Jul – Dec 2022
IT458

Information Retrieval on the Web

Link Analysis and Web Search

Link Analysis Algorithms

- based on analysis of links for computing importance of nodes in a Web graph :
  - Voting based algorithms
  - Link importance based algorithms
  - Learning based algorithms

Link Analysis Algorithms

- Voting based algorithms
  - Hubs and Authorities -- Hypertext Induced Topical Search (HITS)
- Link importance based algorithms
  - PageRank
  - Topic-Specific (Personalized) PageRank
- Learning based algorithms
  - Learning to Rank (LtR)
  - Learning the Ranking function (LtRf)

Voting based Ranking Algorithms

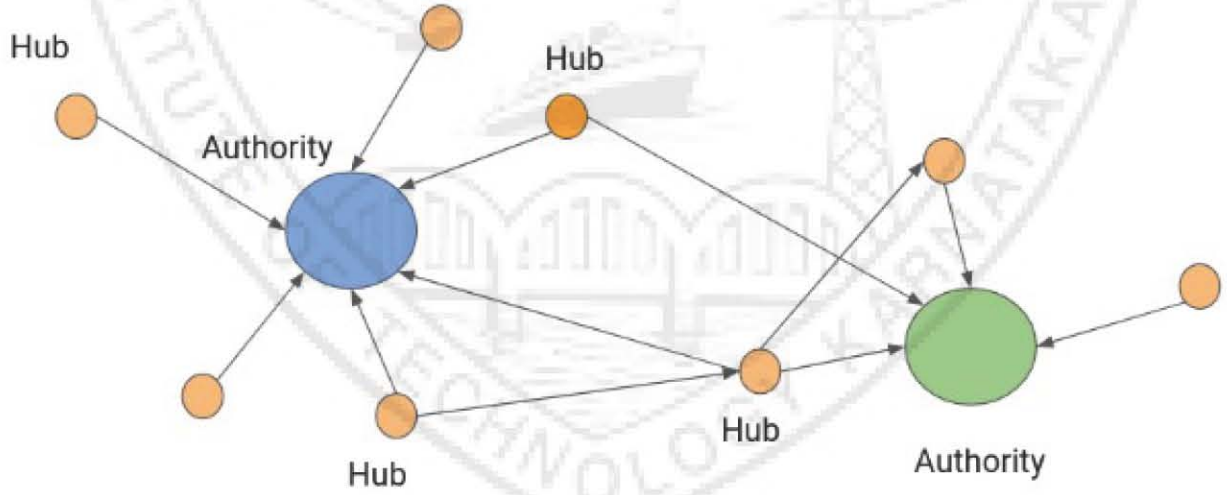
Hubs and Authorities

- Goal :
  - To find the "experts" – pages that link in a coordinated way to "good" content.

- Idea: Links as votes
  - Page is more important if it has more links !!

Hubs and Authorities

- Interesting pages fall into two classes:
  1. **Authorities**: pages containing useful information
     - E.g Newspaper home pages, Course home pages .....
  2. **Hubs**: pages that link to authorities
     - E.g. List of newspapers, University course listings…



HITS Algorithm

- Hypertext-Induced Topical Search
  - Core of early search engines like Altavista, Ask.com etc

- **The Principle of Repeated Improvement**
  - A good hub links to many good authorities
  - A good authority is linked from many good hubs

HITS Algorithm

- Model uses two scores for each node: **Hub score and Authority score**
  - Represented as vectors ***hub(p)*** and ***auth(p)***, where the p^{th} element is the hub/authority score of the p^{th} node
  - Both initially set to 1.

HITS Algorithm

- Voting phases -
  - ***Authority Update Rule:*** For each page p, **update *auth(p)*** to be the **sum of the hub scores** of all pages that point to it.
  - ***Hub Update Rule:*** For each page p, **update *hub(p)*** to be the **sum of the authority scores** of all pages that it points to.

HITS Algorithm - Steps

- Start: hub and authority scores = 1

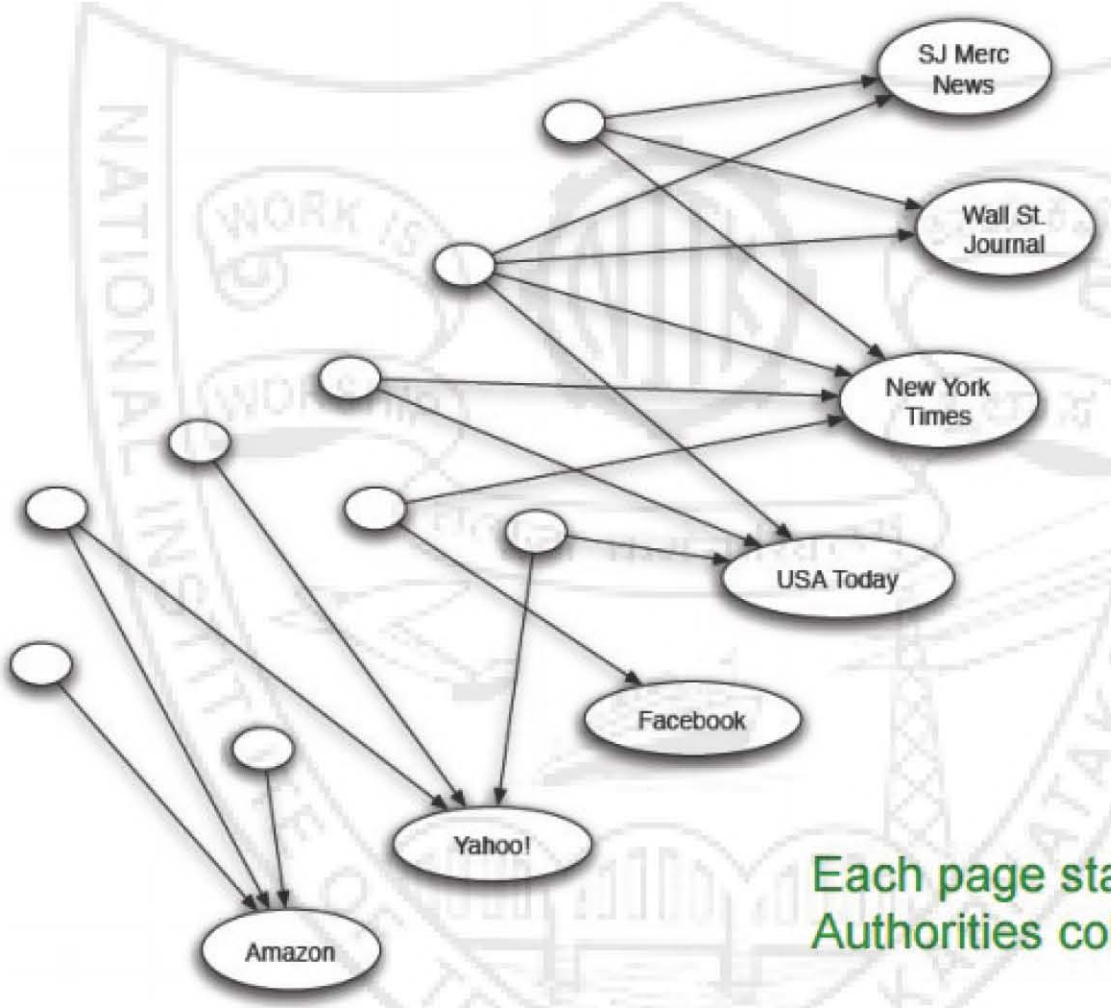
  choose the number of iterative steps k.

