A Proof of Reserves Protocol with Short Proofs and a Method to Estimate Amount Upper Bounds for MimbleWimble

A Seminar Report
Submitted in partial fulfillment of
the requirements for the degree of
Dual Degree (B.Tech & M.Tech) in Electrical Engineering with Specialization in Communication & Signal Processing
by

Suyash Bagad
(Roll No. 15D070007)

Supervisor:
Prof. Saravanan Vijayakumaran

Department of Electrical Engineering
Indian Institute of Technology Bombay
Mumbai 400076 (India)

24 June 2020

Acceptance Certificate

Department of Electrical Engineering
Indian Institute of Technology, Bombay

The dissertation entitled "A Proof of Reserves Protocol with Short Proofs and a Method to Estimate Amount Upper Bounds for MimbleWimble" submitted by Suyash Bagad (Roll No. 15D070007) may be accepted for being evaluated.

Date: 24 June 2020

Prof. Saravanan Vijayakumaran

Declaration

I declare that this written submission represents my ideas in my own words and where others' ideas or words have been included, I have adequately cited and referenced the original sources. I declare that I have properly and accurately acknowledged all sources used in the production of this report. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my submission. I understand that any violation of the above will be a cause for disciplinary action by the Institute and can also evoke penal action from the sources which have thus not been properly cited or from whom proper permission has not been taken when needed.

Suyash Bagad

Date: 24 June 2020 (Roll No. 15D070007)

Abstract

Cryptocurrency exchanges enable customers to buy digital assets without mining them in exchange for fiat currencies. They also provide customers with custodial wallets for trading purposes. However, in cases of the exchanges being hacked or involvement in an exit scam, customer assets are lost leading the customers to lose trust. Proof of Solvency is a technique to prove that an exchange owns assets at least as much as its liabilities, so that in unfortunate cases of hacks, the exchange could reimburse customer funds. If exchanges regularly publish proofs of solvency, they can regain the trust of their customers. A proof of solvency comprises of a proof of reserves and a proof of liabilities. Our work focuses only on designing privacy-preserving proof of reserves protocols for cryptocurrency exchanges as it depends only on publicly available data from the blockchain as against private customer data in the latter.

In this work, we design a novel proof of reserves protocol named RevelioBP for MimbleWimble based cryptocurrencies. Revelio [1], the existing state-of-the-art proof of reserves protocol for MimbleWimble provides privacy to exchange-owned outputs by hiding them in a larger anonymity set. However, it suffers from two major drawbacks: (i) the Revelio proof sizes scale linearly with the anonymity set restricting the frequency of publishing proofs of reserves for auditing, (ii) collusion between multiple exchanges can be detected only if exchanges generate their proofs at the same blockchain state. RevelioBP successfully alleviates both of these drawbacks using a Bulletproofs [2] based framework. The proof size of RevelioBP scales logarithmically in the anonymity set. By linking RevelioBP proof generation to the blockchain state, we enforce the exchanges to generate proofs of reserves simultaneously, enabling robust collusion detection. Further, we use multi-exponentiation technique to boost verification of RevelioBP proofs, making it ~2X faster than proof generation. Shorter proof sizes and faster verification brings great convenience to the customers since they might possess limited computational power. On the other hand, benefits of RevelioBP come with a higher computational cost for the exchanges.

In a parallel research on the outputs in Grin - a MimbleWimble based cryptocurrency, we derive upper bounds on the amounts hidden in Grin outputs. Grin outputs are Pedersen commitments which are perfectly hiding. This implies that given a Grin output, even a computationally unbounded adversary cannot figure out what amount it hides. However, the knowledge of public coinbase reward and the underlying transaction rules might reveal some information about the amounts hidden in Pedersen commitments. In this work, we establish upper bounds on the amounts hidden in Grin outputs leveraging the transaction structure in Grin. In a March 2020 snapshot of the Grin blockchain, we find that out of the 110,149 unspent regular transaction outputs 983 of them have less than 1800 grin (number of coins minted in half an hour) stored in them. On the other hand, 95% of the unspent regular transaction outputs in the snapshot have an upper bound which is at least 90% of the total Grin supply at their respective block heights.

List of Publications

[1] S. Bagad and S. Vijayakumaran, "On the Confidentiality of Amounts in Grin," in *Crypto Valley Conference on Blockchain Technology (CVCBT)*, 2020.

[2] S. Bagad and S. Vijayakumaran, "Performance Trade-offs in Design of MimbleWimble Proofs of Reserves," in *IEEE European Symposium on Security and Privacy Workshops (EuroS&PW)*, 2020.

Table of Contents

List of Figures

List of Tables

Acknowledgements

I would like to express my deepest appreciation and gratitude to my guide Prof. Saravanan Vijayakumaran for his guidance and constant supervision and help throughout the last 18 months. His insightful suggestions and comments have time and again helped me get a more in-depth understanding of the field. His most valuable lesson for me, from amongst many others, was to be persistent especially in times when things are not going as planned. Not only did he offer consistent assistance with regards to the academic work but also gave invaluable suggestions in helping me carve out my career path. I also thank the Bharti Centre for Communication at IIT Bombay and Vibha ma'am particularly for providiting necessary logistical support. Thanks are also due to Arijit Dutta of IIT, Bombay for his many useful remarks and discussions about the project and otherwise. I also acknowledge the help offered by my brother Piyush Bagad (Wadhwani AI) in understanding Python as well as in running simulations. I would also like to thank my dear friend Madhura Pawar for her constant encouragement which helped me work towards my goal even when things weren't working out. Finally, I express gratitude towards my family for giving me the opportunities and freedom in choosing research as my career.

Suyash Bagad
IIT Bombay
24 June 2020

Chapter 1

Introduction

The rise of cryptocurrencies have openend up unending possibilities of how minimal or no trust based systems could be established owing to decentralization. The concept of blockchain was introduced with the founding of Bitcoin by Satoshi Nakamoto [3]. Satoshi Nakamoto proposed and implemented idea of a consensus based, trustless, truly peer-to-peer system for financial transactions. Notwithstanding an intrumental step towards establishing a decentralized and a trustless system, Bitcoin has several practical limitations with regards to privacy, security and scalability [4]. This led to the development of more privacy and anonymity focussed cryptocurrencies like Monero [5] and Zcash [6]. Grin [7] and Beam [8] are two relatively new projects which are backed by the MimbleWimble protocol [9] and claim to promise scalability, anonymity and fungibility all at once. The rise in privacy-centric cryptocurrencies further led to growth in popularity of cryptocurrencies not only among investors but also common people.

1.1 What is a Blockchain?

The concept of a Blockchain originates from the idea of having a decentralized, publicly visible and trust-free ledger. The term *blockchain* was first coined by Satoshi Nakamoto, the creator of Bitcoin [3]. In literal terms, a blockchain implies that blocks containing financial transactions would be added to a publicly verifiable ledger in a timely manner, forming a *chain* of *blocks*. We expand on the need and the basic idea of a decentralized ledger in the following subsection.

1.1.1 Decentralized Ledger

A ledger is a principal book of records which keeps a track of all transactions measured in terms of a monetary unit. Bitcoin is a decentralized, public ledger, decentralized because there is no trusted third party controlling the ledger and public because anyone with bitcoin can participate in the network, receive and send bitcoins, and even hold a copy of this ledger (essentially, the history of all transactions) if they want to. In that sense, the ledger is "non-trusted" and transparent to public. As opposed to such a decentralized ledger setup, traditional banks use centralized ledger system where each bank has it's own ledger visible only to the concerned user.

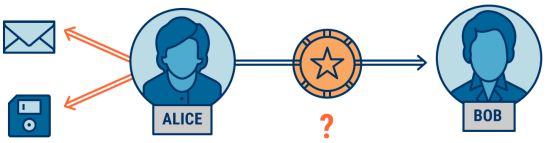
In reference to Figure* 1.1a, in a physical transaction, the Alice hands Bob a physical arcade coin. Bob now has one coin, and Alice has zero. The transaction is complete. If the same transaction is to be performed digitally (Figure 1.1b), Alice would send a string of some bits (corresponding to the desired amount) to Bob. Now, since it is just a string of bits, what is the guarantee that Alice would not reproduce that same string of bits in her mail or hard-disk? If this happens, it would mean that although Alice sent Bob the amount, Alice still possesses the same amount. This clearly is a violation. To tackle this, we must have a ledger or a record of all transactions between Alice and Bob. When Alice gives Bob the digital token, the ledger records the transaction (Figure 1.1c). Bob has the token, and Alice does not. Let us call the one who maintains this ledger as Dave. This setup assumes that Dave is a trusted middleman for maintaining ledger. Now, what if Alice bribes Dave to erase her transaction? What if Dave decides to charge a fee that neither Alice or Bob want to pay? What happens when Alice and Bob cannot trust the third party? This results in a decentralized and public ledger system (Figure 1.1d). When a lot of people have a copy of the same ledger, it becomes very difficult to tamper the system and cheat. This works because everyone is holding a copy of the same digital ledger and more the number of trusted people holding this ledger, the stronger it becomes.

Blockchain technology offers a way for untrusted parties to reach a consensus on a common digital history. The World Bank defines Blockchain as follows [10]:

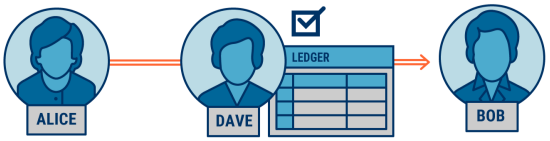
Definition 1.1 (Blockchain) *A 'blockchain' is a particular type of data structure used in some distributed ledgers which stores and transmits data in packages called "blocks" that are connected to each other in a digital 'chain'.*

*Credits: `https://www.cbinsights.com/research/what-is-blockchain-technology/`

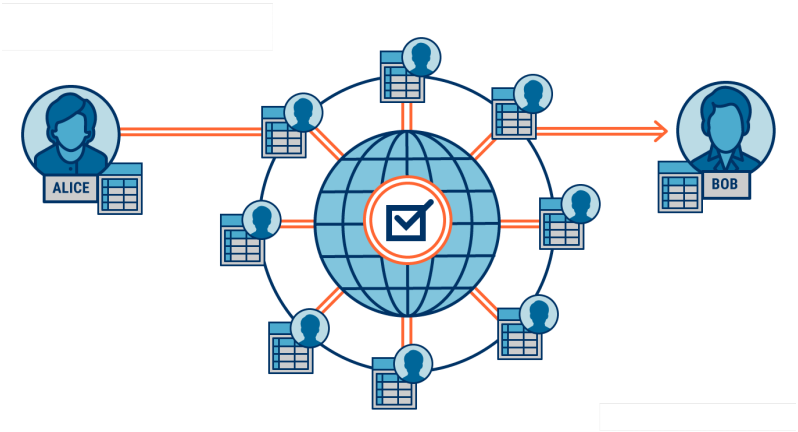
(a) A physical transaction



(b) A digital transaction



(c) A digital transaction with ledger



(d) A decentralized ledger

Figure 1.1: Understanding decentralized ledger

Blockchains employ cryptographic algorithms to record and synchronize data across a network in an unchangeable manner. The *state* of the Blockchain is the current status of the ledger visible to public. In case of Bitcoin, any independent observer can verify the state of the blockchain as well as the validity of all the transactions on the ledger. This is a *serious* limitation for Bitcoin and could possibly prohibit many use cases. As an example, if employees of a company were to receive their salaries in bitcoin, would they agree if their salaries were published on the

public blockchain? This naturally demands for a transaction system which preserves *anonymity* and *confidentiality*.

