IT 300 Assignment 1

Date Page 1

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Topic : Master's Theorem

1) $T(n) = 3T(n/2) + n^2$

⇒ $a = 3, \quad b = 2$

$\log_b a = \log_2 3 = 1.58$

$f(n) = n^2$

∴ in case (3),

(i) $f(n) = \Omega\left(n^{\log_2 3 + \varepsilon}\right)$, $\varepsilon > 0$ is satisfied.

~~Hence~~

(ii) Regularity condition,

$3f(n/2) \leq c\, f(n)$

$\therefore \quad 3\frac{n^2}{4} \leq c\, n^2$

$\therefore \quad \boxed{c \geq 0.75}$, hence satisfied.

∴ Time complexity is $\boxed{\Theta(n^2)}$.

2) $T(n) = 7T(n/2) + n^2$

⇒ $a = 7, \quad b = 2, \quad \log_b a = \log_2 7 = 2.8$

$f(n) = n^2$

It follows case (1),

$f(n) = O(n^{2.8 - \varepsilon})$, $\varepsilon > 0$ ⇒ satisfied

∴ Time complexity is $\boxed{\Theta(n^{\log_2 7})}$.

3) $T(n) = 4T(n/2) + n^2$

⇒ $a = 4, b = 2, \log_b a = 2$

$f(n) = n^2$

It follows case (2) as

$f(n) = \Theta(n^2 \log^k n)$ with $k = 0$.

Hence,

Time complexity is $\boxed{\Theta(n^2 \log n)}$.

4) $T(n) = 9T(n/3) + n$

⇒ $a = 9, b = 3, \log_b a = 2$

$f(n) = n$

It follows case (1) as,

$f(n) = O(n^{2-\varepsilon}) \Rightarrow \varepsilon = 1 > 0$,

Hence

Time complexity is $\boxed{\Theta(n^2)}$.

5) $T(n) = T(2n/3) + 1$

⇒ $a = 1, b = 3/2, \log_b a = 0$

$f(n) = 1$

It follows case (2) as

$f(n) = \Theta(n^0 \log^k n)$ with $k = 0$

Hence,

time complexity is $\boxed{\Theta(\log n)}$

6) $T(n) = 3T(n/4) + n \log n$

⇒ $a = 3, b = 4, \log_b a = 1.26$

$f(n) = n \log n$.

It follows case (3) as,

(i) $f(n) = \Omega(n^{1.26+\varepsilon}) \Rightarrow \varepsilon > 0$

(ii) $3f(n/4) \leq c f(n)$

$\therefore \frac{3n}{4}\log\left(\frac{n}{4}\right) \leq cn\log n \Rightarrow c < 1.$

Hence,
Time complexity is $\boxed{\Theta(n\log n)}$

7) $T(n) = 4T(n/2) + n^2\log n.$

$\Rightarrow$ $a=4,\ b=2,\ \log_b a = 2$
$f(n) = n^2\log n$
It follows case (2),
$f(n) = \Theta(n^2\log^k n) \Rightarrow k=1$
Hence,
Time complexity is $\boxed{\Theta(n^2\log^2 n)}$

8) $T(n) = 4T(n/2) + \log n$

$\Rightarrow$ $a=4,\ b=2,\ \log_b a = 2$
$f(n) = \log n$
It follows case (1),
$f(n) = O(n^{2-\varepsilon}) \Rightarrow \varepsilon = 1 > 0,$
Hence
Time complexity is $\boxed{\Theta(n^2)}$

9) $T(n) = 5T(n/2) + n^2\log n.$

$\Rightarrow$ $a=5,\ b=2,\ \log_b a = 2.32$
$f(n) = n^2\log n$
It follows case (1),
$f(n) = O(n^{2.32-\varepsilon}) \Rightarrow \varepsilon > 0$
Hence
Time complexity is ~~$\Theta(n^2\log$~~ $\boxed{\Theta(n^{\log 5})}$

10) $T(n) = 2T(n/4) + C$

→ $a = 2, b = 4, \log_4 2 = 0.5$

$f(n) = C$

It follows case (1), as

$f(n) = O(n^{0.5 - \varepsilon}) \rightarrow \varepsilon = 0.5 > 0$

Hence,

Time complexity is $\boxed{\theta(n^{0.5})}$

11) $T(n) = T(n/4) + \log n$

→ $a = 1, b = 4, \log_b a = 0$

$f(n) = \log n$

It follows case (2) as,

$f(n) = \theta(n^0 \log^k n) \rightarrow k = 1$

Hence,

Time complexity is $\boxed{\theta(\log^2 n)}$

12) $T(n) = 2T(n/4) + \log n$.

