Introduction to Artificial Neural Network Models

Angshuman Saha

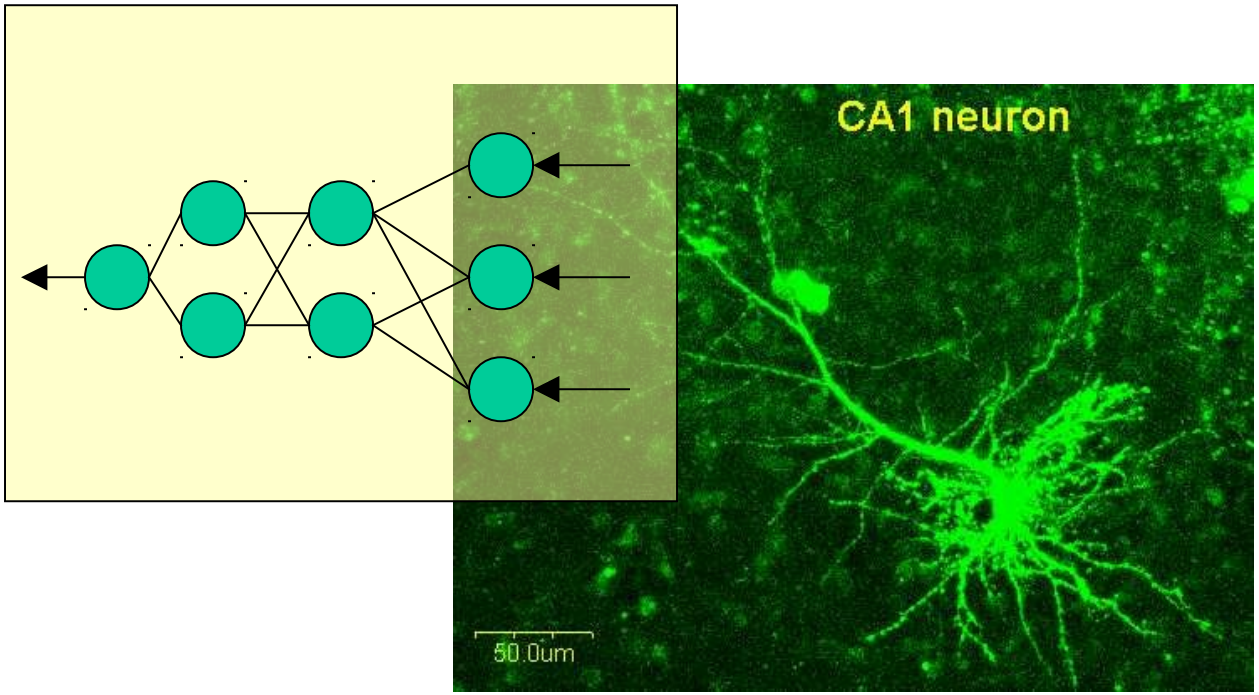

Image Source: ww.physiol.ucl.ac.uk/fedwards/ ca1%20neuron.jpg

Definition

Neural Network

A broad class of models that mimic functioning inside the human brain

There are various classes of NN models.
They are different from each other depending on

- Problem types
  Prediction, Classification , Clustering
- Structure of the model
- Model building algorithm

For this discussion we are going to focus on
Feed-forward Back-propagation Neural Network
(used for Prediction and Classification problems)

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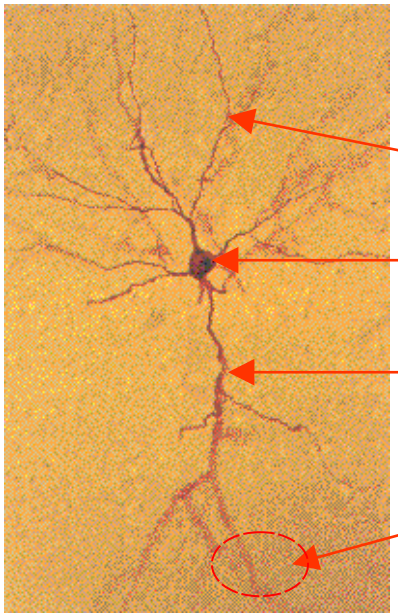
A bit of biology . . .

Most important functional unit in human brain – a class of cells called –

NEURON

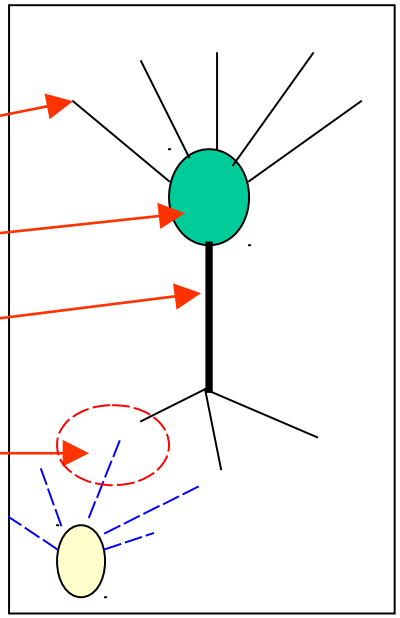


Dendrites

Cell Body

Axon

Synapse



Hippocampal Neurons
Source: heart.cbl.utoronto.ca/ ~berj/projects.html

Schematic

- **Dendrites** – Receive information
- **Cell Body** – Process information
- **Axon** – Carries processed information to other neurons
- **Synapse** – Junction between Axon end and Dendrites of other Neurons