1.2 Notion of Privacy on a Blockchain

The very concept of having all the information about transactions (predominantly financial) public via a distributed ledger or a blockchain demands for privacy and anonymity of the users. Before we talk about privacy on the blockchain, it is necessary to understand what we do mean by terms like *privacy* and *anonymity*. Modern day cryptography is based on the following primary functions [11]:

(i) *Privacy/confidentiality*: To ensure that no one can read or access the message except the intended receiver

(ii) *Authentication*: Proving one's identity

(iii) *Integrity*: Ensuring that the message intended to be received by the receiver is not altered in the path

(iv) *Non-repudiation*: A protocol for checking if a message was actually generated by the sender

(v) *Key exchange*: The protocol which determines how key(s) are shared between the sender and the receiver

All of the above functions form an necessary part of a cryptographic system. A well-designed system ensures all of the above functions are taken care of considering the computational bounds for carrying out any protocol. In a confidential transaction, it is desirable to have *confidentiality* and *anonymity*. This means that in a valid confidential transaction, the identity of the sender and receiver must be confidential and the amounts transferred must also be hidden. The idea of having such private transactions in digital currencies could be traced back to David Chaum's work on blind signatures [12]. A similar concept of privacy is required in the blockchain framework. The digital assets owned by users must not be publicly visible. There should be a mechanism to unlink the identity of a user with his or her digital identity, i.e. public key address. The interaction between the sender and the receiver in a transaction must not reveal anything to them other than the amounts being transferred. A user must not be able to reuse his or her digital assets. Making the transactions digital and public at the same time brings about several challenges

in ensuring that the above requirements are being satisfied. Active research is still being pursued in the direction of not only solving such practical problems but to do them efficiently and in a scalable manner.

1.3 Cryptocurrency Exchanges

The rise of cryptocurrencies began with the inception of Bitcoin in 2009. Since then, several cryptocurrencies with better privacy and security guarantees are being developed. Cryptocurrencies gained popularity among general masses with the establishment of cryptocurrency exchanges. Also known as digital currency exchanges or crypto exchanges, they are essentially businesses that allow customers to trade cryptocurrencies or digital currencies for other assets including conventional fiat money or different digital currencies. From a customer point of view, exchanges not only made owning cryptocurrencies possible to non-miners but also provided them with fast trading platforms for transactions within cryptocurrencies and fiat. Customers were also provided with custodial wallets freeing them from the hassle of storing and remembering private keys. In the early days of cryptocurrency, crypto exchanges were very few and less-known, but not too long ago their number increased dramatically and they became an integral part of the cryptoeconomic ecosystem. They were responsible for the boost in the transaction volumes of the vast majority of the cryptocurrency sales and liquidity.

1.3.1 Hacks and Frauds of Exchanges

The downside of cryptocurrency exchanges is that they are required to store sensitive information of customers like the private keys and account balances. If in case an exchange is hacked, it might result in loss of customer-owned cryptocurrency assets. There have been many high-profile hacks over the years, many of which went unnoticed for some time [13]. In 2014, one of the biggest Bitcoin exchanges Mt.Gox lost almost 750,000 of its customers' bitcoins and around 100,000 of its own bitcoins, totaling around 7% of all bitcoins back then, and worth around $473 million, leading to filing of bankruptcy. There have been cases where exchanges lure customers into buying digital assets in return for fiat currency, but do not allocate any digital assets in reality. Such exit scams by exchanges have led to huge losses of customer and investor funds [14].

Although having a fool-proof method to avoid such hacks might be a difficult task, proof of solvency is one way to uphold the trust of customers. A proof of

solvency is the guarantee by the exchange that it owns reserves at least as much as its total liabilities towards customers. In this way, even after cases of hacking, the exchange could repay its liabilities to the customers.

1.4 Proof of Solvency

An exchange is said to be solvent if it owns assets at least as much as its liabilities. An exchange proving solvency convinces its customers that in an unfortunate case of a hack or a fraud, it could reimburse the lost customer funds through its own assets. A proof of solvency consists of two parts: proof of reserves and proof of liabilities. The former proves the assets owned by the exchange while the latter shows the total liabilities of the exchange towards its customers.

The challenge in designing proof of solvency is to maintain privacy of the exchange as well as the customers. Proof of reserves protocols in [1, 15, 16] publish a Pedersen commitment to the total assets owned by the exchange. Pedersen commitments are a cryptographic way of hiding secret messages or amounts. We will describe Pedersen commitments in detail in the next chapter. Suppose an exchange publishes C_{res} as a Pedersen commitment to its total reserves a_{res}. Similarly, it could publish a Pedersen commitment to its total liabilities a_{liab} as C_{liab} using customer data as described in [15]. However, this proof of reserves in [15] requires each customer to individually verify that his or her amount is included in the proof of liabilities. If a customer fails to verify his or her amount, the exchange in principle could hide the particular customer's data to shrink its liabilities. Chalkias *et al.* [17] recently proposed a scheme called Distributed Auditing Proofs of Liabilities (DAPOL) which addresses this concern. DAPOL uses private information retrieval to ensure the inclusion proof by every customer. Therefore, given the Pedersen commitments to reserves and liabilities of an exchange, it can prove solvency by proving that the quantity $C_{\text{res}} \cdot C_{\text{liab}}^{-1}$ commits to a non-negative amount and thus, $a_{\text{res}} - a_{\text{liab}} \geq 0$. This suffices because $C_{\text{res}} \cdot C_{\text{liab}}^{-1}$ is a Pedersen commitment to $a_{\text{res}} - a_{\text{liab}}$ due to the homomorphic property of Pedersen commitments.

1.4.1 Proof of Reserves

A proof of reserves protocol is used by a cryptocurrency exchange to prove that it owns a certain amount of cryptocurrency. If privacy of the amount or outputs owned by the exchange is not an issue, then proving reserves involves a straightforward proof of the ability to spend the exchange-owned outputs (for example, see

[18]). The simplest way to publish a proof of reserves for an exchange is to reveal all the addresses or account details it owns so that the customers are convinced about the assets owned by the exchange. Another way could be to send all the reserves it owns from all its addresses to a single addresses it owns. If amounts involved in a transaction are public as in the case of Bitcoin, such a self-transaction would be a proof of the exchange's reserves. For example, in 2011, Mt. Gox cryptocurrency exchange transferred 424,242 bitcoins from its wallets to a previously revealed Bitcoin address [19]. Information of an exchange's addresses or accounts and the total assets it owns are crucial for aspects of its business. Thus, non-private proof of reserves protocols are unlikely to be adopted by exchanges as they may reveal business strategy. Privacy-preserving proof of reserves protocols have been proposed for Bitcoin [15, 20], Monero [16], and MimbleWimble [1]. In fact, the protocols proposed by Decker *et al* [20] and Dagher *et al* [15] go one step further and give a privacy-preserving proof of solvency, i.e. they prove that the reserves owned by the exchange exceed its liabilities towards its customers. However, the work in [20] relies on a trusted hardware assumption. And the proof of liabilities protocol in [15] is secure only if every exchange customer checks the proof. In general, it seems that designing proof of reserves protocols is easier than designing proof of liabilities protocols as the former depend only the blockchain state while the latter depend on the exchange's private customer data.

Even without a robust proof of liabilities protocol, a privacy-preserving proof of reserves protocol based on homomorphic commitments is valuable. Exchanges can easily prove that C_{res} is a commitment to an amount which exceeds a base amount a_{base}. While the base amount may not be exactly equal to the total liabilities of the exchange, it can be based on the trade volume data published by the exchange [21]. This technique will help early detection of exchange hacks and exit scams. For example, in Februrary 2019 the Canadian exchange QuadrigaCX claimed that it had lost access to wallets containing customer funds due to the death (in December 2018) of their CEO who had sole custody of the corresponding passwords and keys. But an official investigation found that the wallets had been empty since April 2018, several months before the CEO's death [22, 23]. This discrepancy would have been detected earlier if the exchange had been required to give perioidic proofs of reserves. Realising the importance of proof of reserves as a tool in raising customer confidence in crypto-exchanges leads us to the need for building better proofs in terms of privacy, scalability and performance.

The main challenge in design of proofs of reserves is to preserve privacy and confidentiality of exchanges but at the same time convince customers about an exchange's actual asset ownership. Regaining *trust* of the customers without compromising exchanges' *privacy* is the primary motivation behind the design of better proofs of reserves. Advanced cryptographic techniques make it possible to design proofs of reserves which reveal *nothing* beyond an assertation of the form:

Exchange X owns ? amount of the cryptocurrency Y at time t.

Note that here we do not intend to reveal even the total amount. A publicly verifiable proof backing up such a claim is a cryptographic tool known as a *Non-Interactive Zero-Knowledge* proof.

1.5 Organization of the Thesis

After motivating the problem we are trying to address in this work, we will briefly describe the cryptographic primitives necessary to understand our work in Chapter 2. We briefly discuss the literature in Chapter 3 which is necessary to understand the latter chapters. In Chapter 4, we start with outlining the details of Grin and existing state-of-the-art proof of reserves protocol for MimbleWimble-based cryptocurrencies. We describe RevelioBP - a novel proof of reserves protocol for MimbleWimble-based cryptocurrencies in Chapter 4 along with its security properties. We present the Grin outputs' amount confidentiality analysis in Chapter 5. Finally, we discuss all the security proofs in detail in the Appendix A.

Chapter 2

Cryptographic Preliminaries

Elliptic-curve cryptography (ECC) is essentially a public-key cryptography system, design of which is based on the algebraic structure of elliptic curves over finite fields [24]. ECC enables significant reduction in memory usage as the public-key length in ECC framework is much smaller than other public-key cryptography schemes like RSA for the same security level. It is widely used for many applications like encryption, digital signatures, pseudo-random generators and other tasks.

All cryptocurrencies are built on the Elliptic-curve cryptography framework. The security of such systems depend on the security guarantees provided by the ECC framework. We will see a couple of cryptographic assumptions which promise us the security guarantees for cryptocurrency systems. Thus, before delving into the details of proof of reserves protocols, it is worthwhile spending some time learning about the background math of cryptocurrencies. Note that we assume familiarity of the reader with basic concepts of group theory and modular arithmetic. A short but sufficient primer on both of these topics in present in [25].

