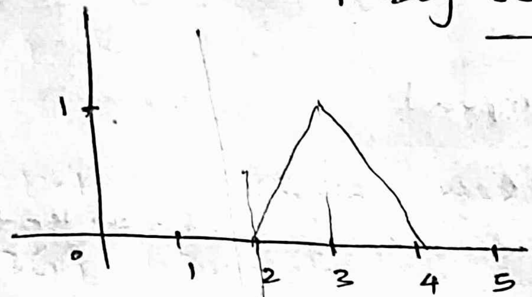
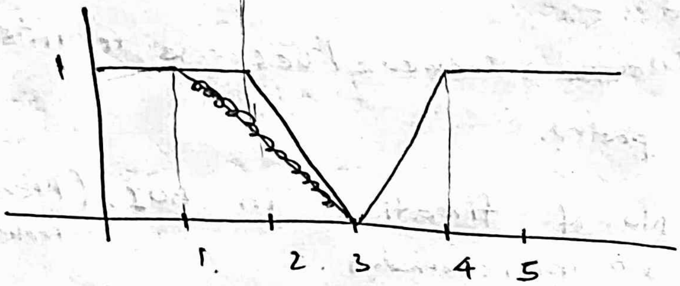
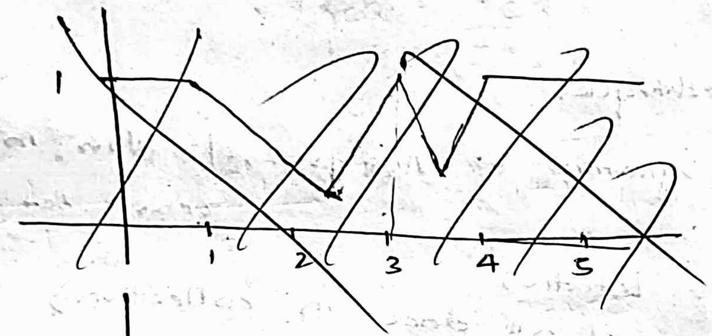
Fuzzy Set Operations

