Computer Vision-IT416

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What is an Active Contour?

Given: Approximate boundary (contour) around the object

Task: Evolve (move) the contour to fit exact object boundary

Image

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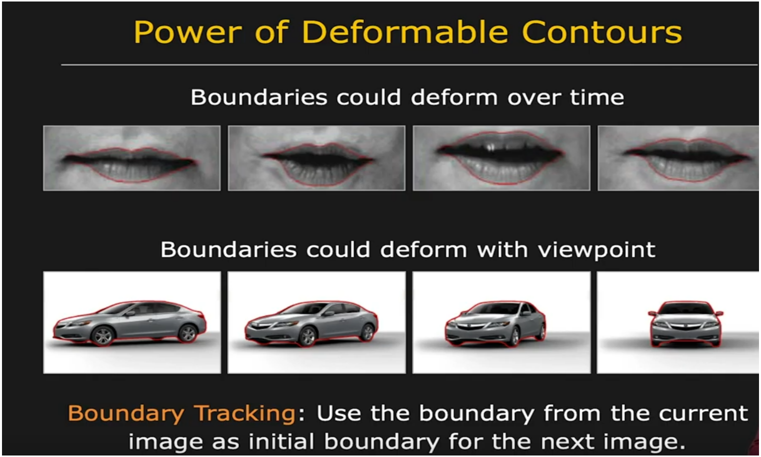
Active Contour:

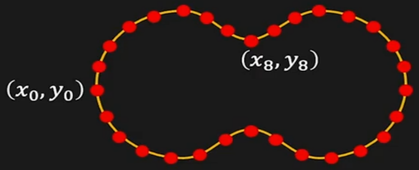
Iteratively "deform" the initial contour so that:

- It is near pixels with high gradient (edges)
- It is smooth

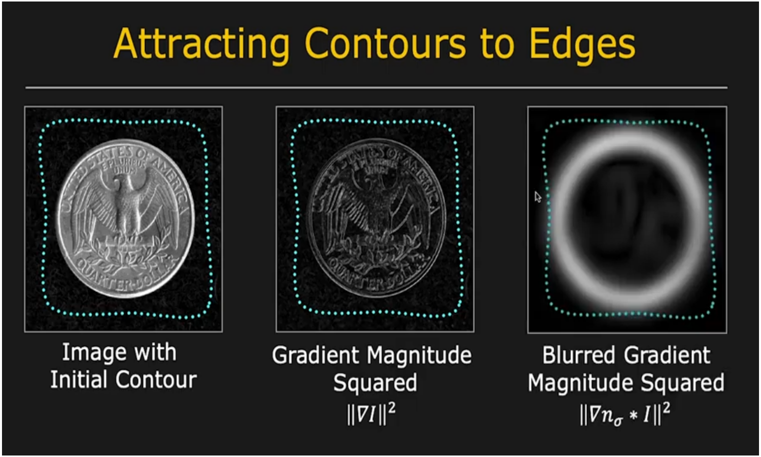

Power of Deformable Contours
Boundaries could deform over time
Boundaries could deform with viewpoint
Boundary Tracking: Use the boundary from the current image as initial boundary for the next image.

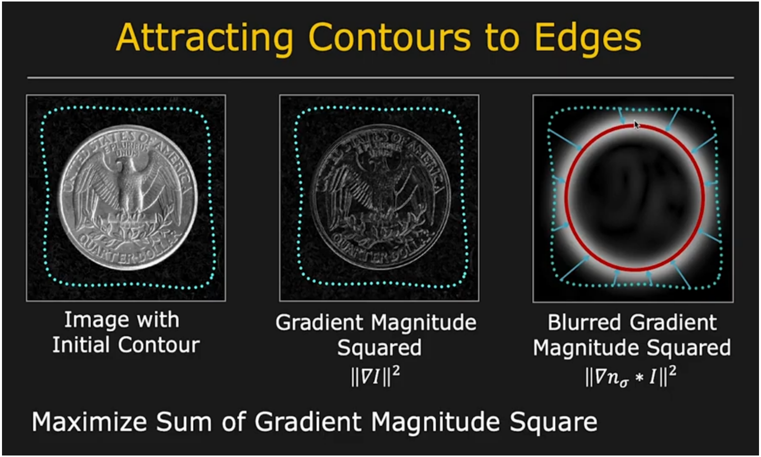
Representing a Contour

Contour **v**: An ordered list of 2D vertices (control points) connected by straight lines of fixed length



$$\mathbf{v} = \{v_i = (x_i, y_i) \mid i = 0, 1, 2, \dots, n-1\}$$


Attracting Contours to Edges
Image with Initial Contour
Gradient Magnitude Squared
$\|\nabla I\|^2$
Blurred Gradient Magnitude Squared
$\|\nabla n_\sigma * I\|^2$


Attracting Contours to Edges
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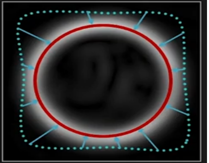
Maximize Sum of Gradient Magnitude Square

Attracting Contours to Edges

Image with Initial Contour

Gradient Magnitude Squared $\|\nabla I\|^2$



Blurred Gradient Magnitude Squared $\|\nabla n_\sigma * I\|^2$

Maximize Sum of Gradient Magnitude Square

≡ Minimize −ve (Sum of Gradient Magnitude Square)

≡ Minimize $\boxed{E_{image} = -\sum_{i=0}^{n-1} \|\nabla n_\sigma * I(v_i)\|^2}$

[redacted]

Contour Deformation: Greedy Algorithm

1. For each contour point v_i $(i = 0, \ldots, n-1)$, move v_i to a position within a window W where the energy function E_{image} for the contour is minimum.

