10

Recurring costs: annual maintenance of $5000 (years 1-4) and $8000 thereafter; upgrade costs $15,000 each 13 years

Step 1



(cont →)

CC Computation - Example

2. CC of nonrecurring costs:

$$CC_1 = -150{,}000 - 50{,}000(P/F,5\%,10) = \$\text{-}180{,}695$$

3. AW of recurring \$15,000 upgrade:

   $AW = -15{,}000(A/F,5\%,13) = \$\text{-}847$ per year

   AW of recurring maintenance costs years 1 to ∞:

   $AW = \$\text{-}5000$ per year forever

4. CC of extra \$3000 maintenance for years 5 to ∞:

$$CC_2 = -3000(P/F,5\%,4)/0.05 = \$\text{-}49{,}362$$

   CC for recurring upgrade and maintenance costs:

$$CC_3 = (-847-5000)/0.05 = \$\text{-}116{,}940$$

5. Total CC obtained by adding all three CC components

$$CC_T = -180{,}695 - 49{,}362 - 116{,}940 = \$\text{-}346{,}997$$

The AW value is the annual cost forever:

$$\mathbf{AW = CC \times i = -346{,}997(0.05) = \$\text{-}17{,}350}$$

CC Evaluation of Alternatives

- For two long-life or infinite-life alternatives:

  SELECT ALTERNATIVE WITH LOWER CC OF COSTS

- For one infinite life and one finite life:

  Determine CC for finite life alternative using AW of 1 life cycle and relation CC = AW/i

  SELECT ALTERNATIVE WITH LOWER CC OF COSTS

CC Evaluation of Alternatives - Example

1 long-term (assumed infinite); 1 finite life

Long-term alternative (LT): $8 million now; $25,000 renewal annual contract

Short-term alternative (ST): $2.75 million now; $120,000 AOC; life is n = 5 years

Select better at MARR = 15% per year

CC_{LT} = -8,000,000 – 25,000/0.15 = $-8.17 million

CC_{ST} = AW/0.15

= [-2,750,000(A/P,15%,5) – 120,000]/0.15

= $-6.27 million

Conclusion: Select ST with lower CC of costs

Annual Worth Analysis

LEARNING OUTCOMES

1. **Advantages of AW**
2. **Capital Recovery and AW values**
3. **AW analysis**
4. **Perpetual life**
5. **Life-Cycle Cost analysis**

Advantages of AW Analysis

AW calculated for only one life cycle

Assumptions:

- Services needed for *at least the LCM* of lives of alternatives
- Selected alternative *will be repeated* in succeeding life cycles in same manner as for the first life cycle
- All cash flows *will be same* in every life cycle (i.e., will change by only inflation or deflation rate)

Alternatives usually have the following cash flow estimates

- **Initial investment, P – First cost of an asset**
- **Salvage value, S – Estimated value of asset at end of useful life**
- **Annual amount, A – Cash flows associated with asset, such as annual operating cost (AOC), etc.**

Relationship between AW, PW and FW

AW = PW(A/P,i%,n) = FW(A/F,i%,n)

n is years for equal-service comparison (value of LCM or specified study period)

Calculation of Annual Worth

AW for one life cycle is the *same for all* life cycles!!

An asset has a first cost of \$20,000, an annual operating cost of \$8000 and a salvage value of \$5000 after 3 years. Calculate the AW for one and two life cycles at i = 10%

AW_{one} = - 20,000(A/P,10%,3) – 8000 + 5000(A/F,10%,3)

= **\$-14,532**

AW_{two} = - 20,000(A/P,10%,6) – 8000 – 15,000(P/F,10%,3)(A/P,10%,6) + 5000(A/F,10%,6)

= **\$-14,532**

Capital Recovery and AW

Capital recovery (CR) is the equivalent annual amount that an asset, process, or system must earn each year to just recover the first cost and a stated rate of return over its expected life. Salvage value is considered when calculating CR.

$$\mathbf{CR = -P(A/P,i\%,n) + S(A/F,i\%,n)}$$

Use previous example:(note: AOC not included in CR)

CR = -20,000(A/P,10%,3) + 5000(A/F,10%,3) = $ – 6532 per year

Now **AW = CR + A**

AW = – 6532 – 8000 = $ – 14,532

Selection Guidelines for AW Analysis

One alternative: If AW ≥ 0, the requested MARR is met or exceeded and the alternative is economically justified.

Two or more alternatives: Select the alternative with the AW that is ~~**numerically largest,**~~ that is, less negative or more positive. This indicates a lower AW of cost for cost alternatives or a larger AW of net cash flows for revenue alternatives.

ME Alternative Evaluation by AW

Not necessary to use LCM for different life alternatives

A company is considering two machines. Machine X has a first cost of $30,000, AOC of $18,000, and S of $7000 after 4 years.
Machine Y will cost $50,000 with an AOC of $16,000 and S of $9000 after 6 years.
Which machine should the company select at an interest rate of 12% per year?

Solution:

$AW_X = -30{,}000(A/P,12\%,4) - 18{,}000 + 7{,}000(A/F,12\%,4)$
$= \$-26{,}412$

$AW_Y = -50{,}000(A/P,12\%,6) - 16{,}000 + 9{,}000(A/F,12\%,6)$
$= \$-27{,}052$

Select Machine X; it has the numerically larger AW value

AW of Permanent Investment

Use A = Pi for AW of ***infinite*** life alternatives
Find AW over ***one life cycle*** for ***finite*** life alternatives

Compare the alternatives below using AW and i = 10% per year

	C	D
First Cost, $	-50,000	-250,000
Annual operating cost, $/year	-20,000	-9,000
Salvage value, $	5,000	75,000
Life, years	5	∞

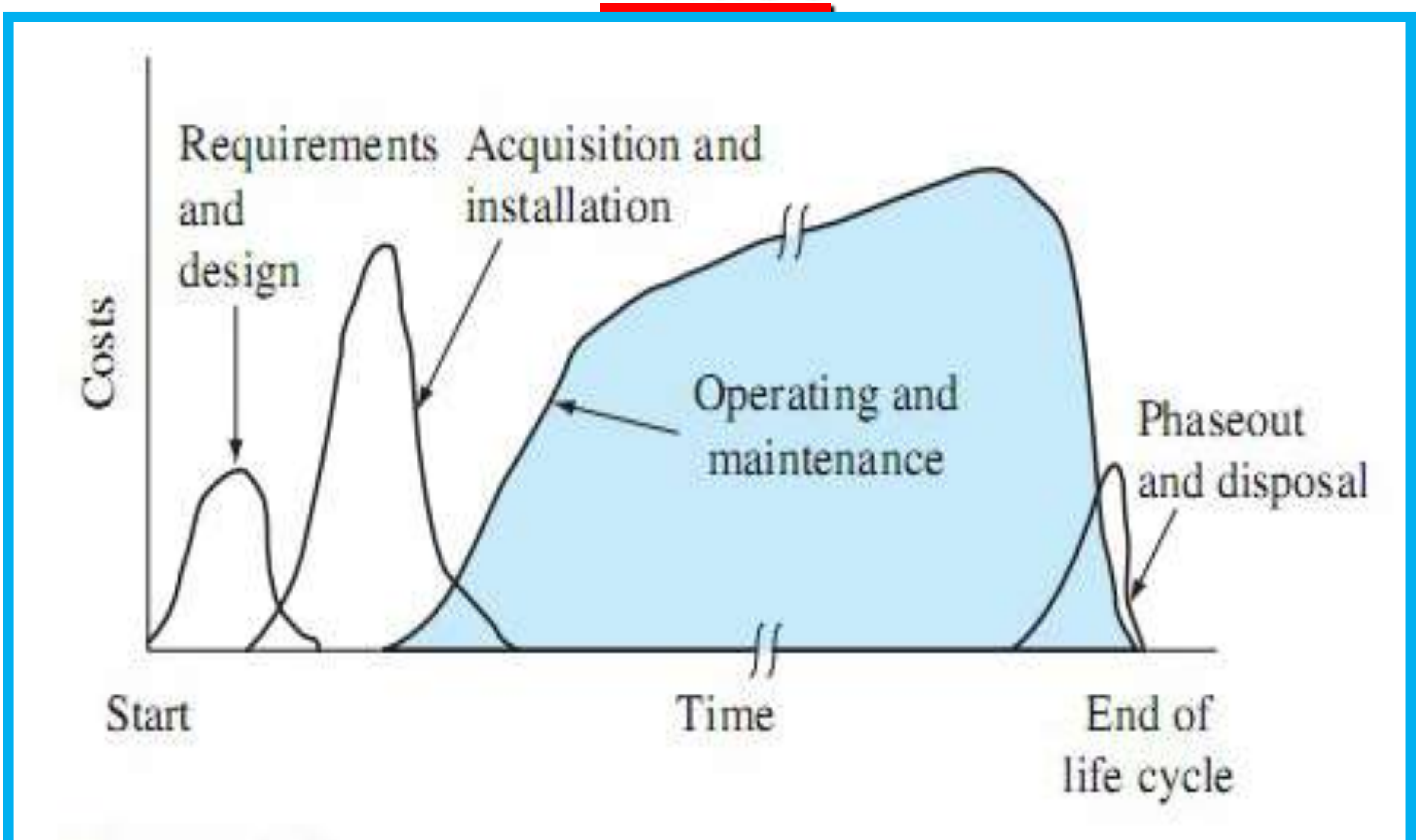
Solution: Find AW of C over 5 years and AW of D using relation A = P*i*

AW_C = -50,000(A/P,10%,5) – 20,000 + 5,000(A/F,10%,5)
= $-32,371

AW_D = P*i* + AOC = -250,000(0.10) – 9,000
= $-34,000

Select alternative C

Typical Life-Cycle Cost Distribution by Phase



Life-Cycle Cost Analysis

LCC analysis includes *all* costs for *entire* life span, from concept to disposal

Best when large percentage of costs are *M&O*

Includes phases of *acquisition, operation, & phaseout*

- ✓ Apply the AW method for LCC analysis of 1 or more cost alternatives
- ✓ Use PW analysis if there are revenues and other benefits considered

Summary of Important Points

- AW method converts all cash flows to *annual value at MARR*
- Alternatives can be *mutually exclusive, independent, revenue, or cost*
- AW comparison is *only one life cycle of each alternative*
- For infinite life alternatives, annualize *initial cost as A = P(i)*
- Life-cycle cost analysis includes *all costs* over a project's life span

Rate of Return for Single Project

LEARNING OUTCOMES

1. Understand meaning of ROR
2. Calculate ROR for cash flow series
3. Understand difficulties of ROR
4. Determine multiple ROR values
5. Calculate External ROR (EROR)
6. Calculate r and i for bonds

Interpretation of ROR

Rate paid on *unpaid balance* of borrowed money such that final payment brings balance to exactly zero with interest considered

ROR equation can be written in terms of PW, AW, or FW

Use trial and error solution by *factor*

Numerical value can range from *-100% to infinity*

ROR Calculation and Project Evaluation

- To determine ROR, find the *i** value in the relation

$$PW = 0 \quad \text{or} \quad AW = 0 \quad \text{or} \quad FW = 0$$

- Alternatively, a relation like the following finds i*

$$PW_{outflow} = PW_{inflow}$$

- For evaluation, a project is economically viable if

$$i^* \geq MARR$$

ROR Calculation Using PW, FW or AW Relation

ROR is the unique i* rate at which a PW, FW, or AW relation equals exactly 0

Example: An investment of $20,000 in new equipment will generate income of $7000 per year for 3 years, at which time the machine can be sold for an estimated $8000. If the company's MARR is 15% per year, should it buy the machine?

Solution:: The ROR equation, based on a PW relation, is:

$$0 = -20{,}000 + 7000(P/A,i^*,3) + 8000(P/F,i^*,3)$$

Solve for i* by trial and error: i* = 18.2% per year

Since i* > MARR = 15%, *the company should buy the machine*

Special Considerations for ROR

- May get *multiple i* values* (discussed later)
- i* assumes *reinvestment* of positive cash flows earn *at i* rate* (may be unrealistic)
- *Incremental analysis* necessary for multiple alternative evaluations (discussed later)

Multiple ROR Values

Multiple i* values may exist when there is more than one sign change in net cash flow (CF) series.
Such CF series are called non-conventional

Two tests for multiple i* values:

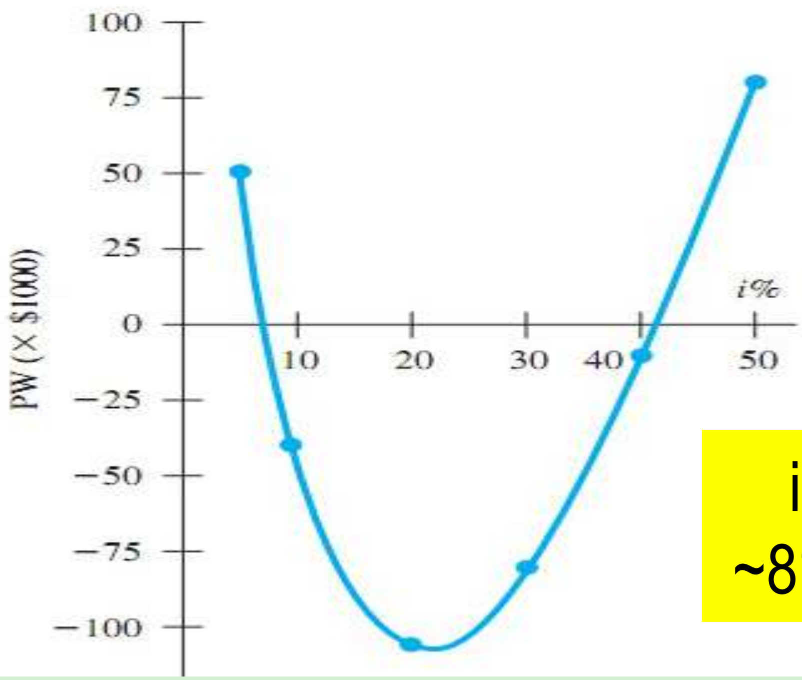
Descarte's rule of signs: total number of real i* values is ≤ the number of sign changes in *net cash flow series.*

Norstrom's criterion: if the *cumulative cash flow* starts off negatively and has only *one sign change*, there is only one positive root .

Plot of PW for CF Series with Multiple ROR Values

Year	Cash Flow ($1000)	Sequence Number	Cumulative Cash Flow ($1000)
0	+2000	S_0	+2000
1	−500	S_1	+1500
2	−8100	S_2	−6600
3	+6800	S_3	+200

i%	5	10	20	30	40	50
PW ($1000)	+51.44	−39.55	−106.13	−82.01	−11.83	+81.85



i* values at ~8% and ~41%

Example: Multiple i* Values

Determine the maximum number of i* values for the cash flow shown below

Year	Expense	Income	**Net cash flow**	**Cumulative CF**
0	-12,000	-	-12,000	-12,000
1	-5,000	+ 3,000	-2,000	-14,000
2	-6,000	+9,000	+3,000	-11,000
3	-7,000	+15,000	+8,000	-3,000
4	-8,000	+16,000	+8,000	+5,000
5	-9,000	+8,000	-1,000	+4,000

Solution:

The sign on the net cash flow changes twice, indicating two possible i* values

The cumulative cash flow begins negatively with *one sign change*

Therefore, there is only one i* value (i* = 8.7%)

Removing Multiple i* Values

Two new interest rates to consider:

- ***Investment rate*** i_i **– rate at which extra funds are *invested* *external* to the project**
- ***Borrowing rate*** i_b **– rate at which funds are borrowed *from an external source* to provide funds to the project**

Two approaches to determine External ROR (EROR)

- (1) Modified ROR (MIRR)
- (2) Return on Invested Capital (ROIC)

Modified ROR Approach (MIRR)

Four step Procedure:

- Determine PW in *year 0* of all negative CF at i_b
- Determine FW in *year n* of all positive CF at i_i
- Calculate EROR = i' by FW = PW(F/P,i',n)
- If i' ≥ MARR, project is economically justified

Example: EROR Using MIRR Method

For the NCF shown below, find the EROR by the MIRR method if MARR = 9%, i_b = 8.5%, and i_i = 12%

Year	0	1	2	3
NCF	+2000	-500	-8100	+6800

Solution:

$$PW_0 = -500(P/F,8.5\%,1) - 8100(P/F,8.5\%,2)$$
$$= \$-7342$$

$$FW_3 = 2000(F/P,12\%,3) + 6800$$
$$= \$9610$$

$$PW_0(F/P,i',3) + FW_3 = 0$$
$$-7342(1 + i')^3 + 9610 = 0$$

i' = 0.939 (9.39%)

Since i' > MARR of 9%, project is justified

Return on Invested Capital Approach

- Measure of how effectively project uses funds that ***remain internal to project***
- ROIC rate, i”, is determined using ***net-investment procedure***

Three step Procedure

(1) Develop series of FW relations for each year t using:

$$F_t = F_{t-1}(1 + k) + NCF_t$$

where: $k = i_i$ if $F_{t-1} > 0$ and $k = i''$ if $F_{t-1} < 0$

(2) Set future worth relation for last year n equal to 0 (i.e., $F_n = 0$); solve for i”

(3) If i” ≥ MARR, *project is justified*; otherwise, *reject*

ROIC Example

For the NCF shown below, find the EROR by the ROIC method if MARR = 9% and i_i = 12%

Year	0	1	2	3
NCF	+2000	-500	-8100	+6800

Solution:

Year 0: $F_0 = \$+2000$ — $F_0 > 0$; invest in year 1 at $i_i = 12\%$

Year 1: $F_1 = 2000(1.12) - 500 = \$+1740$ — $F_1 > 0$; invest in year 2 at $i_i = 12\%$

Year 2: $F_2 = 1740(1.12) - 8100 = \$-6151$ — $F_2 < 0$; use i'' for year 3

Year 3: $F_3 = -6151(1 + i'') + 6800$ — Set $F_3 = 0$ and solve for i''

$$-6151(1 + i'') + 6800 = 0$$
$$i'' = 10.55\%$$

Since i'' > MARR of 9%, project is justified

Important Points to Remember

About the computation of an EROR value

- EROR values are dependent upon the selected investment and/or borrowing rates
- Commonly, multiple i* rates, i’ from MIRR and i’’ from ROIC have different values

About the method used to decide

- For a definitive economic decision, set the MARR value and *use the PW or AW method* to determine economic viability of the project

ROR of Bond Investment

Bond is ***IOU*** with face value **(V)**, coupon rate **(b)**, no. of payment periods/year **(c)**, dividend **(I)**, and maturity date **(n)**. Amount paid for the bond is **P.**

$$I = Vb/c$$

General equation for i*: $0 = -P + I(P/A,i^*,n \times c) + V(P/F,i^*,n \times c)$

A $10,000 bond with 6% interest payable quarterly is purchased for $8000. If the bond matures in 5 years, what is the ROR (a) per quarter, (b) per year?