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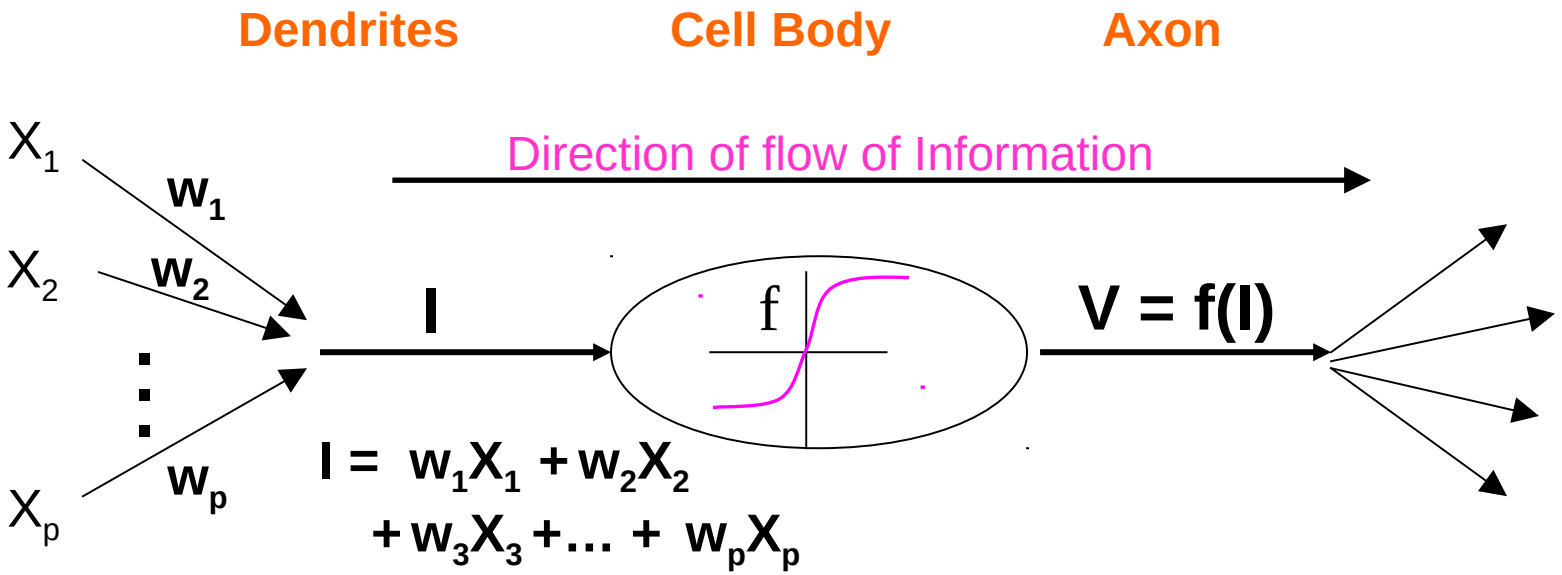
An Artificial Neuron



- Receives Inputs X_1 X_2 … X_p from other neurons or environment
- Inputs fed-in through connections with ‘weights’
- Total Input = Weighted sum of inputs from all sources
- Transfer function (Activation function) converts the input to output
- Output goes to other neurons or environment

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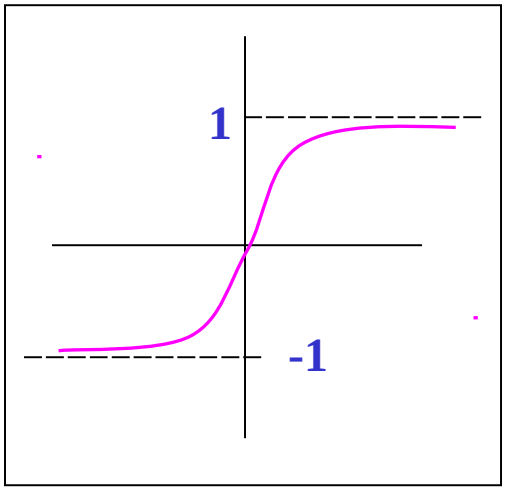
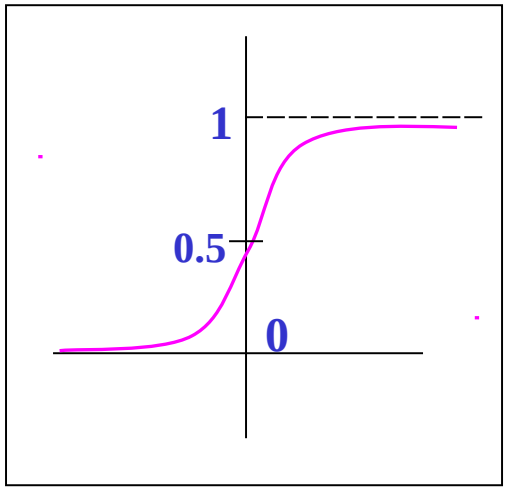
Transfer Functions

