ACCIO ALPHA - KALMAN FILTERING MAGIC ASSIGNMENT
WEEK - 1
DATA ANALYSIS

1 Questions

1.1 Q1

Suppose we have the following information:
1. There is a 60 percent chance that it will rain today.
2. There is a 50 percent chance that it will rain tomorrow.
3. There is a 30 percent chance that it does not rain either day.
Find the following probabilities:
a. The probability that it will rain today or tomorrow.
b. The probability that it will rain today and tomorrow.
c. The probability that it will rain today but not tomorrow.
d. The probability that it either will rain today or tomorrow, but not both.

1.2 Q2

I roll a fair die twice and obtain two numbers: X1= result of the first roll, and X2= result of the second roll. Write down the sample space S, and assuming that all outcomes are equally likely (because the die is fair), find the probability of the event A defined as the event that X1+X2=8.

1.3 Q3

A family has n children. We pick one of them at random and find out that she is a girl. What is the probability that all their children are girls?

1.4 Q4

Here is one way to think about a mixed random variable. Suppose that we have a discrete random variable X_d with (generalized) PDF and CDF $f_d(x)$ and $F_d(x)$, and a continuous random variable X_c with PDF and CDF $f_c(x)$ and $F_c(x)$. Now we create a new random variable X in the following way. We have a coin with $P(H) = p$. We toss the coin once. If it lands heads, then the value of X is determined according to the probability distribution of X_d. If the coin lands tails, the value of X is determined according to the probability distribution of X_c.

1. Find the CDF of X, $F_X(x)$.
2. Find the PDF of X, $f_X(x)$.
3. Find $\mathbb{E}[X]$.
4. Find $\mathrm{Var}(X)$.

1.5 Q5

Let X and Y be two independent $N(0,1)$ random variables and

$$Z = 1 + X + XY^2, \qquad W = 1 + X.$$

Find $\mathrm{Cov}(Z, W)$.

1.6 Q6

Your friend tells you that he had four job interviews last week. He says that based on how the interviews went, he thinks he has a 20chance of receiving an offer from each of the companies he interviewed with. Nevertheless, since he interviewed with four companies, he is 90sure that he will receive at least one offer. Is he right?

1.7 Q7

In a communication system each data packet consists of 1000 bits. Due to the noise, each bit may be received in error with probability 0.1. It is assumed bit errors occur independently. Find the probability that there are more than 120 errors in a certain data packet.

1.8 Q8

You have invited 64 guests to a party. You need to make sandwiches for the guests. You believe that a guest might need 0, 1, or 2 sandwiches with probabilities $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$ respectively. You assume that the number of sandwiches each guest needs is independent from other guests. How many sandwiches should you make so that you are 95% sure that there is no shortage?

1.9 Q9

Let

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right).$$

Find:

1. $\mathbb{E}[X]$ and $\mathbb{E}[Y]$,
2. $\mathrm{Var}(X)$ and $\mathrm{Var}(Y)$,
3. $\mathrm{Cov}(X, Y)$.

1.10 Q10

Let

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim N\left(\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 & 1 \\ 1 & 3 \end{pmatrix}\right).$$

Find the conditional distribution of $X \mid Y = y$ (its mean and variance).

1.11 Q11

Let (X, Y) be jointly Gaussian with

$$\mathbb{E}[X] = 0, \quad \mathbb{E}[Y] = 0, \quad \mathrm{Var}(X) = 1, \quad \mathrm{Var}(Y) = 4, \quad \mathrm{Cov}(X, Y) = 1.$$

Define the new variable

$$Z = 3X - 2Y.$$

Find the distribution of Z and compute $\mathrm{Corr}(Z, X)$.

1.12 Q12

Let

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 & 2 & 1 \\ 2 & 3 & -1 \\ 1 & -1 & 5 \end{pmatrix}\right).$$

Compute the conditional mean and variance of X given $(Y = y,\, Z = z)$.