→ $a = 2, b = 4, \log_b a = 0.5$

$f(n) = \log n$

It follows case (1) as

$f(n) = O(n^{0.5 - \varepsilon}) \Rightarrow \varepsilon > 0$,

Hence,

Time complexity is $\boxed{\theta(n^{0.5})}$

13) $T(n) = 3T(n/3) + n \log n$

→ $a = 3, b = 3, \log_b a = 1$

$f(n) = n \log n$

It follows case (2) as

$f(n) = \theta(n \log^k n) \rightarrow k = 1$

Hence

Time complexity is $\boxed{\theta(n \log^2 n)}$

14) $T(n) = 2T(n/4) + \sqrt{n}$

⇒ $a = 2$, $b = 4$, $\log_b a = 0.5$

$f(n) = \sqrt{n}$

It follows case (2) as

$f(n) = \Theta(n^{0.5} \log^k n) \Rightarrow k = 0$

Hence,

Time complexity is $\boxed{\Theta(\sqrt{n} \log n)}$

15) $T(n) = 2T(n/4) + n^{0.51}$

⇒ $a = 2$, $b = 4$, $\log_b a = 0.5$

$f(n) = n^{0.51}$

It follows case (3) as

$f(n) = \Omega(n^{0.5+\varepsilon}) \Rightarrow \varepsilon > 0$

Hence,

Time complexity is $\boxed{\Theta(n^{0.51})}$

16) $T(n) = 16T(n/4) + n!$

⇒ $a = 16$, $b = 4$, $\log_b a = 2$

$f(n) = n!$

It follows case (3) as

$f(n) = \Omega(n^{2+\varepsilon}) \Rightarrow \varepsilon > 0$

Hence,

Time complexity is $\boxed{\Theta(n!)}$

17) $T(n) = 3T(n/2) + n$

⇒ $a = 3$, $b = 2$, $\log_b a = 1.58$

$f(n) = n$

It follows case (1) as

$f(n) = O(n^{\log 3 - \varepsilon}) \Rightarrow \varepsilon > 0$

Hence,

Time complexity is $\boxed{\Theta(n^{\log_2 3})}$

18) $T(n) = 4T(n/2) + cn$

⇒ $a = 4,\ b = 2\ \log_b a = 2$

$f(n) = cn$

It follows case(1) as,

$f(n) = O(n^{2-\varepsilon}) \Rightarrow \varepsilon > 0$

Hence,

Time complexity is $\boxed{\Theta(n^2)}$

19) $T(n) = 3T(n/3) + n/2$

⇒ $a = 3,\ b = 3,\ \log_b a = 1$

$f(n) = n/2$

It follows case(2) as,

$f(n) = \Theta(n \log^k n) \Rightarrow k = 0$

Hence,

Time complexity is $\boxed{\Theta(n \log n)}$

20) $T(n) = 4T(n/2) + n/\log n$

→ $a = 4,\ b = 2,\ \log_b a = 2$

$f(n) = n/\log n$

It follows case (1) as,

$f(n) = O(n^{2-\varepsilon}) \Rightarrow \varepsilon > 0$

Hence,

Time complexity is $\boxed{\Theta(n^2)}$

21) $T(n) = 7T(n/3) + n^2$

⇒ $a = 7,\ b = 3,\ \log_b a = 1.77$

$f(n) = n^2$

It follows case(3) as,

$f(n) = O(n^{1.77+\varepsilon}) \Rightarrow \varepsilon > 0$

Hence,

Time complexity is ~~$\Theta(n^{\log_3 7})$~~ $\boxed{\Theta(n^2)}$

22) $T(n) = 8T(n/3) + 2^n$

$\Rightarrow$ $a = 8$, $b = 3$, $\log_b a = 1.89$

$f(n) = 2^n$

It follows case (3) as,

$f(n) = \Omega(n^{1.89+\epsilon}) \Rightarrow \epsilon > 0$

Time complexity is $\boxed{\theta(2^n)}$

23) $T(n) = 16T(n/4) + n$.

$\Rightarrow$ $a = 16$, $b = 4$, $\log_b a = 2$.

$f(n) = n$

It follows case (1) as,

$f(n) = O(n^{2-\epsilon}) \Rightarrow \epsilon > 0$

Hence,

Time complexity is $\boxed{\theta(n^2)}$