- Perform a sequence of k hub-authority updates.
  1. Apply the Authority Update Rule to the current set of scores.
  2. Then, apply the Hub Update Rule to the resulting set of scores.

- After k iterations, normalize all resulting scores.
  1. Divide each authority score by the sum of all authority scores.
  2. Divide each hub score by the sum of all hub scores.

- Repeat from the second step as necessary.

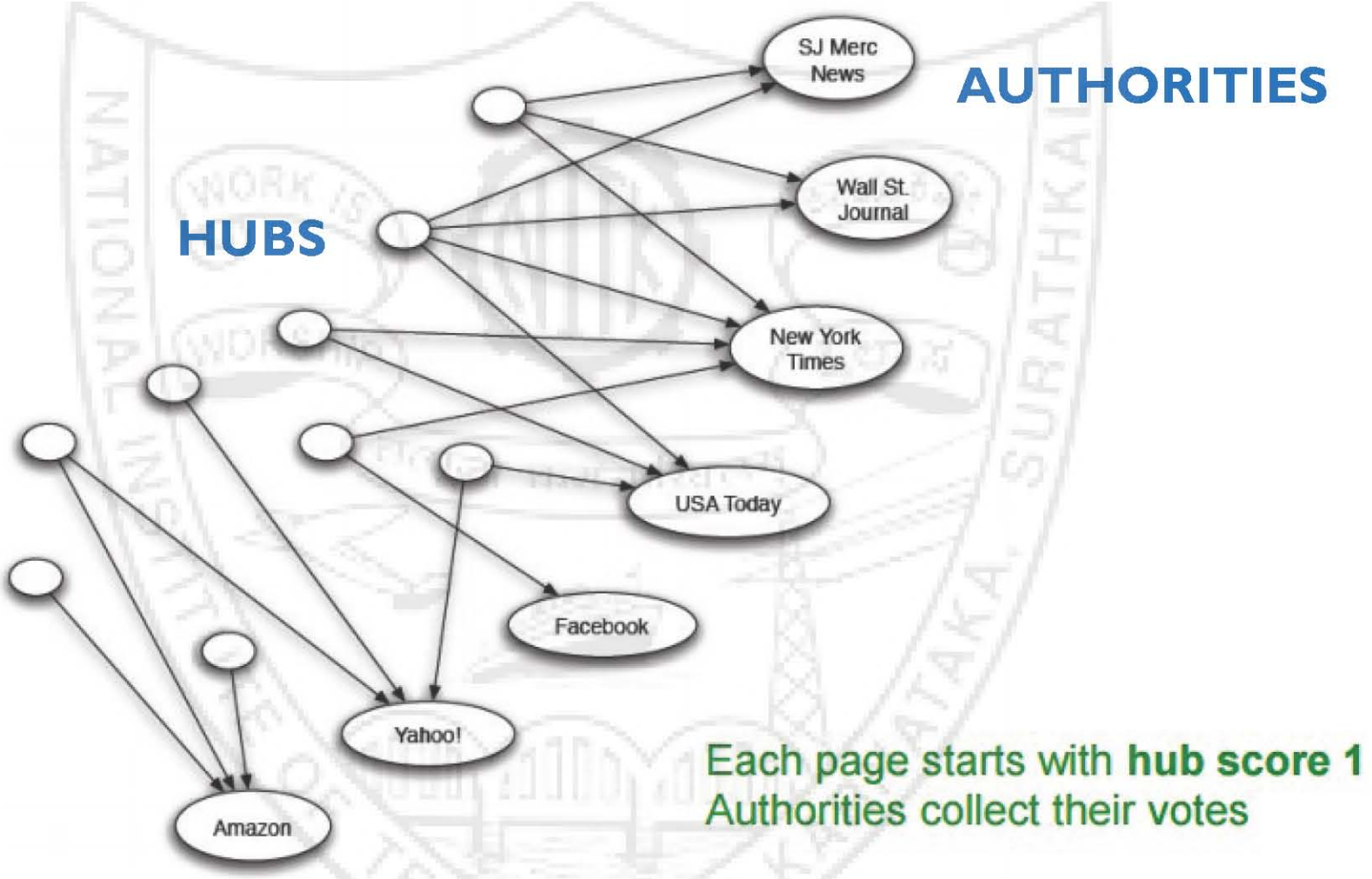
Counting in-links: Authority



Each page starts with **hub score 1**
Authorities collect their votes

(Note this is idealized example. In reality graph is not bipartite and each page has both the hub and authority score)