Solution: **(a)** I = 10,000(0.06)/4 = $150 per quarter

ROR equation is: $0 = -8000 + 150(P/A,i^*,20) + 10{,}000(P/F,i^*,20)$

By trial and error or spreadsheet: $i^* = 2.8\%$ per quarter

(b) Nominal i* per year = 2.8(4) = 11.2% per year

Effective i* per year = $(1 + 0.028)^4 - 1$ = 11.7% per year

Summary of Important Points

- ROR equations can be written in terms of PW, FW, or AW and usually require *trial and error solution*
- i* assumes *reinvestment* of positive cash flows *at i* rate*
- More than 1 sign change in NCF may cause *multiple i* values*
- Descarte's rule of signs and Norstrom's criterion useful when *multiple i* values* are suspected
- EROR can be calculated using MIRR or ROIC approach. Assumptions about investment and borrowing rates is required.
- General ROR equation for bonds is

$$0 = -P + I(P/A, i^*, n_x c) + V(P/F, i^*, n_x c)$$

ROR of Bond Investment

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- General ROR equation for bonds is

$$0 = -P + I(P/A,i^*,n_x c) + V(P/F,i^*,n_x c)$$

Rate of Return Multiple Alternatives

LEARNING OUTCOMES

1. Why incremental analysis is required in ROR
2. Incremental cash flow (CF) calculation
3. Interpretation of ROR on incremental CF
4. Select alternative by ROR based on PW relation
5. Select alternative by ROR based on AW relation
6. Select best from several alternatives using ROR method

Why Incremental Analysis is Necessary

- Selecting the alternative with highest ROR may not yield highest return on ***available capital***
- Must consider ***weighted average*** of total capital available
- Capital ***not*** invested in a project is assumed to ***earn at MARR***

Example: Assume $90,000 is available for investment and MARR = 16% per year. If alternative A would earn 35% per year on investment of $50,000, and B would earn 29% per year on investment of $85,000, the weighted averages are:

Overall ROR_A = [50,000(0.35) + 40,000(0.16)]/90,000 = 26.6%

Overall ROR_B = [85,000(0.29) + 5,000(0.16)]/90,000 = 28.3%

Which investment is better, economically?

Why Incremental Analysis is Necessary

If selection basis is higher ROR:

Select alternative A (wrong answer)

If selection basis is higher overall ROR:

Select alternative B

Conclusion: Must use an incremental ROR analysis to make a consistently correct selection

Unlike PW, AW, and FW values, if not analyzed correctly, ROR values can lead to an incorrect alternative selection. This is called the ranking inconsistency problem (discussed later)

Calculation of Incremental CF

Incremental cash flow = cash flow$_B$ – cash flow$_A$
where *larger initial investment* is Alternative B

Example: Either of the cost alternatives shown below can be used in a grinding process. Tabulate the incremental cash flows.

	A	B	B - A
First cost, $	-40,000	- 60,000	-20,000
Annual cost, $/year	-25,000	-19,000	+6000
Salvage value, $	8,000	10,000	+2000

The incremental CF is shown in the (B-A) column

The ROR on the extra $20,000 investment in B determines which alternative to select (as discussed later)

Interpretation of ROR on Extra Investment

Based on concept that any *avoidable investment* that does not yield at least the MARR should not be made.

Once a lower-cost alternative *has been economically* justified, the ROR on the *extra investment* *(i.e., additional amount of money* associated with a higher first-cost alternative) must also yield a ROR ≥ MARR (because the extra investment *is avoidable* by selecting the economically-justified lower-cost alternative).

This incremental ROR is identified as Δi^*

For independent projects, select all that have ROR ≥ MARR
(no incremental analysis is necessary)

ROR Evaluation for Two ME Alternatives

(1) Order alternatives by *increasing initial investment cost*

(2) *Develop incremental CF series* using LCM of years

(3) Draw incremental *cash flow diagram*, if needed

(4) Count sign changes to see if *multiple Δi^* values exist*

(5) Set up PW, AW, or FW = 0 relation and *find Δi^*_{B-A}*

Note: Incremental ROR analysis requires equal-service comparison. The LCM of lives must be used in the relation

(6) If Δi^*_{B-A} < MARR, *select A*; otherwise, select B

If multiple Δ**i*** values exist, ***find EROR*** using either MIRR or ROIC approach.

Example: Incremental ROR Evaluation

Either of the cost alternatives shown below can be used in a chemical refining process. If the company's MARR is 15% per year, determine which should be selected on the basis of ROR analysis?

	A	B
First cost ,$	-40,000	-60,000
Annual cost, $/year	-25,000	-19,000
Salvage value, $	8,000	10,000
Life, years	5	5

Initial observations: ME, cost alternatives with equal life estimates and no multiple ROR values indicated

Example: ROR Evaluation of Two Alternatives

Solution, using procedure:

	A	B	B - A
First cost , $	-40,000	-60,000	-20,000
Annual cost, $/year	-25,000	-19,000	+6000
Salvage value, $	8,000	10,000	+2000
Life, years	5	5	

Order by first cost **and** ***find incremental cash flow*** **B - A**

Write ROR equation **(in terms of PW, AW, or FW) on incremental CF**

$$0 = -20{,}000 + 6000(P/A,\Delta i^*,5) + 2000(P/F,\Delta i^*,5)$$

Solve **for Δi^* and** ***compare*** **to MARR**

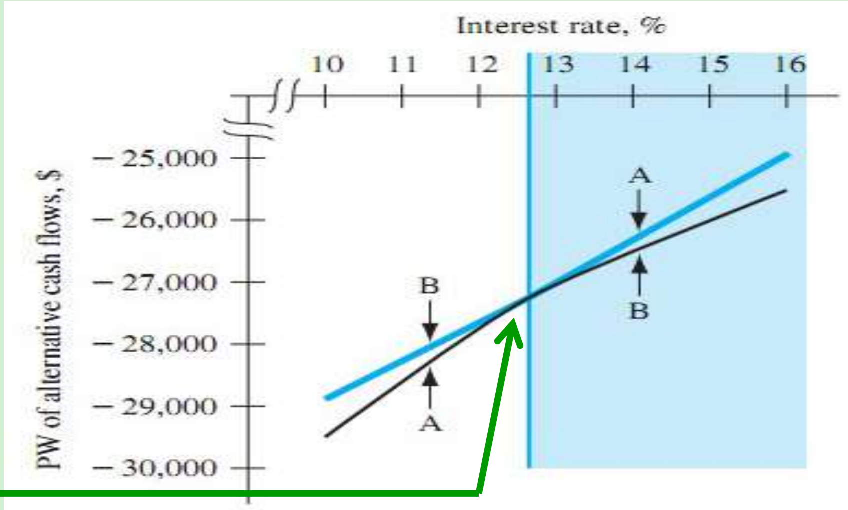
$\Delta i^*_{B\text{-}A} = 17.2\% > \text{MARR of } 15\%$

ROR on $20,000 extra investment is acceptable: **Select B**

Breakeven ROR Value

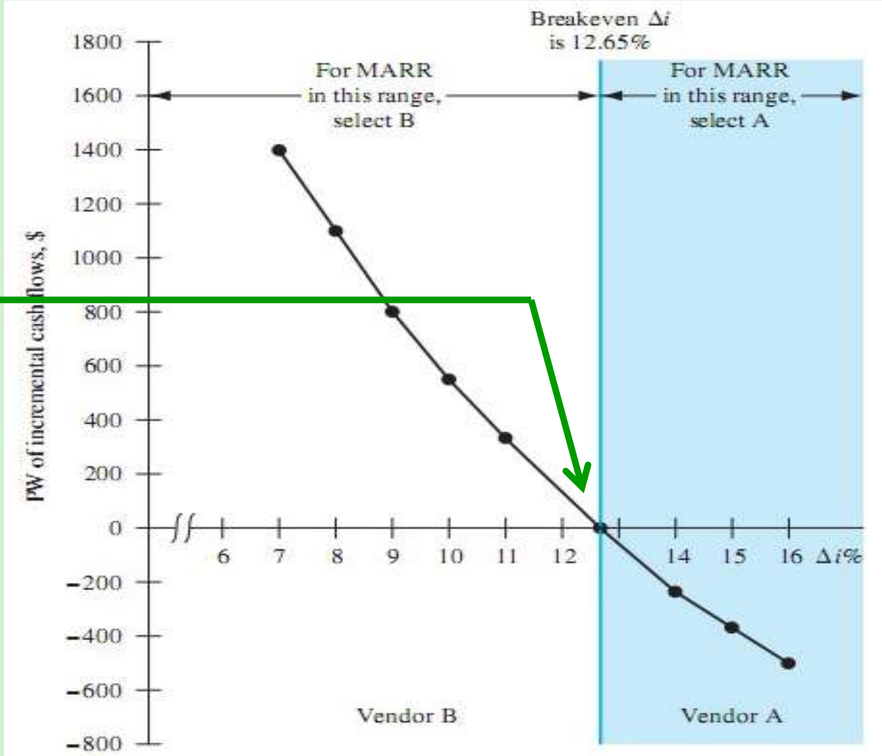
An ROR at which the PW, AW or FW values:

- Of cash flows for two alternatives are exactly equal. This is the i* value



- Of *incremental* cash flows between two alternatives are exactly equal.

  This is the Δi* value



If MARR > breakeven ROR, select lower-investment alternative

ROR Analysis – Multiple Alternatives

Six-Step Procedure for Mutually Exclusive Alternatives

(1) Order alternatives from *smallest to largest initial investment*

(2) For revenue alts, calculate **i*** (vs. DN) and *eliminate all with i* < MARR*; remaining alternative with lowest cost is ***defender*. F**or cost alternatives, go to step (3)

(3) Determine incremental CF between ***defender*** and ***next lowest-cost*** alternative (known as the *challenger*). Set up ROR relation

(4) Calculate Δ**i*** on incremental CF between *two alternatives from step (3)*

(5) If Δ**i* ≥ MARR,** *eliminate defender* and *challenger becomes new defender* against next alternative on list

(6) Repeat steps (3) through (5) *until only one alternative* remains. ***Select it***.

For Independent Projects

Compare each alternative vs. DN and select ***all with ROR ≥ MARR***

Example: ROR for Multiple Alternatives

The five mutually exclusive alternatives shown below are under consideration for improving visitor safety and access to additional areas of a national park. If all alternatives are considered to last indefinitely, determine which should be selected on the basis of a rate of return analysis using an interest rate of 10%.

	A	B	C	D	E
First cost, $ millions	-20	-40	-35	-90	-70
Annual M&O cost, $ millions	-2	-1.5	-1.9	-1.1	-1.3

Solution: **Rank on the basis of initial cost: A,C,B,E,D; calculate CC values**

C vs. A: 0 = -15 + 0.1/0.1 **Δi* = 6.7%** (eliminate C)

B vs. A: 0 = -20 + 0.5/0.1 **Δi* = 25%** (eliminate A)

E vs. B: 0 = -30 + 0.2/0.1 **Δi* = 6.7%** (eliminate E)

D vs. B: 0 = -50 + 0.4/0.1 **Δi* = 8%** (eliminate D)

Select alternative B

Summary of Important Points

- Must consider ***incremental cash flows*** for mutually exclusive alternatives

  Incremental cash flow = cash flow$_B$ – cash flow$_A$

  where alternative with ***larger*** initial investment is **Alternative B**

- Eliminate **B** if incremental ROR **Δi* < *MARR*;** *otherwise,* **eliminate A**

- ***Breakeven ROR*** is i* between ***project cash flows*** of two alternatives, or Δi* between ***incremental cash flows*** of two alternatives

- For multiple mutually exclusive alternatives, compare two at a time and eliminate alternatives until ***only one remains***

- For independent alternatives, compare each against **DN** and ***select all that have ROR ≥ MARR***

Assignment

1. A city corporation is considering two alternatives of waste disposal systems for managing city waste. System A has an initial investment of Rs. 5 crores, an annual maintenance cost of Rs. 5 Lakhs which increases by Rs. 25,000 from 3rd year onwards, salvage value of Rs. 75 lakhs and a life of 10 years. System B has an initial investment of Rs. 10 crores, an annual maintenance cost of Rs. 2 Lakhs which increases by Rs. 10,000 from 5th year onwards, salvage value of Rs. 1.5 crore and a life of 15 years. If the effective annual interest rate is 8%, which alternative should be selected based on the present worth analysis?

Benefit/Cost Analysis

LEARNING OUTCOMES

1. Explain difference in public vs. private sector projects
2. Calculate B/C ratio for single project
3. Select better of two alternatives using B/C method
4. Select best of multiple alternatives using B/C method
5. Use cost-effectiveness analysis (CEA) to evaluate service sector projects
6. Describe how ethical compromises may enter public sector projects

Differences: Public vs. Private Projects

Characteristic	Public	Private
Size of Investment	Large	Small, medium, large
Life	Longer (30 – 50+ years)	Shorter (2 – 25 years)
Annual CF	No profit	Profit-driven
Funding	Taxes, fees, bonds, etc.	Stocks, bonds, loans, etc.
Interest rate	Lower	Higher
Selection criteria	Multiple criteria	Primarily ROR
Environment of evaluation	Politically inclined	Economic

Types of Contracts

Contractors does not share project risk

- **Fixed price** - lump-sum payment
- **Cost reimbursable** - Cost plus, as negotiated

Contractor shares in project risk

- **Public-private partnerships (PPP)**, such as:
  - **Design-build projects** - Contractor responsible from design stage to operations stage
  - **Design-build-operate-maintain-finance (DBOMF) projects** - Turnkey project with contractor managing **financing** (manage cash flow); government obtains **funding** for project

Cash Flow Classifications and B/C Relations

Must identify each cash flow as either benefit, disbenefit, or cost

Benefit (B) -- Advantages to the *public*

Disbenefit (D) -- Disadvantages to the *public*

Cost (C) -- Expenditures by the *government*

Note: Savings to government are subtracted from costs

Conventional B/C ratio = (B–D) / C

Modified B/C ratio = [(B–D) – C] / Initial Investment

Profitability Index = NCF / Initial Investment

Note 1: All terms must be expressed in *same units*, i.e., PW, AW, or FW

Note 2: *Do not use minus sign* ahead of costs

Decision Guidelines for B/C and PI

Benefit/cost analysis

If $B/C \geq 1.0$, project **is** economically justified at discount rate applied

If $B/C < 1.0$, project **is not** economically acceptable

Profitability index analysis of revenue projects

If $PI \geq 1.0$, project **is** economically justified at discount rate applied

If $PI < 1.0$, project **is not** economically acceptable

B/C Analysis – Single Project

$$\text{Conventional B/C ratio} = \frac{B - D}{C}$$

$$\text{Modified B/C ratio} = \frac{B - D - M\&O}{C}$$

If B/C ≥ 1.0,
accept project;
otherwise, reject

$$PI = \frac{\text{PW of } NCF_t}{\text{PW of initial investment}}$$

Denominator is
initial investment

If PI ≥ 1.0,
accept project;
otherwise, reject

Example: B/C Analysis – Single Project

A flood control project will have a first cost of \$1.4 million with an annual maintenance cost of \$40,000 and a 10 year life. Reduced flood damage is expected to amount to \$175,000 per year. Lost income to farmers is estimated to be \$25,000 per year. At an interest rate of 6% per year, should the project be undertaken?

Solution: Express all values in AW terms and find B/C ratio

B = \$175,000

D = \$25,000

C = 1,400,000(A/P,6%,10) + \$40,000 = \$230,218

B/C = (175,000 – 25,000)/230,218

= 0.65 < 1.0

Do not build project

Defender, Challenger and Do Nothing Alternatives

When selecting from two or more ME alternatives, there is a:

- ✓ **Defender –** in-place system or currently selected alternative
- ✓ **Challenger –** Alternative challenging the defender
- ✓ **Do-nothing option –** Status quo system

General approach for incremental B/C analysis of two ME alternatives:

- Lower total cost alternative is first compared to ***Do-nothing (DN)***
- If B/C for the lower cost alternative is < 1.0, the DN option is compared to ΔB/C of the higher-cost alternative
- If both alternatives lose out to DN option, DN prevails, unless overriding needs requires selection of one of the alternatives

Alternative Selection Using Incremental B/C Analysis – Two or More ME Alternatives

Procedure similar to ROR analysis for multiple alternatives

(1) **Determine *equivalent total cost* for each alternative**
(2) Order alternatives by increasing total cost
(3) **Identify *B and D for each alternative*, if given, or go to step 5**
(4) Calculate B/C for each alternative and eliminate all with B/C < 1.0
(5) Determine incremental costs and benefits for first two alternatives
(6) Calculate ΔB/C; if >1.0, higher cost alternative becomes defender
(7) Repeat steps 5 and 6 until only one alternative remains

Example: Incremental B/C Analysis

Compare two alternatives using i = 10% and B/C ratio

Alternative	X	Y
First cost, $	320,000	540,000
M&O costs, $/year	45,000	35,000
Benefits, $/year	110,000	150,000
Disbenefits, $/year	20,000	45,000
Life, years	10	20

Solution: First, calculate equivalent total cost

AW of costs$_X$ = 320,000(A/P,10%,10) + 45,000 = $97,080

AW of costs$_Y$ = 540,000(A/P,10%,20) + 35,000 = $98,428

Order of analysis is X, then Y

X vs. DN: (B-D)/C = (110,000 – 20,000) / 97,080 = **0.93** **Eliminate X**

Y vs. DN: (150,000 – 45,000) / 98,428 = **1.07** **Eliminate DN**

Select Y

Example: ΔB/C Analysis; Selection Required

Must select one of two alternatives using i = 10% and ΔB/C ratio

Alternative	X	Y
First cost, $	320,000	540,000
M&O costs, $/year	45,000	35,000
Benefits, $/year	110,000	150,000
Disbenefits, $/year	20,000	45,000
Life, years	10	20

Solution: **Must select X or Y; DN not an option, *compare Y to X***

AW of costs_X = $97,080　　　AW of costs_Y = $98,428

Incremental values: ΔB = 150,000 – 110,000 = $40,000

ΔD = 45,000 – 20,000 = $25,000

ΔC = 98,428 – 97,080 = $1,348

Y vs. X: (ΔB - ΔD) / ΔC = (40,000 – 25,000) / 1,348 = 11.1　　**Eliminate X**

Select Y

B/C Analysis of Independent Projects

- Independent projects comparison does **not require incremental analysis**
- Compare each alternative's overall B/C with **DN option**

+ **No budget limit:** **Accept all** alternatives with **B/C ≥ 1.0**
+ **Budget limit specified:** capital budgeting problem; selection follows different procedure.

Cost Effectiveness Analysis

Service sector projects primarily involve **intangibles**, **not physical facilities**; examples include health care, security programs, credit card services, etc.

Cost-effectiveness analysis (CEA) combines monetary cost estimates with ***non-monetary*** benefit estimates to calculate the

Cost-effectiveness ratio (CER)

$$\text{CER} = \frac{\text{Equivalent total costs}}{\text{Total effectiveness measure}}$$

$$= C/E$$

CER Analysis for Independent Projects

Procedure is as follows:

(1) Determine equivalent total cost **C**, total effectiveness measure **E** and **CER**

(2) **Order projects by smallest to largest CER**

(3) Determine cumulative cost of projects and compare to budget limit ***b***

(4) Fund all projects such that ***b* is not exceeded**

Example: The effectiveness measure ***E*** is the number of graduates from adult training programs. For the CERs shown, determine which ***independent*** programs should be selected; b = $500,000.