There are various choices for Transfer / Activation functions



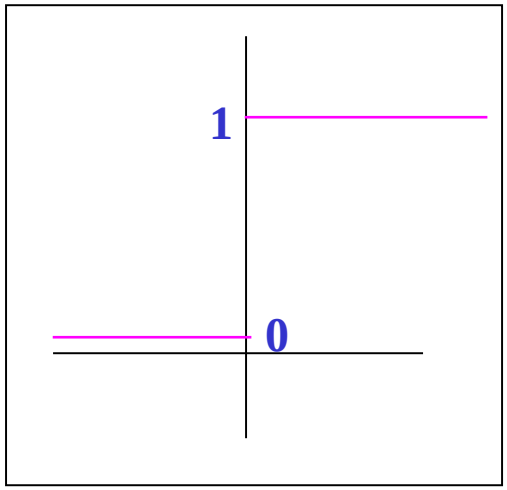
Tanh

$f(x) = (e^{x} – e^{-x}) / (e^{x} + e^{-x})$



Logistic

$f(x) = e^{x} / (1 + e^{x})$



Threshold

$$f(x) = \begin{cases} 0 \text{ if } x < 0 \\ 1 \text{ if } x >= 1 \end{cases}$$

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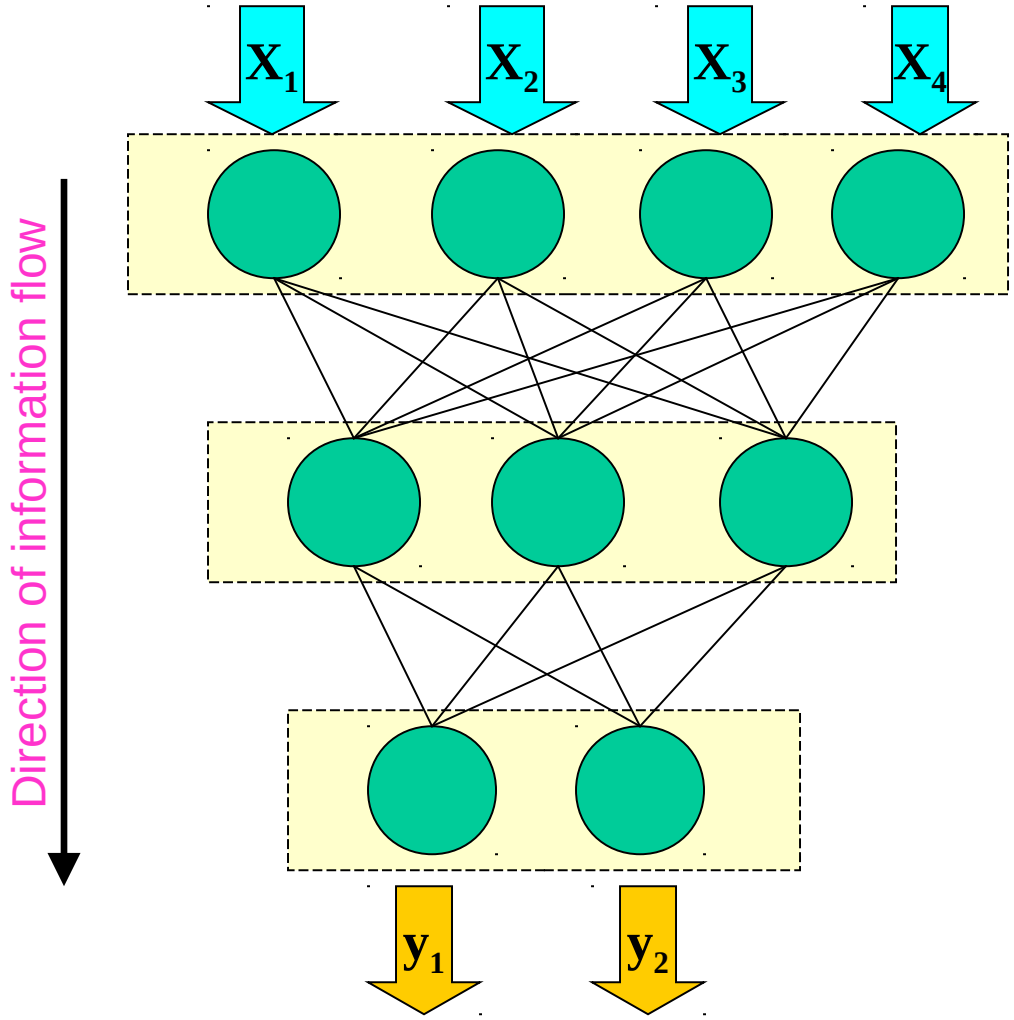
ANN – Feed-forward Network

A collection of neurons form a ‘Layer’