Counting in-links: Authority



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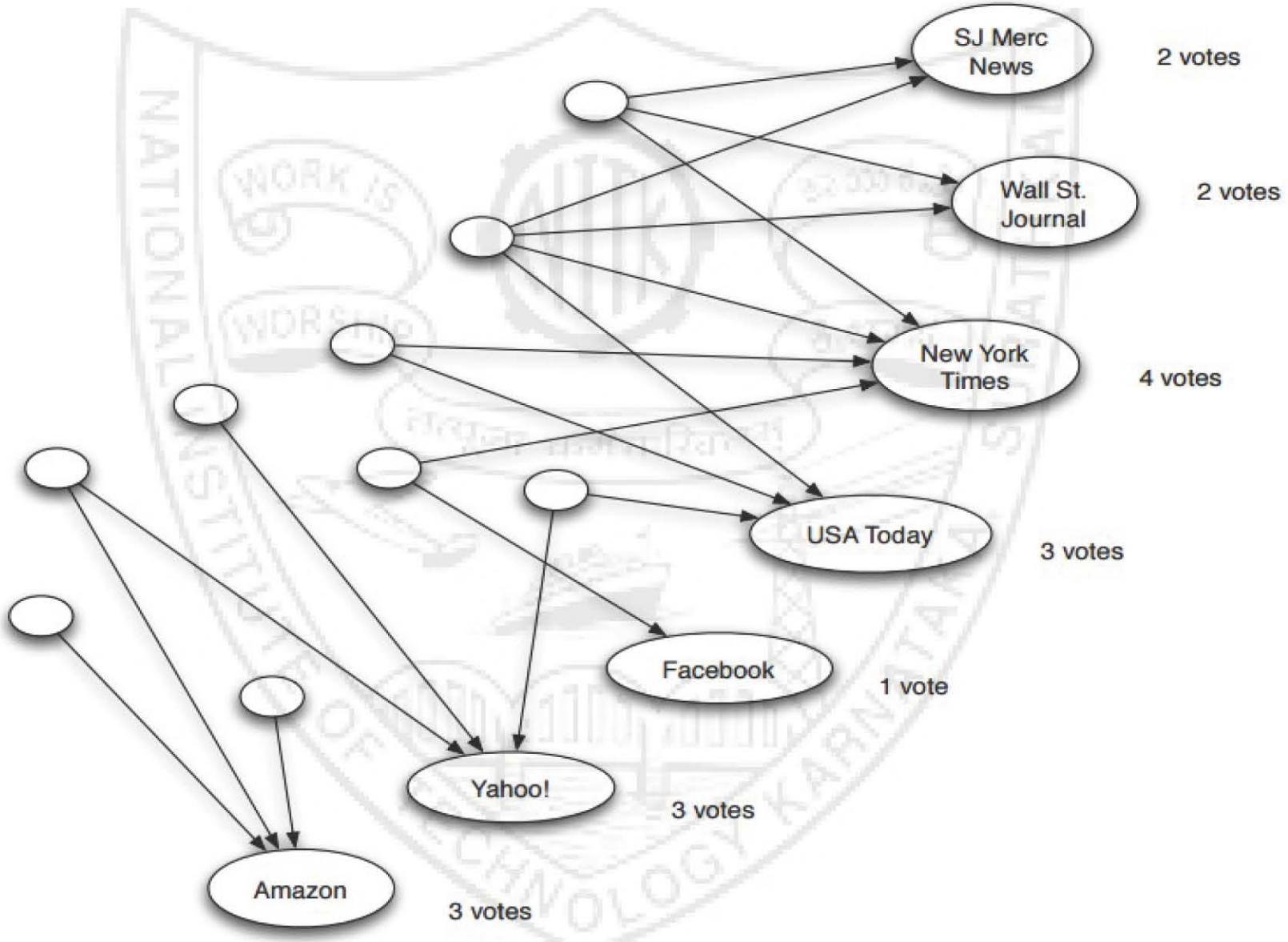