Program	CER, $/graduate	Program Cost, $
A	1203	305,000
B	752	98,000
C	2010	126,000
D	1830	365,000

Example: CER for Independent Projects

First, rank programs according to increasing CER:

Program	CER, $/graduate	Program Cost, $	Cumulative Cost, $
B	752	98,000	98,000
A	1203	305,000	403,000
D	1830	365,000	768,000
C	2010	126,000	894,000

Next, select programs until budget is not exceeded

Select programs B and A at total cost of $403,000

Note: To expend the entire $500,000, accept as many additional individuals as possible from D at the per-student rate

CER Analysis for Mutually Exclusive Projects

Procedure is as follows

(1) **Order alternatives smallest to largest by effectiveness measure *E***

(2) Calculate **CER for first alternative** (defender) and compare to DN option

(3) Calculate incremental cost (ΔC), effectiveness (ΔE), and incremental measure **ΔC/E for challenger** (next higher *E* measure)

(4) If $\Delta C/E_{challenger} < C/E_{defender}$ challenger becomes defender **(dominance);** otherwise, **no dominance** is present and both alternatives are retained

(5) **Dominance present:** Eliminate defender and compare next alternative to new defender per steps (3) and (4).

Dominance not present: Current challenger becomes new defender against next challenger, **but old defender remains viable**

(6) Continue steps (3) through (5) until only **1 alternative remains** or only **non-dominated alternatives remain**

(7) Apply budget limit or other criteria to **determine which of remaining non-dominated alternatives** can be funded

Example: CER for ME Service Projects

The effectiveness measure **E is wins per person**. From the cost and effectiveness values shown, determine which alternative to select.

Program	Cost (C) $/person	Effectiveness (E) wins/person	CER $/win
A	2200	4	550
B	1400	2	700
C	6860	7	980

Example: CER for ME Service Projects

Solution:

Order programs according to increasing effectiveness measure E

Program	Cost (C) $/person	Effectiveness (E) wins/person	CER $/win
B	1,400	2	700
A	2,200	4	550
C	6,860	7	980

B vs. DN: $C/E_B = 1400/2 = 700$

A vs. B: $\Delta C/E = (2200 - 1400)/(4 - 2) = 400$ Dominance; eliminate B

C vs. A: $\Delta C/E = (6860 - 2200)/(7 - 4) = 1553$ No dominance; retain C

Must use other criteria to select either A or C

Ethical Considerations

Engineers are routinely involved in two areas where ethics may be compromised:

Public policy making – **Development of strategy**, e.g., water system management (supply/demand strategy; ground vs. surface sources)

Public planning - **Development of projects**, e.g., water operations (distribution, rates, sales to outlying areas)

Engineers must maintain integrity and impartiality and *always* adhere to Code of Ethics

Summary of Important Points

- B/C method used in ***public sector*** project evaluation
- Can use PW, AW, or FW for incremental B/C analysis, but must ***be consistent*** with units for B,C, and D estimates
- For multiple mutually exclusive alternatives, compare two at a time and eliminate alternatives until ***only one remains***
- For independent alternatives with no budget limit, compare each against **DN** and select ***all alternatives that have B/C ≥ 1.0***
- **CEA analysis** for service sector projects combines cost and ***nonmonetary measures***
- Ethical dilemmas are ***especially prevalent*** in public sector projects

Breakeven and Payback Analysis

LEARNING OUTCOMES

1. **Breakeven point – one parameter**
2. **Breakeven point – two alternatives**
3. **Payback period analysis**

Breakeven Point

Value of a parameter that makes two elements equal

The parameter (or variable) can be an amount of revenue, cost, supply, demand, etc. for one project or between two alternatives

- **One project** - Breakeven point is identified as Q_{BE}. Determined using linear or non-linear math relations for revenue and cost
- **Between two alternatives** - Determine one of the parameters **P, A, F, i, or n** with others constant

Solution is by one of two methods:

- **Direct solution of relations**
- **Trial and error**

Cost-Revenue Model — One Project

Quantity, Q — An amount of the variable in question, e.g., units/year, hours/month

Breakeven value is Q_{BE}

Fixed cost, FC — Costs not directly dependent on the variable, e.g., buildings, fixed overhead, insurance, minimum workforce cost

Variable cost, VC — Costs that change with parameters such as production level and workforce size. These are labor, material and marketing costs. Variable cost per unit is v

Total cost, TC — Sum of fixed and variable costs, TC = FC + VC

Revenue, R — Amount is dependent on quantity sold

Revenue per unit is r

Profit, P — Amount of revenue remaining after costs

P = R – TC = R – (FC+VC)

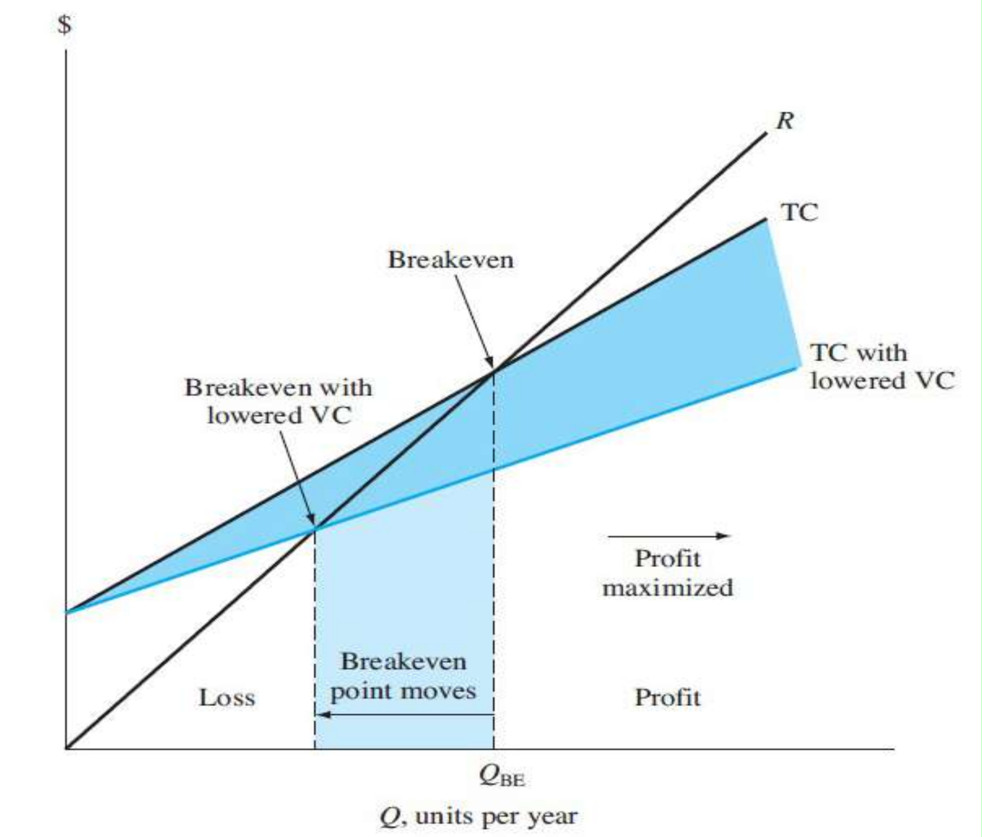
Breakeven for linear R and TC

Set R = TC and solve for $Q = Q_{BE}$

$$R = TC$$
$$rQ = FC + vQ$$

$$Q_{BE} = \frac{FC}{r - v}$$

When variable cost, v, is lowered, Q_{BE} decreases (moves to left)



Example: One Project Breakeven Point

A plant produces 15,000 units/month. Find breakeven level if FC = $75,000 /month, revenue is $8/unit and variable cost is $2.50/unit. Determine expected monthly profit or loss.

Solution: Find Q_{BE} and compare to 15,000; calculate Profit

$$Q_{BE} = 75{,}000 / (8.00-2.50) = 13{,}636 \text{ units/month}$$

Production level is above breakeven → **Profit**

Profit = R – (FC + VC)

= rQ – (FC + vQ) = (r-v)Q – FC

= (8.00 – 2.50)(15,000) – 75,000

= $ 7500/month

Breakeven Between Two Alternatives

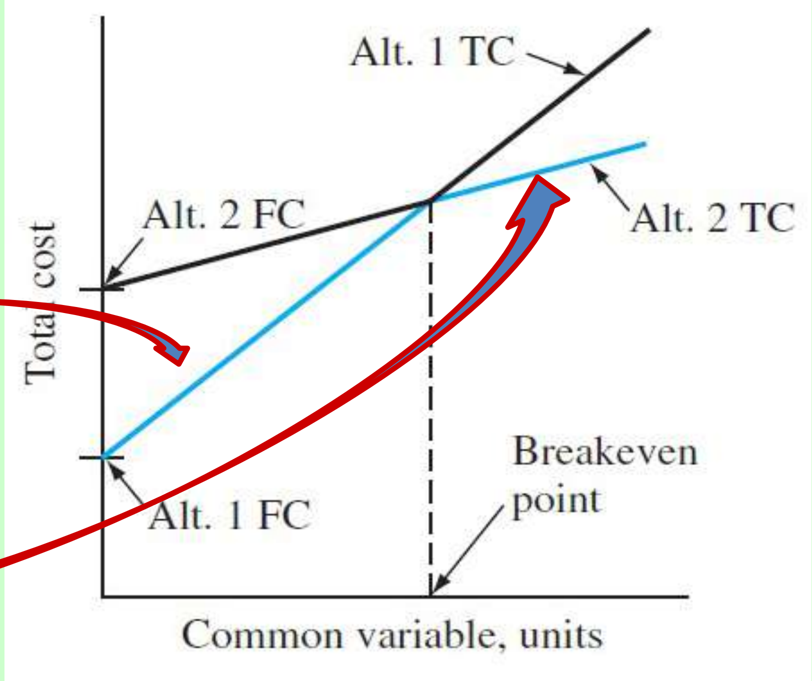
To determine value of common variable between 2 alternatives, do the following:

1. Define the common variable
2. Develop equivalence PW, AW or FW relations as function of common variable for each alternative
3. Equate the relations; solve for variable. **This is breakeven value**

Selection of alternative is based on anticipated value of common variable:

✓ Value BELOW breakeven;
select higher variable cost

✓ Value ABOVE breakeven;
select lower variable cost



Example: Two Alternative Breakeven Analysis

Perform a make/buy analysis where the common variable is X, the number of units produced each year. AW relations are:

$$AW_{make} = -18{,}000(A/P,15\%,6) + 2{,}000(A/F,15\%,6) - 0.4X$$

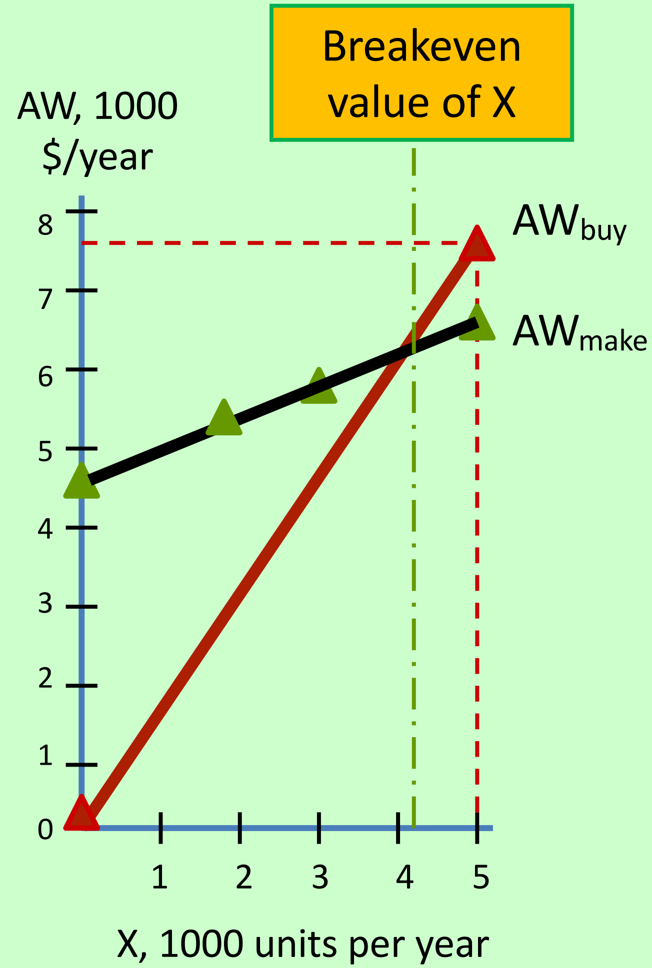
$$AW_{buy} = -1.5X$$

Solution: Equate AW relations, solve for X

$$-1.5X = -4528 - 0.4X$$

$$X = 4116 \text{ per year}$$

If anticipated production > 4116, select make alternative (lower variable cost)



Payback Period Analysis

Payback period: **Estimated amount of time (n_p) for cash inflows to recover an initial investment (P) plus a stated return of return (i%)**

Types of payback analysis: **No-return** and **discounted** payback

1. **No-return payback** means rate of return is ZERO (i = 0%)
2. **Discounted payback** considers time value of money (i > 0%)

Caution: Payback period analysis is a good initial screening tool, rather than the primary method to justify a project or select an alternative (Discussed later)

Payback Period Computation

Formula to determine payback period (n_p) varies with type of analysis.

NCF = Net Cash Flow per period t

No return, $i = 0\%$; NCF_t varies annually:	$0 = -P + \sum_{t=1}^{t=n_p} NCF_t$	**Eqn. 1**
No return, $i = 0\%$; annual uniform NCF:	$n_p = \frac{P}{NCF}$	**Eqn. 2**
Discounted, $i > 0\%$; NCF_t varies annually:	$0 = -P + \sum_{t=1}^{t=n_p} NCF_t(P/F,i,t)$	**Eqn. 3**
Discounted, $i > 0\%$; annual uniform NCF:	$0 = -P + NCF(P/A,i,n_p)$	**Eqn. 4**

Points to Remember About Payback Analysis

- **No-return payback neglects time value of money**, so no return is expected for the investment made
- **No cash flows after the payback period are considered** in the analysis. Return may be higher if these cash flows are expected to be positive.

- Approach of payback analysis is different from PW, AW, ROR and B/C analysis. A different alternative may be selected using payback.
- Rely on payback as a **supplemental tool**; use PW or AW at the MARR for a reliable decision
- Discounted payback ($i > 0\%$) gives a good sense of the **risk** involved

Example: Payback Analysis

	System 1	System 2
First cost, $	12,000	8,000
NCF, $ per year	3,000	1,000 (year 1-5)
		3,000 (year 6-14)
Maximum life, years	7	14

Problem: Use (a) no-return payback, (b) discounted payback at 15%, and (c) PW analysis at 15% to select a system. Comment on the results.

Solution: (a) Use Eqns. 1 and 2

n_{p1} = 12,000 / 3,000 = **4 years**

n_{p2} = -8,000 + 5(1,000) + 1(3,000) = **6 years**

Select system 1

Example: Payback Analysis (continued)

	System 1	System 2
First cost, \$	12,000	8,000
NCF, \$ per year	3,000	1,000 (year 1-5)
		3,000 (year 6-14)
Maximum life, years	7	14

Solution: (b) Use Eqns. 3 and 4

System 1: $0 = -12{,}000 + 3{,}000(P/A,15\%,n_{p1})$

$n_{p1} =$ **6.6 years**

System 2: $0 = -8{,}000 + 1{,}000(P/A,15\%,5) + 3{,}000(P/A,15\%,n_{p2} - 5)(P/F,15\%,5)$

$n_{p2} =$ **9.5 years**

Select system 1

(c) Find PW over LCM of 14 years

$PW_1 =$ **\$663**

$PW_2 =$ **\$2470**

Select system 2

Comment: PW method considers cash flows after payback period. Selection changes from system 1 to 2

Summary of Important Points

- ***Breakeven*** **amount is a** ***point of indifference*** **to accept or reject a project**
- **One project breakeven:** ***accept if quantity is > Q_{BE}***
- **Two alternative breakeven: if** ***level > breakeven*****, select lower variable cost alternative** ***(smaller slope)***
- **Payback estimates time to recover investment. Return can be i = 0% or i > 0%**
- **Use** ***payback as supplemental*** **to PW or other analyses, because n_p neglects cash flows after payback, and if i = 0%, it neglects time value of money**
- ***Payback*** **is useful to sense the** ***economic risk*** **in a project**

Depreciation Methods

Depreciation Terminology

Definition: ***Book (noncash) method*** to represent **decrease in value** of a tangible asset over time

Two types: book depreciation and tax depreciation

Book depreciation: used for *internal accounting* to track value of assets

***Tax depreciation*:** used to determine *taxes due* based on tax laws

In USA only, **tax depreciation** *must be calculated* ***using MACRS***; **book depreciation** *can be calculated* ***using any method***

Common Depreciation Terms

First cost *P* or unadjusted basis *B*: Total installed cost of asset

Book value BV_t: Remaining undepreciated capital investment in year t

Recovery period *n*: Depreciable life of asset in years

Market value *MV*: Amount realizable if asset were sold on open market

Salvage value *S*: Estimated trade-in or MV at end of asset's useful life

Depreciation rate d_t: Fraction of first cost or basis removed each year t

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Personal property: Possessions of company used to conduct business

Real property: Real estate and all improvements **(land is not depreciable)**

Half-year convention: Assumes assets are placed in service in midyear

Straight Line Depreciation

➡ **Book value decreases** ***linearly with time***

$$D_t = \frac{B - S}{n}$$

Where: D_t = annual depreciation charge
t = year
B = first cost or unadjusted basis
S = salvage value
n = recovery period

$$BV_t = B - tD_t$$

Where: BV_t = book value after t years

SL depreciation rate is **constant** for each year: $d = d_t = 1/n$

Example: SL Depreciation

An argon gas processor has a first cost of \$20,000 with a \$5,000 salvage value after 5 years. Find (a) D_3 and (b) BV_3 for year three. (c) Plot book value vs. time.

Solution:

(a) $D_3 = (B - S)/n$

$= (20{,}000 - 5{,}000)/5$

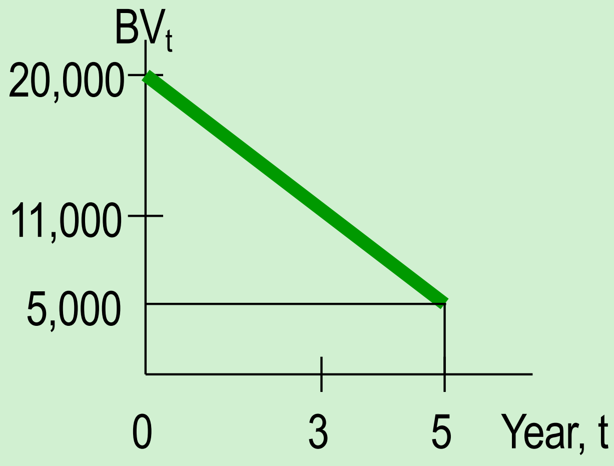
$= \$3{,}000$

(b) $BV_3 = B - tD_t$

$= 20{,}000 - 3(3{,}000)$

$= \$11{,}000$

(c) Plot BV vs. time



Declining Balance (DB) and Double Declining Balance (DDB) Depreciation

- **Determined by multiplying BV at beginning of year *by fixed percentage d***
  - **Max rate for d is twice straight line rate, i.e., $d \le 2/n$**
  - **Cannot depreciate below salvage value**

Depreciation for year t is obtained by either relation:

$$D_t = dB(1 - d)^{t-1} = dBV_{t-1}$$

Where: D_t = depreciation for year t
d = uniform depreciation rate (2/n for DDB)
B = first cost or unadjusted basis
BV_{t-1} = book value at end of previous year

Book value for year t is given by:

$$BV_t = B(1 - d)^t$$

Example: Double Declining Balance

A depreciable construction truck has a first cost of \$20,000 with a \$4,000 salvage value after 5 years. Find the (a) depreciation, and (b) book value after 3 years using DDB depreciation.