Input Layer

- Each neuron gets ONLY one input, directly from outside

Hidden Layer

- Connects Input and Output layers

Output Layer

- Output of each neuron directly goes to outside

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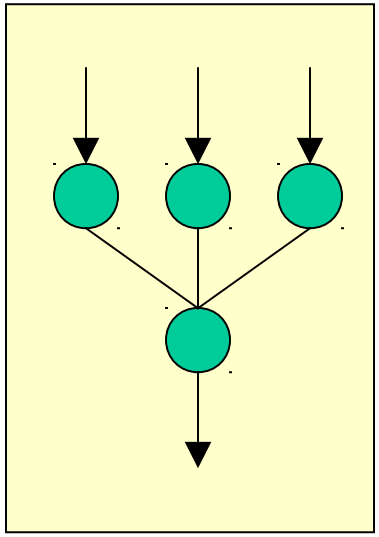
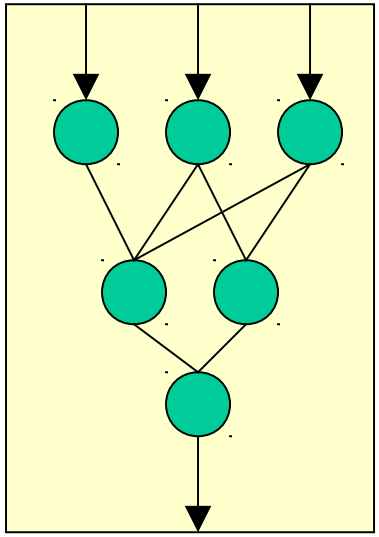
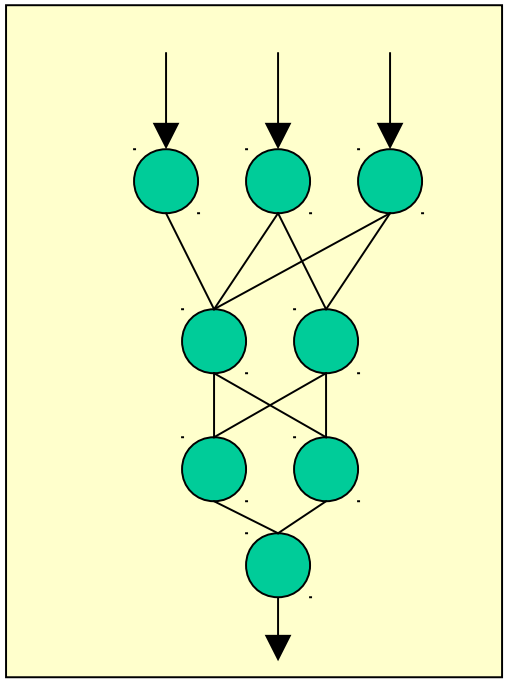
ANN – Feed-forward Network

Number of hidden layers can be **None** **One** **More**







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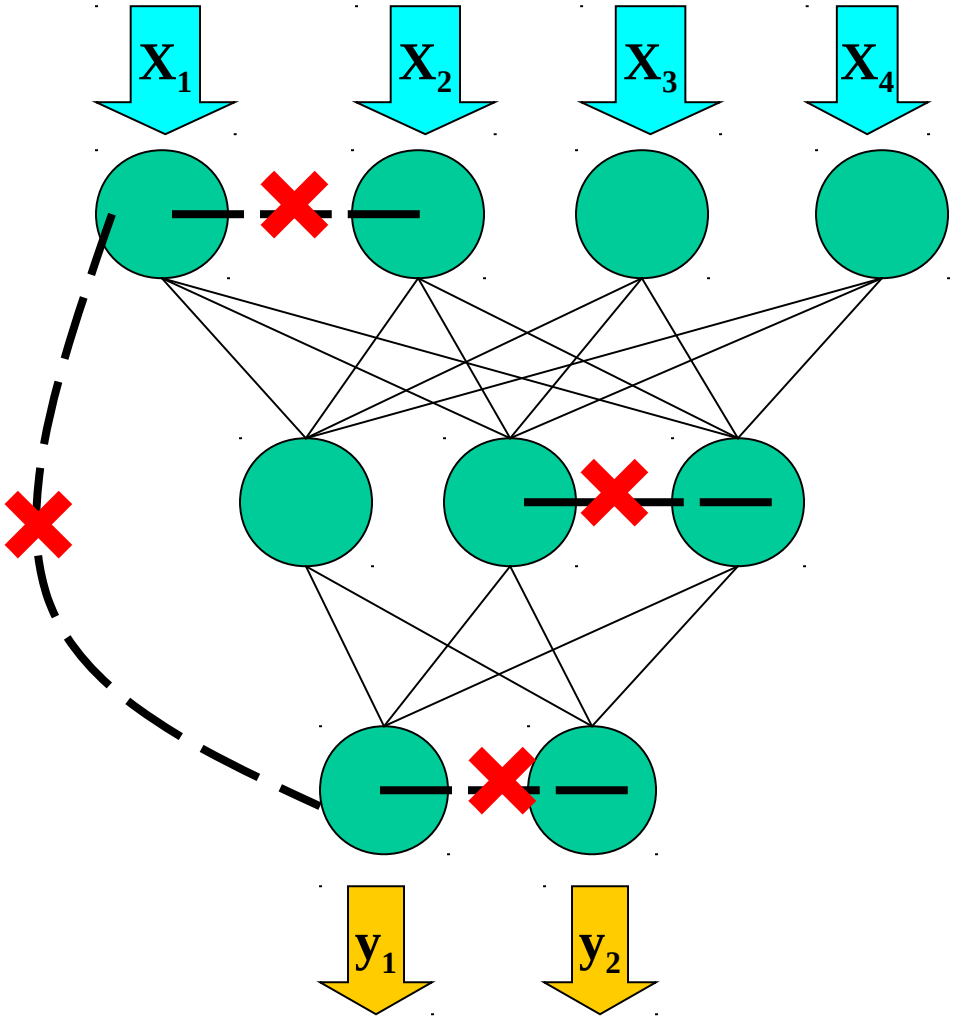
ANN – Feed-forward Network

Couple of things to note



- Within a layer neurons are NOT connected to each other.
- Neuron in one layer is connected to neurons ONLY in the NEXT layer. (Feed-forward)
- Jumping of layer is NOT allowed

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One particular ANN model

What do we mean by ‘ A particular Model ‘ ?

Input: X_1 X_2 X_3 Output: Y Model: $Y = f(X_1 X_2 X_3)$

For an ANN : Algebraic form of f(.) is too complicated to write down.