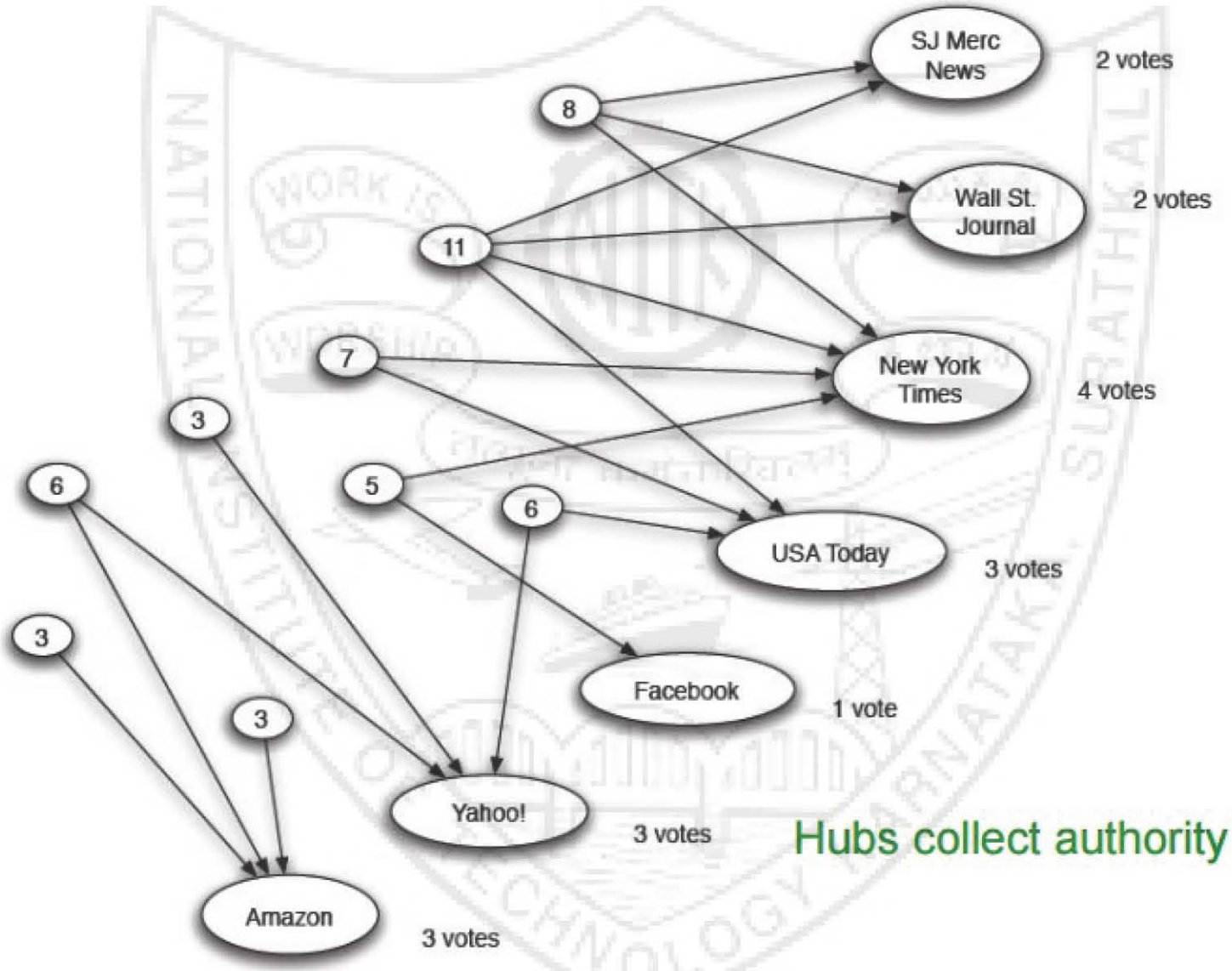
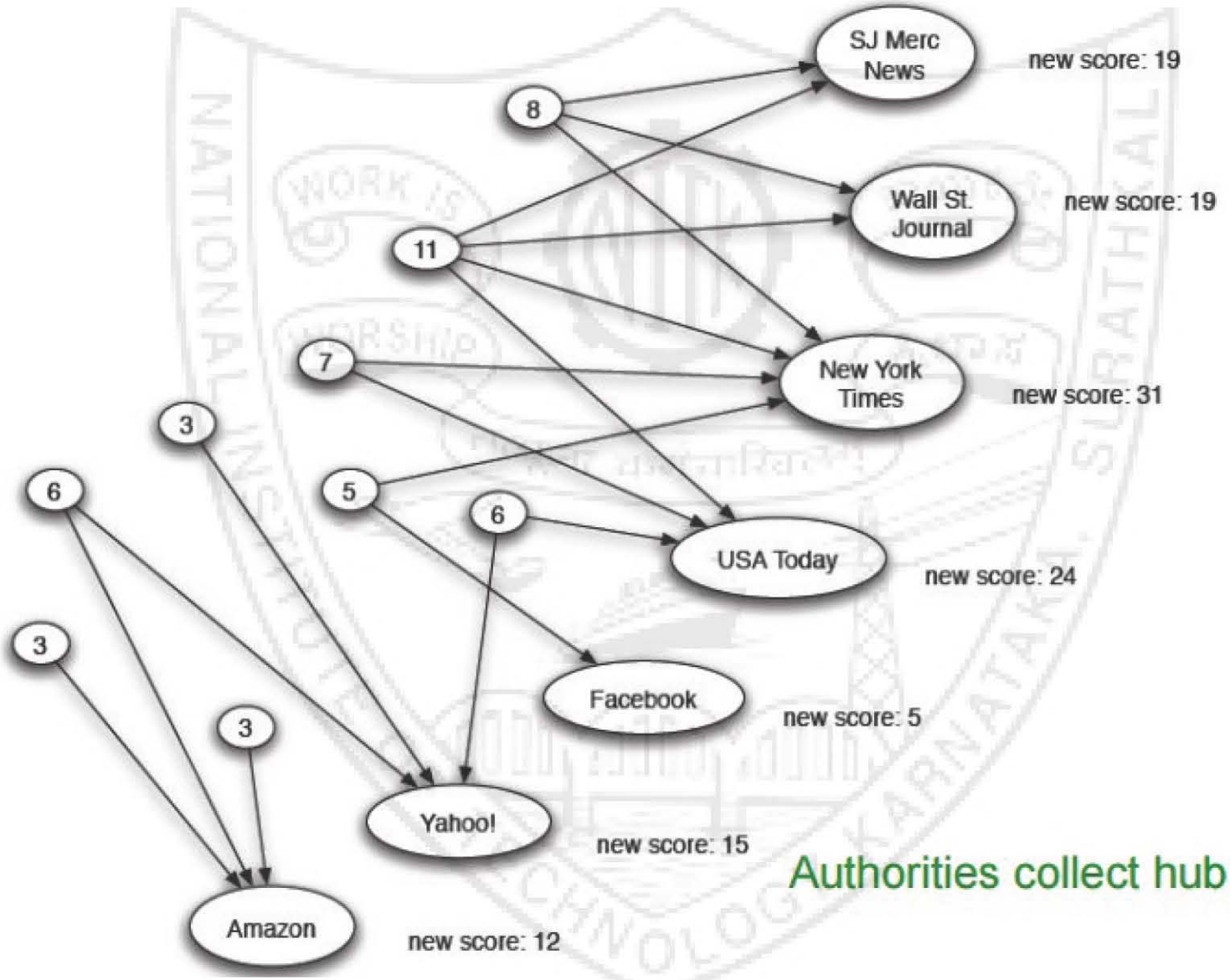
Votes by other hubs: Authority