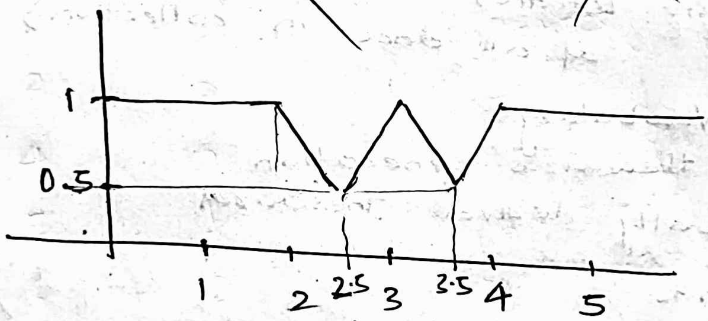


A

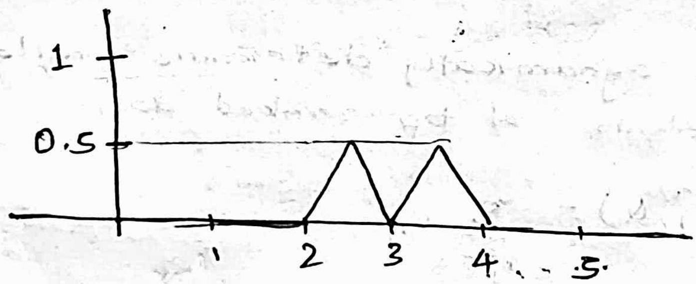


B

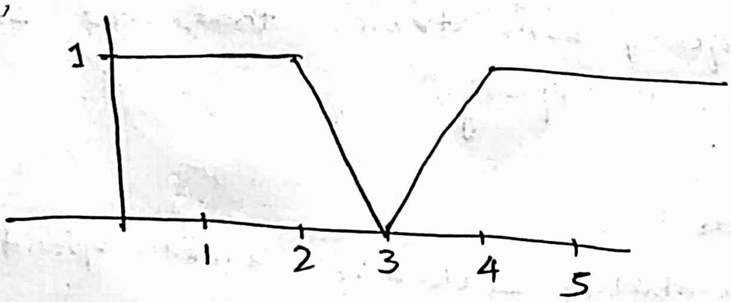




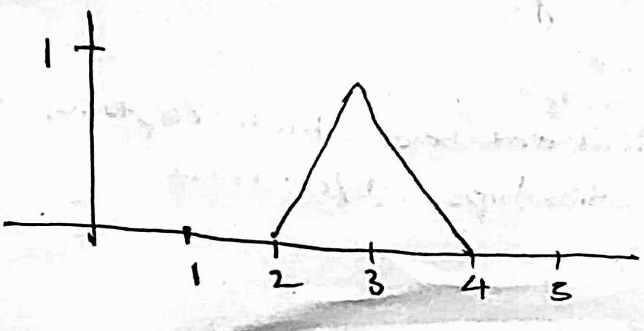
A ∪ B



A ∩ B



$\overline{A}$



$\overline{B}$

21 Oct. SC

→ A & B → 2 Fuzzy set
μ_A μ_B → Membership values

$\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$

$\mu_{A \cap B}(x) = \min($ " $)$

$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$

→ ~~Prod~~ $\mu_{AB}(x) = \mu_A(x) \cdot \mu_B(x)$

$\mu_A(x) = \mu_B(x) \Rightarrow$ A & B are equal.

→ $\mu_{aA}(x) = a \cdot \mu_A(x)$. a → Crisp Value

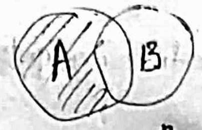
→ $\mu_{A^\alpha}(x) = (\mu_A(x))^\alpha$

$A = \{ (x_1, 0.4), (x_2, 0.2) \dots \}$

$\alpha = 2$

$A^\alpha = A^2 = \{ (x_1, 0.16), (x_2, 0.04), (x_3, 0.49) \}$

→ $A - B = (A \cap \bar{B})$



Back

$= \min(\mu_A, 1 - \mu_B)$

→ Disjunctive Sum:

$A \oplus B = (\bar{A} \cap B) \cup (A \cap \bar{B})$

$= \max(\min(1 - \mu_A, \mu_B), \min(\mu_A, 1 - \mu_B))$

Properties

Commutative
$A \cup B = B \cup A$
$A \cap B = B \cap A$

Associative
$(A \cup B) \cup C = A \cup (B \cup C)$
(same for $\cap$).

Distributive
Idempotence
Transitivity
De Morgans Law

Crisp Set
$A \cup \bar{A} = U$
$A \cap \bar{A} = \phi$

But in

Fuzzy set
$A \cup \bar{A} \neq U$
$A \cap \bar{A} \neq \phi$

Crisp Vs Fuzzy Relation

Crisp relation

A, B → 2 sets

$A \times B = \{ (a,b) \mid a \in A, b \in B \}$

eg → $A = \{a, b\}$ $B = \{1, 2\}$

$A \times B = \{ (a,1), (a,2), (b,1) (b,2) \}$

$R(X, Y)$ is a 2 dim matrix where $x \to$ rows, $y \to$ cols.

$R(i, j) = 1$ if $(x_i, y_j) \in R$
$R(i, j) = 0$ if $(x_i, y_j) \notin R$ } $R \subset X \times Y$

$X = \{x_1, x_2, \dots x_n\}$
$Y = \{y_1, y_2, \dots y_n\}$

eg → $X = \{1, 2, 3, 4\}$
$R = \{ (x, y) \mid y = x + 1, \; x, y \in X \}$
$R = \{(1, 2), (2, 3), (3, 4)\}$

R =	1	2	3	4
1	0	1	0	0
2	0	0	1	0
3	0	0	0	1
4	0	0	0	0

Composition of relan

R : Relation on (X, Y). ⇒ $R \cdot S$ is composition on X, Z.
S : " " (Y, Z) $T = R \cdot S$

Q) R on. $\{1, 3, 5\} \times \{1, 3, 5\}$
$R = \{ (x, y) \mid y = x + 2 \}$ $S = \{(x, y) \mid x < y\}$

find $R \cdot S$

Ans) $\{1, 3, 5\} \times \{1, 3, 5\}$
$= \{ (1,1) (1,3) (1,5) (3,1) (3,3) (3,5) (5,1) (5,3) (5,5) \}$