However it is characterized by

- # Input Neurons
- # Hidden Layers
- # Neurons in each Hidden Layer
- # Output Neurons
- WEIGHTS for all the connections

‘ Fitting ‘ an ANN model = Specifying values for all those parameters

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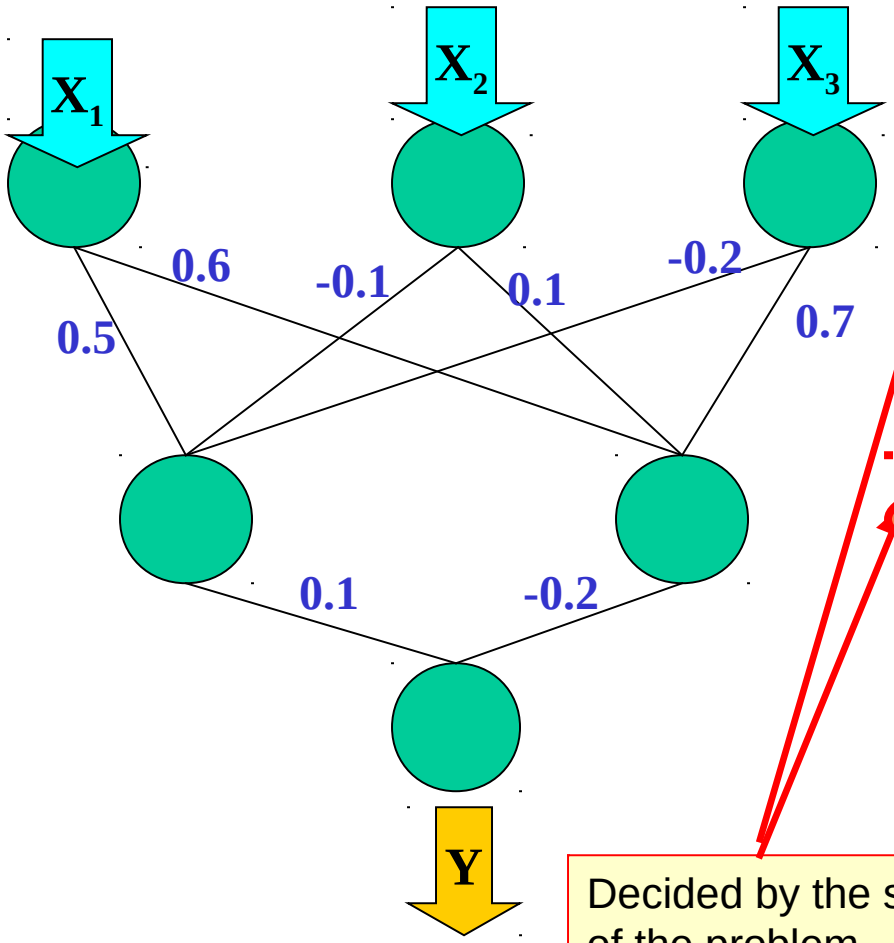
One particular Model – an Example

Input: X_1 X_2 X_3 **Output:** Y **Model:** $Y = f(X_1\ X_2\ X_3)$



Parameters	Example
# Input Neurons	3
# Hidden Layers	1
# Hidden Layer Size	3
# Output Neurons	3
Weights	Specified

Decided by the structure of the problem
Input Nrns = # of X's
Output Nrns = # of Y's

Free parameters

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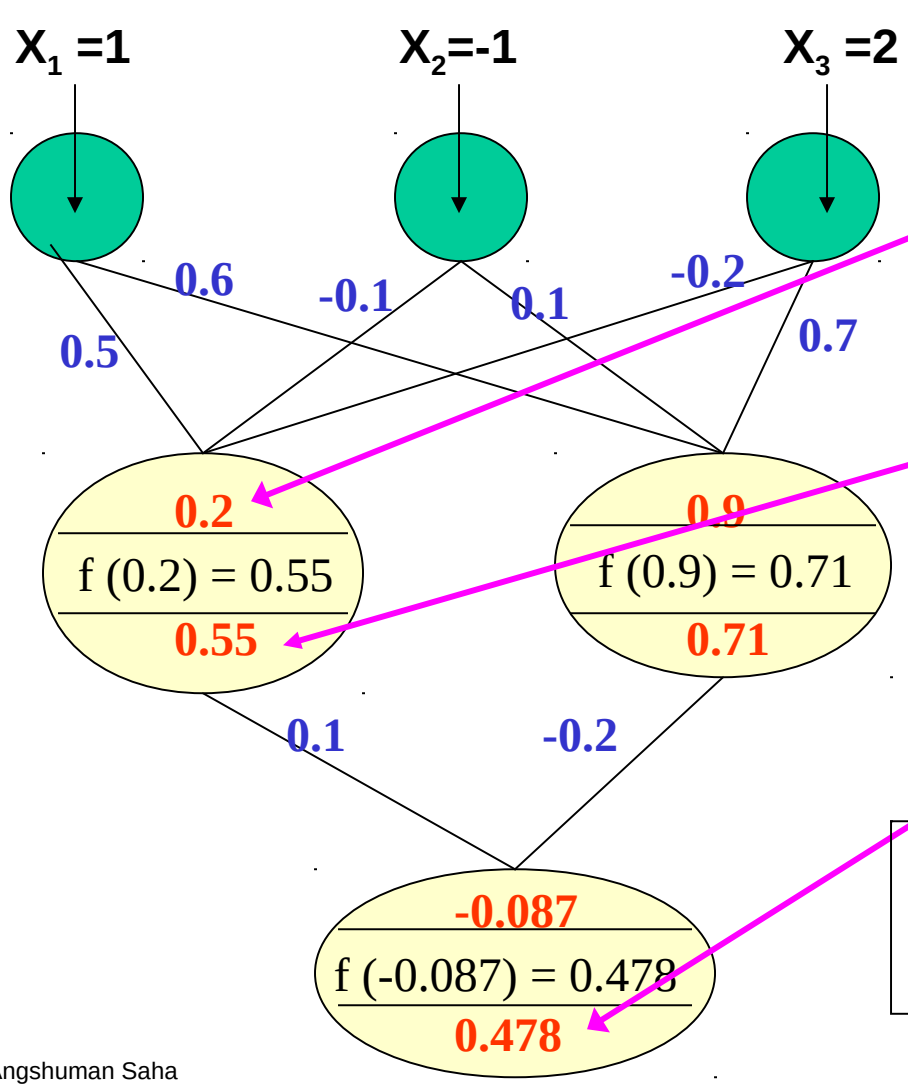
Prediction using a particular ANN Model