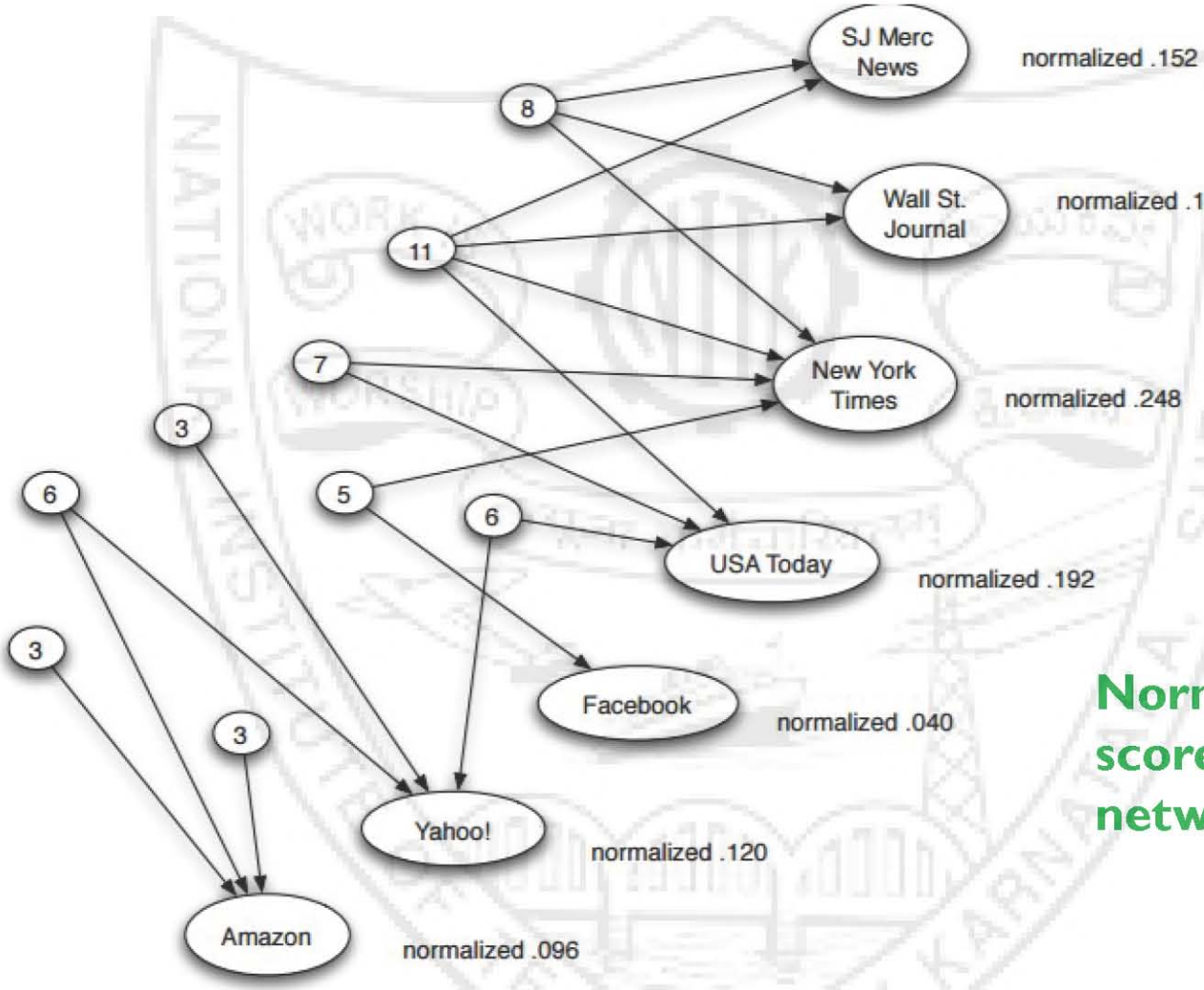
Expert Quality: Hub



Reweighting



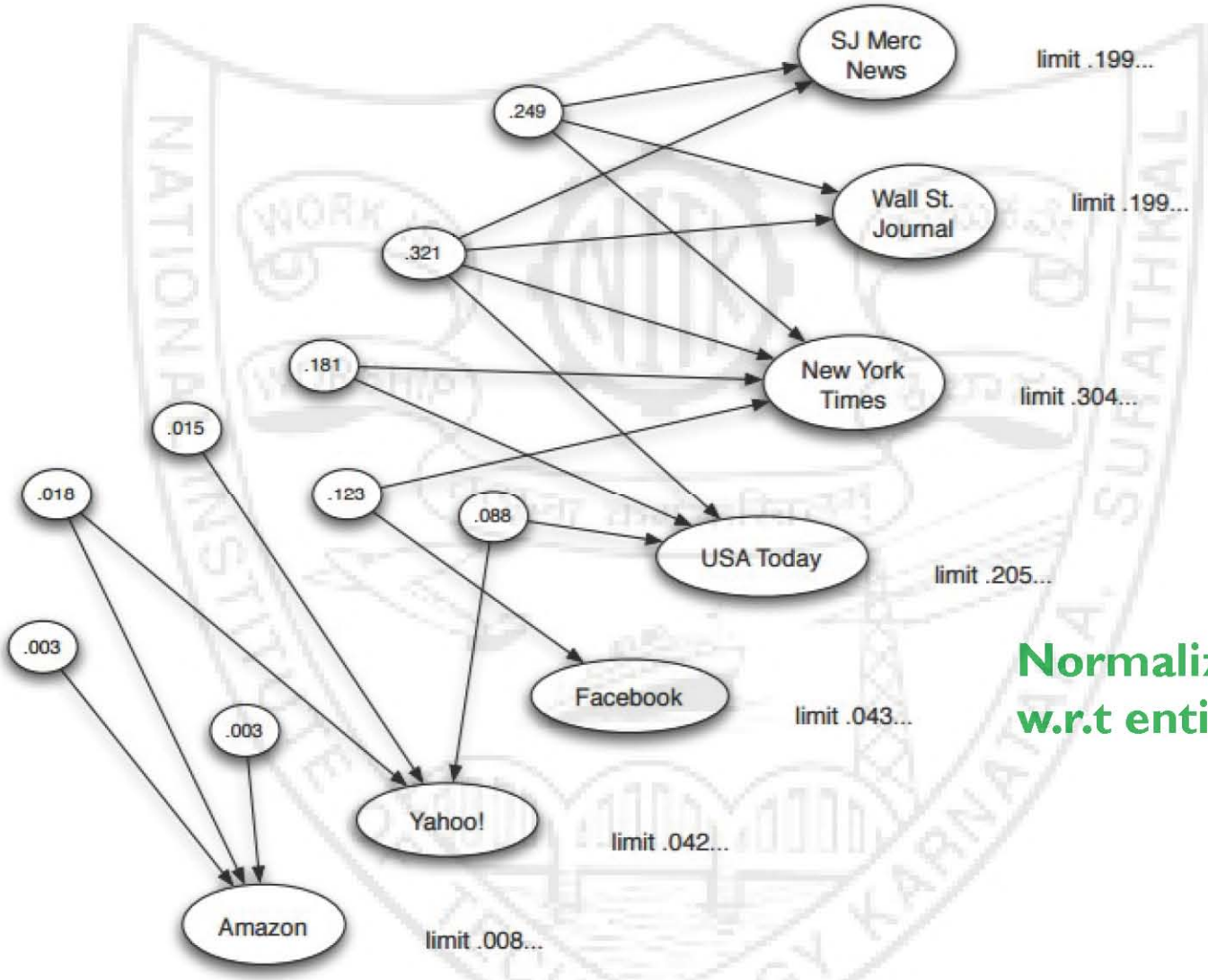
Normalizing Authority score



Normalizing authority scores w.r.t entire network

Divide each authority score by the sum of all authority scores

Normalizing Hub score



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HITS - Summary

- Merits:
  - Simple way of calculating relative importance of connected pages
- Demerits:
  - it is executed at query time, not at indexing time
    - High latency due to query-time processing.
  - processed on a small subset of 'relevant' documents, not all documents
    - PageRank addresses this issue.

Link Importance based Ranking Algorithms

Dr. Sowmya Kamath S, Dept of IT, NITK Surathkal

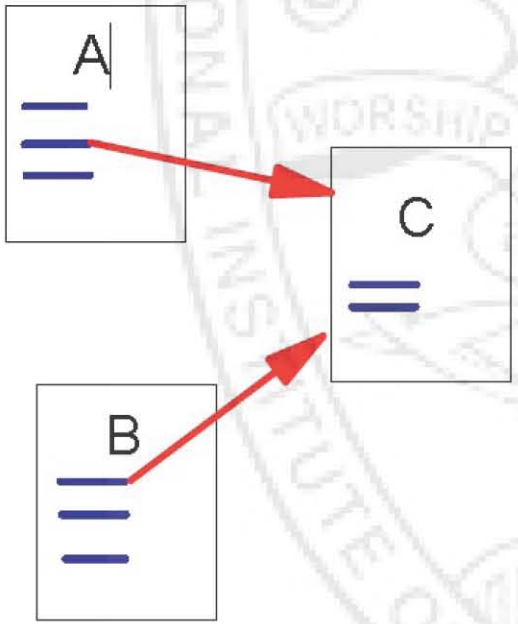
PageRank

- PageRank
  - rates the importance of **web pages** objectively and continuously using the **link structure** of the web.

- developed by Larry Page (hence the name *Page*-Rank) and Sergey Brin.
  - used by the Google Internet search engine.
  - Patent issued to Stanford University!

PageRank Algorithm

- Exploits the link structure of the Web.
- *Stats*: More than 1.98 billion websites on the Internet → Trillions of links.



Back (incoming) links and *Forward (outgoing)* links:

- A and B are C's incoming links
- C is A and B's outgoing link

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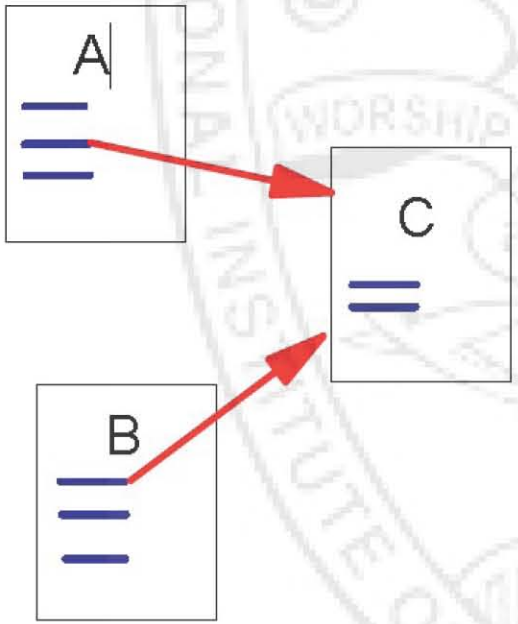


Back (incoming) links and *Forward (outgoing)* links:

- A and B are C's incoming links
- C is A and B's outgoing link

* Intuitively, a webpage is important if it has a lot of incoming links, that keep changing over time.