$R = \{ (1, 3) (3, 5) \}$
$S = \{ (1, 3) (1, 5) (3, 5) \}$

R =	1	3	5
1	0	1	0
3	0	0	1
5	0	0	0

S =	1	3	5
1	0	1	1
3	0	0	1
5	0	0	0

$R \cdot S =$ ~~matmul (R, S)~~

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$RS(1, 1) = \text{Max}(\text{Min}(0, 0), \min(1, 0), \min(0, 0)) = 0$

28th Oct (SC)

→. Max min relation for crisp sets. (Something like that)

Fuzzy Relations

let A & B be 2 fuzzy sets defined on X & Y respectively.
A × B results in a fuzzy relation R.

$$R = A \times B \subset X \times Y$$

where R has its membership function given by

$$\mu_R(x,y) = \mu_{A\times B}(x,y) = \min(\mu_A(x), \mu_B(y)).$$

AB

Q) $A = \{(x_1, 0.2), (x_2, 0.7), (x_3, 0.4)\}$
$B = \{(y_1, 0.5), (y_2, 0.6)\}$

$$R = A \times B = \begin{array}{c|cc} & y_1 & y_2 \\ \hline x_1 & 0.2 & 0.2 \\ x_2 & 0.5 & 0.6 \\ x_3 & 0.4 & 0.4 \end{array}$$

Operations on Fuzzy Relations.

Let R & S be 2 Fuzzy Relⁿ on X × Y.

$\mu_{R\cup S} =$ Max $(\mu_R(x,y), \mu_S(x,y))$

$\mu_{R\cap S} =$ Min (")

$\mu_{\bar{R}}(x,y) = 1 - \mu_R(x,y)$.

Composition of FR

R is FR on X × Y
S " " " Y × Z.

⇒ R∘S is FR on X × Z.

$$\mu_{R\cdot S}(x,z) = \underline{\text{Max}(\text{Min}(\mu_R(x,y), \mu_S(y,z)))}$$

Q) $X = \{x_1 \ldots x_3\}$ $Y = \{y_1, y_2\}$ $Z = \{z_1, \ldots z_3\}$

$$R = \begin{array}{c|cc} & y_1 & y_2 \\ \hline x_1 & 0.5 & 0.1 \\ x_2 & 0.2 & 0.9 \\ x_3 & 0.3 & 0.6 \end{array} \qquad S = \begin{array}{c|ccc} & z_1 & z_2 & z_3 \\ \hline y_1 & 0.6 & 0.4 & 0.7 \\ y_2 & 0.5 & 0.6 & 0.9 \end{array}$$

$$R\cdot S = \begin{array}{c|ccc} & z_1 & z_2 & z_3 \\ \hline x_1 & 0.5 & 0.4 & 0.5 \\ x_2 & 0.5 & 0.6 & 0.9 \\ x_3 & 0.5 & 0.6 & 0.6 \end{array}$$

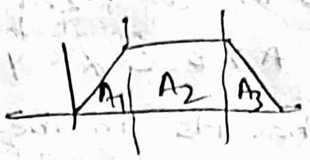
$x_1 z_1 \rightarrow \max(\min(0.5, 0.6), \min(0.1, 0.5))$
$= \underline{0.5}$

Fuzzification & Defuzzification

Converting crisp to fuzzy using Membership f^n is called Fuzzification.

Fuzzy to crisp → Deffuzification

① • Centroid Method → $\dfrac{\Sigma A(\bar{x})}{\Sigma(A)}$ $\bar{x}$ → Centroid, A → Area of segments



• Center of Area Method

② . Mean of Maximum
Consider crisp value with highest degree of membership

$$x = \frac{\sum_{x_i \in M} x_i}{|M|}$$

$M = \{x_i \mid M(x_i) = \text{height of fuzzy set}\}$

$|M|$ = Cardinality ratio

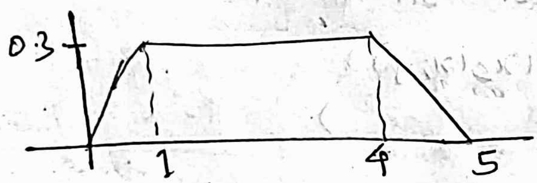
③ Weighted avg. method

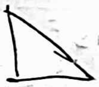
$$x = \frac{\Sigma \mu_c(\bar{z}) \cdot \bar{z}}{\Sigma \mu_c(\bar{z})}$$

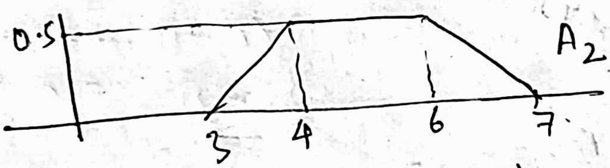
$\bar{z}$ → Centroid of Membership fn.