Input: X_1 X_2 X_3 **Output:** Y **Model:** $Y = f(X_1\ X_2\ X_3)$



$0.2 = 0.5 * 1 - 0.1*(-1) - 0.2 * 2$

$f(x) = e^x / (1 + e^x)$

$f(0.2) = e^{0.2} / (1 + e^{0.2}) = 0.55$

Predicted Y = 0.478

Suppose Actual Y = 2

Then

Prediction Error = (2-0.478) =1.522

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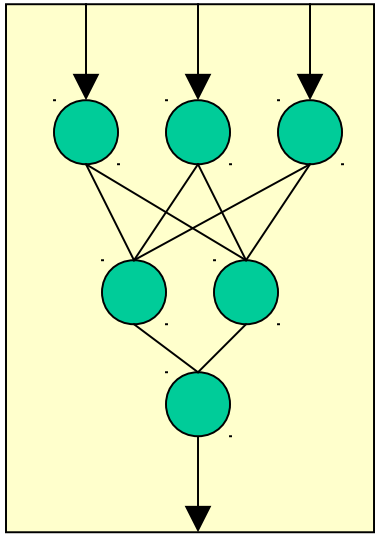
Building ANN Model

How to build the Model ?

Input: $X_1\ X_2\ X_3$ **Output:** Y **Model:** $Y = f(X_1\ X_2\ X_3)$

Input Neurons = # Inputs = **3** # Output Neurons = # Outputs = **1**

# Hidden Layer = ???	Try **1**	No fixed strategy.
# Neurons in Hidden Layer = ???	Try **2**	By trial and error



Architecture is now defined … How to get the weights ???

Given the Architecture There are 8 weights to decide.

$\underline{\mathbf{W}} = (W_1, W_2, \ldots, W_8)$

Training Data: (Yi , X1i, X2i, …, Xpi) i= 1,2,…,n

Given a particular choice of $\underline{\mathbf{W}}$, we will get predicted Y's

$(V_1, V_2, \ldots, V_n)$

They are *function* of $\underline{\mathbf{W}}$.

Choose $\underline{\mathbf{W}}$ such that over all prediction error ***E*** is minimized

$$E = \Sigma\ (Yi - Vi)^2$$

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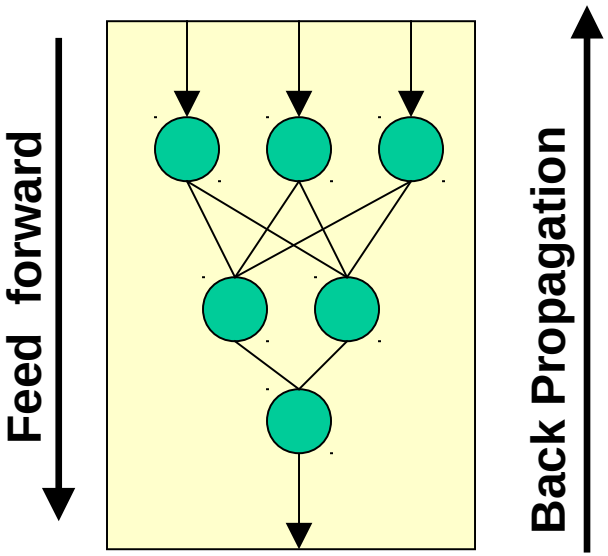
Training the Model

How to train the Model ?



$$E = \Sigma (Yi - Vi)^2$$

- Start with a random set of weights.
- Feed forward the first observation through the net
  $X_1 \rightarrow$ Network $\rightarrow$ V_1 ; Error $= (Y_1 - V_1)$
- Adjust the weights so that this error is reduced
  (network fits the first observation well)
- Feed forward the second observation.
  Adjust weights to fit the second observation well
- Keep repeating till you reach the last observation
- This finishes one CYCLE through the data
- Perform many such training cycles till the overall prediction error ***E*** is small.

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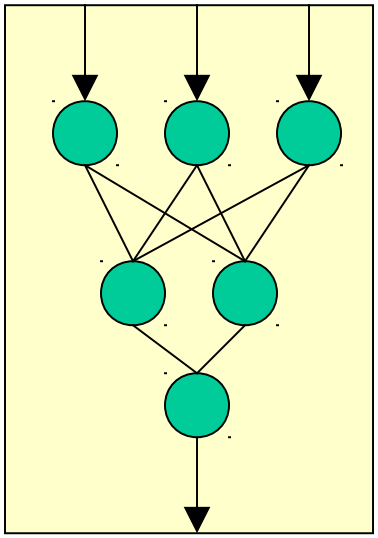
Back Propagation

Bit more detail on Back Propagation



Each weight ‘Shares the Blame’ for prediction error with other weights.

Back Propagation algorithm decides how to distribute the blame among all weights and adjust the weights accordingly.