Solution:

(a) $d = 2/n = 2/5 = 0.4$

$$D_3 = dB(1 - d)^{t-1}$$
$$= 0.4(20{,}000)(1 - 0.40)^{3-1}$$
$$= \$2880$$

(b) $$BV_3 = B(1 - d)^t$$
$$= 20{,}000(1 - 0.4)^3$$
$$= \$4320$$

Switching Between Depreciation Methods

- **Switch between methods to maximize PW of depreciation**

$$PW_D = \sum_{t=1}^{t=n} D_t\,(P/F,i\%,t)$$

- A switch from **DDB to SL in latter part of life** is usually better
- Can switch only one time during recovery period

Procedure to switch from DDB to SL:

1) Each year *t* compute DDB and SL depreciation using the relations

$$D_{DDB} = d(BV_{t-1}) \qquad \text{and} \qquad D_{SL} = BV_{t-1} / (n-t-1)$$

2) Select larger depreciation amount, i.e., $D_t = \max[D_{DDB}, D_{SL}]$
3) If required, calculate PW_D

Example

1. A mining company has purchased an earth mover with a cost basis of Rs. 1 Crore and a depreciable life of 15 years. The machine has Rs. 20 Lakhs salvage value at the end of its depreciable life. Calculate the amount of depreciation and book value for every year in the depreciable life by applying 200% decline balance method with switch to straight line depreciation.

MACRS Depreciation

Required method to use for **tax depreciation** in **USA only**

Originally developed to offer accelerated depreciation for economic growth

$$D_t = d_t B$$

Where: D_t = depreciation charge for year t
B = first cost or unadjusted basis
d_t = depreciation rate for year t (decimal)

Get value for d_t from IRS table for MACRS rates

$$BV_t = B - \sum_{j=1}^{j=t} D_j$$

Where: D_j = depreciation in year j
$\sum D_j$ = all depreciation through year t

MACRS Depreciation

- Always depreciates to **zero**; no salvage value considered
- Incorporates **switching from DDB to SL** depreciation
- ***Standardized recovery periods*** (n) are tabulated
- MACRS recovery time is always **n+1 years**;
  half-year convention assumes purchase in midyear

Example: MACRS Depreciation

A finishing machine has a first cost of $20,000 with a $5,000 salvage value after 5 years. Using MACRS, find (a) D and (b) BV for year 3.

Solution: **(a)** From table, d_3 = 19.20

$$D_3 = 20{,}000(0.1920)$$
$$= \$3{,}840$$

(b) $BV_3 = 20{,}000 - 20{,}000(0.20 + 0.32 + 0.1920)$
$$= \$5{,}760$$

Note: Salvage value S = $5,000 is not used by MACRS and $BV_6 = 0$

MACRS Recovery Period

Recovery period (n) is function of ***property class***

Two systems for determining recovery period:

- **general depreciation system (GDS)** – **fastest write-off allowed**
- **alternative depreciation system (ADS)** – **longer recovery; uses SL**

IRS publication 946 gives n values for an asset. For example:

Asset description	MACRS n value: GDS	MACRS n value: ADS range
Special manufacturing devices, racehorses, tractors	3	3 - 5
Computers, oil drilling equipment, autos, trucks, buses	5	6 - 9.5
Office furniture, railroad car, property not in another class	7	10 – 15
Nonresidential real property (not land itself)	39	40

Unit-of-Production (UOP) Depreciation

- Depreciation based on ***usage of equipment***, not time
- Depreciation for year t obtained by relation

$$D_t = \frac{\text{actual usage for year } t}{\text{expected total lifetime usage}}(B - S)$$

Example: A new mixer is expected to process 4 million yd^3 of concrete over 10-year life time. Determine depreciation for year 1 when 400,000 yd^3 is processed. Cost of mixer was \$175,000 with no salvage expected.

Solution:

$$D_1 = \frac{400{,}000}{4{,}000{,}000}(175{,}000 - 0) = \$17{,}500$$

Depletion Methods

Depletion: **book (noncash) method to represent decreasing value of *natural resources***

Two methods: *cost* depletion (CD) and ***percentage*** depletion (PD)

Cost depletion: Based on level of activity to remove a natural resource

- Calculation: Multiply factor CD_t by amount of resource removed

  Where: $\mathbf{CD_t}$ **= first cost / resource capacity**

- Total depletion can not exceed first cost of the resource

Percentage depletion: Based on gross income (GI) from resource

- Calculation: Multiply GI by standardized rate (%) from table
- Annual depletion can not exceed 50% of company's taxable income (TI)

Example: Cost and Percentage Depletion

A mine purchased for $3.5 million has a total expected yield of one million ounces of silver. Determine the depletion charge in year 4 when 300,000 ounces are mined and sold for $30 per ounce using (a) cost depletion, and (b) percentage depletion. (c) Which is larger for year 4?

Solution: Let depletion amounts equal CDA_4 and PDA_4

(a) Factor, CD_4 = 3,500,000/ 1,000,000 = $3.50 per ounce

CDA_4 = 3.50(300,000) = **$1,050,000**

(b) Percentage depletion rate for silver mines is 0.15

PDA_4 = (0.15)(300,000)(30) = **$1,350,000**

(c) Claim **percentage depletion** amount, provided it is ≤ 50% of TI

Summary of Important Points

- **Two types** for depreciation: **tax and book**
- Classical methods are **straight line** and **declining balance**
- In USA only, **MACRS** method is ***required*** for ***tax depreciation***
- Determine MACRS recovery period using either **GDS** or **ADS**
- Switching between methods is allowed; MACRS switches automatically from DDB to SL to maximize write-off
- **Depletion** (instead of depreciation) used for ***natural resources***
- ***Two methods*** *of depletion:* ***cost*** *(amount resource removed* × CD_t *factor) and* ***percentage*** *(gross income* × *tabulated %)*

Demand Forecasting

What is Forecasting?

- ☑ Process of predicting a future event
- ☑ Underlying basis of all business decisions
  - ☑ Production
  - ☑ Inventory
  - ☑ Personnel
  - ☑ Facilities

Forecasting Time Horizons

- ☑ Short-range forecast
  - ☑ Up to 1 year, generally less than 3 months
  - ☑ Purchasing, job scheduling, workforce levels, job assignments, production levels
- ☑ Medium-range forecast
  - ☑ 3 months to 3 years
  - ☑ Sales and production planning, budgeting
- ☑ Long-range forecast
  - ☑ 3^{+} years
  - ☑ New product planning, facility location, research and development

Distinguishing Differences

- ☑Medium/long range forecasts deal with more comprehensive issues and support management decisions regarding planning and products, plants and processes
- ☑Short-term forecasting usually employs different methodologies than longer-term forecasting
- ☑Short-term forecasts tend to be more accurate than longer-term forecasts

Influence of Product Life Cycle

Introduction – Growth – Maturity – Decline

- ☑ Introduction and growth require longer forecasts than maturity and decline
- ☑ As product passes through life cycle, forecasts are useful in projecting
  - ☑ Staffing levels
  - ☑ Inventory levels
  - ☑ Factory capacity

Product Life Cycle

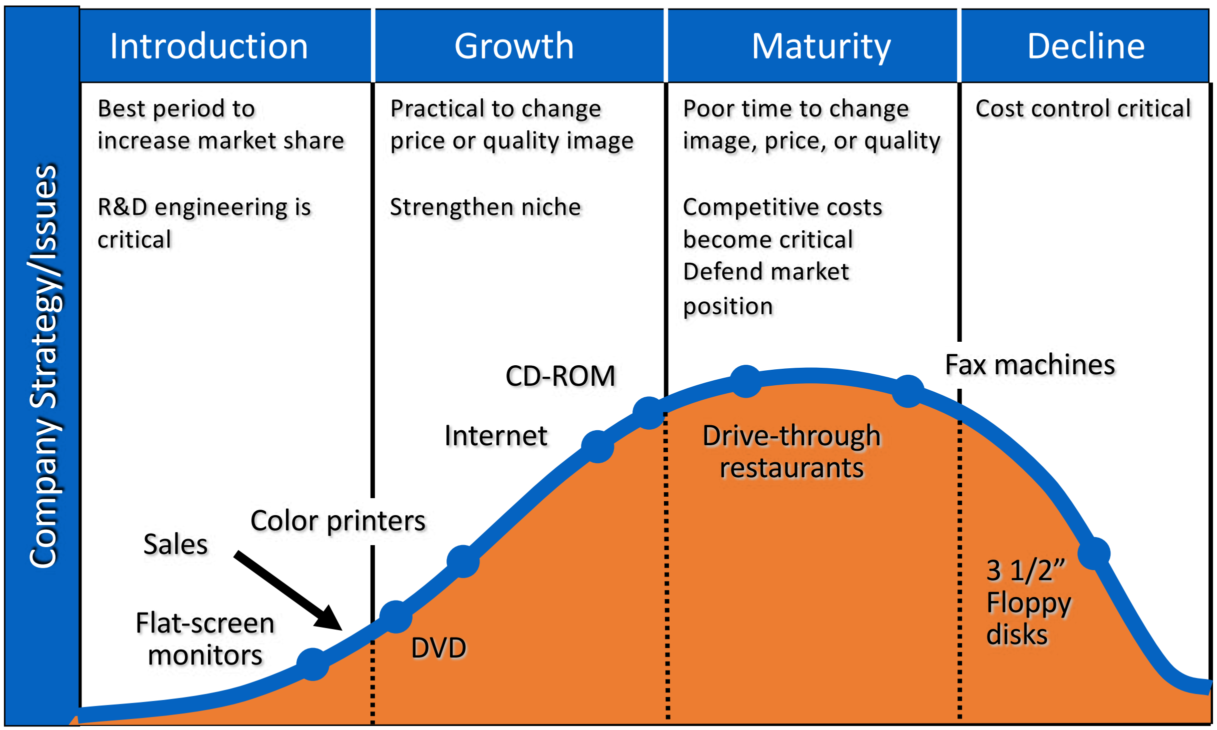


Figure 2.5

Types of Forecasts

- ☑ Economic forecasts
  - ☑ Address business cycle – inflation rate, money supply, housing stats, etc.
- ☑ Technological forecasts
  - ☑ Predict rate of technological progress
  - ☑ Impacts development of new products
- ☑ Demand forecasts
  - ☑ Predict sales of existing product

Strategic Importance of Forecasting

- ☑ Human Resources – Hiring, training, laying off workers
- ☑ Capacity – Capacity shortages can result in undependable delivery, loss of customers, loss of market share
- ☑ Supply-Chain Management – Good supplier relations and price advance

Seven Steps in Forecasting

- ☑ Determine the use of the forecast
- ☑ Select the items to be forecasted
- ☑ Determine the time horizon of the forecast
- ☑ Select the forecasting model(s)
- ☑ Gather the data
- ☑ Make the forecast
- ☑ Validate and implement results

The Realities!

- ☑ Forecasts are seldom perfect
- ☑ Most techniques assume an underlying stability in the system
- ☑ Product family and aggregated forecasts are more accurate than individual product forecasts

Forecasting Approaches

Qualitative Methods

- ☑ Used when situation is vague and little data exist
  - ☑ New products
  - ☑ New technology
- ☑ Involves intuition, experience
  - ☑ e.g., forecasting sales on Internet

Forecasting Approaches

Quantitative Methods

- ☑ Used when situation is ‘stable’ and historical data exist
  - ☑ Existing products
  - ☑ Current technology
- ☑ Involves mathematical techniques
  - ☑ e.g., forecasting sales of color televisions

Overview of Qualitative Methods

- ☑ Jury of executive opinion
  - ☑ Pool opinions of high-level executives, sometimes augment by statistical models
- ☑ Delphi method
  - ☑ Panel of experts, queried iteratively

Overview of Qualitative Methods

- ☑ Sales force composite
  - ☑ Estimates from individual salespersons are reviewed for reasonableness, then aggregated
- ☑ Consumer Market Survey
  - ☑ Ask the customer

Jury of Executive Opinion

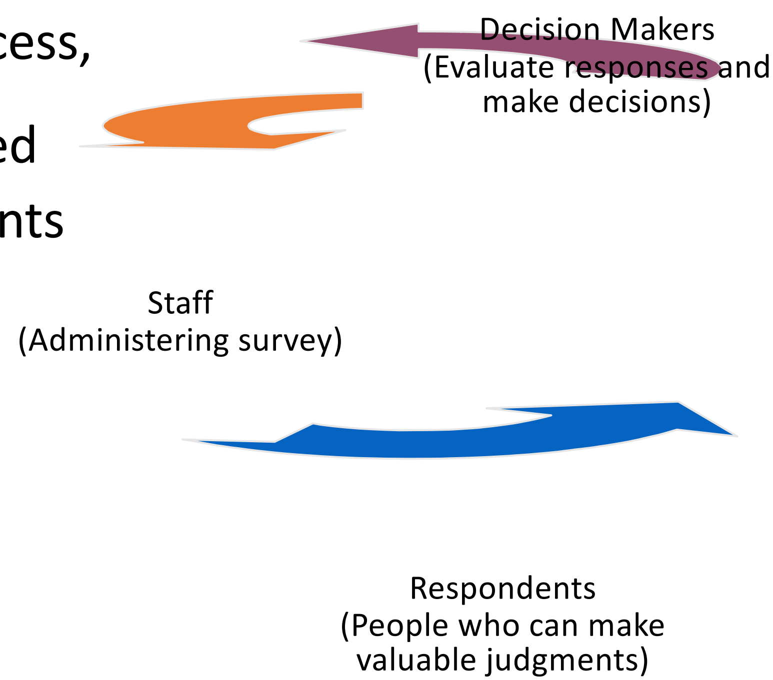
- ☑ Involves small group of high-level managers
- ☑ Group estimates demand by working together
- ☑ Combines managerial experience with statistical models
- ☑ Relatively quick
- ☑ 'Group-think' disadvantage

Sales Force Composite

- ☑ Each salesperson projects his or her sales
- ☑ Combined at district, state and national levels
- ☑ Sales reps know customers' wants
- ☑ Tends to be overly optimistic

Delphi Method

- ☑ Iterative group process, continues until consensus is reached
- ☑ 3 types of participants
  - ☑ Decision makers
  - ☑ Staff
  - ☑ Respondents



Consumer Market Survey

- ☑ Ask customers about purchasing plans
- ☑ What consumers say, and what they actually do are often different
- ☑ Sometimes difficult to answer
- ☑ Suitable for very short term

Overview of Quantitative Approaches

1. Naive approach
2. Moving averages
3. Exponential smoothing
4. Trend projection

Time-Series Models

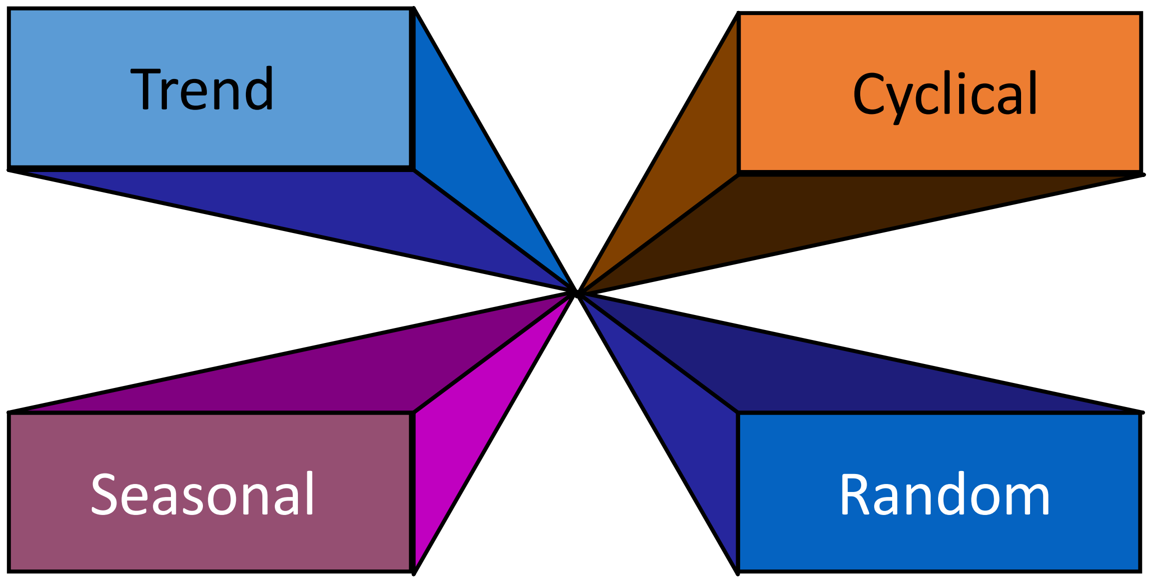
5. Linear regression

Associative Model

Time Series Forecasting

- ☑ Set of evenly spaced numerical data
  - ☑ Obtained by observing response variable at regular time periods
- ☑ Forecast based only on past values
  - ☑ Assumes that factors influencing past and present will continue to influence in future