2.1 Notation

Let $\mathcal{G} = \{\mathbb{G}, q, g\}$ be the description of a cyclic group $\mathbb{G}$ of prime order q with generator g of $\mathbb{G}$. Let $h \in \mathbb{G}$ be another random generator of $\mathbb{G}$ such that the discrete logarithm relation between g and h is not known. Let $\mathbb{G}^n$ and $\mathbb{Z}_q^n$ be the n-ary Cartesian powers of sets $\mathbb{G}$ and $\mathbb{Z}_q$ respectively. Group elements which are Pedersen commitments are denoted by uppercase letters and randomly chosen group elements are denoted by lowercase letters. Bold font denotes vectors. Inner product of two vectors $\mathbf{a}, \mathbf{b} \in \mathbb{Z}_q^n$ is defined as $\langle \mathbf{a}, \mathbf{b} \rangle \coloneqq \sum_{i=1}^{n} a_i \cdot b_i$ where $\mathbf{a} = (a_1, \ldots, a_n), \mathbf{b} = (b_1, \ldots, b_n)$. Further, Hadamard and Kronecker products are defined respectively

as, $\mathbf{a} \circ \mathbf{b} \coloneqq (a_1 \cdot b_1, \ldots, a_n \cdot b_n) \in \mathbb{Z}_q^n$, $\mathbf{a} \otimes \mathbf{c} \coloneqq (a_1\mathbf{c}, \ldots, a_n\mathbf{c}) \in \mathbb{Z}_q^{nm}$ where $\mathbf{c} \in \mathbb{Z}_q^m$. For a base vector $\mathbf{g} = (g_1, \ldots, g_n) \in \mathbb{G}^n$, vector exponentiation is defined as $\mathbf{g}^{\mathbf{a}} = \prod_{i=1}^n g_i^{a_i} \in \mathbb{G}$. For a scalar $u \in \mathbb{Z}_q^*$, we denote its consecutive powers in the form of a vector $\boldsymbol{u}^n \coloneqq (1, u, u^2, \ldots, u^{n-1})$. To represent the exponentiation of all components of a vector $\mathbf{a}$ by the same scalar $k \in \mathbb{Z}_q$, we use $\mathbf{a}^{\circ k}$ to mean $(a_1^k, a_2^k, \ldots, a_n^k)$. If an element a is chosen uniformly from a set A, such a choice is denoted by $a \xleftarrow{\$} A$. We denote the relation *Relation* using the specified input and witness as $\{(Public\,Input;\, Witness) : Relation\}$. We refer to $\mathcal{A}$ as a PPT adversary which is a probabilistic Turing Machine that runs in polynomial time in the security parameter λ. An *interactive proof* for the decision problem π is described as follows:

1. There are two participants, a prover $\mathcal{P}$ and a verifier $\mathcal{V}$.
2. The proof consists of a specified number of rounds.
3. In the beginning, both participants get the same input.
4. In each round, the verifier challenges the prover, and the prover responds to the challenge.
5. Both the verifier and the prover can perform some private computation.
6. At the end, the verifier states whether he was convinced or not.

2.2 Basics of Elliptic Curves

Let $a, b \in \mathbb{R}$ such that $4a^3 + 27b^2 \neq 0$. Let E be the set of solutions $(x, y) \in \mathbb{R}^2$ to the equation

$$y^2 = x^3 + ax + b. \tag{2.1}$$

An elliptic curve over $\mathbb{R}$ is given by the set $E \cup \{\mathcal{O}\}$ where $\mathcal{O}$ is known as the *point at infinity*. An example of an elliptic curve over $\mathbb{R}$ is given in Figure 2.1. Note that the condition $4a^3 + 27b^2 \neq 0$ ensures that the curve does not have repeating roots. This condition is necessary in the discussion of elliptic curve groups.

2.2.1 Point Addition in Elliptic Curves

The set $E \cup \{\mathcal{O}\}$ needs to be an algebraic group for us to be able to define operations on it. Thus, we define a group operation over $E \cup \{\mathcal{O}\}$ known as *point addition*. Suppose we have two points $P = (x_1, y_1), Q = (x_2, y_2) \in \mathbb{R} \times \mathbb{R}$ such that $x_1 \neq x_2$. We show them by blue and yellow coloured points in Figure 2.2(a). We

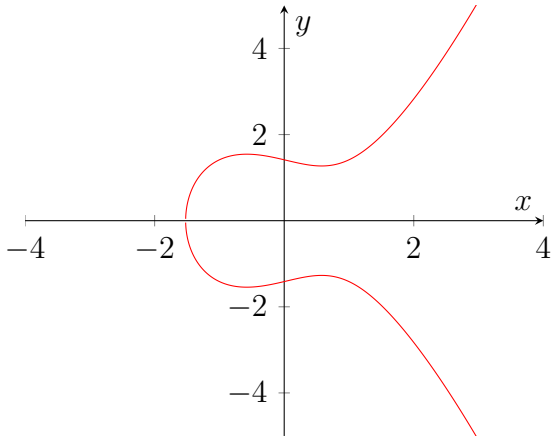


Figure 2.1: An elliptic curve given by the equation $y^2 = x^3 - x + 2$ over $\mathbb{R}$

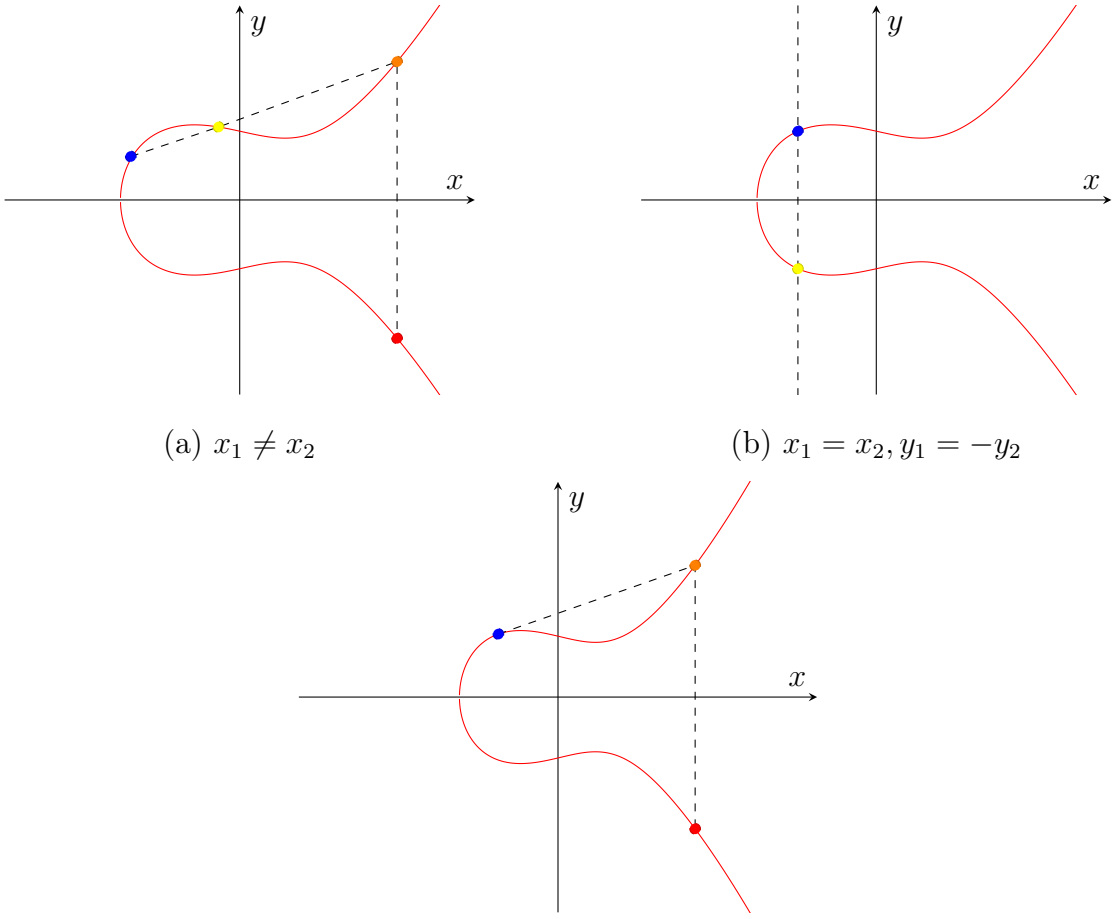
draw a line passing through P and Q. As the degree of an elliptic curve equation is 3, any non-tangent line must intersect the curve in 3 distinct points. Suppose the line passing through P and Q intersects the curve at point $R' = (x_3, -y_3) \in \mathbb{R} \times \mathbb{R}$, shown in orange colour. We define the result of point addition of points P and Q to be the point R which is the mirror reflection of R'. Therefore, we have $R = (x_3, y_3)$ shown in red colour. Simple calculation shows that given $P = (x_1, y_1), Q = (x_2, y_2)$ and $x_1 \neq x_2$, point addition gives us $P + Q = (x_3, y_3)$ such that

$$x_3 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)^2 - x_1 - x_2, \;\; y_3 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x_1 - x_3) - y_1. \tag{2.2}$$

Now if P and Q are such that $x_1 = x_2$ but $y_1 = -y_2$, the line passing through P and Q is vertical and possibly intersects the curve at infinity. In this case, we define the point addition operation as $P + Q = \mathcal{O}$. Therefore, the special point $\mathcal{O}$ acts as an identity point in the group $E \cup \{\mathcal{O}\}$. Further, if we have $P = Q$, i.e. $x_1 = x_2, y_1 = y_2 \neq 0$, we draw tangent to the curve at point P. As the degree of the curve is 3, any tangent will intersect the curve at one and only one point, say point R'. The result R in this case is again the mirror reflection of point R' about the x-axis. The addition of a point to itself is known as *point doubling*. The explicit formula for point doubling of point $P = (x_1, y_1)$ can be written as $P + P = 2P = (x_2, y_2)$ such that

$$x_2 = \left(\frac{3x_1^2 + a}{2y_1}\right)^2 - 2x_1, \;\; y_2 = \left(\frac{3x_1^2 + a}{2y_1}\right)(x_1 - x_2) - y_1. \tag{2.3}$$

The point addition operation is closed in the set $E \cup \{\mathcal{O}\}$ by construction. Further, $\mathcal{O}$ acts as an identity element in the set $E \cup \{\mathcal{O}\}$. For each point $P \in E$, we can find its *inverse* $Q \in E$ as its reflection about the x-axis as for such cases, we have $P + Q = \mathcal{O}$. Therefore, the set $E \cup \{\mathcal{O}\}$ is a group under point addition.



(a) $x_1 \neq x_2$

(b) $x_1 = x_2, y_1 = -y_2$

(c) $x_1 = x_2, y_1 = y_2 \neq 0$

Figure 2.2: Point addition in elliptic curves over $\mathbb{R}$

In practice, we use elliptic curves over finite fields instead of real numbers. The group operations are now defined over an underlying finite field F. The division by a non-zero field element $x \in F$ is interpreted as multiplication by its multiplicative inverse x^{-1}. Similarly, subtraction of a field element $x \in F$ is interpreted as addition by its additive inverse $-x$. This is the basis of the elliptic curve cryptography used in modern day systems except that of For example, Bitcoin and Grin cryptocurrency systems use the prime-ordered elliptic curve `secp256k1` [26].

Note that from hereon we use multiplicative notation to denote group operation on elliptic curves $\mathbb{G}$ on finite fields $\mathbb{F}_q$ for a large prime q. Lastly, addition of a point $P \in \mathbb{G}$ for k times is shown as scalar multiplication in additive notation and exponentiation in multiplicative respectively.

$$\underbrace{P + P + \cdots + P}_{k \text{ times}} = kP \in \mathbb{G}.$$

Similarly, in multiplicative notation, such an operation is called as exponentiation and is shown below

$$\underbrace{P \cdot P \cdot \ldots \cdot P}_{k \text{ times}} = P^k \in \mathbb{G}.$$

2.3 Cryptographic Assumptions

Every practical cryptographic system is built on certain hardness assumptions. For example, the widely popular RSA digital signature algorithm was based on the assumption that it computationally hard to factorize big primes [27]. Elliptic curve cryptography is similarly based on the assumption that the *Discrete Log Problem* is difficult to be solved by a computationally bounded adversary.

Definition 2.3.1 (Discrete Log Relation) *For all PPT adversaries $\mathcal{A}$ and for all $n \geq 2$, $\exists$ a negligible function $\mu(\lambda)$ s.t*

$$\Pr\left[\begin{array}{ll} \mathbb{G} = Setup(1^\lambda), g_1, \ldots, g_n \leftarrow \mathbb{G} \ ; & \\ a_1, \ldots, a_n \in \mathbb{Z}_p \leftarrow \mathcal{A}(\mathbb{G}, g_1, \ldots, g_n) & \because \exists a_i \neq 0 \wedge \prod_{i=1}^n g_i^{a_i} = 1 \end{array}\right] \leq \mu(\lambda)$$

We say $\prod_{i=1}^n g_i^{a_i} = 1$ is a non trivial discrete log relation between $g_1, \ldots, g_n$. If the Discrete Log Relation assumption stands, it implies that no PPT adversary can find a non-trivial relation between randomly chosen group elements. This is known as the Discrete Log Problem.