PageRank Algorithm (contd.)

- Salient features –
  - interprets a link from page A to page B as a vote by page A for page B.
  - All votes don't weigh the same.
  - does not rank the whole website.
  - PR of page A is a recursive function defined by the PR of all those pages which link to page A.

PageRank's Random Surfer Model

PageRank can be thought of as a model of user behavior.

- *"We assume there is a "random surfer" who when given a web page at random, keeps clicking on links, never hitting "back" but eventually gets bored and starts on another random page. The probability that the random surfer visits a page is its* ***PageRank****."*

- *"And, the* ***damping factor d*** *(teleportation probability) is the probability the "random surfer" will get bored and request another random page."*

Brin, Sergey, and Lawrence Page. "The anatomy of a large-scale hypertextual web search engine." *Computer networks and ISDN systems* 30.1-7 (1998): 107-117.

The original PageRank™ algorithm

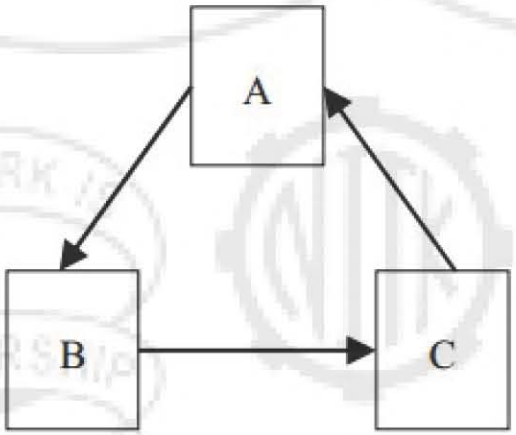
The PageRank (PR) of a page *u* is given as:

$$PR(u) = (1-d) \times (1/N) + d \times \sum (PR(v)/C(v))$$

- Assume page *u* has set of pages *v* which point to it (i.e., are votes).
  - PR(u) - PageRank of page *u*,
  - PR(v) - PageRank of the set of pages *v* that link to page *u*,
  - C(*v*) - number of links going out of pages *v*.
  - *N* - number of pages in the network.
  - *d* - damping factor which can be set between 0 and 1.
    - d is set to 0.7 - 0.85, if not otherwise specified.

Calculating a Webpage's PageRank (PR)
Examples

Consider an example:



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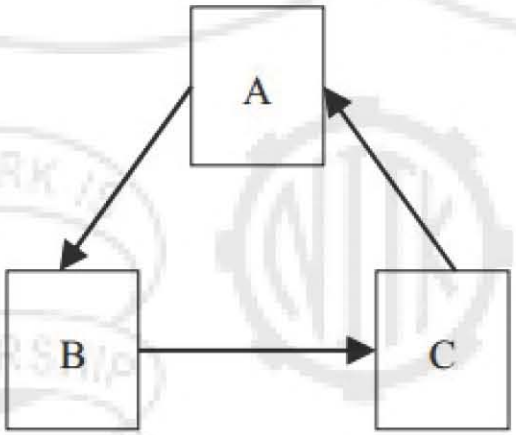

The number of web pages N = 3 ;

Let the damping parameter be d = 0.7

Calculating a Webpage's PageRank (PR)
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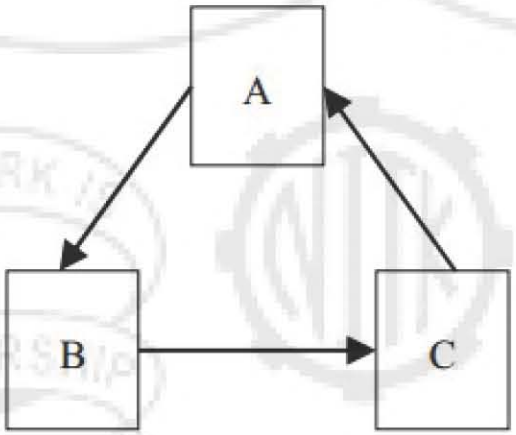


The number of web pages N = 3 ;

Let the damping parameter be d = 0.7

PageRank PR(u) = (1 - d) × (1/N) + d × ∑ (PR(v)/C(v))

Where u can be A, B or C.

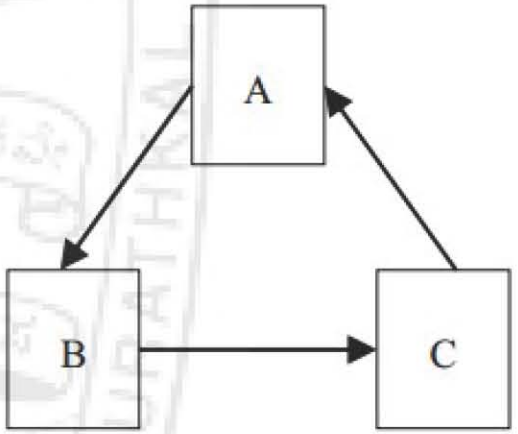
Calculating a Webpage's PageRank (PR)

So,

$$PR(A) = (1 - d) \times (1/N) + d \times (PR(C) / 1)$$

$$PR(B) = (1 - d) \times (1/N) + d \times (PR(A) / 1)$$

$$PR(C) = (1 - d) \times (1/N) + d \times (PR(B) / 1)$$



Substituting value of d and N ,

$$PR(A) = 0.1 + 0.7 \times PR(C) \quad \text{---- (i)}$$

$$PR(B) = 0.1 + 0.7 \times PR(A) \quad \text{---- (ii)}$$

$$PR(C) = 0.1 + 0.7 \times PR(B) \quad \text{---- (iii)}$$

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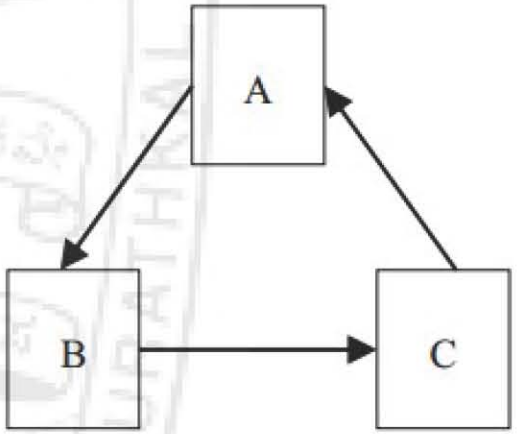