Time Series Components



Components of Demand

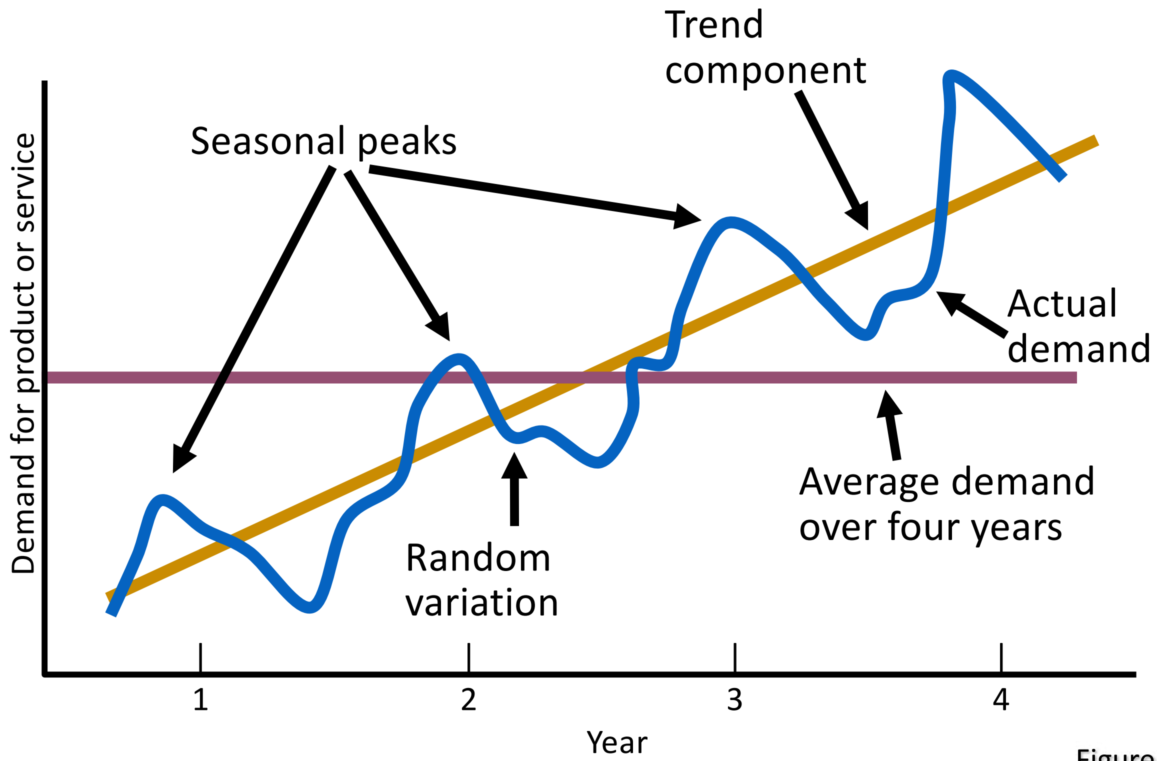
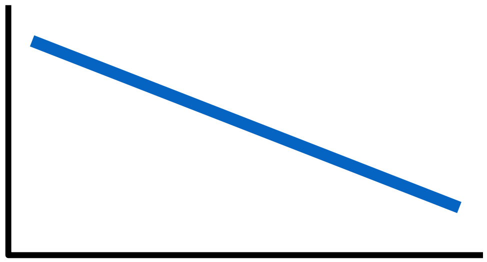


Figure 4.1

Trend Component

- ☑ Persistent, overall upward or downward pattern
- ☑ Changes due to population, technology, age, etc.
- ☑ Typically several years duration



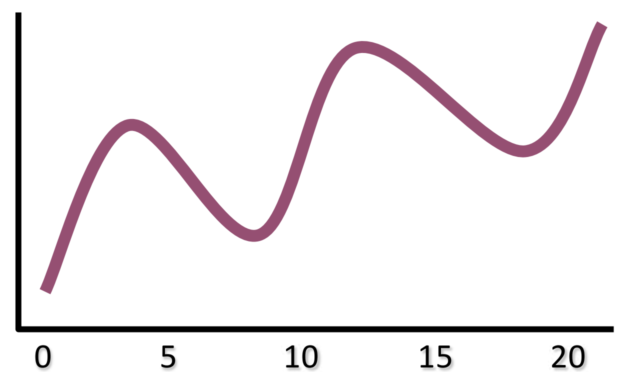
Seasonal Component

- ☑ Regular pattern of up and down fluctuations
- ☑ Due to weather, customs, etc.
- ☑ Occurs within a single year

Period	Length	Number of Seasons
Week	Day	7
Month	Week	4-4.5
Month	Day	28-31
Year	Quarter	4
Year	Month	12
Year	Week	52

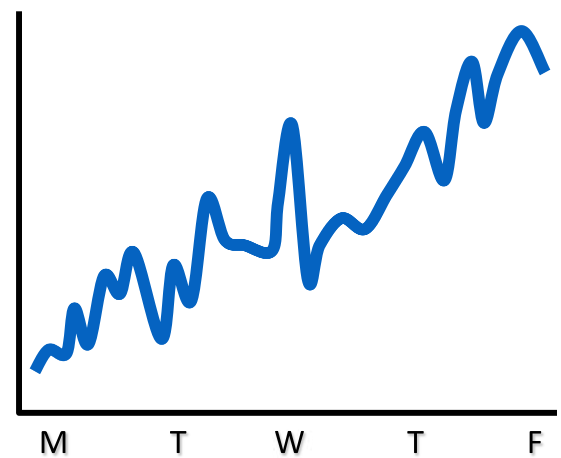
Cyclical Component

- ☑ Repeating up and down movements
- ☑ Affected by business cycle, political, and economic factors
- ☑ Multiple years duration
- ☑ Often causal or associative relationships



Random Component

- ☑ Erratic, unsystematic, 'residual' fluctuations
- ☑ Due to random variation or unforeseen events
- ☑ Short duration and nonrepeating



Naive Approach

- ☑ Assumes demand in next period is the same as demand in most recent period
  - ☑ e.g., If May sales were 48, then June sales will be 48
- ☑ Sometimes cost effective and efficient

Moving Average Method

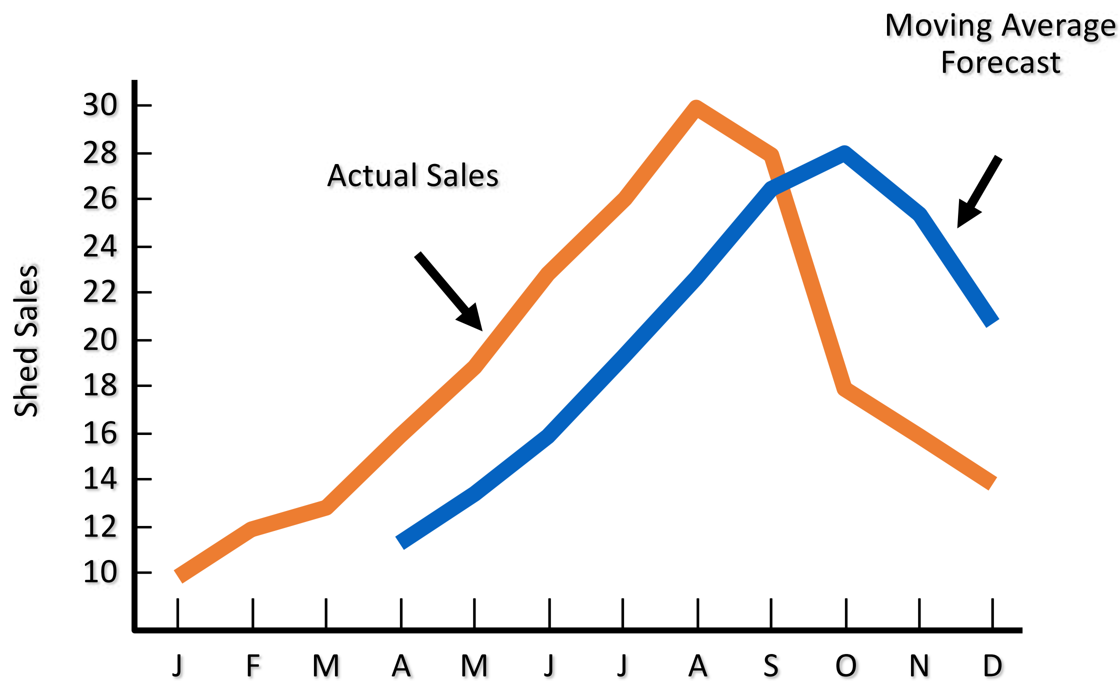
- ☑ MA is a series of arithmetic means
- ☑ Used if little or no trend
- ☑ Used often for smoothing
  - ☑ Provides overall impression of data over time

$$\text{Moving average} = \frac{\sum \text{demand in previous } n \text{ periods}}{n}$$

Moving Average Example

Month	Actual Shed Sales	3-Month Moving Average
January	10	
February	12	
March	13	
April	16	$(10 + 12 + 13)/3 = 11\ ^2/_3$
May	19	$(12 + 13 + 16)/3 = 13\ ^2/_3$
June	23	$(13 + 16 + 19)/3 = 16$
July	26	$(16 + 19 + 23)/3 = 19\ ^1/_3$

Graph of Moving Average



Weighted Moving Average

- ☑ Used when trend is present
  - ☑ Older data usually less important
- ☑ Weights based on experience and intuition

$$\text{Weighted moving average} = \frac{\sum (\text{weight for period } n) \times (\text{demand in period } n)}{\sum \text{weights}}$$

Weighte

Weights Applied	Period
3	Last month
2	Two months ago
1	Three months ago
6	Sum of weights

Month	Actual Shed Sales	3-Month Weighted Moving Average
January	10	
February	12	
March	13	
April	16	$[(3 \times 13) + (2 \times 12) + (10)]/6 = 12^1/_6$
May	19	$[(3 \times 16) + (2 \times 13) + (12)]/6 = 14^1/_3$
June	23	$[(3 \times 19) + (2 \times 16) + (13)]/6 = 17$
July	26	$[(3 \times 23) + (2 \times 19) + (16)]/6 = 20^1/_2$

Potential Problems With Moving Average

- ☑ Increasing ‘n’ smooths the forecast but makes it less sensitive to changes
- ☑ Do not forecast trends well
- ☑ Require extensive historical data

Moving Average And Weighted Moving Average

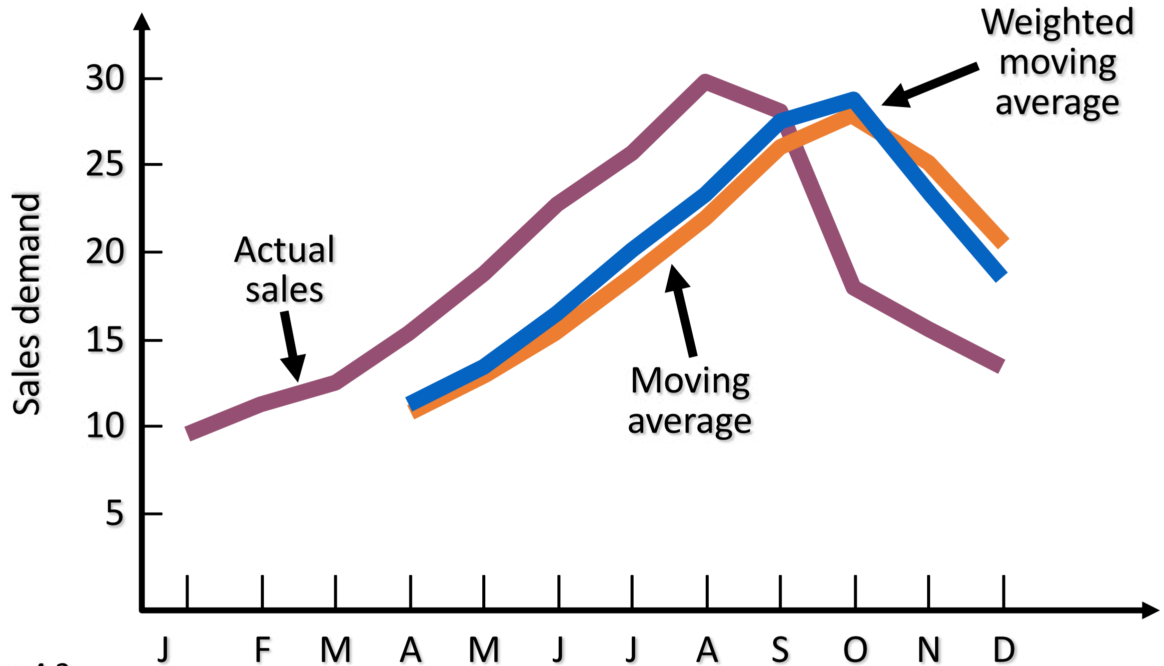


Figure 4.2

Exponential Smoothing

- ☑ Form of weighted moving average
  - ☑ Weights decline exponentially
  - ☑ Most recent data weighted most
- ☑ Requires smoothing constant (α)
  - ☑ Ranges from 0 to 1
  - ☑ Subjectively chosen
- ☑ Involves little record keeping of past data

Exponential Smoothing

New forecast = last period's forecast
+ α (last period's actual demand
– last period's forecast)

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$$

where F_t = new forecast

F_{t-1} = previous forecast

α = smoothing (or weighting) constant ($0 \leq \alpha \leq 1$)

Exponential Smoothing Example

Predicted demand = 142 Ford Mustangs
Actual demand = 153
Smoothing constant α = .20

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Predicted demand = 142 Ford Mustangs
Actual demand = 153
Smoothing constant α = .20

New forecast $= 142 + .2(153 – 142)$

Exponential Smoothing Example

Predicted demand = 142 Ford Mustangs
Actual demand = 153
Smoothing constant α = .20

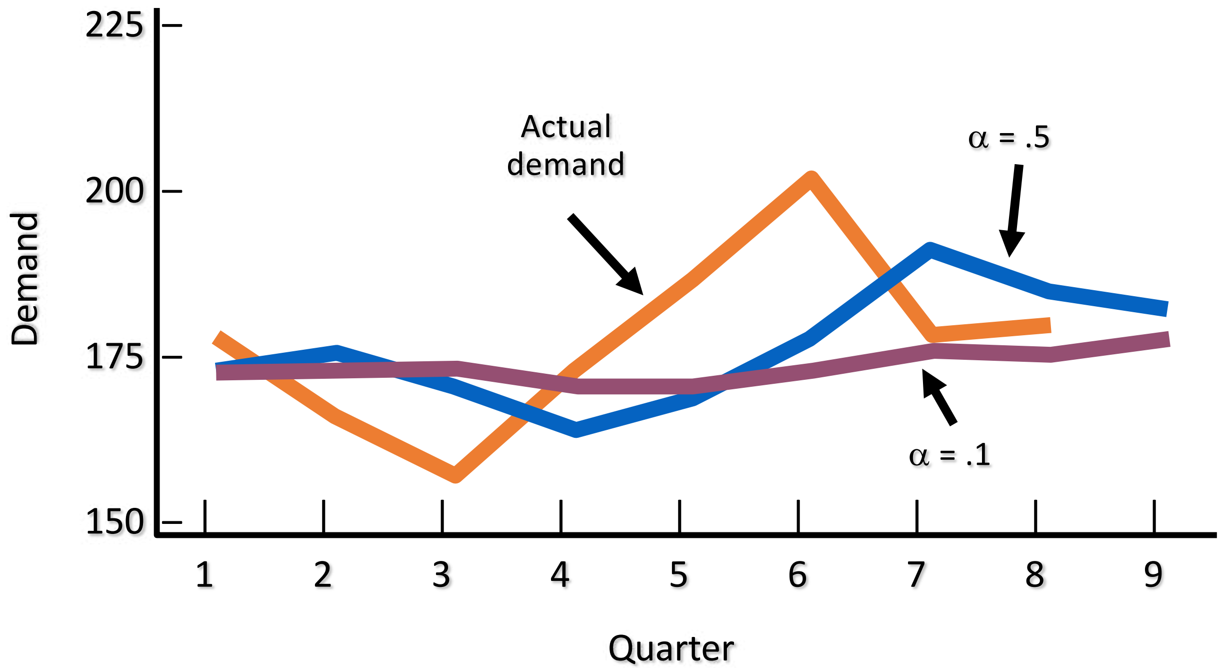
$$
\begin{aligned}
\text{New forecast} &= 142 + .2(153 - 142) \\
&= 142 + 2.2 \\
&= 144.2 \approx 144 \text{ cars}
\end{aligned}
$$

Effect of Smoothing Constants

Weight Assigned to

Smoothing Constant	Most Recent Period (α)	2nd Most Recent Period $\alpha(1 - \alpha)$	3rd Most Recent Period $\alpha(1 - \alpha)^2$	4th Most Recent Period $\alpha(1 - \alpha)^3$	5th Most Recent Period $\alpha(1 - \alpha)^4$
$\alpha = .1$	.1	.09	.081	.073	.066
$\alpha = .5$	.5	.25	.125	.063	.031

Impact of Different α



Demand Forecasting

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– last period's forecast)

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$$

where

F_t = new forecast

F_{t-1} = previous forecast

α = smoothing (or weighting) constant ($0 \le \alpha \le 1$)

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New forecast = 142 + .2(153 – 142)

Exponential Smoothing Example

Predicted demand = 142 Ford Mustangs
Actual demand = 153
Smoothing constant $\alpha = .20$

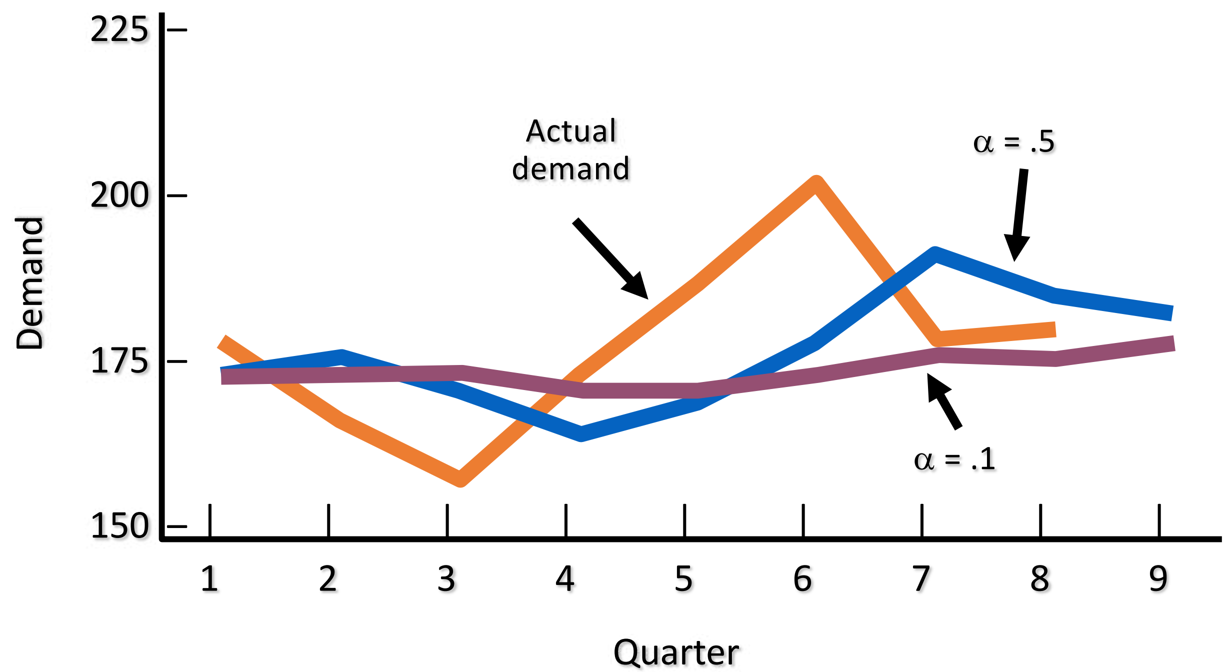
New forecast $= 142 + .2(153 - 142)$
$= 142 + 2.2$
$= 144.2 \approx 144$ cars

Effect of Smoothing Constants

Weight Assigned to

Smoothing Constant	Most Recent Period (α)	2nd Most Recent Period $\alpha(1 - \alpha)$	3rd Most Recent Period $\alpha(1 - \alpha)^2$	4th Most Recent Period $\alpha(1 - \alpha)^3$	5th Most Recent Period $\alpha(1 - \alpha)^4$
$\alpha = .1$	.1	.09	.081	.073	.066
$\alpha = .5$	.5	.25	.125	.063	.031

Impact of Different α



Choosing α

The objective is to obtain the most accurate forecast no matter the technique

We generally do this by selecting the model that gives us the lowest forecast error