We use additional cryptographic assumptions such as Decisional Diffie-Hellman and its variants as described in [28].

2.4 Cryptographic Commitments

Cryptographic commitments are an important preliminary widely used to anonymise data like amounts. We also briefly discuss some key properties of commitments of our interest.

Definition 2.4.1 (Commitments) *A non-interactive commitment consists of two PPT algorithms* (Setup, Com)*. For a message $x \in \mathbf{M}_{pp}$ (message space), the algorithm proceeds as follows:*

1. *public parameters $pp \leftarrow Setup(1^\lambda)$ for security paramter λ*
2. *$\mathrm{Com}_{pp} : \mathbf{M}_{pp} \times \mathbf{R}_{pp} \rightarrow \mathbf{C}_{pp}$, where $\mathbf{R}_{pp}$ is randomness space*
3. *$r \leftarrow \mathbf{R}_{pp}$ and compute $\mathbf{com} = \mathrm{Com}_{pp}(x; r)$*

Definition 2.4.2 (Homomorphic Commitments) *A homomorphic commitment is a non-interactive commitment such that* $\mathbf{M}_{pp}$, $\mathbf{R}_{pp}$, $\mathbf{C}_{pp}$ *are all abelian groups, and* $\forall\ x_1, x_2 \in \mathbf{M}_{pp}$, $r_1, r_2 \in \mathbf{R}_{pp}$, *we have*

$$\mathrm{Com}(x_1; r_1) + \mathrm{Com}(x_2; r_2) = \mathrm{Com}(x_1 + x_2; r_1 + r_2)$$

Definition 2.4.3 (Hiding Commitment) *A commitment scheme is said to be hiding if for all PPT adversaries* $\mathcal{A}$, $\exists \mu(\lambda)$, *a negligible function such that,*

$$\left| \Pr\left[b\text{'}{=}b \;\middle|\; \begin{array}{l} pp \leftarrow \mathrm{Setup}(1^\lambda); \\ (x_0, x_1) \in \mathbf{M}_{pp}^2 \leftarrow \mathcal{A}(pp),\ b \leftarrow \{0,1\},\ r \leftarrow \mathbf{R}_{pp}, \\ \mathbf{com} = Com(x_b; r),\ b' \leftarrow \mathcal{A}(pp,\ \mathbf{com}) \end{array} \right] - \frac{1}{2} \right| \leq \mu(\lambda)$$

where the probability is over $b', r, Setup$ *and* $\mathcal{A}$. *For perfectly hiding schemes,* $\mu(\lambda) = 0$.

In simple words, a commitment scheme is *hiding* if it is impossible for a computationally bounded adversary to find what the message is hidden in a commitment or what randomness was used in computing the commitment.

Definition 2.4.4 (Binding Commitment) *A commitment scheme is said to be binding if for all PPT adversaries* $\mathcal{A}$, $\exists \mu(\lambda)$, *a negligible function such that,*

$$\Pr\left[\mathrm{Com}(x_0; r_0) = \mathrm{Com}(x_1; r_1) \wedge x_0 \neq x_1 \;\middle|\; \begin{array}{c} pp \leftarrow \mathrm{Setup}(1^\lambda), \\ x_0, x_1, r_0, r_1 \leftarrow \mathcal{A}(pp) \end{array} \right] \leq \mu(\lambda)$$

where the probability is over Setup and $\mathcal{A}$. *Again, if* $\mu(\lambda) = 0$ *then we say the scheme is perfectly binding.*

A commitment scheme is known as *binding* if it is impossible for a computationally bounded adversary to change the message a commitment commits to once it has published the commitment to the original message.

Definition 2.4.5 (Pedersen Commitment) $\mathbf{M}_{pp}, \mathbf{R}_{pp} = \mathbb{Z}_p$, $\mathbf{C}_{pp} = \mathbb{G}$ *of order p.*

1. Setup*:* $g, h \leftarrow \mathbb{G}$
2. $\mathrm{Com}(x; r) = (g^x h^r)$

Definition 2.4.6 (Pedersen Vector Commitment) $\mathbf{M}_{pp} = \mathbb{Z}_p^n$, $\mathbf{R}_{pp} = \mathbb{Z}_p$, $\mathbf{C}_{pp} = \mathbb{G}$ *of order p.*

1. Setup*:* $\boldsymbol{g} = (g_1, \ldots, g_n), h \leftarrow \mathbb{G}$

2. $\mathrm{Com}(\boldsymbol{x} = (x_1, \ldots, x_n); r) = (h^r \boldsymbol{g}^{\boldsymbol{x}})$

The Pedersen vector commitment is *perfectly hiding* and *computationally binding* under the discrete logarithm assumption. This means that no matter how much computational power an adversary possesses, he cannot find the message hidden in a Pedersen commitment. Further, for a computationally bounded adversary, it is infeasible to find another opening to a Pedersen commitment once he has committed it to original message.

2.5 Zero-Knowledge Arguments of Knowledge

Zero knowledge proofs are a powerful cryptographic tool which allow a prover to prove the validity of an assertion without revealing any other information about the secret owned. We give the formal definitions of zero knowledge proofs and their properties along with an example. In a formal context, a *proof* implies security guarantees against any adversary while security of an *argument* stands valid only for a computationally bounded adversary. From hereon, we will interchangeably use the terms *proof* and *argument* as we consider only PPT adversaries.

2.5.1 Zero-Knowledge Arguments

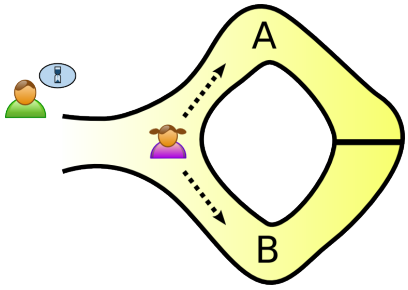
A protocol in which a prover convinces a verifier that a statement is true *without* revealing any information about why it holds is known as a Zero-knowledge argument. An argument is a proof only if the prover is computationally bounded and some computational hardness holds. Hereafter, we use the terms *proof* and *argument* interchangeably.

We illustrate the idea of zero-knowledge arguments of proof using the example of *Ali-Baba's secret cave** [29].

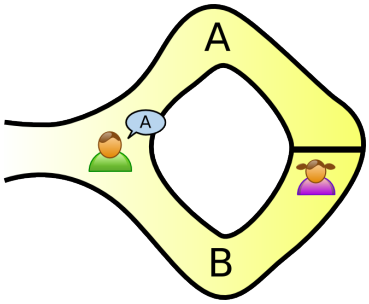
In the above example, Peggy knows the secret word used to open a mysterious door in a cave. The cave is shaped like a horse-hoe. The entrance is on one side and the magic door blocking the opposite side. Victor wants to know whether Peggy knows the secret word; but Peggy, does not want to reveal her knowledge (the secret word) to Victor or to reveal the fact of her knowledge to anyone in the world.

Peggy and Victor run the protocol described in figure 2.3. Provided she really does know the magic word, and the path she enters and path Victor asks her to come from are same, then it's trivial for Peggy to succeed and Victor to believe

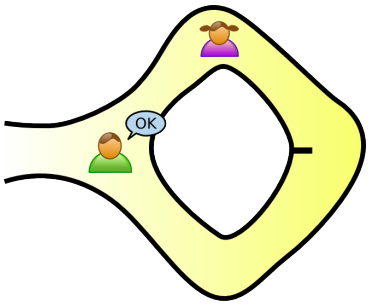
*Figure courtesy: `https://en.wikipedia.org/wiki/Zero-knowledge_proof`.



(a) Peggy chooses a path uniformly from A, B without Victor knowing.



(b) Victor asks her to come out of the cave from path A.



(c) If Peggy had entered from path A, she returns trivially. Otherwise, she could open the door using the secret key and return from path A.

Figure 2.3: Example of a zero knowledge proof

that she actually knows the secret key. Further, if the chosen path by Peggy and asked by Victor doesn't match, even then she could open the door and return from a desired path. If they were to repeat this protocol many times, say 15 times in a row, her chance of successfully "guessing" all of Victor's requests would become exponentially small (about three in a lakh).

For zero-knowledge arguments presented in this report, we will consider arguments consisting of three interactive probabilistic polynomial time algorithms $(\text{Setup}, \mathcal{P}, \mathcal{V})$. These algorithms are described by:

1. Setup: $\sigma \leftarrow \text{Setup}(1^\lambda)$, σ is common reference string
2. $\mathcal{P}$: prover, $\mathcal{V}$: verifier
3. Transcript $tr \leftarrow \langle \mathcal{P}, \mathcal{V} \rangle$

4. $\langle \mathcal{P}, \mathcal{V} \rangle = b$, $b = 0$ if the verifier rejects or $b = 1$ accepts

Further, we define the relation $\mathcal{R}$ and the CRS-dependent language as:

$$\mathcal{R} := \{(\sigma, u, w) \in \{0,1\}^* \times \{0,1\}^* \times \{0,1\}^* : \ w \text{ is a witness for } u \mid \sigma\}$$
$$\mathcal{L}_\sigma := \{x \mid \exists w \in (\sigma, u, w) \in \mathcal{R}\}$$

So, $\mathcal{L}_\sigma$ is essentially the set of statements x that have a witness w in the relation $\mathcal{R}$.

2.5.2 Defining Zero-Knowledge Arguments of Knowledge

To mathematically define the notion of zero-knowledge and zero-knowledge arguments, we will provide the necessary definitions below.

Definition 2.5.1 (Argument of Knowledge) *The triple $(Setup, \mathcal{P}, \mathcal{V})$ is called an argument of knowledge for relation $\mathcal{R}$ if it is perfectly complete and has computational witness-extended emulation.*

Definition 2.5.2 (Perfect completeness) *$(Setup, \mathcal{P}, \mathcal{V})$ has perfect completeness if for all non-uniform polynomial time adversaries $\mathcal{A}$*

$$\Pr\left[\ (\sigma, u, w) \notin \mathcal{R} \text{ or } \langle \mathcal{P}(\sigma, u, w), \mathcal{V}(\sigma, u) \rangle = 1 \ \middle| \ \begin{array}{l} \sigma \leftarrow Setup(1^\lambda) \\ (u, w) \leftarrow \mathcal{A}(\sigma) \end{array} \right] = 1$$

Perfect completeness implies that if a statement is actually true, then an honest verifier is convinced with probability 1 about the truth of the statement by an honest prover.