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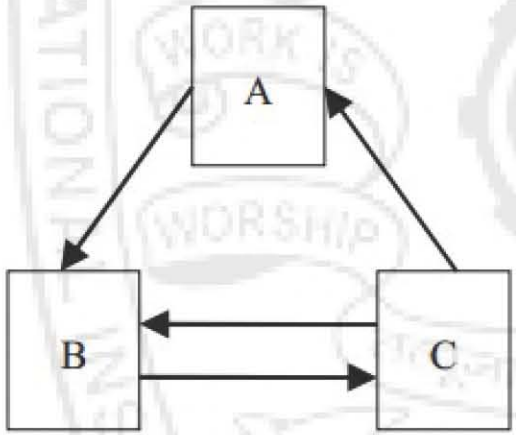
By solving the resulting system of linear equations, we get

PR(A) = 1/3 = 0.33 **PR(B) = 1/3 = 0.33** **PR(C) = 1/3 = 0.33**

Calculating a Webpage's PageRank (PR)

Consider another example:



Substituting value of d and N, and by solving the resulting systems of linear equations, we get

PR(A) = **PR(B) =** **PR(C) =**

Calculating a Webpage's PageRank (PR)

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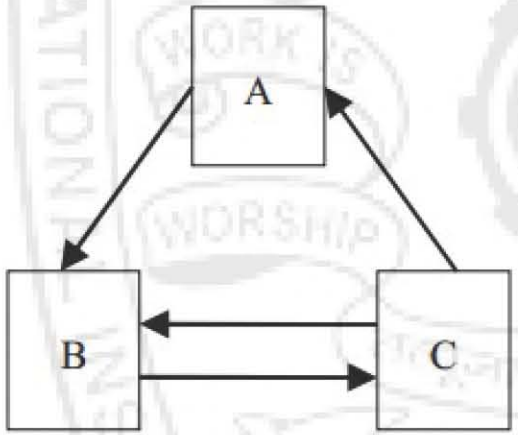

Substituting value of d and N, and by solving the resulting systems of linear equations, we get

PR(A) = 0.2314 **PR(B) = 0.3933** **PR(C) = 0.3753**

Calculating a Webpage's PageRank (PR)

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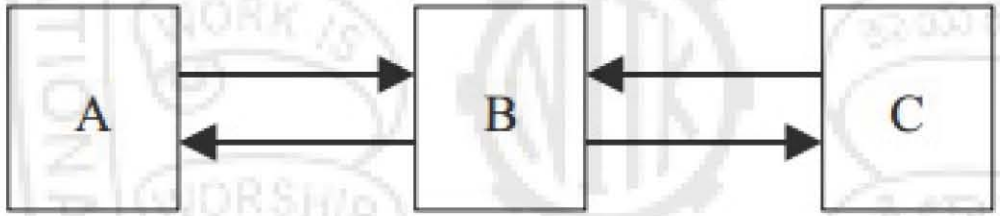

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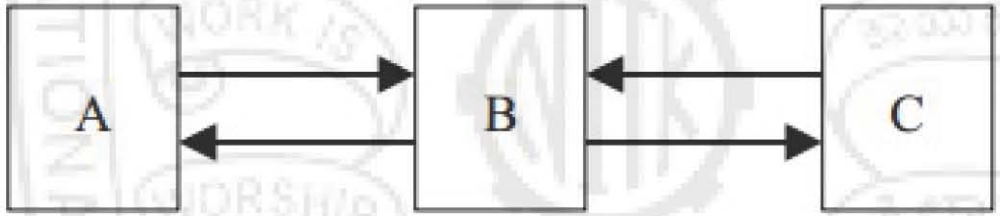
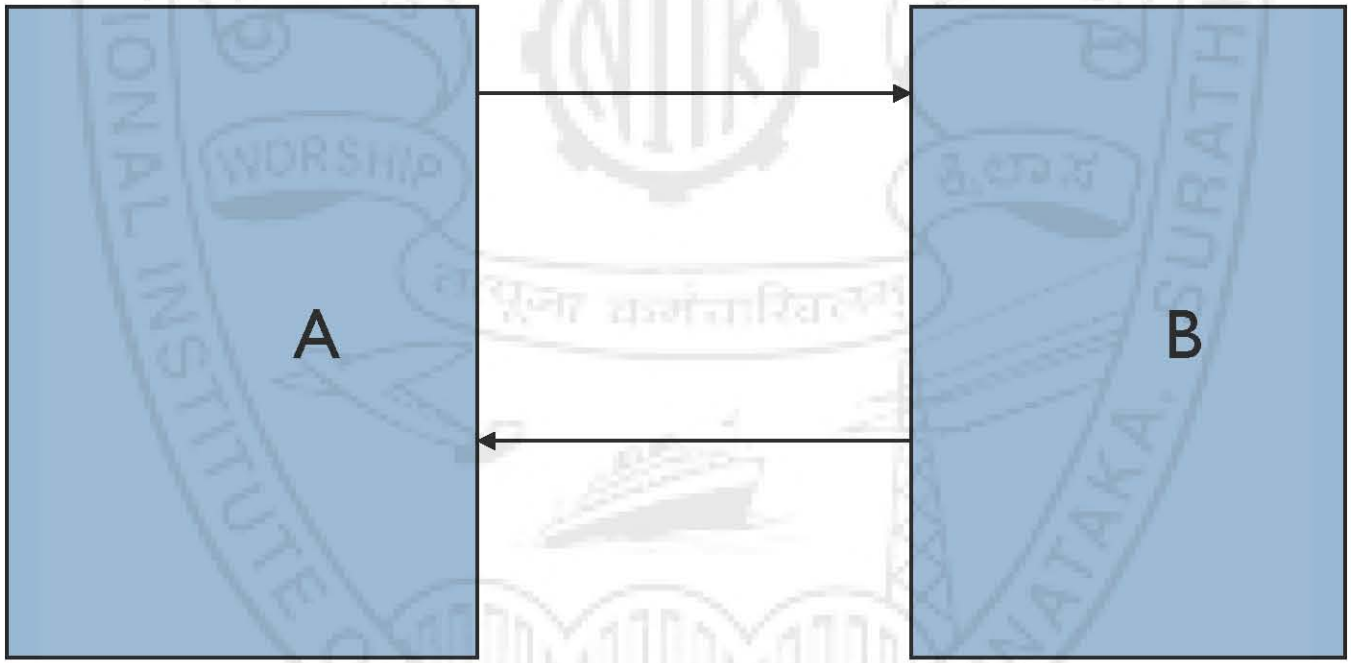


Substituting value of d and N and by solving the resulting systems of linear equations, we get

PR(A) = 0.2647 **PR(B) = 0.4706** **PR(C) = 0.2647**

Calculating a Webpage's PageRank (PR)
Iterative Computation

Consider two pages A and B, pointing to each other.