Forecast error = Actual demand - Forecast value

$= A_t - F_t$

Common Measures of Error

Mean Absolute Deviation (MAD)

$$\text{MAD} = \frac{\sum |\text{actual - forecast}|}{n}$$

Mean Squared Error (MSE)

$$\text{MSE} = \frac{\sum (\text{forecast errors})^2}{n}$$

Common Measures of Error

Mean Absolute Percent Error (MAPE)

$$\text{MAPE} = \frac{100 \sum_{i=1}^{n} |\text{actual}_i - \text{forecast}_i| / \text{actual}_i}{n}$$

Comparison of Forecast Error

Quarter	Actual Tonnage Unloaded	Rounded Forecast with $\alpha = .10$	Absolute Deviation for $\alpha = .10$	Rounded Forecast with $\alpha = .50$	Absolute Deviation for $\alpha = .50$
1	180	175	5	175	5
2	168	176	8	178	10
3	159	175	16	173	14
4	175	173	2	166	9
5	190	173	17	170	20
6	205	175	30	180	25
7	180	178	2	193	13
8	182	178	4	186	4
			84		100

Comparison of Forecast Error

$$\text{MAD} = \frac{\sum |\text{deviations}|}{n}$$

For α = .10

$= 84/8 = 10.50$

For α = .50

$= 100/8 = 12.50$

				Rounded Forecast with α = .50	Absolute Deviation for α = .50
				175	5
				178	10
				173	14
				166	9
				170	20
				180	25
				193	13
8	182	178	4	186	4
			84		100

Comparison of Forecast Error

$$MSE = \frac{\sum (\text{forecast errors})^2}{n}$$

For $\alpha = .10$

$= 1{,}558/8 = 194.75$

For $\alpha = .50$

$= 1{,}612/8 = 201.50$

				Rounded Forecast with $\alpha = .50$	Absolute Deviation for $\alpha = .50$
				175	5
				178	10
				173	14
				166	9
				170	20
				180	25
				193	13
8	182	178	4	186	4
			84		100
		MAD	10.50		12.50

Comparison of Forecast Error

$$\text{MAPE} = \frac{100 \sum_{i=1}^{n} |\text{deviation}_i| / \text{actual}_i}{n}$$

For $\alpha = .10$

$= 45.62/8 = 5.70\%$

For $\alpha = .50$

$= 54.8/8 = 6.85\%$

					Absolute Deviation for $\alpha = .50$
					5
					10
					14
					9
					20
					25
					13
8	182	178	4	186	4
			84		100
		MAD	10.50		12.50
		MSE	194.75		201.50

Comparison of Forecast Error

Quarter	Actual Tonnage Unloaded	Rounded Forecast with $\alpha = .10$	Absolute Deviation for $\alpha = .10$	Rounded Forecast with $\alpha = .50$	Absolute Deviation for $\alpha = .50$
1	180	175	5	175	5
2	168	176	8	178	10
3	159	175	16	173	14
4	175	173	2	166	9
5	190	173	17	170	20
6	205	175	30	180	25
7	180	178	2	193	13
8	182	178	4	186	4
			84		100
		MAD	10.50		12.50
		MSE	194.75		201.50
		MAPE	5.70%		6.85%

Exponential Smoothing with Trend Adjustment

When a trend is present, exponential smoothing must be modified

Forecast including trend (FIT_t) = exponentially smoothed forecast (F_t) + exponentially smoothed trend (T_t)

Exponential Smoothing with Trend Adjustment

$$F_t = \alpha(A_{t-1}) + (1 - \alpha)(F_{t-1} + T_{t-1})$$

$$T_t = \beta(F_t - F_{t-1}) + (1 - \beta)T_{t-1}$$

Step 1: Compute F_t

Step 2: Compute T_t

Step 3: Calculate the forecast $FIT_t = F_t + T_t$

Exponential Smoothing with Trend Adjustment Example

Month(t)	Actual Demand (A_t)	Smoothed Forecast, F_t	Smoothed Trend, T_t	Forecast Including Trend, FIT_t
1	12	11	2	13.00
2	17			
3	20			
4	19			
5	24			
6	21			
7	31			
8	28			
9	36			
10				

Table 4.1

Exponential Smoothing with Trend Adjustment Example

Month(t)	Actual Demand (A_t)	Smoothed Forecast, F_t	Smoothed Trend, T_t	Forecast Including Trend, FIT_t
1	12	11	2	13.00
2	17			
3	20			
4	19			
5	24			
6	21			
7	31			
8	28			
9	36			
10				

Table 4.1

Step 1: Forecast for Month 2

$F_2 = \alpha A_1 + (1 - \alpha)(F_1 + T_1)$

$F_2 = (.2)(12) + (1 - .2)(11 + 2)$

$= 2.4 + 10.4 = 12.8$ units

Exponential Smoothing with Trend Adjustment Example

Month(t)	Actual Demand (A_t)	Smoothed Forecast, F_t	Smoothed Trend, T_t	Forecast Including Trend, FIT_t
1	12	11	2	13.00
2	17	12.80		
3	20			
4	19			
5	24			
6	21			
7	31			
8	28			
9	36			
10				

Table 4.1

Step 2: Trend for Month 2

$$T_2 = \beta(F_2 - F_1) + (1 - \beta)T_1$$

$$T_2 = (.4)(12.8 - 11) + (1 - .4)(2)$$

$$= .72 + 1.2 = 1.92 \text{ units}$$

Exponential Smoothing with Trend Adjustment Example

Month(t)	Actual Demand (A_t)	Smoothed Forecast, F_t	Smoothed Trend, T_t	Forecast Including Trend, FIT_t
1	12	11	2	13.00
2	17	12.80	1.92	
3	20			
4	19			
5	24			
6	21			
7	31			
8	28			
9	36			
10				

Table 4.1

Step 3: Calculate FIT for Month 2

$$FIT_2 = F_2 + T_1$$
$$FIT_2 = 12.8 + 1.92$$
$$= 14.72 \text{ units}$$

Exponential Smoothing with Trend Adjustment Example

Month(t)	Actual Demand (A_t)	Smoothed Forecast, F_t	Smoothed Trend, T_t	Forecast Including Trend, FIT_t
1	12	11	2	13.00
2	17	12.80	1.92	14.72
3	20	15.18	2.10	17.28
4	19	17.82	2.32	20.14
5	24	19.91	2.23	22.14
6	21	22.51	2.38	24.89
7	31	24.11	2.07	26.18
8	28	27.14	2.45	29.59
9	36	29.28	2.32	31.60
10		32.48	2.68	35.16

Table 4.1

Exponential Smoothing with Trend Adjustment Example

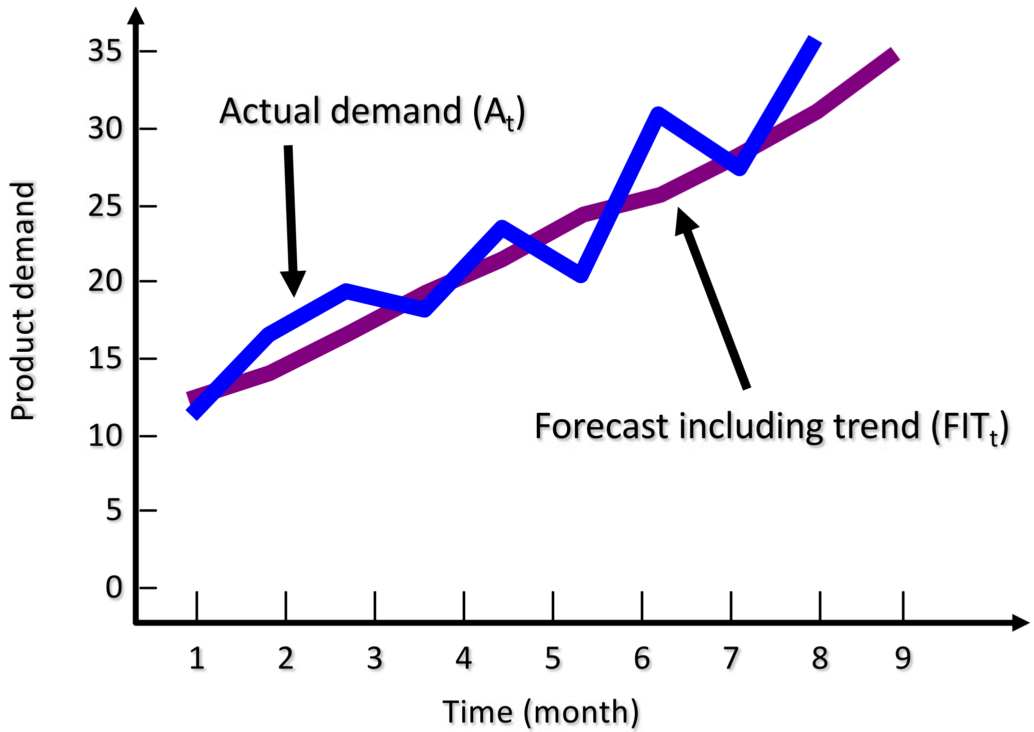


Figure 4.3

Replacement & Retention

LEARNING OUTCOMES

1. Explain replacement terminology and basics
2. Determine economic service life (ESL)
3. Perform replacement/retention study
4. Understand special situations in replacement
5. Perform replacement study over specified time
6. Calculate trade-in value of defender

Replacement Study Basics

Reasons for replacement study

1. **Reduced performance**
2. **Altered requirements**
3. **Obsolescence**

Terminology

Defender – ***Currently installed*** asset

Challenger – ***Potential replacement*** for defender

Market value (MV) – Value of defender if ***sold in open market***

Economic service life – No. of years at which ***lowest AW*** of cost occurs

Defender first cost – ***MV of defender***; used as its first cost (P) in analysis

Challenger first cost – ***Capital to recover*** for challenger (usually its P value)

Sunk cost – Prior expenditure ***not recoverable from challenger cost***

Nonowner's viewpoint – ***Outsider's (consultant's) viewpoint*** for objectivity

Example: Replacement Basics

An asset purchased 2 years ago for $40,000 is harder to maintain than expected. It can be sold now for $12,000 or kept for a maximum of 2 more years, in which case its operating cost will be $20,000 each year, with a salvage value of $9,000 two years from now. A suitable challenger will have a first cost of $60,000 with an annual operating cost of $4,100 per year and a salvage value of $15,000 after 5 years. Determine the values of P, A, n, and S for the defender and challenger for an AW analysis.

Solution:

Defender: P = \$-12,000; A = \$-20,000; n = 2; S = \$9,000

Challenger: P = \$-60,000; A = \$-4,100; n = 5; S = \$15,000

Overview of a Replacement Study

- Replacement studies are applications of the AW method
- Study periods (planning horizons) are either specified or unlimited
- Assumptions for unlimited study period:
  1. Services provided for indefinite future
  2. Challenger is best available now and for future, and will be repeated in future life cycles
  3. Cost estimates for each life cycle for defender and challenger remain the same
- If study period is specified, assumptions do not hold
- Replacement study procedures differ for the two cases

Economic Service Life

Economic service life (ESL) refers to the asset retention time (n) that yields its *lowest equivalent AW*

Determined by calculating AW for 1, 2, 3,…n years

General equation is:

Total AW = capital recovery – AW of annual operating costs
= CR – AW of AOC

Example: Economic Service Life

Determine the ESL of an asset which has the costs shown below. Let i = 10%

Year	Cost,$/year	Salvage value,$
0	- 20,000	-
1	-5,000	10,000
2	-6,500	8,000
3	- 9,000	5,000
4	-11,000	5,000
5	-15,000	3,000

Solution:

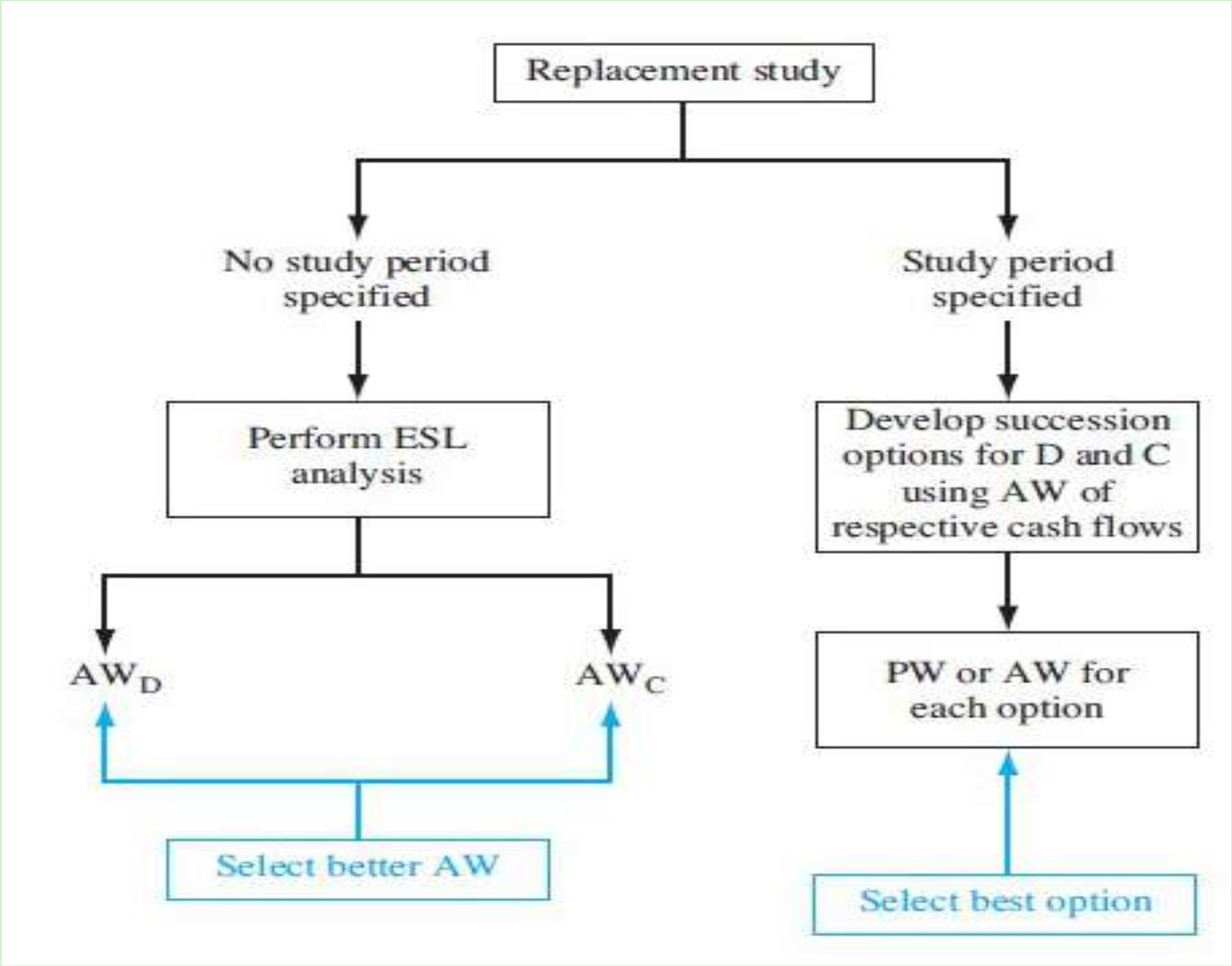
AW_1 = - 20,000(A/P,10%,1) – 5000(P/F,10%,1)(A/P,10%,1) + 10,000(A/F,10%,1) = $ -17,000

AW_2 = - 20,000(A/P,10%,2) – [5000(P/F,10%,1) + 6500(P/F,10%,2)](A/P,10%,2)
+ 8000(A/F,10%,2)
= $ -13,429

Similarly, AW_3 = $ -13,239 $\boxed{AW_4 = \$ -12,864}$ AW_5 = $ -13,623

Economic service life is 4 years

Performing a Replacement Study



Performing a Replacement Study -- Unlimited Study Period

1. **Calculate AW_D and AW_C** ***based on their ESL;*** **select lower AW**
2. **If AW_C was selected** in step (1), **keep for n_C** years (i.e., economic service life of challenger); **if AW_D** was selected, keep defender **one more year** and then **repeat analysis** (i.e., one-year-later analysis)
3. As long as all estimates **remain current** in succeeding years, **keep defender** until n_D is reached, and then replace defender with best challenger
4. **If any estimates change** before n_D is reached, **repeat steps (1) through (4)**

Note: If study period is specified, perform steps (1) through (4) *only through end of study period* (discussed later)

Example: Replacement Analysis

An asset purchased 2 years ago for $40,000 is harder to maintain than expected. It can be sold now for $12,000 or kept for a maximum of 2 more years, in which case its operating cost will be $20,000 each year, with a salvage value of $10,000 after 1 year or $9000 after two years. A suitable challenger will have an annual worth of $-24,000 per year. At an interest rate of 10% per year, should the defender be replaced now, one year from now, or two years from now?

Solution: **First, determine ESL for defender**

AW_{D1} = -12,000(A/P,10%,1) – 20,000 + 10,000(A/F,10%,1) = $-23,200

AW_{D2} = -12,000(A/P,10%,2) – 20,000 + 9,000(A/F,10%,2) = $-22,629

ESL is n = 2 years; AW_D = $-22,629

AW_C = $-24,000

Lower AW = $-22,629 **Replace defender in 2 years**

Note: *conduct one-year-later analysis next year*

Additional Considerations

Opportunity cost approach is the procedure that was previously presented for obtaining **P for the defender.** The opportunity cost is the **money foregone by keeping the defender** (i.e., not selling it). This approach is always correct

Cash flow approach subtracts income received from sale of defender from first cost of challenger.

Potential problems with cash flow approach:

- ⇨ Provides ***falsely low value*** for capital recovery of challenger
- ⇨ Can't be used if ***remaining life of defender is not same*** as that of challenger

Replacement Analysis Over Specified Study Period

Same procedure as before, except *calculate AW values over study period* instead of over ESL years of n_D and n_C

- **It is necessary to develop *all viable defender-challenger combinations* and calculate AW or PW for each one *over study period***

- **Select option with lowest cost or highest income**

Example: Replacement Analysis; Specified Period

An asset purchased 2 years ago for $40,000 is harder to maintain than expected. It can be sold now for $12,000 or kept for a maximum of 2 more years, in which case its operating cost will be $20,000 each year, with a salvage value of $10,000 after 1 year or $9000 after two. A suitable challenger will have an annual worth of $-24,000 per year. At an interest rate of 10% per year and over a study period of exactly 2 years, determine when the defender should be replaced.

Solution: From previous analysis, AW_D for 1 and 2 years, and AW_C are:

AW_{D1} = $-23,200 AW_{D2} = $-22,629 AW_C = $-24,000

Option	Year 1, $	Year 2, $	Year 3, $	AW, $
1 (C, C, C)	-24,000	-24,000	-24,000	-24,000
2 (D, C, C)	-23,200	-24,000	-24,000	-23,708
3 (D, D, C)	-23,200	-22,629	-24,000	-23,042

Decision: Option 3; **Keep D for 2 years**, then replace

Replacement Value

***Replacement value* (RV) is market/trade-in value of defender that renders AW_D and AW_C equal to each other**

Set up equation $\mathbf{AW_D = AW_C}$ except **use RV in place of P** for the defender; **then solve for RV**

If defender can be sold for amount > RV, *challenger is the better option,* because it will have a lower AW value

Example: Replacement Value

An asset purchased 2 years ago for $40,000 is harder to maintain than expected. It can be sold now for $12,000 or kept for a maximum of 2 more years, in which case its operating cost will be $20,000 each year, with a salvage value of $10,000 at the end of year two. A suitable challenger will have an initial cost of $65,000, an annual cost of $15,000, and a salvage value of $18,000 after its 5 year life. Determine the RV of the defender that will render its AW equal to that of the challenger, using an interest rate of 10% per year. Recommend a course of action.