Definition 2.5.3 (Computational Witness-Extended Emulation) *$(Setup, \mathcal{P}, \mathcal{V})$ has witness-extended emulation if for all deterministic polynomial time P^* there exists an expected polynomial time emulator $\mathcal{E}$ such that for all pairs of interactive adversaries $\mathcal{A}_1, \mathcal{A}_2$ there exists a negligible function $\mu(\lambda)$ such that*

$$\left| \Pr\left[\ \mathcal{A}_1(tr) = 1 \ \middle| \ \begin{array}{l} \sigma \leftarrow Setup(1^\lambda,) \\ (u, s) \leftarrow \mathcal{A}_2(\sigma), \\ tr \leftarrow \langle (\mathcal{P}^*(\sigma, u, s), V(u, s) \rangle \end{array} \right] - \right.$$
$$\left. \Pr\left[\ \begin{array}{l} \mathcal{A}_1(tr) = 1 \wedge \\ \\ (tr \text{ accepted } \implies (\sigma, u, w) \in \mathcal{R}) \end{array} \ \middle| \ \begin{array}{l} \sigma \leftarrow Setup(1^\lambda,) \\ (u, s) \leftarrow \mathcal{A}_2(\sigma), \\ (tr, w) \leftarrow \mathcal{E}^{\mathcal{O}}(\sigma, u) \end{array} \right] \right| \leq \mu(\lambda)$$

where the oracle is given by $\mathcal{O} = \langle(\mathcal{P}^*(\sigma, u, s), V(u, s)\rangle$*, and permits rewinding to a specific point and resuming with fresh randomness for the verifier from this point onwards. We can also define com- putational witness-extended emulation by restricting to non-uniform polynomial time adversaries* $\mathcal{A}_1$ *and* $\mathcal{A}_2$*.*

Computational witness-extended emulation implies that when an adversary produces an argument to convince the verifier with some probability, then we have a corresponding emulator producing identically distributed argument with same probability, but also a witness.

Definition 2.5.4 (Public coin) *An argument of knowledge* $(Setup, \mathcal{P}, \mathcal{V})$ *is called public coin if all messages sent from the verifier to the prover are chosen uniformly at random and independent of the prover's messages, i.e., the challenges correspond to the verifier's randomness* ρ*.*

Definition 2.5.5 (Zero Knowledge Argument of Knowledge) *An argument of knowledge* $(Setup, \mathcal{P}, \mathcal{V})$ *is zero knowledge if it reveals no information about* w *apart from what could be deduced from the fact that* $(\sigma, u, w) \in \mathcal{R}$*.*

An argument of knowledge is zero knowledge if it does not leak information about w apart from what can be deduced from the fact that $(\sigma, u, w) \in \mathcal{R}$. More explicitly, we note that, a zero knowledge argument of knowledge ensures that no PPT adversary (or verifier) can ever recover w given it's relation with σ, u.

Definition 2.5.6 (Perfect Special Honest-Verifier Zero-Knowledge) *A public coin argument of knowledge* $(Setup, \mathcal{P}, \mathcal{V})$ *is a perfect special honest verifier zero knowledge (SHVZK) argument of knowledge for* $\mathcal{R}$ *if there exists a probabilistic polynomial time simulator* $\mathcal{S}$ *such that for all pairs of interactive adversaries* $\mathcal{A}_1, \mathcal{A}_2$

$$\Pr\left[\begin{array}{l|l} (\sigma, u, w) \in \mathcal{R} \wedge \mathcal{A}_1(tr) = 1 & \begin{array}{l}\sigma \leftarrow Setup(1^\lambda,) \\ (u, w, \rho) \leftarrow \mathcal{A}_2(\sigma), \\ tr \leftarrow \langle(\mathcal{P}(\sigma, u, s), V(\sigma, u; \rho)\rangle\end{array}\end{array}\right]$$

$$= \Pr\left[\begin{array}{l|l} (\sigma, u, w) \in \mathcal{R} \wedge \mathcal{A}_1(tr) = 1 & \begin{array}{l}\sigma \leftarrow Setup(1^\lambda,) \\ (u, w, \rho) \leftarrow \mathcal{A}_2(\sigma), \\ tr \leftarrow \mathcal{S}(u, \rho)\end{array}\end{array}\right]$$

PSHVZK AoK implies that even if an adversary chooses a distribution over statements and witnesses, it isn't able to distinguish between simulated transcript and honestly generated transcript for $u \in \mathcal{L}_\sigma$.

We will be using these definitions of Zero knowledge argument and its properties in the discussion further without redefining them, unless explicitly stated.

Chapter 3

Literature Survey

In this chapter, we briefly discuss the MimbleWimble protocol and Grin, technical details of recently proposed protocols like Improved Inner Product argument, Bulletproofs [2] and Omniring [28] which inspire the design of RevelioBP protocol. We also describe Revelio protocol which is the current state-of-the-art proof of reserves protocol for MimbleWimble based cryptocurrencies. Finally, we will discuss a few limitations of Revelio which motivated us to design a better proof of reserves protocol.

3.1 Improved Inner-Product Argument

We will define Inner-Product argument which is a proof construction for proving to a verifier that the prover knows two vectors which are hidden in a Pedersen commitment and the the inner product of those two vectors is known.

The inputs to the inner-product argument are independent generators $\mathbf{g}, \mathbf{h} \in \mathbb{G}^n$, $P \in \mathbb{G}$ and a scalar $c \in \mathbb{Z}_q$. P is a binding vector commitment to $\mathbf{a}, \mathbf{b}$. The argument lets the prover convince a verifier that the prover knows two vectors $\mathbf{a}, \mathbf{b} \in \mathbb{Z}_q^n$ such that

$$P = \mathbf{g}^{\mathbf{a}}\mathbf{h}^{\mathbf{b}} \ \wedge \ c = \langle \mathbf{a}, \mathbf{b} \rangle$$

The inner product argument is an efficient proof system for the language:

$$\mathcal{L}_{\mathsf{IP}} = \{(\underbrace{\mathbf{g}, \mathbf{h} \in \mathbb{G}^n, P \in \mathbb{G}, c \in \mathbb{Z}_q}_{crs}; \ \underbrace{\mathbf{a}, \mathbf{b} \in \mathbb{Z}_q^n}_{wit}) : \ \underbrace{P = \mathbf{g}^{\mathbf{a}}\mathbf{h}^{\mathbf{b}} \wedge c = \langle \mathbf{a}, \mathbf{b} \rangle}_{stmt}\} \tag{3.1}$$

Here, *crs, stmt, wit* are common reference string, statement and witness respectively as defined in Section 2.1. Clearly, the simplest proof system for (3.1) is: $\mathcal{P}$ sends $\mathcal{V}$ $(\mathbf{a}, \mathbf{b}) \in \mathbb{Z}_q^n$, requiring to send $2n$ elements to $\mathcal{V}$. We wish to build an efficient proof which requires much less elements to be exchanged. Recall, the communication

cost directly affects *efficiency* of a protocol.

Note that the witness-CRS relation in (3.1) is essentially an AND of two different relations. To simplify things to start with the inner-product argument, we propose to design a proof system for the language:

$$\mathcal{L}_{\mathsf{IP\_mod}} = \{(\underbrace{\mathbf{g}, \mathbf{h} \in \mathbb{G}^n, u, P \in \mathbb{G}}_{crs}; \underbrace{\mathbf{a}, \mathbf{b} \in \mathbb{Z}_q^n}_{wit}) : \underbrace{P = \mathbf{g}^{\mathbf{a}}\mathbf{h}^{\mathbf{b}} \cdot u^{\langle \mathbf{a}, \mathbf{b} \rangle}}_{stmt}\} \tag{3.2}$$

We will first show a protocol or a proof system for language defined in (3.2) and then prove that the same proof system gives a proof system for language in (3.1) with same complexity. Define a hash function $H : \mathbb{Z}_q^{2n+1} \to \mathbb{G}$ such that for input vectors $\mathbf{a}_1, \mathbf{a}'_1, \mathbf{b}_1, \mathbf{b}'_1 \in \mathbb{Z}_q^{n/2}, c \in \mathbb{Z}_q$

$$H(\mathbf{a}_1, \mathbf{a}'_1, \mathbf{b}_1, \mathbf{b}'_1, c) := \mathbf{g}_{[:n/2]}^{\mathbf{a}_1} \cdot \mathbf{g}_{[n/2:]}^{\mathbf{a}'_1} \cdot \mathbf{h}_{[:n/2]}^{\mathbf{b}_1} \cdot \mathbf{h}_{[n/2:]}^{\mathbf{b}'_1} \cdot u^c \ \in \mathbb{G} \tag{3.3}$$

Further, we notice that in (3.2), we can write P as:

$$n' = n/2, \ \mathbf{a} = (\mathbf{a}_{[:n']}, \mathbf{a}_{[n':]}), \ \mathbf{b} = (\mathbf{b}_{[:n']}, \mathbf{b}_{[n':]}),$$
$$P = H(\mathbf{a}_{[:n']}, \mathbf{a}_{[n':]}, \mathbf{b}_{[:n']}, \mathbf{b}_{[n':]}, \langle \mathbf{a}, \mathbf{b} \rangle)$$

We also note that H is additively homomorphic in its inputs, i.e

$$H(\mathbf{a}_1, \mathbf{a}'_1, \mathbf{b}_1, \mathbf{b}'_1, c_1) \cdot H(\mathbf{a}_2, \mathbf{a}'_2, \mathbf{b}_1, \mathbf{b}'_2, c_2) =$$
$$\left(\mathbf{g}_{[:n/2]}^{\mathbf{a}_1} \cdot \mathbf{g}_{[n/2:]}^{\mathbf{a}'_1} \cdot \mathbf{h}_{[:n/2]}^{\mathbf{b}_1} \cdot \mathbf{h}_{[n/2:]}^{\mathbf{b}'_1} \cdot u^{c_1}\right) \cdot \left(\mathbf{g}_{[:n/2]}^{\mathbf{a}_2} \cdot \mathbf{g}_{[n/2:]}^{\mathbf{a}'_2} \cdot \mathbf{h}_{[:n/2]}^{\mathbf{b}_2} \cdot \mathbf{h}_{[n/2:]}^{\mathbf{b}'_2} \cdot u^{c_2}\right)$$

$$\begin{aligned} \implies H(\mathbf{a}_1, \mathbf{a}'_1, \mathbf{b}_1, \mathbf{b}'_1, c_1) \cdot & H(\mathbf{a}_2, \mathbf{a}'_2, \mathbf{b}_1, \mathbf{b}'_2, c_2) \\ &= \left(\mathbf{g}_{[:n/2]}^{\mathbf{a}_1+\mathbf{a}_2} \cdot \mathbf{g}_{[n/2:]}^{\mathbf{a}'_1+\mathbf{a}'_2} \cdot \mathbf{h}_{[:n/2]}^{\mathbf{b}_1+\mathbf{b}_2} \cdot \mathbf{h}_{[n/2:]}^{\mathbf{b}'_1+\mathbf{b}'_2} \cdot u^{c_1+c_2}\right) \\ &= H(\mathbf{a}_1 + \mathbf{a}_2, \mathbf{a}'_1 + \mathbf{a}'_2, \mathbf{b}_1 + \mathbf{b}_2, \mathbf{b}'_1 + \mathbf{b}'_2, c_1 + c_2) \end{aligned}$$

We now describe a protocol for the language $\mathcal{L}_{\mathsf{IP\_mod}}$ (3.2) in argument of knowledge 3.1.

3.1.1 Inner-Product Argument

Figure 3.1: Argument of knowledge for $\mathcal{L}_{\mathsf{IP\_mod}}$

We are denoting the first and second halves of a vector $\mathbf{a}$ by color coding to simplify visualization, $\mathbf{a}_{[:n']}$ as the first half and $\mathbf{a}_{[n':]}$ by second half.