Calculating a Webpage's PageRank (PR)

Iterative Computation

Lets start with $PR(A) = PR(B) = 10$

1st iteration:

$$PR(A) = (1\text{-}d)*1/N + d*(PR(B)/C(B))$$
$$= 0.15*0.5 + 0.85 * (10/1)$$
$$= 8.58$$
$$PR(B) = (1\text{-}d) *1/N + d*(PR(A)/C(A))$$
$$= 0.15*0.5 + 0.85 * (8.58/1)$$
$$= 7.36$$

Calculating a Webpage's PageRank (PR)

Iterative Computation

After 2nd iteration:

$$PR(A) = (1\text{-}d)*1/N + d*(PR(B)/C(B))$$
$$= 0.15*0.5 + 0.85 * (7.36/1)$$
$$= 6.331$$

$$PR(B) = (1\text{-}d)*1/N + d*(PR(A)/C(A))$$
$$= 0.15*0.5 + 0.85 * (6.331/1)$$
$$= 5.456$$

And so on….. till?

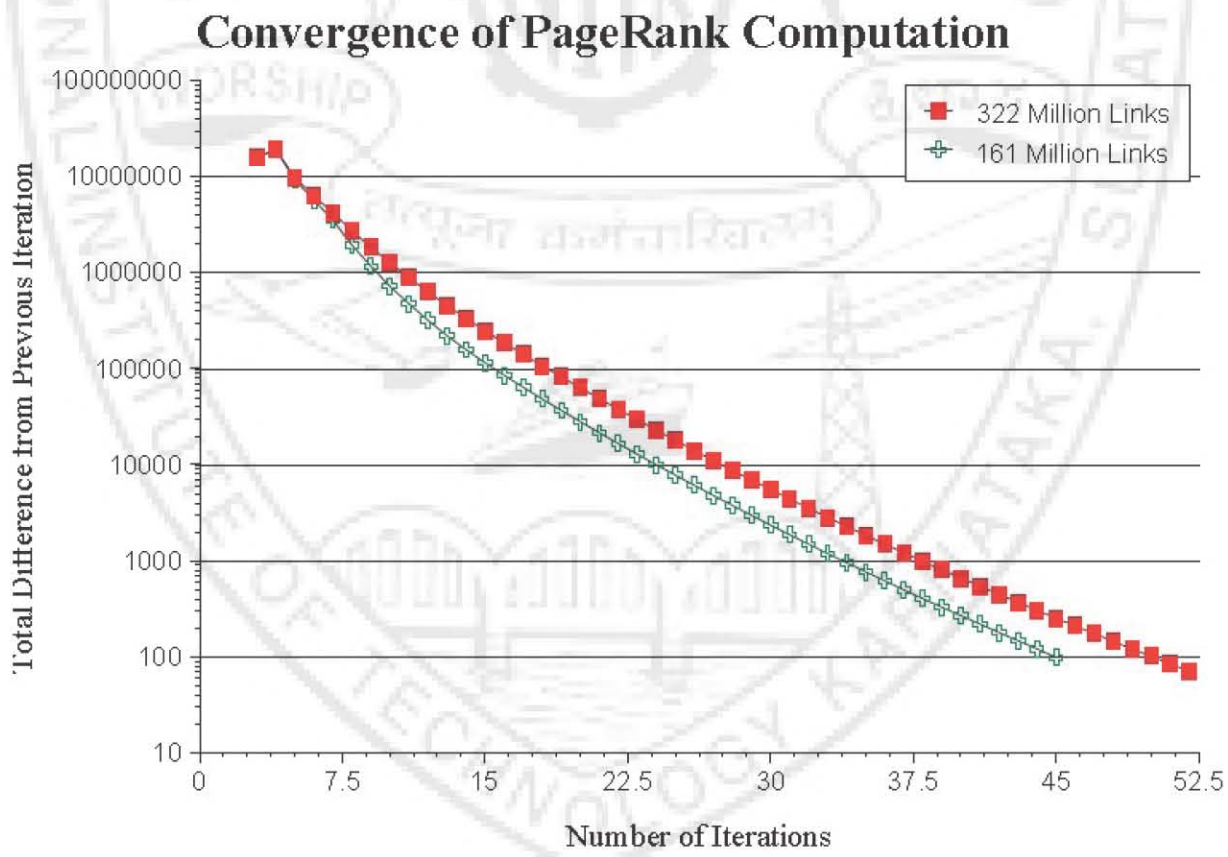
Calculating a Webpage's PageRank (PR)
Iterative Computation

→ Iterations should be repeated till PR values converge…

i.e., start with any value of PR, and repeat iterations till PR values converge.

PageRank: Convergence Behavior

- PR (322 Million Links): 52 iterations
- PR (161 Million Links): 45 iterations
  - Scaling factor is roughly linear !



Personalised PageRank

- Similar to PageRank, but more biased towards user's preferences.
- Instead of teleporting uniformly to any page, the jump is "biased" to prefer some specific topics/pages over others.

$$PR(P) = dv + (1-d)\left(\frac{PR(W_1)}{O(W_1)} + \frac{PR(W_2)}{O(W_2)} + \ldots + \frac{PR(W_n)}{O(W_n)}\right)$$

Personalised PageRank

- For example -
  - v has higher value for your preferences (e.g. most visited page/frequent topic/liked article genre) and 0 otherwise.
  - *i.e.*, algorithm *gives preference* to the topics you are interested in (obtained from Query log analysis/Clickstream analysis).

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- Typically used in recommender systems.

PageRank - Summary

- Concepts considered:
  - A page is important if it has *many* links (incoming? outgoing?)
  - Are all links *equally* important?
  - A page is important if it is cited by other *important* pages.
  - A page is important if it is linked to (highly ranked) *topically related* pages.

Learning to Rank

Learning to Rank Concept

- Apply machine learning techniques to **learn the ranking of the results**
  - a learning algorithm fed with training data that contains ranking information
- Define a loss function to be minimized.

Learning to Rank - Process

- Uses query chains – i.e., a sequence of reformulated queries
  - Automatically detect query chains from large-scale search logs
  - Use query chains to infer relevance of results in each query and between results from all queries in the chain
  - Use ML models (SVM/Genetic algorithm/swarm intelligence…..) to learn a retrieval function from the results.

Inferring Relevance from Query Chains

- Some strategies for generating feedback from query chains
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  - **Click >q'Top Two Earlier Query:** If nothing was clicked in this query, the clicked document from earlier query is more relevant than the top two from this query.

Generating relevance training data

- Given query Q, three types of training data are then generated:
  - **pointwise**: a set of relevant pages for Q
  - **pairwise**: a set of pairs of relevant pages indicating the ranking relation between the two pages
  - **listwise**: a set of ordered relevant pages: p1 $\succ$ p2 $\succ$ $\cdots$ $\succ$ pm

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Generating relevance training data

- Training data may also be originated from editorial judgements made by human experts.
  - ML classifiers are trained so that ranking can be improved over time.

Learning the Ranking Function

- An effort to learn the ranking function, rather than the ranking order.

- E.g. using a genetic algorithm based approach.
  - members of the population are function instances over a given set of ranking features.
  - at every step of the genetic algorithm, different functions are mutated or mixed.
  - the goodness of each learning function is evaluated through a set of ground truth or training data.
  - after many iterations, the fittest ranking function is selected.

More reading...

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