Solution: Set $AW_D = AW_C$

$$- RV(A/P,10\%,2) - 20,000 + 10,000(A/F,10\%,2) = - 65,000(A/P,10\%,5) -15,000 +18,000(A/F,10\%,5)$$

RV = $24,228

Thus, if market value of defender > $24,228, *select challenger*

Summary of Important Points

- In replacement study, **P** for presently-owned asset is its ***market value***
- **Economic service life** is the n value that yields lowest AW
- In replacement study, if **no study period** is specified, *calculate AW over the respective life of each alternative*
- ***Opportunity cost approach*** is correct, it recognizes **money foregone by keeping the defender,** not by reducing challenger's first cost
- When study period is specified, *must consider all viable defender-challenger combinations* in analysis
- **Replacement value (RV)** is *P value for defender that renders its AW equal to that of challenger*

Location Decisions

Locating in a linear market

- Assume there is a kilometer beach and two ice cream vendors. Market is distributed uniformly. Where do they locate?
- Whether the selected location is optimal?
- Should govt. intervene in selecting deciding location?

Features

- The objective is to maximize the benefit of location to the firm
- Increasingly global in nature
- Significant impact on fixed and variable costs
- Decisions made relatively infrequently
- Once committed to a location, many resource and cost issues are difficult to change
- Long-term decisions

Location Decisions- Country Selection

1. Political risks, government rules, attitudes, incentives
2. Cultural and economic issues
3. Location of markets
4. Labor talent, attitudes, productivity, costs
5. Availability of supplies, communications, energy
6. Exchange rates and currency risks

Location Decisions- Region Selection

- Political risks in a federal structure
- Attractiveness of region
- Labor availability and costs
- Availability of utilities
- Environmental regulations
- Government incentives and fiscal policies
- Proximity to raw materials and customers
- Land/construction costs

Location Decisions- Site Selection

- Site size and cost
- Air, rail, highway, and waterway systems
- Zoning restrictions
- Proximity of services/ supplies needed
- Environmental impact issues

Methods for Location Selection

- Factor Rating Model
- Locational Break Even Analysis
- Center of Gravity Model
- Geographic Information System

Factor-Rating Method

- ☑ Popular because a wide variety of factors can be included in the analysis
- ☑ Six steps in the method
    1. Develop a list of relevant factors called critical success factors
    2. Assign a weight to each factor
    3. Develop a scale for each factor
    4. Score each location for each factor
    5. Multiply score by weights for each factor for each location
    6. Recommend the location with the highest point score

Factor-Rating Example

Critical Success Factor	Weight	Scores (out of 100) France	Scores (out of 100) Denmark	Weighted Scores France	Weighted Scores Denmark
Labor availability and attitude	.25	70	60	(.25)(70) = 17.5	(.25)(60) = 15.0
People-to car ratio	.05	50	60	(.05)(50) = 2.5	(.05)(60) = 3.0
Per capita income	.10	85	80	(.10)(85) = 8.5	(.10)(80) = 8.0
Tax structure	.39	75	70	(.39)(75) = 29.3	(.39)(70) = 27.3
Education and health	.21	60	70	(.21)(60) = 12.6	(.21)(70) = 14.7
Totals	1.00			70.4	68.0

Locational Break-Even Analysis

- ☑ Method of cost-volume analysis used for industrial locations
- ☑ Three steps in the method
  1. Determine fixed and variable costs for each location
  2. Plot the cost for each location
  3. Select location with lowest total cost for expected production volume

Locational Break-Even Analysis Example

Three locations:

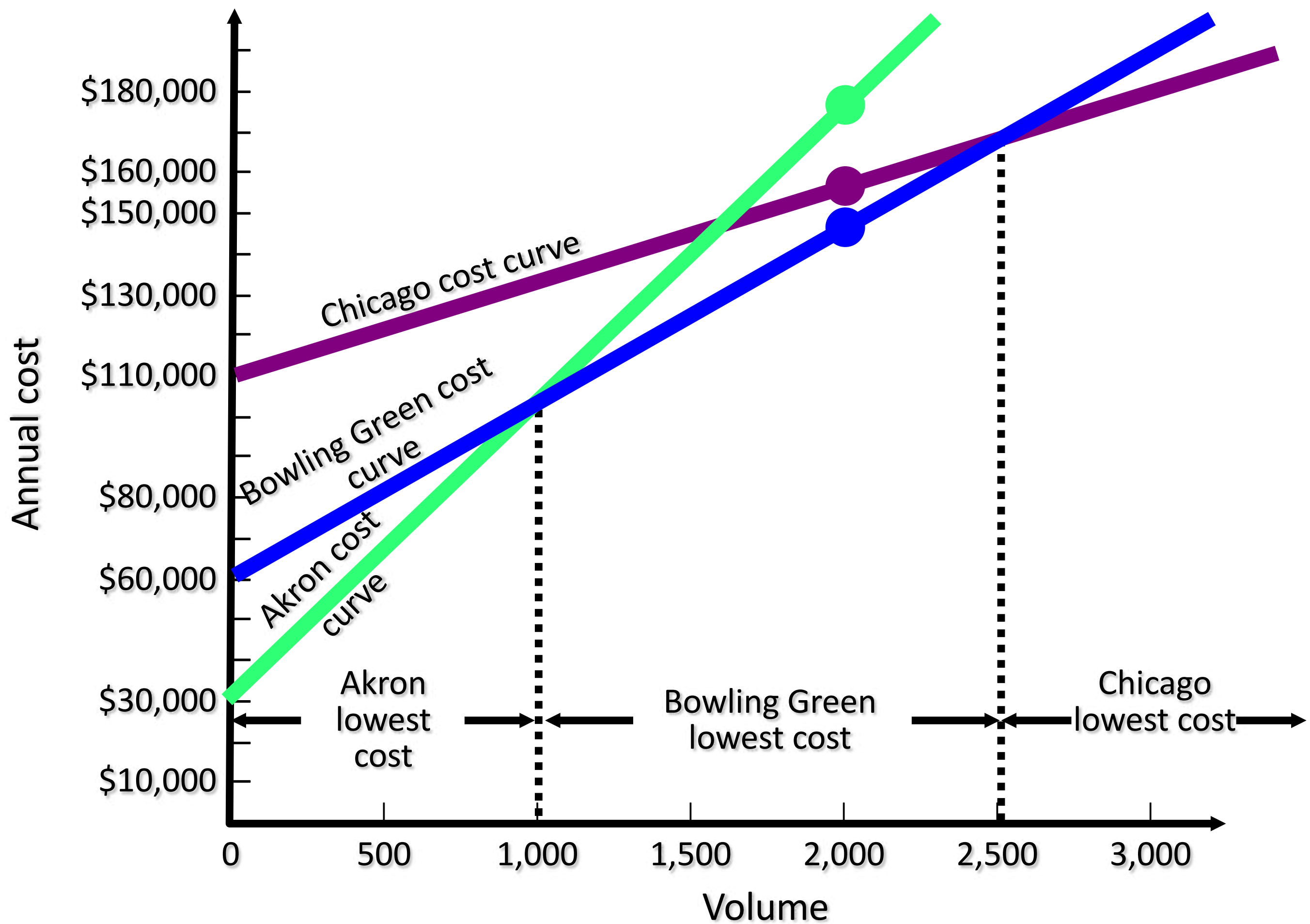
City	Fixed Cost	Variable Cost	Total Cost
Akron	\$30,000	\$75	\$180,000
Bowling Green	\$60,000	\$45	\$150,000
Chicago	\$110,000	\$25	\$160,000

Selling price = \$120

Expected volume = 2,000 units

Total Cost = Fixed Cost + Variable Cost x Volume

Locational Break-Even Analysis Example



Center-of-Gravity Method

- ☑ Finds location of distribution center that minimizes distribution costs
- ☑ Considers
  - ☑ Location of markets
  - ☑ Volume of goods shipped to those markets
  - ☑ Shipping cost (or distance)

Center-of-Gravity Method

- ☑ Place existing locations on a coordinate grid
  - ☑ Grid origin and scale is arbitrary
  - ☑ Maintain relative distances
- ☑ Calculate X and Y coordinates for ‘center of gravity’
  - ☑ Assumes cost is directly proportional to distance and volume shipped

Center-of-Gravity Method

$$x\text{ - coordinate} = \frac{\sum_i d_{ix} Q_i}{\sum_i Q_i}$$

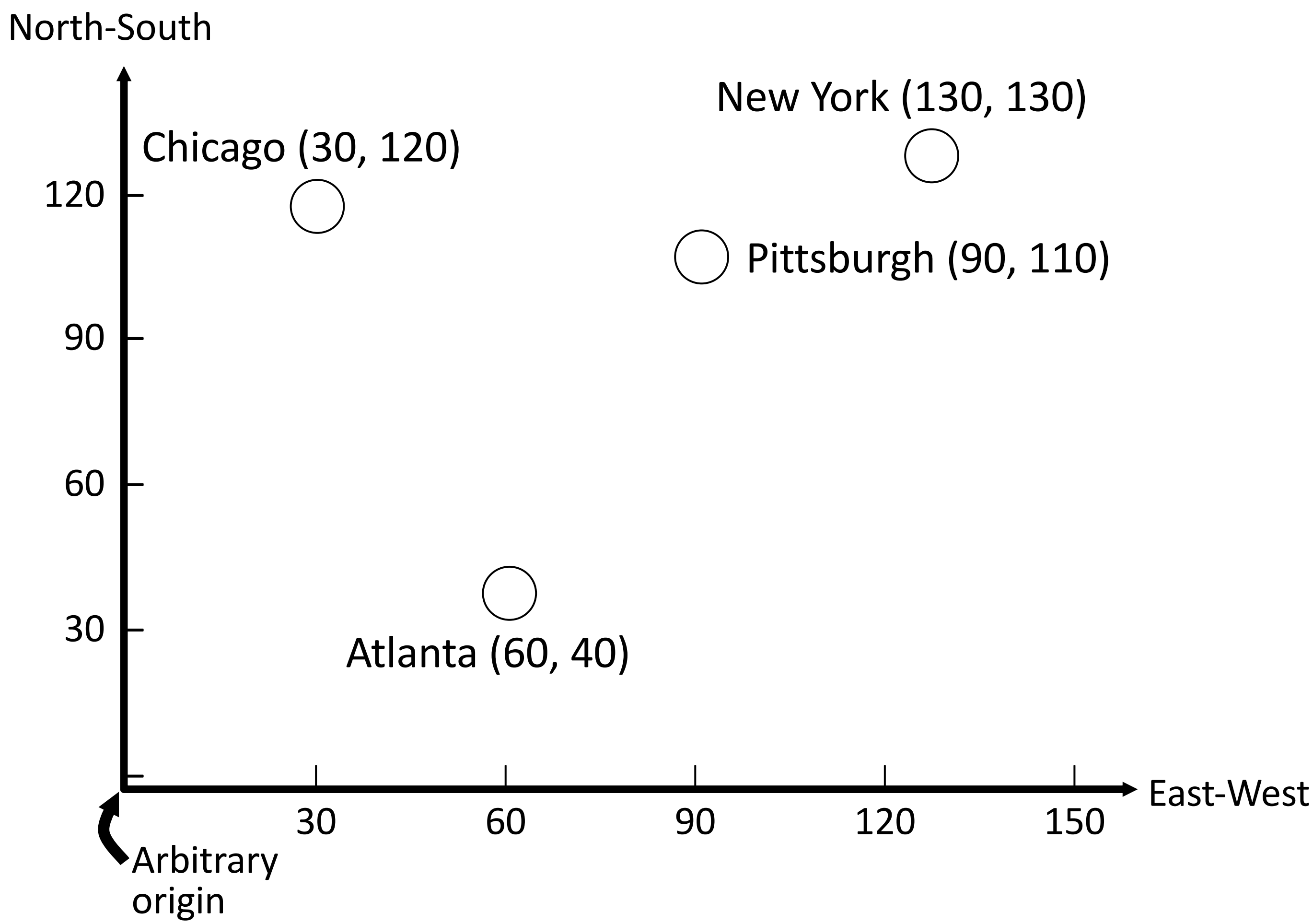
$$y\text{ - coordinate} = \frac{\sum_i d_{iy} Q_i}{\sum_i Q_i}$$

where d_{ix} = x-coordinate of location i

d_{iy} = y-coordinate of location i

Q_i = Quantity of goods moved to or from location i

Center-of-Gravity Method



Center-of-Gravity Method

Store Location	Number of Containers Shipped per Month
Chicago (30, 120)	2,000
Pittsburgh (90, 110)	1,000
New York (130, 130)	1,000
Atlanta (60, 40)	2,000

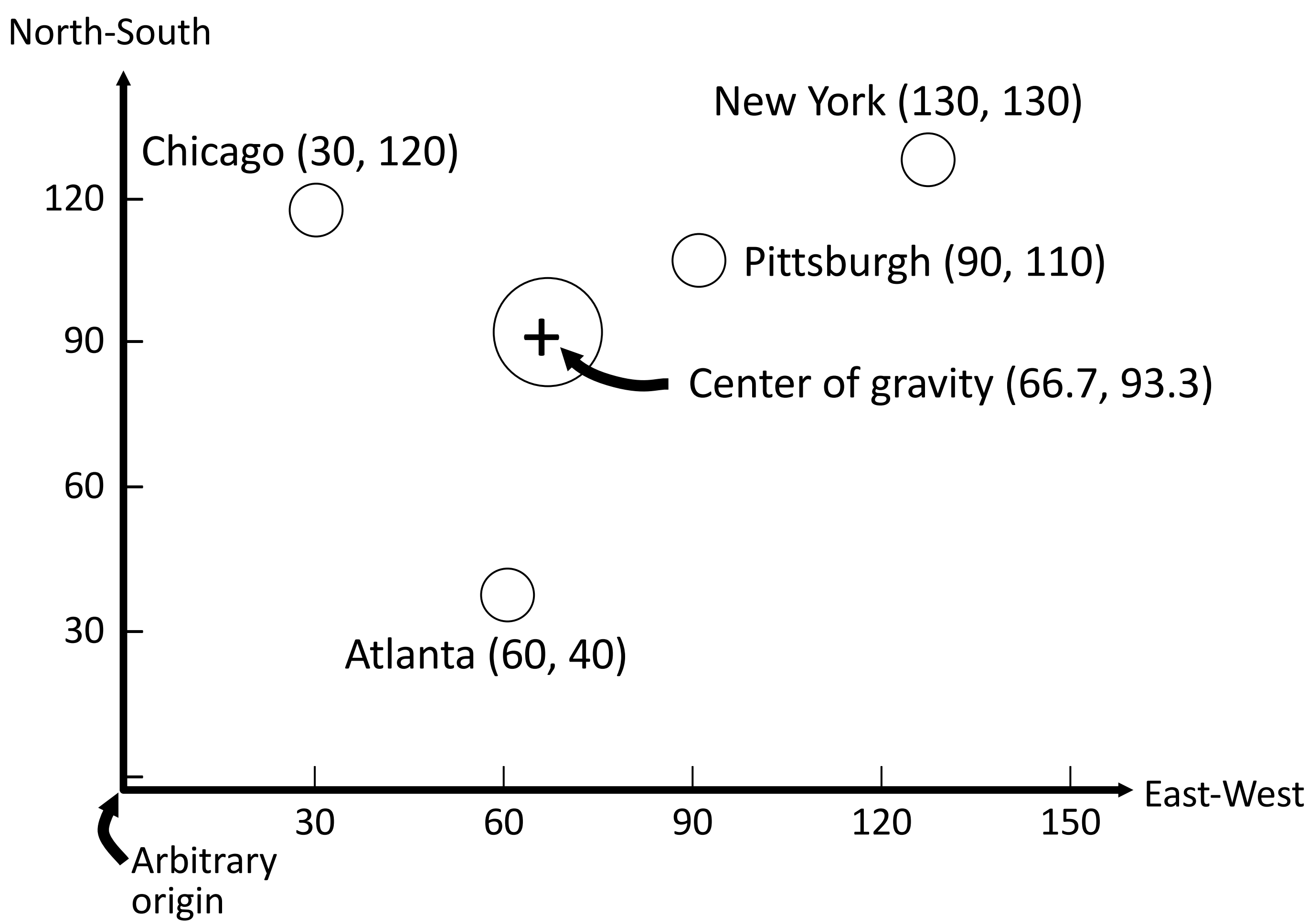
$$\text{x-coordinate} = \frac{(30)(2000) + (90)(1000) + (130)(1000) + (60)(2000)}{2000 + 1000 + 1000 + 2000}$$

$$= 66.7$$

$$\text{y-coordinate} = \frac{(120)(2000) + (110)(1000) + (130)(1000) + (40)(2000)}{2000 + 1000 + 1000 + 2000}$$

$$= 93.3$$

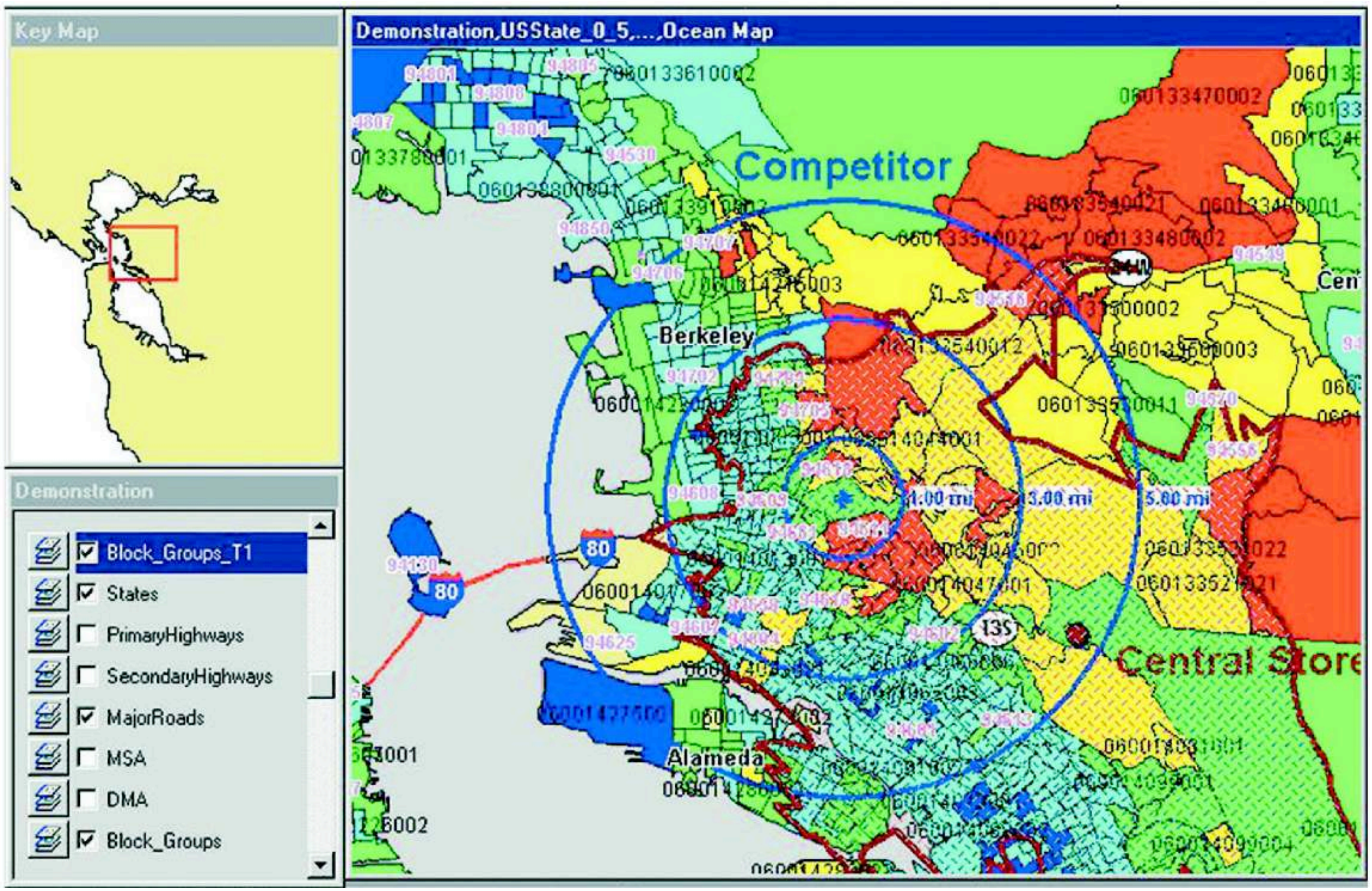
Center-of-Gravity Method



Geographic Information Systems (GIS)