$Setup(\lambda, \mathcal{L}_{\mathsf{IP\_mod}})$:

Generate following elements randomly from $\mathbb{G}$: $\mathbf{g} \xleftarrow{\$} \mathbb{G}^n, \mathbf{h} \xleftarrow{\$} \mathbb{G}^n, u \xleftarrow{\$} \mathbb{G}$

$crs = (\mathbb{G}, q, \mathbf{g}, \mathbf{h}, u, P)$, $wit = (\mathbf{a}, \mathbf{b}) \in \mathbb{Z}_q^n$, $stmt$: $P = \mathbf{g}^{\mathbf{a}} \cdot \mathbf{h}^{\mathbf{b}} \cdot u^{\langle \mathbf{a}, \mathbf{b} \rangle}$

$\langle \mathcal{P}(crs,\ stmt,\ wit), \mathcal{V}(crs,\ stmt) \rangle$:

$\mathcal{P}$:

(i) $n' = n/2$

(ii) $L = H(\mathbf{0}^{n'}, \mathbf{a}_{[:n']}, \mathbf{b}_{[n':]}, \mathbf{0}^{n'}, \langle \mathbf{a}_{[:n']}, \mathbf{b}_{[n':]} \rangle)$

(iii) $R = H(\mathbf{a}_{[n':]}, \mathbf{0}^{n'}, \mathbf{0}^{n'}, \mathbf{b}_{[n':]}, \langle \mathbf{a}_{[n':]}, \mathbf{b}_{[n':]} \rangle)$

$\mathcal{P} \longrightarrow \mathcal{V}$: $L, R \in \mathbb{G}$

$\mathcal{V}$: $x \xleftarrow{\$} \mathbb{Z}_q$, $\mathcal{V} \longrightarrow \mathcal{P}$: x

$\mathcal{P}$:

(i) $\mathbf{a}' = x \cdot \mathbf{a}_{[:n']} + x^{-1} \cdot \mathbf{a}_{[n':]}$

(ii) $\mathbf{b}' = x^{-1} \cdot \mathbf{b}_{[:n']} + x \cdot \mathbf{b}_{[n':]}$

$\mathcal{P} \longrightarrow \mathcal{V}$: $\mathbf{a}', \mathbf{b}' \in \mathbb{Z}_q^{n'}$

$\mathcal{V}$:

(i) $P' = L^{(x^2)} \cdot P \cdot R^{(x^{-2})}$

(ii) $P' \stackrel{?}{=} H(x^{-1}\mathbf{a}', x\mathbf{a}', x\mathbf{b}', x^{-1}\mathbf{b}', \langle \mathbf{a}', \mathbf{b}' \rangle)$ *// Verification equation*

The verifier is convinced because indeed the left hand size of the verification equation (1) by the verifier can be written as:

$$L^{x^2} \cdot P \cdot R^{x^{-2}} = H(\mathbf{a}_{[:n']} + x^{-2}\mathbf{a}_{[n':]}, x^2\mathbf{a}_{[:n']} + \mathbf{a}_{[n':]}, x^2\mathbf{b}_{[n':]} + \mathbf{b}_{[:n']}, \mathbf{b}_{[n':]} + x^{-2}\mathbf{b}_{[:n']}, \langle \mathbf{a}', \mathbf{b}' \rangle)$$

The key features of this approach are listed below.

1. This proof system requires sending $n + 2$ elements.
2. To extract a valid witness $\mathbf{a}, \mathbf{b} \in \mathbb{Z}_q^n$ from a successful prover, we need to rewind the prover three times. After the prover sends L, R we rewind the prover three

times to obtain three tuples $(x_i, \mathbf{a}'_i, \mathbf{b}'_i)$ for $i = 1, 2, 3$ such that for all distinct x_i:

$$L^{(x_i^2)} \cdot P \cdot L^{(-x_i^2)} = H(x^{-1}\mathbf{a}'_i, x_i\mathbf{a}'_i, x_i\mathbf{b}', x_i^{-1}\mathbf{b}'_i, \langle \mathbf{a}', \mathbf{b}'_i \rangle) \tag{3.4}$$

3. We can now find $\nu_1, \nu_2, \nu_3 \in \mathbb{Z}_q$ such that,

$$\sum_{i=1}^{3} x_i^2 \nu_i = 0, \quad \sum_{i=1}^{3} \nu_i = 1, \quad \sum_{i=1}^{3} x_i^{-2} \nu_i = 0$$

4. Thus, $\mathbf{a}, \mathbf{b}$ can be found by computing:

$$\mathbf{a} = \sum_{i=1}^{3} (\nu_i \cdot x_i^{-1}\mathbf{a}'_i \parallel \nu_i \cdot x_i^1 \mathbf{a}'_i) \in \mathbb{Z}_q^n$$
$$\mathbf{b} = \sum_{i=1}^{3} (\nu_i \cdot x_i^{-1}\mathbf{b}'_i \parallel \nu_i \cdot x_i^1 \mathbf{b}'_i) \in \mathbb{Z}_q^n$$

5. Can we still improve? Observe that the test in the verification equation (1) is equivalent to:

$$P' \stackrel{?}{=} \left(\mathbf{g}_{[:n']}^{x^{-1}} \circ \mathbf{g}_{[n':]}^{x}\right)^{\mathbf{a}'} \cdot \left(\mathbf{h}_{[:n']}^{x} \circ \mathbf{h}_{[n':]}^{x^{-1}}\right)^{\mathbf{b}'} \cdot u^{\langle \mathbf{a}', \mathbf{b}' \rangle}$$

6. Thus the prover can recursively engage in an inner-product argument for P' with respect to generators:

$$(\mathbf{g}_{[:n']}^{x^{-1}} \circ \mathbf{g}_{[n':]}^{x}, \mathbf{h}_{[:n']}^{x} \circ \mathbf{h}_{[n':]}^{x^{-1}}, u)$$

7. We hope to get a total communication of Protocol 2 to be only $2\lceil \log_2(n) \rceil$ elements in $\mathbb{G}$ plus 2 elements in $\mathbb{Z}_q$. Let's see how we go about doing it in the next section.

3.1.2 Recursive Inner-Product Argument

Figure 3.2: Improved Inner-Product Argument for $\mathcal{L}_{\mathsf{IP\_mod}}$

Setup$(\lambda, \mathcal{L}_{\mathsf{IP\_mod}})$:

Generate following elements randomly from $\mathbb{G}$: $\mathbf{g} \xleftarrow{\$} \mathbb{G}^n, \mathbf{h} \xleftarrow{\$} \mathbb{G}^n, u \xleftarrow{\$} \mathbb{G}$

crs $= (\mathbb{G}, q, \mathbf{g}, \mathbf{h}, u, P)$, *wit* $= (\mathbf{a}, \mathbf{b}) \in \mathbb{Z}_q^n$, *stmt*: $P = \mathbf{g}^{\mathbf{a}} \cdot \mathbf{h}^{\mathbf{b}} \cdot u^{\langle \mathbf{a}, \mathbf{b} \rangle}$

$\langle \mathscr{P}(crs,\ stmt,\ wit), \mathcal{V}(crs,\ stmt) \rangle$:

$\mathscr{P}$: If $n' = 1$:

$\mathcal{P} \longrightarrow \mathcal{V}$: $a, b \in \mathbb{Z}_q$

$\mathcal{V}$:

(i) $c = a \cdot b$

(ii) $P \stackrel{?}{=} g^a h^b u^c$ *// Single exponentiation verification equation*

$\mathcal{P}$: Else $n' > 1$:

(i) $n' = n/2$ where $n = |\mathbf{g}|$

(ii) $c_L = \langle \mathbf{a}_{[:n']}, \mathbf{b}_{[n':]} \rangle \in \mathbb{Z}_q$,

(iii) $c_R = \langle \mathbf{a}_{[n':]}, \mathbf{b}_{[n':]} \rangle \in \mathbb{Z}_q$

(iv) $L = \mathbf{g}_{[n':]}^{\mathbf{a}_{[:n']}} \cdot \mathbf{h}_{[:n']}^{\mathbf{b}_{[n':]}} \cdot u^{c_L} \in \mathbb{G}$

(v) $R = \mathbf{g}_{[:n']}^{\mathbf{a}_{[n':]}} \cdot \mathbf{h}_{[n':]}^{\mathbf{b}_{[n':]}} \cdot u^{c_R} \in \mathbb{G}$

$\mathcal{P} \longrightarrow \mathcal{V}$: $L, R \in \mathbb{G}$

$\mathcal{V}$: $x \xleftarrow{\$} \mathbb{Z}_q$, $\mathcal{V} \longrightarrow \mathcal{P}$: x

$\mathcal{P}$, $\mathcal{V}$:

(i) $\mathbf{g}' = \mathbf{g}_{[:n']}^{x^{-1}} \circ \mathbf{g}_{[n':]}^{x} \in \mathbb{G}'$

(ii) $\mathbf{h}' = \mathbf{h}_{[:n']}^{x} \circ \mathbf{h}_{[n':]}^{x^{-1}} \in \mathbb{G}'$

(iii) $P' = L^{x^2} P R^{x^{-2}} \in \mathbb{G}$

$\mathcal{P}$:

(i) $\mathbf{a}' = x \cdot \mathbf{a}_{[:n']} + x^{-1} \cdot \mathbf{a}_{[n':]} \in \mathbb{Z}_q^{n'}$

(ii) $\mathbf{b}' = x^{-1} \cdot \mathbf{b}_{[:n']} + x \cdot \mathbf{b}_{[n':]} \in \mathbb{Z}_q^{n'}$

Set *crs'* $= (\mathbb{G}, q, \mathbf{g}', \mathbf{h}', u, P')$, *wit'* $= (\mathbf{a}', \mathbf{b}') \in \mathbb{Z}_q^n$, *stmt'*: $P' = (\mathbf{g}')^{\mathbf{a}'} \cdot (\mathbf{h}')^{\mathbf{b}'} \cdot u^{\langle \mathbf{a}', \mathbf{b}' \rangle}$

Run Protocol 3.2 with $\langle \mathcal{P}(crs', stmt', wit'), \mathcal{V}(crs', stmt') \rangle$ *// Recursion step*

In the above recursive inner-product protocol, the resulting depth is of the order of $\log_2(n)$. The total communication in this protocol is $2 \times \lceil \log_2(n) \rceil$ elements in $\mathbb{G}$ and 2 elements in $\mathbb{Z}_q$, i.e the prover sends the following in the specified order:

$$(L_1, R_1), \ldots, (L_{\log_2(n)}, R_{\log_2(n)}), a, b$$

Now, coming back to the language defined in (3.1), we now are in an position to design a proof system for (3.1). The improved inner product protocol for the relation (3.1) is presented in Figure 3.3.

Figure 3.3: Improved Inner-Product Argument for $\mathcal{L}_{\mathsf{IP}}$ (3.1)

$Setup(\lambda, \mathcal{L}_{\mathsf{IP}})$:

Generate following elements randomly from $\mathbb{G}$: $\mathbf{g} \xleftarrow{\$} \mathbb{G}^n, \mathbf{h} \xleftarrow{\$} \mathbb{G}^n, u \xleftarrow{\$} \mathbb{G}$

$crs = (\mathbb{G}, q, \mathbf{g}, \mathbf{h}, u, P)$, $wit = (\mathbf{a}, \mathbf{b}) \in \mathbb{Z}_q^n$, $stmt$: $P = \mathbf{g}^{\mathbf{a}} \cdot \mathbf{h}^{\mathbf{b}} \cdot u^{\langle \mathbf{a}, \mathbf{b} \rangle}$

$\langle \mathcal{P}(crs,\ stmt,\ wit), \mathcal{V}(crs,\ stmt) \rangle$:

$\mathcal{V}$: $x \xleftarrow{\$} \mathbb{Z}_q$, $\mathcal{V} \longrightarrow \mathcal{P}$: x

$\mathcal{P}$:

(i) $P' = P \cdot U^{x \cdot c}$

Set $crs' = (\mathbb{G}, q, \mathbf{g}, \mathbf{h}, u^x, P')$, $wit' = (\mathbf{a}, \mathbf{b}) \in \mathbb{Z}_q^n$

Run Protocol 3.2 with $\langle \mathcal{P}(crs',\ stmt,\ wit'), \mathcal{V}(crs',\ stmt) \rangle$

Therefore, the above is the improved inner product argument for the language defined in (3.1). We now state the Inner-Product Argument which shows that Protocol 3.3 is a proof system for (3.1).