Small portion of blame leads to small adjustment.
Large portion of the blame leads to large adjustment.

$$E = \Sigma (Yi - Vi)^2$$

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Weight adjustment during Back Propagation

Weight adjustment formula in Back Propagation

V_i – the prediction for *i*th observation –
is a function of the network weights vector $\underline{W} = (W_1, W_2, \ldots)$
Hence, E, the total prediction error is also a function of W

$$E(\underline{W}) = \Sigma [Yi - Vi(\underline{W})]^2$$

Gradient Descent Method :

For every individual weight W_i, updation formula looks like

$$W_{new} = W_{old} + \alpha * (\partial E / \partial W) |_{Wold}$$

α = Learning Parameter (between 0 and 1)

Another slight variation is also used sometimes

$$W_{(t+1)} = W_{(t)} + \alpha * (\partial E / \partial W) |_{W(t)} + \beta * (W_{(t)} - W_{(t-1)})$$

β = Momentum (between 0 and 1)

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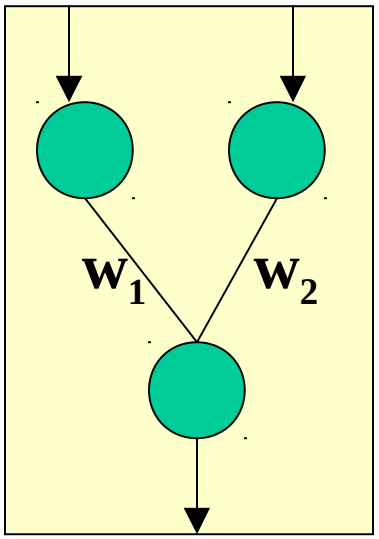
Geometric interpretation of the Weight adjustment

Consider a very simple network with 2 inputs and 1 output. No hidden layer. There are only two weights whose values needs to be specified.



$$E(w_1, w_2) = \Sigma [Yi - Vi(w_1, w_2)]^2$$

- A pair (w_1, w_2) is a point on 2-D plane.
- For any such point we can get a value of E.
- Plot E vs (w_1, w_2) - a 3-D surface - '**Error Surface**'
- Aim is to identify that pair for which E is minimum
- That means – identify the pair for which the height of the error surface is minimum.

Gradient Descent Algorithm

- Start with a random point (w_1, w_2)
- Move to a 'better' point (w'_1, w'_2) where the height of error surface is lower.
- Keep moving till you reach (w^*_1, w^*_2), where the error is minimum.

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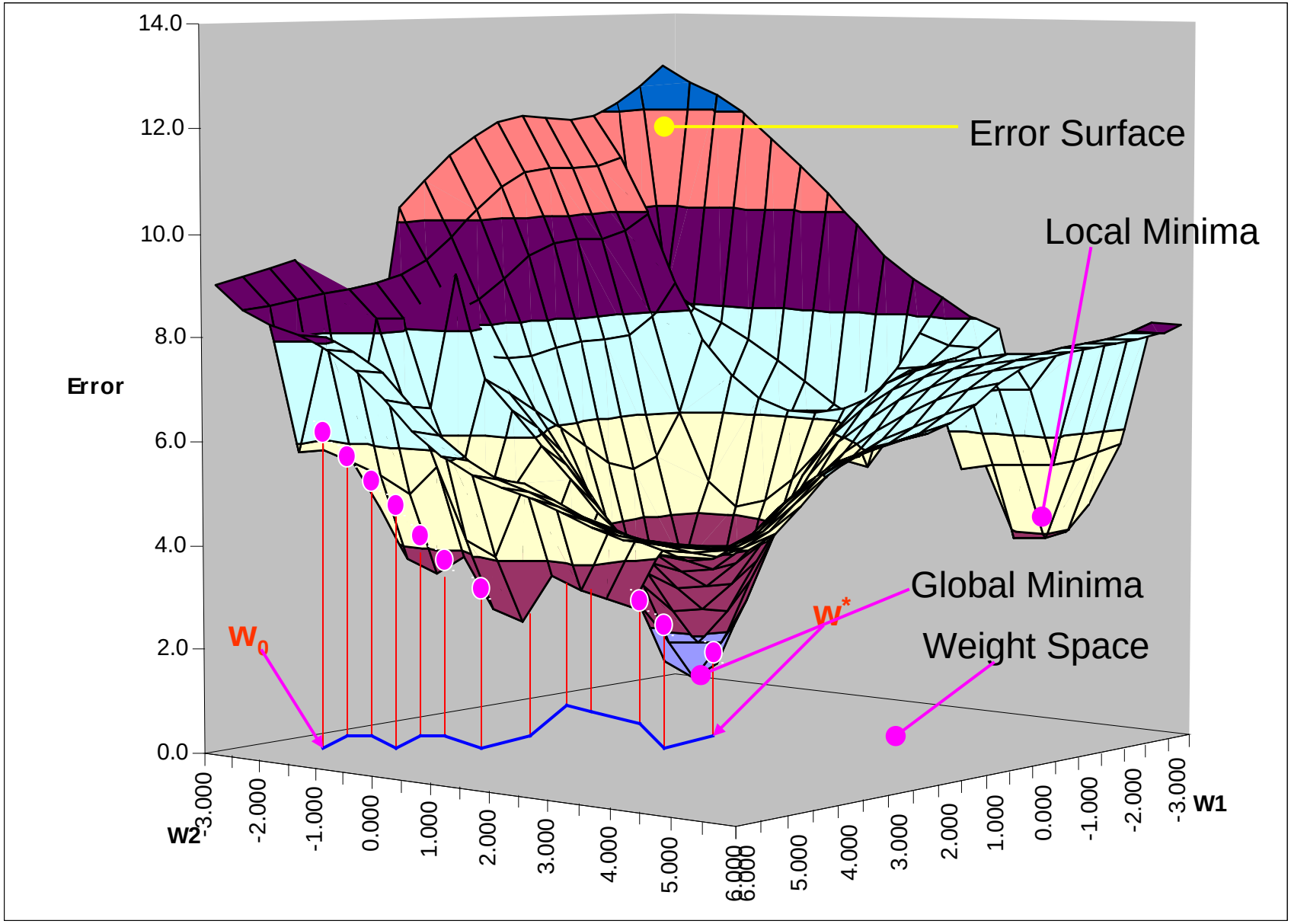
Crawling the Error Surface