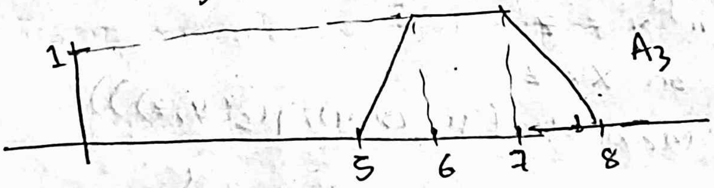
$\mu_c(\bar{z})$ → Maximum membership value

Q)



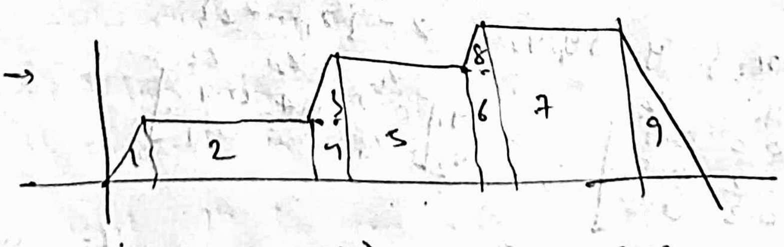






①

Aggregate →



$$x = \frac{\Sigma A(\bar{x})}{\Sigma A} = \frac{18.353}{3.715} = 4.9$$

② Mean of maxi

$$x = \frac{6+7}{2} = 6.5$$

③ Weighted avg

$$x = \frac{0.3 \times 2.5 + 0.5 \times 5 + 1 \times 6.5}{0.3 + 0.5 + 1} = 5.41$$

4th Nov (SC)

Fuzzy If then rules

- Convenient way to represent knowledge
- Highly interpretable

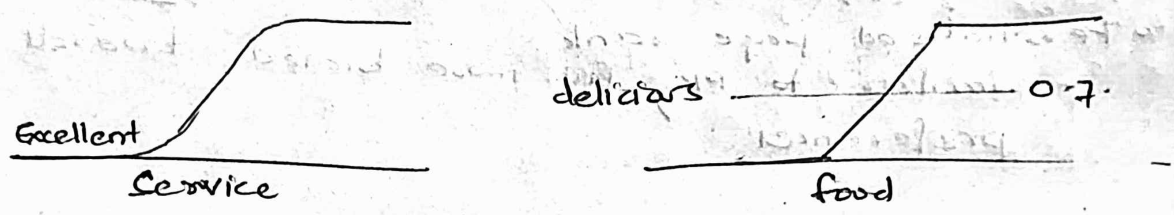
e.g → If x is A then y is B.
→ A & B are linguistic (fuzzy) values
→ x & y are fuzzy sets.

- If service is good then tip is average
- If height becomes tall, then weight becomes heavy.

Interpreting fuzzy values

① Resolve all fuzzy statements in the antecedent (If part) to a degree of membership b/w 0 & 1.

e.g → If $\boxed{\text{service is excellent}}$ OR $\boxed{\text{food is delicious}}$, then tip is high.
→ 2 Fuzzy sets.



μ (Excellent = 0) μ (delicious = 0.7)

μ = 0.7 (OR)