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Greedy solution might be suboptimal and slow.

Sensitivity to Noise and Initialization

Contour fitted to
gradient magnitude

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Contour fitted to
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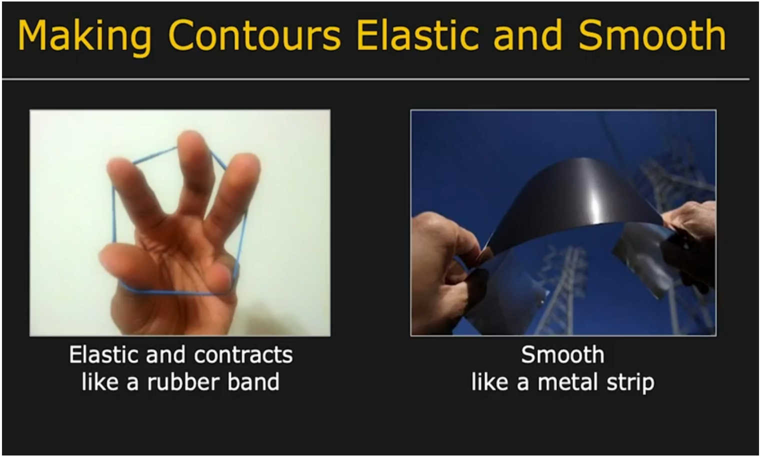
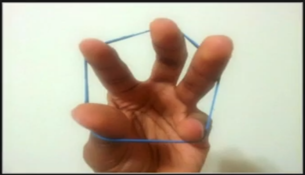
Solution: Add constraints that make the contour
contract and remain smooth


Making Contours Elastic and Smooth
Elastic and contracts
like a rubber band
Smooth
like a metal strip

Making Contours Elastic and Smooth



Elastic and contracts
like a rubber band

Smooth
like a metal strip

Minimize Internal Bending Energy of the Contour:

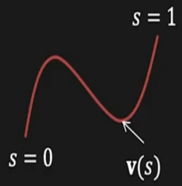
$$\boxed{E_{contour} = \alpha E_{elastic} + \beta E_{smooth}}$$

(α, β): Control the influence of elasticity and smoothness

Elasticity and Smoothness

For point $0 \le s \le 1$ on continuous contour $\mathbf{v}(s) = (x(s), y(s))$:

$$\boxed{E_{elastic} = \left\| \frac{d\mathbf{v}}{ds} \right\|^2} \qquad \boxed{E_{smooth} = \left\| \frac{d^2\mathbf{v}}{ds^2} \right\|^2}$$



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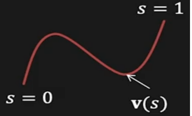

Discrete approximations at control point $\mathbf{v}_i$:

$$E_{elastic}(\mathbf{v}_i) = \left\| \frac{d\mathbf{v}}{ds} \right\|^2 \approx \|\mathbf{v}_{i+1} - \mathbf{v}_i\|^2 = (x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2$$

$$E_{smooth}(\mathbf{v}_i) = \left\| \frac{d^2\mathbf{v}}{ds^2} \right\|^2 \approx \|(\mathbf{v}_{i+1} - \mathbf{v}_i) - (\mathbf{v}_i - \mathbf{v}_{i-1})\|^2$$

$$= (x_{i+1} - 2x_i + x_{i-1})^2 + (y_{i+1} - 2y_i + y_{i-1})^2$$

Elasticity and Smoothness

Internal bending energy along the entire contour:

$$\boxed{E_{contour} = \alpha\, E_{elastic} + \beta\, E_{smooth}}$$

where:

$$E_{elastic} = \sum_{i=0}^{n-1} [(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2]$$

$$E_{smooth} = \sum_{i=0}^{n-1} [(x_{i+1} - 2x_i + x_{i-1})^2 + (y_{i+1} - 2y_i + y_{i-1})^2]$$

Combining the Forces

Image Energy, E_{image}: Measure of how well the contour latches on to edges

Internal Energy, $E_{contour}$: Measure of elasticity and smoothness

Total Energy of Active Contour:

$$\boxed{E_{total} = E_{image} + E_{contour}}$$

Minimize the Total Energy

Contour Deformation: Greedy Algorithm

1. Uniformly sample the contour to get n contour points.

2. For each contour point v_i $(i = 0, \ldots, n-1)$, move v_i to a position within a window W where the energy function E_{total} for the entire contour is minimum.

$$\boxed{E_{total} = E_{image} + E_{contour}}$$

3. If the sum of motions of all the contour points is less than a threshold, stop. Else go to Step 1.

Result: Effect of Contour Constraint

Without contour constraint

$$E_{total} = E_{image}$$

With contour constraint

$$E_{total} = E_{image} + E_{contour}$$

Result: Boundary Around Two Objects

Large α
(More like a rubber band)

Small α
(Less like a rubber band)