- ☑ New tool to help in location analysis
- ☑ Enables more complex demographic analysis
- ☑ Available data bases include
  - ☑ Detailed census data
  - ☑ Detailed maps
  - ☑ Utilities
  - ☑ Geographic features
  - ☑ Locations of major services

Geographic Information Systems (GIS)



Cost Estimation and Indirect Costs

LEARNING OUTCOMES

1. **Approaches to estimation**
2. **Unit method**
3. **Cost indexes**
4. **Cost-capacity equations**
5. **Factor method**
6. **Indirect cost rates and allocation**
7. **ABC allocation**
8. **Ethical considerations**

Direct and Indirect Cost Estimates

Direct cost examples

- Physical assets
- Maintenance and operating costs (M&O)
- Materials
- Direct human labor (costs and benefits)
- Scrapped and reworked product
- Direct supervision of personnel

Indirect cost examples

- Utilities
- IT systems and networks
- Purchasing
- Management
- Taxes
- Legal functions
- Warranty and guarantees
- Quality assurance
- Accounting functions
- Marketing and publicity

What Direct Cost Estimation Includes

Direct costs are more commonly estimated than revenue in an engineering environment. Preliminary decisions required are:

- ✓ What **cost components** should be estimated?
- ✓ What **approach** to estimation is best to apply?
- ✓ How **accurate** should the estimates be?
- ✓ What **technique(s)** will be applied to estimate costs?

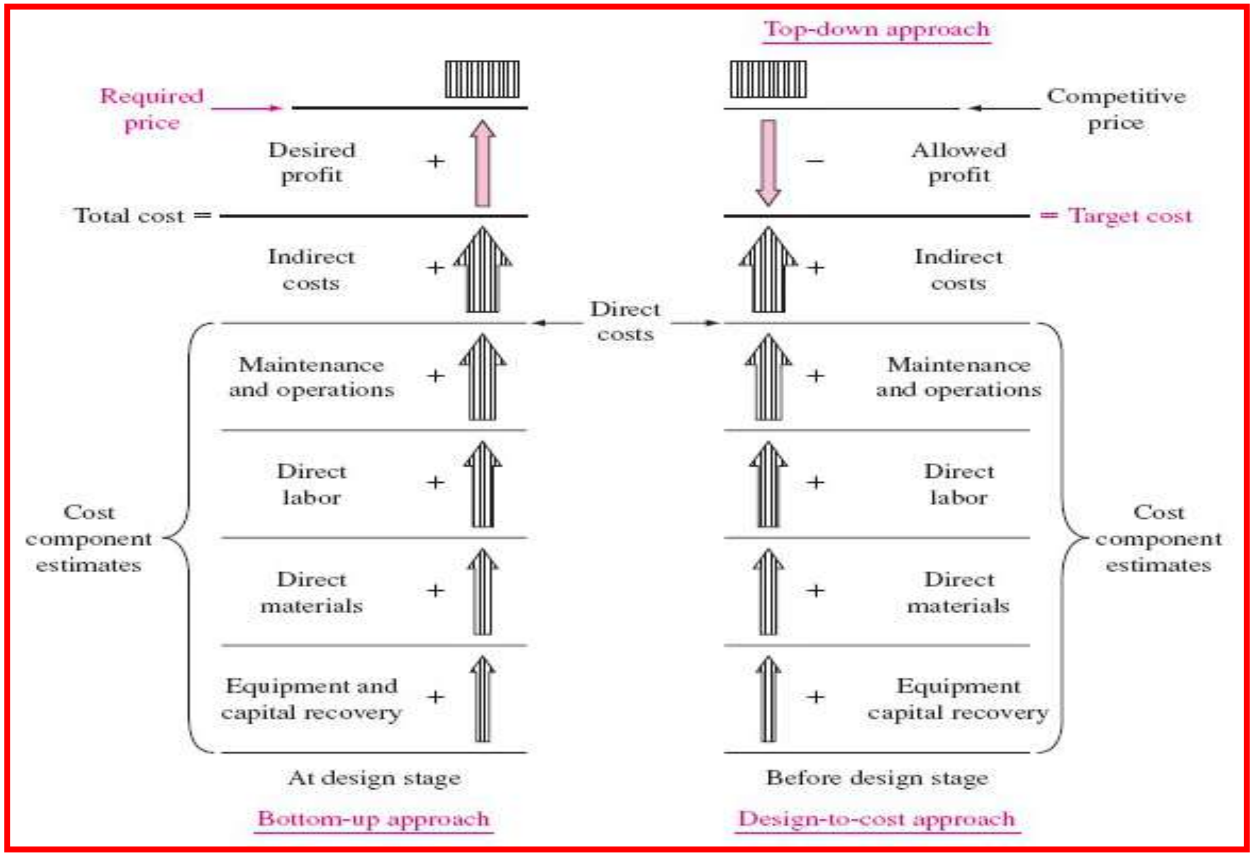
Sample direct cost components: first costs and its elements (P); annual costs (AOC or M&O); salvage/market value (S)

Approaches: bottom-up; design-to-cost (top down)

Accuracy: feasibility stage through detailed design estimates require more exacting estimates

Some techniques: unit; factor; cost estimating relations (CER)

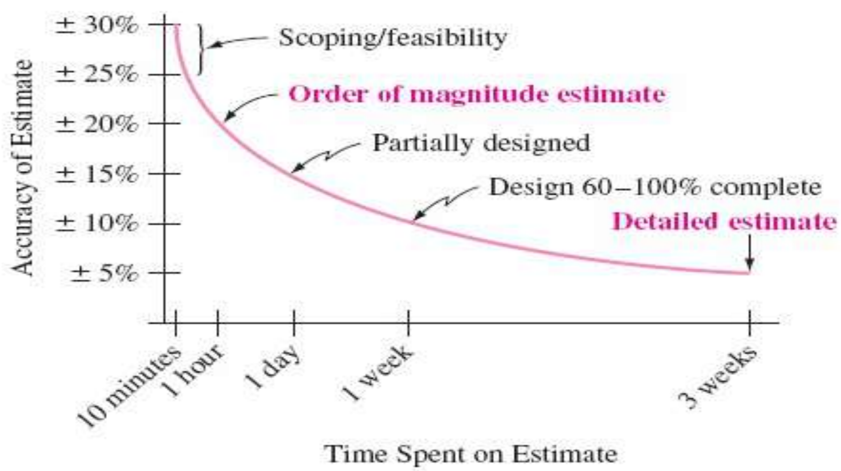
Different Approaches to Cost Estimation



Accuracy of Cost Estimates

	General guidelines for accuracy	
	Conceptual/Feasibility stage – order-of-magnitude estimates are in range of ±20% of actual costs **Detailed design stage** - Detailed estimates are in range of ±5% of actual costs	

Characteristic curve of accuracy vs. time to make estimates



Unit Method

- Commonly used technique for ***preliminary design stage*** estimates
- Total cost estimate C_T is **per unit cost (u)** times **number of units (N)**

$$C_T = u \times N$$

- **Example uses:**

➢ Cost to operate a car at 60¢/mile for 500 miles: **$C_T = 0.60 \times 500 = \$300$**
➢ Cost to build a 250 m^2 house at \$2250/m^2: **$C_T = 2250 \times 250 = \$562,500$**

- **Cost factors must be updated periodically to remain timely**

When several components are involved, estimate cost of each component and add to determine total cost estimate C_T

Cost Indexes

❖ **Definition: Cost Index is ratio of cost today to cost in the past**

- Indicates change in cost **over time**; therefore, they account for the **impact of inflation**
- Index is dimensionless
- CPI (Consumer Price Index) is a good example

Formula for total cost is

$$C_t = C_0\left(\frac{I_t}{I_0}\right)$$

C_t = estimated cost at present time t

C_0 = cost at previous time t_0

I_t = index value at time t

I_0 = index value at base time 0

Example: Cost Index Method

Problem: Estimate the total cost of labor today in US dollars for a maritime construction project using data from a similar project in Europe completed in 1998.

Labor index, 1998: 789.6
Labor index, current: 1165.8

Cost in 1998: €3.9 million
Currently, 1 € = 1.5 US$

Solution: Let t = today and 0 = 1998 base

$$C_t = 3.9 \text{ million} \times (1165.8/789.6) = €5.76 \text{ million}$$
$$= €5.76 \times 1.5 = \$8.64 \text{ million}$$

Finding Cost Indexes

Cost indexes are maintained in areas such as construction, chemical and mechanical industries

- Updated monthly and annually; many include regionalized and international project indexes
- Indexes in these areas are often subdivided into smaller components and can be used in preliminary, as well as detailed design stages

Examples are:

✓ Chemical Engineering Plant Cost Index (CEPCI)
www.che.com/pci

✓ McGraw-Hill Construction Index
www.construction.com

✓ US Department of Labor, Bureau of Labor Statistics
www.bls.gov

Cost-Estimating Relationships (CER)

- CER equations are **used in early design stages** to estimate plant, equipment and construction costs
- CERs are generically different from index relations, because they estimate based on **design variables** (weight, thrust, force, pressure, speed, etc.)

Two commonly used CERs

- **Cost-capacity equation** (relates cost to capacity)
- **Factor method** (total plant cost estimator, including indirect costs)

Cost-Capacity Equation

Also called ***power law and sizing model***

$$C_2 = C_1\left(\frac{Q_2}{Q_1}\right)^x$$

Exponent defines relation between capacities

$$C_1 = \text{cost at capacity } Q_1$$
$$C_2 = \text{cost at capacity } Q_2$$
$$x = \text{correlating exponent}$$

x = 1, relationship is **linear**
x < 1, economies of scale (larger capacity is **less costly than linear**)
x > 1, diseconomies of scale

Cost-Capacity Combined with Cost Index

Multiply the cost-capacity equation by a cost index (I_t/I_0) to adjust for time differences and obtain estimates of current cost (in constant-value dollars)

$$C_2 = C_1\left(\frac{Q_2}{Q_1}\right)^x\left(\frac{I_t}{I_0}\right)$$

Example: A 100 hp air compressor costs $3000 five years ago when the cost index was 130. Estimate the cost of a 300 hp compressor today when the cost index is 255. The exponent for a 300 hp air compressor is 0.9.

Solution: Let C_{300} represent the cost estimate today

$$C_{300} = 3000(300/100)^{0.9}(255/130)$$
$$= \$15,817$$

Factor Method

- Factor method is especially useful in estimating **total plant cost** in processing industries
- Both **direct and indirect costs** can be included

Total plant cost estimate C_T is **overall cost factor (h)** times **total cost of major equipment items (C_E)**

$$C_T = h \times C_E$$

Overall cost factor h is determined using one of two bases:

- Delivered-equipment cost (purchase cost of major equipment)
- Installed-equipment cost (equipment cost plus all make-ready costs)

Cost Factor h

The cost factor is commonly the sum of a direct cost component and an indirect cost component, that is,

$$h = 1 + \Sigma f_i$$

for i = 1, 2, …, n components, including indirect costs

Example: Equipment is expected to cost \$20 million delivered to a new facility. A cost factor for direct costs of 1.61 will make the plant ready to operate. An indirect cost factor of 0.25 is used. What will the plant cost?

Solution: $h = 1 + 1.61 + 0.25 = 2.86$

$C_T = 20$ million $(2.86) =$ **\$57.2 million**

Cost Factor h

If indirect costs are charged separately against all direct costs, the indirect cost component is added separately, that is,

$$h = 1 + \Sigma f_i \qquad \text{(direct costs components)}$$

and

$$C_T = hC_E(1 + f_{indirect})$$

Example: Conveyor delivered-equipment cost is \$1.2 million. Factors for installation costs (0.4) and training (0.2) are determined. An **indirect cost factor of 0.3** is applied to all direct costs. Estimate total cost.

Solution:

$$h = 1 + 0.4 + 0.2 = 1.6$$

$$C_T = hC_E(1 + f_{indirect})$$

$$= 1.6(1.2 \text{ million})(1 + 0.3) = \$2.5 \text{ million}$$

Indirect Costs

Indirect costs (IDC) are incurred in production, processes and service delivery that are not easily tracked and assignable to a specific function.

- **Indirect costs (IDC) are shared by many functions because they are necessary to perform the overall objective of the company**
- **Indirect costs make up a significant percentage of the overall costs in many organizations – 25 to 50%**

Sample indirect costs

- **IT services**
- **Quality assurance**
- **Human resources**
- **Management**
- **Safety and security**
- **Purchasing; contracting**
- **Accounting; finance; legal**

Indirect Cost Allocation - Traditional Method

- **Cost center --** Department, function, or process used by the cost accounting system to collect both direct and indirect costs
- **Indirect-cost rate –** Traditionally, a predetermined rate is used to allocate indirect costs to a cost center using a **specified basis**. General relation is:

$$\text{Indirect-cost rate} = \frac{\text{Estimated total indirect costs}}{\text{Estimated basis level}}$$

Example:

Cost Source	Allocation Basis	Estimated Activity Level
Machine 1	Direct labor cost	\$100,000
Machine 2	Direct labor hours	2000 hours
Machine 3	Direct material cost	\$250,000

Allocation rates for \$50,000 to each machine

Machine 1: Rate = \$50,000/100,000 = \$0.50 per DL \$
Machine 2: Rate = \$50,000/2,000 = \$25 per DL hour
Machine 3: Rate = \$50,000/250,000 = \$0.20 per DM \$

Example: AW Analysis - Traditional IDC Allocation

MAKE/BUY DECISION

Buy: AW = $-2.2 million per year

Make: P = $-2 million S = $50,000 n = 10 years MARR = 15%

- Direct costs of $800,000 per year are detailed below
- Indirect cost rates are established by department

Department	Indirect Costs: Basis, Hours	Indirect Costs: Rate Per Hour	Indirect Costs: Allocated Hours	Direct Material Cost	Direct Labor Cost
A	Labor	$10	25,000	$200,000	$200,000
B	Machine	5	25,000	50,000	200,000
C	Labor	15	10,000	50,000	100,000
				$300,000	$500,000

Example: Indirect Cost Analysis - Traditional Method

INDIRECT COST ALLOCATION FOR MAKE ALTERNATIVE

Dept A: Basis is -- Direct labor hours — **25,000(10) = $250,000**
Dept B: Basis is -- Machine hours — **25,000(5) = $125,000**
Dept C: Basis is -- Direct labor hours — **10,000(15) = $150,000**

$525,000

ECONOMIC COMPARISON AT MARR = 15%

AOC_{make} = direct labor + direct materials + indirect allocation

= 500,000 + 300,000 + 525,000 = $1.325 M

AW_{make} = - 2 M(A/P,15%,10) + 50,000(A/F,15%,10) - 1.325 M

= $-1.72 M

AW_{buy} = $-2.2 M

Conclusion: Cheaper to **make**

ABC Allocation

- **Activity-Based Costing —** Provides excellent **allocation strategy and analysis of costs** for more advanced, high overhead, technologically-based systems
- **Cost Centers (cost pools) —** Final products/services that **receive allocations**
- **Activities —** Support departments that **generate indirect costs** for distribution to cost centers (maintenance, engineering, management)
- **Cost drivers —** These are the **volumes that drive consumption** of shared resources (# of POs, # of machine setups, # of safety violations, # of scrapped items)

Steps to implement ABC:

1. Identify each ***activity*** and its ***total cost*** (e.g., maintenance at $5 million/year)
2. Identify ***cost drivers*** and expected ***volume*** (e.g., 3,500 requested repairs and 500 scheduled maintenances per year)
3. Calculate cost rate for each activity using the relation:

   ABC rate = total activity cost/volume of cost driver

4. Use ***ABC rate*** to allocate IDC to ***cost centers*** for each activity

Example: ABC Allocation

Use ABC to allocate safety program costs to plants in US and Europe

Cost centers: US and European plants
Activity and cost: Safety program costs $200,200 per year
Cost driver: # of accidents
Volume: 560 accidents; 425 in US plants and 135 in European plants

Solution:

ABC rate for accident basis = 200,200/560 = $357.50/accident

US allocation: 357.50(425) = $151,938
Europe allocation: 357.50(135) = $48,262

Example: Traditional Allocation Comparison

Use traditional rates to allocate safety costs to US and EU plants

Cost centers: US and European plants
Activity and cost: Safety program costs $200,200 per year
Basis: # of employees
Volume: 1400 employees; 900 in US plants and 500 in European plants

Solution:

Rate for employee basis = 200,200/1400 = $143/employee

US allocation: 143(900) = $128,700
Europe allocation: 143(500) = $71,500

Comparison: US allocation went down;
European allocation increased

Traditional vs. ABC Allocation

- Traditional method is easier to set up and use
- Traditional method is usually better when making cost estimates
- ABC is more accurate when process is in operation
- ABC is more costly, but provides more information for cost analysis and decision making
- **Traditional and ABC methods complement each other:**
  - **Traditional is good for cost estimation and allocation**
  - **ABC is better for cost tracking and cost control**

Ethics and Cost Estimating

Unethical practices in estimation may be the result of:

- ❑ Personal gain motivation
- ❑ Bias
- ❑ Deception
- ❑ Favoritism toward an individual or organization
- ❑ Intentional poor accuracy
- ❑ Pre-arranged financial favors (bribes, kickbacks)

When making any type of estimates, always comply with the

Code of Ethics for Engineers

Avoid deceptive acts