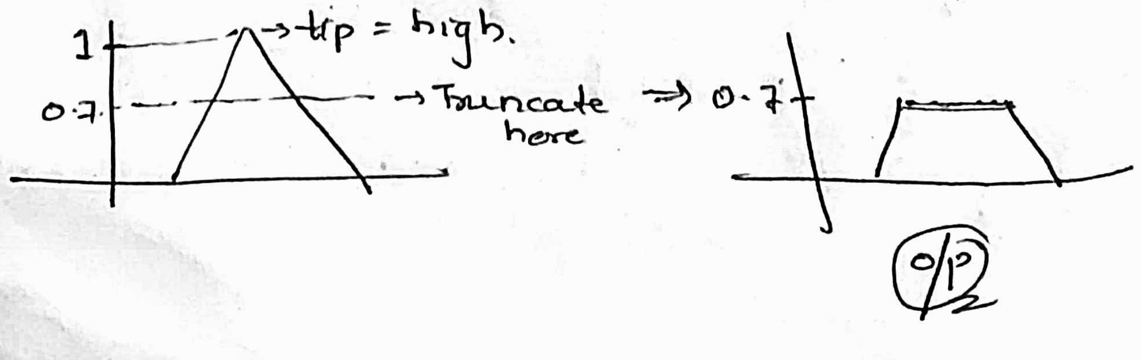
② Apply fuzzy operator & find degree of support of rule.

OR ⇒ Support of rule = 0.7

③ Apply implication method.

Truncate (cut) the o/p Fuzzy set at the level of degree of support for rule

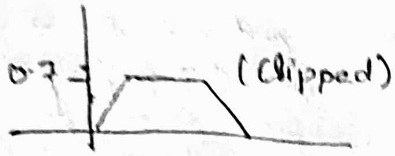
Suppose output fn for example is:



Implication Methods

③ Mandani method

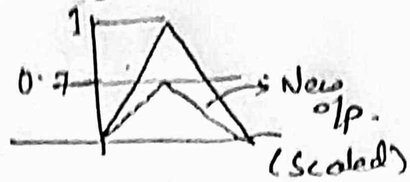
- $\mu_A \wedge \mu_B$
  ↓
  Min operator



- output shape isn't preserved.

③ Larsen method.

- $\mu_A \cdot \mu_B$
  ↓
  Algebraic Product



- o/p shape is preserved

④ If required, apply defuzzification on o/p Fuzzy set.

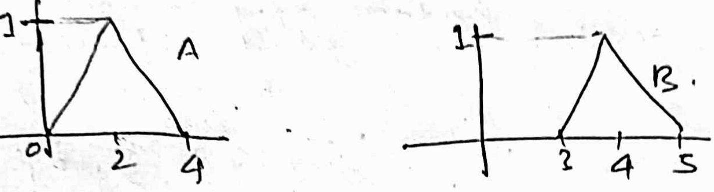
Q). Fuzzy rule R: If u is A then V is B where A = (0, 2, 4) & B(3, 4, 5) are triangular Fuzzy sets.

① what is the o/p. B, if I/p. is crisp value $u_0 = 3$

② " " " " " " " " fuzzy set A = (0, 1, 2)

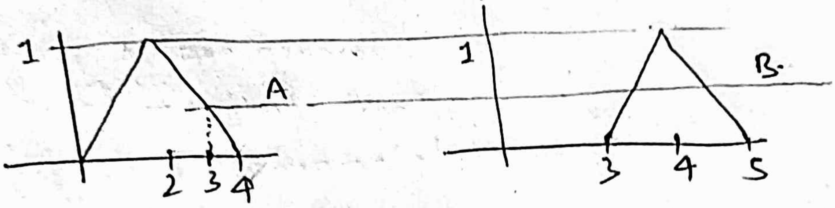
(Use Mandani method)

Ans)

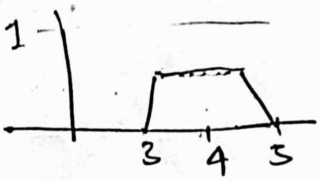


①

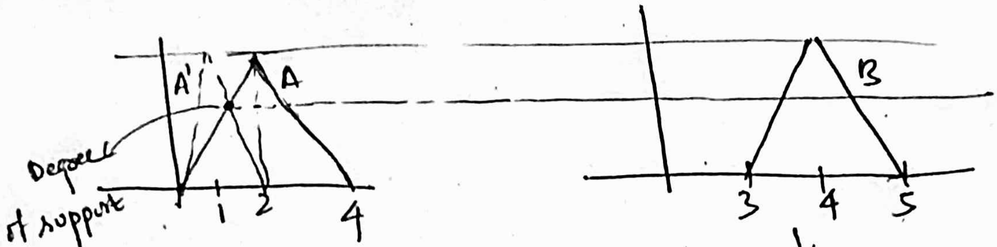
$u_0 = 3$



o/p =



②



↓

o/p :