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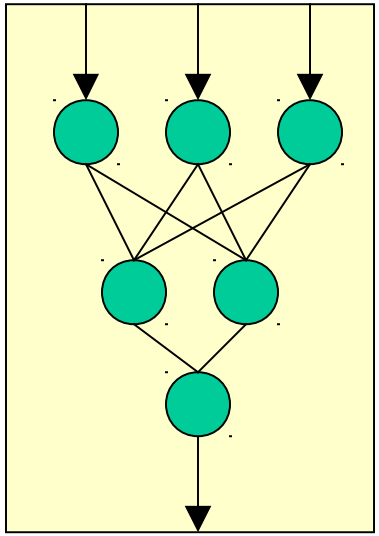
Training Algorithm



$$E = \Sigma (Yi - Vi)^2$$

Decide the Network architecture
(# Hidden layers, #Neurons in each Hidden Layer)

Decide the Learning parameter and Momentum

Initialize the Network with random weights

- Do till Convergence criterion is not met
  - For I = 1 to # Training Data points
    - Feed forward the I-th observation thru the Net
      Compute the prediction error on I-th observation
    - Back propagate the error and adjust weights
  - Next I
  - Check for Convergence
- End Do

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Convergence Criterion

When to stop training the Network ?

Ideally – when we reach the global minima of the error surface

How do we know we have reached there ? We don't …

Suggestion:

1. Stop if the decrease in total prediction error (since last cycle) is *small*.
2. Stop if the overall changes in the weights (since last cycle) are *small*.

Drawback:

Error keeps on decreasing. We get a very good fit to training data.
BUT … The network thus obtained have poor generalizing power on unseen data

The phenomenon is also known as - **Over fitting** of the Training data
The network is said to **Memorize** the training data.

- so that when an X in training set is given,
the network faithfully produces the corresponding Y.

-However for X's which the network didn't see before, it predicts poorly.

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Convergence Criterion

Modified Suggestion:

Partition the training data into **Training set** and **Validation set**

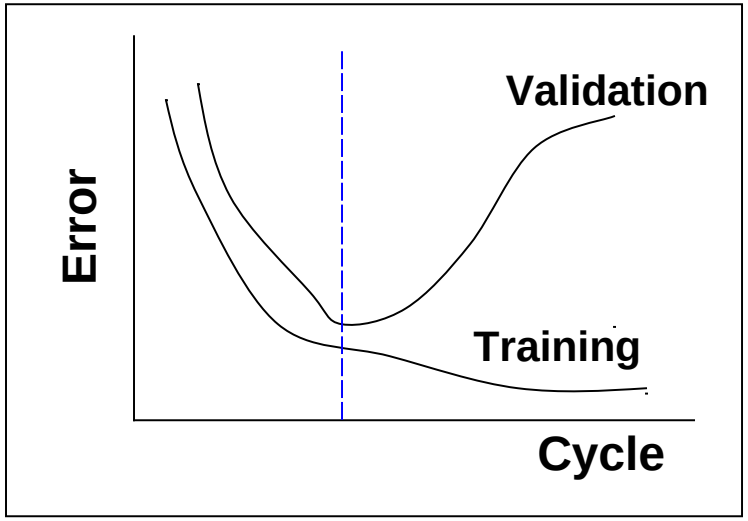
Use Training set - build the model

Validation set - test the performance of the model on unseen data

Typically as we have more and more training cycles

Error on Training set keeps on decreasing.

Error on Validation set keeps first decreases and then increases.



Stop training when the error on Validation set starts increasing

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Choice of Training Parameters

Learning Parameter and Momentum

- needs to be supplied by user from outside. Should be between 0 and 1

What should be the optimal values of these training parameters ?

- No clear consensus on any fixed strategy.
- However, effects of wrongly specifying them are well studied.

Learning Parameter

Too big – Large leaps in weight space – risk of missing global minima.

Too small –

- Takes long time to converge to global minima
- Once stuck in local minima, difficult to get out of it.

<u>Suggestion</u>

Trial and Error – Try various choices of Learning Parameter and Momentum

See which choice leads to minimum prediction error

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Wrap Up

- Artificial Neural network (ANN)
  - A class of models inspired by biological Neurons
- Used for various modeling problems – Prediction, Classification, Clustering, ..
- One particular subclass of ANN's – Feed forward Back propagation networks
  - Organized in layers – Input, hidden, Output
  - Each layer is a collection of a number of artificial Neurons
  - Neurons in one layer in connected to neurons in next layer
  - Connections have weights
- Fitting an ANN model is to find the values of these weights.
- Given a training data set – weights are found by Feed forward Back propagation algorithm, which is a form of Gradient Descent Method – a popular technique for function minimization.
- Network architecture as well as the training parameters are decided upon by trial and error. Try various choices and pick the one that gives lowest prediction error.

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