Theorem 3.1 (Improved Inner-Product Argument) *The argument presented in Figure 3.3 for the relation (3.1) has perfect completeness and statistical witness-extended-emulation for either extracting a non-trivial discrete logarithm relation between $\boldsymbol{g}, \boldsymbol{h}, u$ or extracting a valid witness $\boldsymbol{a}, \boldsymbol{b}$.*

Proof: The proof for above theorem is given in Appendix B in [2].

3.2 Bulletproofs

Bulletproofs [2] is the state-of-art range proof with logarithmic communication size. A range proof is a zero-knowledge proof showing that a given number lies in a particular range without revealing the number itself. In this section, we review the logarithmic range proof protocol presented in Bulletproofs paper.

We wish to design a proof system for the following relation which is equivalent to the range proof language

$$\mathcal{L}_{\mathsf{BP}} = \Big\{(\underbrace{g, h, V \in \mathbb{G},\ n \in \mathbb{N}}_{crs};\ \underbrace{v, \gamma \in \mathbb{Z}_q}_{wit}) :\ \underbrace{V = g^v h^\gamma \wedge v \in [0, 2^n)}_{stmt}\Big\} \tag{3.5}$$

Let $\mathbf{a}_L = (a_1, \ldots, a_n) \in \{0,1\}^n$ be the vector containing the bits of v, $v = \langle \mathbf{a}_L, \mathbf{2}^n \rangle$. Recall that $\mathbf{2}^n = (1, 2^1, 2^2, \ldots, 2^{n-1}) \in \mathbb{Z}_q^n$. Prover $\mathcal{P}$ convinces the verifier that $v \in [0, 2^n - 1]$ by proving that:

1. It knows $\mathbf{a}_L \in \mathbb{Z}_q^n,\ v, \gamma \in \mathbb{Z}_q$ such that $V = g^v h^\gamma$
2. $\langle \mathbf{a}_L, \mathbf{2}^n \rangle = v$ and $\mathbf{a}_L \circ \mathbf{a}_R = \mathbf{0}^n$ and $\mathbf{a}_R = \mathbf{a}_L - \mathbf{1}^n$

To do so, we take a random linear combination (chosen by the verifier) of the constraints to use inner-product argument. Note that $\langle \mathbf{b}, \mathbf{y}^n \rangle = 0, y \in \mathbb{Z}_q \implies \mathbf{b} = \mathbf{0}^n$. For randomly chosen $y, z \in \mathbb{Z}_q$, we can write:

$$z^2 \cdot \langle \mathbf{a}_L, \mathbf{2}^n \rangle + z \cdot \langle \mathbf{a}_L - \mathbf{1}^n - \mathbf{a}_R, \mathbf{y}^n \rangle + \langle \mathbf{a}_L, \mathbf{a}_R \circ \mathbf{y}^n \rangle = z^2 \cdot v \tag{3.6}$$

$$\implies \Big\langle \underbrace{\mathbf{a}_L - z \cdot \mathbf{1}^n}_{\text{left secret}},\ \underbrace{\mathbf{y}^n \circ (\mathbf{a}_R + z \cdot \mathbf{1}^n) + z^2 \cdot \mathbf{2}^n}_{\text{right secret}} \Big\rangle = z^2 + \delta(y, z) \tag{3.7}$$

$$\delta(y, z) = (z - z^2) \cdot \langle \mathbf{1}^n, \mathbf{y}^n \rangle - z^3 \langle \mathbf{1}^n, \mathbf{2}^n \rangle$$

Here, $\delta(y, z) \in \mathbb{Z}_q$ is a quantity that the verifier can easily calculate since $y, z \in \mathbb{Z}_q^\star$ are the challenges generated by the verifier himself. So now, if the prover could send to the verifier the two vectors in the inner product in (3.7), then the verifier could check (3.7) using the commitment V to v. Thus the verifier would be convinced that $v \in [0, 2^n - 1]$. But there's an issue in this approach. The verifier can easily extract $\mathbf{a}_L$ from the first vector which the prover sent for calculation of inner product.

To address this issue, by introducing two additional blinding terms $\mathbf{s}_L, \mathbf{s}_R \in \mathbb{Z}_q^n$ to blind these vectors. Thus, we define two linear vector polynomials $l(X), r(X) \in \mathbb{Z}_q^n[X]$ and $t(X) \in \mathbb{Z}_q$ as follows:

$$\begin{aligned} l(X) &= \mathbf{a}_L - z \cdot \mathbf{1}^n + \mathbf{s}_L \cdot X \\ r(X) &= \mathbf{y}^n \circ (\mathbf{a}_R + z \cdot \mathbf{1}^n + \mathbf{s}_R \cdot X) + z^2 \cdot \mathbf{2}^n \\ t(X) &= \langle l(X), r(X) \rangle = t_0 + t_1 \cdot X + t_2 \cdot X^2 \end{aligned}$$

where

$$t_0 = \left\langle \mathbf{a}_L - z \cdot \mathbf{1}^n,\ \mathbf{y}^n \circ (\mathbf{a}_R + z \cdot \mathbf{1}^n) + z^2 \cdot \mathbf{2}^n \right\rangle$$
$$t_1 = \left\langle \mathbf{a}_L - z \cdot \mathbf{1}^n, \mathbf{y}^n \circ \mathbf{s}_R \right\rangle + \left\langle \mathbf{s}_L, \left(\mathbf{y}^n \circ (\mathbf{a}_R + z \cdot \mathbf{1}^n) + z^2 \cdot \mathbf{2}^n \right) \right\rangle$$
$$t_2 = \left\langle \mathbf{s}_L, \mathbf{y}^n \circ \mathbf{s}_R \right\rangle$$

The constant term of $\langle l(X), r(X) \rangle$ are the required inner product vectors in (3.7). Now, the prover can publish $l(x)$ and $r(x)$ for one $x \in \mathbb{Z}_q$. The constant term of $t(X)$, denoted t_0, is the result of the inner product. The prover needs to convince the verifier that this t_0 satisfies $t_0 = z^2 \cdot v + \delta(y, z)$. To so do, the prover commits to the remaining coefficients of $t(X)$, $t_1, t_2 \in \mathbb{Z}_q$.

Figure 3.4: Inner Product Range Proof for $\mathcal{L}_{\mathsf{BP}}$

Setup$(\lambda, \mathcal{L}_{\mathsf{BP}})$:

Generate following elements randomly from $\mathbb{G}$: $h \xleftarrow{\$} \mathbb{G},\ \mathbf{g}, \mathbf{h} \xleftarrow{\$} \mathbb{G}^n$

Set: (*crs, stmt, wit*) as defined in the language $\mathcal{L}_{\mathsf{BP}}(\mathbb{G}, q, \mathbf{g}, \mathbf{h}, h)$ in (3.5)

$\langle \mathcal{P}$(*crs, stmt, wit*)$, \mathcal{V}$(*crs, stmt*)$\rangle$:

$\mathcal{P}$:

(i) Set $\mathbf{a}_L \in \{0,1\}^n$ such that $\mathbf{a}_L, \mathbf{2}^n = v$

(ii) Set $\mathbf{a}_R = \mathbf{a}_L - \mathbf{1}^n \in \mathbb{Z}_q^n$

(iii) $\alpha \leftarrow \mathbb{Z}_q$

(iv) $A = h^\alpha \mathbf{g}^{\mathbf{a}_L} \mathbf{h}^{\mathbf{a}_R}$

(v) $\mathbf{s}_L, \mathbf{s}_R \leftarrow \mathbb{Z}_q^n$

(vi) $\rho \leftarrow \mathbb{Z}_q$

(vii) $S = h^\rho \mathbf{g}^{\mathbf{s}_L} \mathbf{h}^{\mathbf{s}_R}$

(viii) $\tau_1, \tau_2 \leftarrow \mathbb{Z}_q$

(ix) $T_i = g^{t_i} h^{\tau_i},\ i \in \{1, 2\}$

$\mathcal{P} \longrightarrow \mathcal{V}$: $A, S, T_1, T_2 \in \mathbb{G}$

$\mathcal{V}$: $x, y, z \xleftarrow{\$} \mathbb{Z}_q$, $\mathcal{V} \longrightarrow \mathcal{P}$: x, y, z

$\mathcal{P}$:

(i) $\mathbf{l} = l(x) = \mathbf{a}_L - z \cdot \mathbf{1}^n + \mathbf{s}_L \cdot x \ \in \mathbb{Z}_q^n$

(ii) $\mathbf{r} = r(x) = \mathbf{y}^n \circ (\mathbf{a}_R + z \cdot \mathbf{1}^n + \mathbf{s}_R \cdot X) + z^2 \cdot \mathbf{2}^n \ \in \mathbb{Z}_q^n$

(iii) $\hat{t} = \langle \mathbf{l}, \mathbf{r} \rangle \in \mathbb{Z}_q$

(iv) $\tau_x = \tau_2 \cdot x^2 + \tau_1 \cdot x + z^2 \cdot \gamma \in \mathbb{Z}_q$

(v) $\mu = \alpha + \rho \cdot x \in \mathbb{Z}_q$

$\mathcal{P} \longrightarrow \mathcal{V}$: $\mathbf{l}, \mathbf{r} \in \mathbb{Z}_q^n,\ \tau_x, \mu, \hat{t} \in \mathbb{Z}_q$

$\mathcal{V}$:

i $h'_i = h_i^{(y^{-i+1})} \in \mathbb{G}, \forall i$

(ii) $P = A \cdot S^x \cdot \mathbf{g}^{-z} \cdot (\mathbf{h}')^{z \cdot \mathbf{y}^n + z^2 \cdot \mathbf{2}^n} \in \mathbb{G}$

(iii) $\hat{t} \stackrel{?}{=} \langle \mathbf{l}, \mathbf{r} \rangle \in \mathbb{Z}_q$ // *$\hat{t}$ was computed correctly*

(iv) $g^{\hat{t}} h^{\tau_x} \stackrel{?}{=} V^{z^2} \cdot g^{\delta(y,z)} \cdot T_1^x \cdot T_2^{x^2}$ // *$\hat{t}$ satisfies $t_0 + t_1 x + t_2 x^2$*

(v) $P \stackrel{?}{=} h^\mu \cdot \mathbf{g}^{\mathbf{l}} \cdot (\mathbf{h}')^{\mathbf{r}}$ // *Check if $\boldsymbol{\ell} = l(x)$ and $\mathfrak{r} = r(x)$*

Some key observations from the above protocol in Figure 3.4 are:

1. In this protocol, $\mathcal{P}$ sends $(\mathbf{l}, \mathbf{r})$, whose size is linear in n.
2. The total communication cost in this protocol is $2n + 4$ elements in $\mathbb{G}$ and 3 elements in $\mathbb{Z}_q$.
3. The verifier, to check if the received $\mathbf{l}, \mathbf{r}$ are actually $l(x), r(x)$ and also check if $t(x) = \langle \mathbf{l}, \mathbf{r} \rangle$, first, switches the generators of the commitment from $\mathbf{h} \in \mathbb{G}^n$ to $\mathbf{h}' = \mathbf{h}^{(\mathbf{y}^{-n})}$.
4. Now, A becomes a Pedersen vector commitment to $(\mathbf{a}_L, \mathbf{a}_R \circ \mathbf{y}^n)$ w.r.t $(\mathbf{g}, \mathbf{h}', h)$. Similarly, S is now a Pedersen vector commitment to $(\mathbf{s}_L, \mathbf{s}_R \circ \mathbf{y}^n)$.
5. If all three conditions marked by red rectangle result in a positive answer to the verifier, (s)he accepts and thus prover succeeds.

Theorem 3.2 (Range Proof) *The range proof presented above has perfect completeness, perfect special honest verifier zero-knowledge, and computational witness extended emulation.*

Proof: The range proof is a special case of the aggregated range proof with $m = 1$. Refer Appendix C in [2] for the proof of the theorem 3.2.

In the above protocol, prover had to transfer $\mathbf{l} \in \mathbb{Z}_q^n$ and $\mathbf{r} \in \mathbb{Z}_q^n$ resulting in proof size proporional to $2n$. For a proof whose size is logarithmic in n, we can eliminate the transfer of $\mathbf{l}$ and $\mathbf{r}$ using the inner-product argument. We also observe that vectors $(\mathbf{l}, \mathbf{r})$ are not secret and hence a protocol that only provides soundness* is sufficient.

Concretely, observe that verifying first and third checks of the protocol in Figure 3.4 is the same as verifying that the witness $(\mathbf{l}, \mathbf{r})$ satisfies the inner product relation (3.1) on public input $(\mathbf{g}, \mathbf{h}', Ph^{-\mu}, \hat{t})$, where P is a Pedersen vector commitment to vectors $\mathbf{l}, \mathbf{r} \in \mathbb{Z}_q^n$ whose inner product is $\hat{t}$.

$$P \stackrel{?}{=} h^{\mu} \cdot \mathbf{g}^{\mathbf{l}} \cdot (\mathbf{h}')^{\mathbf{r}}$$

$$\hat{t} \stackrel{?}{=} \langle \mathbf{l}, \mathbf{r} \rangle \in \mathbb{Z}_q$$

$$\{(\mathbf{g}, \mathbf{h} \in \mathbb{G}^n, P \in \mathbb{G}, c \in \mathbb{Z}_q;\ \mathbf{a}, \mathbf{b} \in \mathbb{Z}_q^n) :\ P = \mathbf{g}^{\mathbf{a}}\mathbf{h}^{\mathbf{b}} \wedge c = \langle \mathbf{a}, \mathbf{b} \rangle\}$$

Thus, using the inner product argument, the total communication cost reduces down to $2\lceil \log_2(n) \rceil + 4$ elements in $\mathbb{G}$ and 5 elements in $\mathbb{Z}_q$.

3.3 Omniring

Omniring [28] was proposed as a novel RingCT scheme which (i) did not require any trusted setup, (ii) had a proof size logarithmic in the size of the ring, and (iii) allowed to share the same ring between all source addressed in a transaction, thereby enabling significantly improved privacy. Omniring provides key insights in the direction of generalising Bulletproofs [2] framework for RingCT applications.

3.3.1 Main Idea

We start with a ring $\mathcal{R} = (P_1, P_2, \ldots, P_n) \in \mathbb{G}^n$ of public keys of the form $P_i = g^{x_i}$ where $g \in \mathbb{G}$ is the group generator and $x_i \in \mathbb{Z}_q$ is the secret key, for all $i \in [n]$. Suppose we own the public keys (addresses) at indices $(i_1, i_2, \ldots, i_s)$ such that each $i_j \in [n]$ for all $j \in [s]$ and $s < n$. This implies that we know the secret keys $\mathbf{x} = (x_{i_1}, x_{i_2}, \ldots, x_{i_s})$. We would like to prove the knowledge of tuples (i_j, x_{i_j}) for $j \in [s]$. For all $j \in [s]$, let us define unit vectors $\mathbf{e}_j \in \{0, 1\}^n$ such that it has 1

Soundness is the property of only being able to prove "true" things. *Completeness* is the property of being able to prove all true things. [30]

only in position i_j. Therefore,

$$1 = g^{-x_j}\mathcal{R}^{\mathbf{e}_j} \text{ for all } j \in [s].$$

Combining the above equations using powers of a scalar challenge $u \in \mathbb{Z}_q$, we get

$$\implies 1 = g^{-\langle \boldsymbol{u}^s, \mathbf{x}\rangle} \cdot \mathcal{R}^{\sum_{j=1}^{s} u^{j-1}\mathbf{e}_j}. \tag{3.8}$$

We will refer to equation (3.8) as the *main equality*. By embedding the bases $(g\|\mathcal{R})$ and secrets $\mathbf{a} = (\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_s, x_{i_1}, x_{i_2}, \ldots, x_{i_s})$ in the main equality in form of a Pedersen vector commitment, we can build an inner product relationship between the secrets and challenges to use the Bulletproofs framework in building a RingCT. However, the soundness of Bulletproofs is guaranteed because the elements in the base vectors used in the Pedersen vector commitment are independent and uniformly random. In this case, the prover might know the discrete log relationship between some elements in $\mathcal{R}$ since he owns multiple addresses. To overcome this, Lai et al. [28] proposed using base vector of the form:

$$\mathbf{g}_w \coloneqq (g\|\mathcal{R})^{\circ w} \circ \mathbf{p},$$

where $w \in \mathbb{Z}_q$ and $\mathbf{p} \xleftarrow{\$} \mathbb{G}^{n+1}$. A PPT adversary cannot find the discrete log relation between the elements of the newly constructed base $\mathbf{g}_w$ (refer to Lemma 4.1). Note that $\mathbf{g}_w^{\mathbf{a}} = \mathbf{g}_{w'}^{\mathbf{a}}$ for any $w, w' \in \mathbb{Z}_q$ because of the main equality. Therefore, using the new base vector for two different challenges $w, w' \in \mathbb{Z}_q$, we can run a Bulletproofs-like protocol twice and extract the secret vectors. This ensures that the Bulletproofs-like protocol preserves soundness.

3.4 Overview of MimbleWimble & Grin

MimbleWimble [9] is a blockchain protocol relying only on elliptic curve cryptography and promises to provide scalability, privacy and fungibility all at once. Interestingly, MimbleWimble does not have any addresses or accounts and the amounts also are hidden using Pedersen commitments. The simple design of MimbleWimble coupled with its privacy and scalability guarantees (which most existing blockchain implementations fail to address collectively) is what makes it popular for use in decentralized systems. Grin [7] and Beam [8] are cryptocurrency systems which are powered by the MimbleWimble protocol.

3.4.1 Outputs in Grin

Amounts in MimbleWimble are hidden in Pedersen commitments known as *outputs*. An output containing amount $a \in \{0, 1, \ldots, 2^{64} - 1\}$ is of the form[†] $P = kG + aH$ where $r \in \mathbb{Z}_q$ and G, H are generators of $\mathbb{G}$. Note that the discrete log relation between G and H is assumed to be unknown. The quantity k is called as the *blinding factor* and it serves as the secret key of output P. Knowledge of k implies ownership of the output. At the time of creation of new outputs, they are accompanied by a range proof which proves that the amounts hidden in them lie in a finite range.

3.4.2 Transactions in Grin

A Grin transaction consists of some inputs which are being spent and some outputs in which the funds would be deposited. Note that the inputs in a transaction are essentially outputs which were generated in some block in the past. Grin transactions are of two types: (i) coinbase transactions, which transfer the block mining rewards to miners and (ii) regular transactions, which makes up for amount transfers in non-miners. Coinbase transactions do not contain any inputs and typically have just one output known as *coinbase* output. A regular Mimblewimble transaction [31] includes the following:

(i) A set of inputs, that reference and spend a set of previous outputs,

(ii) A set of new outputs each with a range proof,

(iii) An transaction fee,

(iv) A Schnorr signature whose private key is computed by taking the excess amount (the sum of all output amounts plus the fee, minus the input amounts),

(v) The public key published with the Schnorr signature is known as *kernel excess* and is of the form $rG,\ r \in \mathbb{Z}_q$.

We will refer to the collection of transaction fee, public key and signature as a *kernel*. Note that a transaction can easily be validated by determining that the kernel excess is a valid public key.

[†]We use additive notation in this section only to be consistent with the original MimbleWimble protocol in [9].

3.4.3 Transaction Aggregation

Transactions in a Grin block are aggregated before the block is added to the blockchain to ensure unlinkability in inputs and outputs. Consider the following example where we intend to aggregate two given regular transactions.

	Inputs	Outputs	Excesses	Fee
Tx#1	I_1, I_2	O_1	K_1	f_1
Tx#2	I_3	O_2	K_2	f_2
Aggregated Tx	I_1, I_2, I_3	O_1, O_2	K_1, K_2	$f_1 + f_2$

Table 3.1: Example of Transaction Aggregation

However, there is a slight issue here. It is possible to try all combinations of inputs and outputs to recover one of the transactions where the equality

$$\sum(\text{Ouputs}) + \sum(\text{Fees})H - \sum(\text{Inputs}) = \text{Kernel excess}$$

satisfies. To mitigate this, the kernel excess is redefined from rG to $(r - k_{\text{offset}})G$ where r is the sum of output amounts plus fees minus input amounts and k_{offset} is a scalar in Z_q. Thus, The kernel offset k_{offset} is thus a blinding factor that needs to be added to the excess value to ensure the commitments sum to zero as:

$$\sum(\text{Ouputs}) + \sum(\text{Fees})H - \sum(\text{Inputs}) = \text{Kernel excess} + k_{\text{offset}}G$$

Further, if in a same block, some outputs generated in a transaction are spent in the same block in another transaction, we can drain those outputs and inputs from the block as the structure of each transaction does not actually matter. As long as the sum of inputs and outputs cancels off, removing matching outputs would not matter. This is known as *cut-through*.

Going a step further, all the blocks on the Grin blockchain can be considered as individual transactions. Thus, the outputs generated at block height h_1 which are spent as inputs in block h_2 also could be removed from the blockchain as they can be considered intermediate transactions by the same logic as above. This significantly improves scalability of MimbleWimble-based blockchains.

3.4.4 Grin Blocks

Let $\mathbb{G}$ be the secp256k1 elliptic curve group of order n. In Grin, coins are stored in Pedersen commitments of the form $C = kG + aH$ where $k, a \in \mathbb{F}_n$ are scalars and

$G, H \in \mathbb{G}$ are the generators of $\mathbb{G}$ with an unknown discrete logarithm with respect to each other. The quantity a is the amount stored in C and k is a randomly chosen scalar known as the blinding factor. A Grin block consists of the following:

(i) A block header from which a scalar $k_{\text{off}} \in \mathbb{F}_n$ called the *kernel offset* can be derived. The other header fields are not relevant to our discussion.

(ii) A list of L input commitments $I_1, I_2, \ldots, I_L$. This list is empty for blocks without regular transactions. Each input commitment refers to an output commitment in a previous block.

(iii) A list of M output commitments $O_1, O_2, \ldots, O_M$ where $M \geq 1$. Each output commitment is tagged as either a coinbase output or a regular transaction output. Each output commitment is also accompanied by a range proof to prove that it commits to an amount in the range $\{0, 1, 2, \ldots, 2^{64} - 1\}$.

(iv) A list of N *transaction kernels* each of which contains a fee amount $f_i \in \mathbb{F}_n$ and a curve point $X_i \in \mathbb{G}$ called the *kernel excess*. Each kernel also contains a Schnorr signature proving that X_i is of the form $x_i G$ for some $x_i \in \mathbb{F}_n$. Each transaction kernel is also tagged either as a coinbase kernel or a regular transaction kernel.

Let $\mathcal{I}_{\text{cb}} \subset \{1, 2, \ldots, M\}$ be the set of indices corresponding to coinbase outputs and $\mathcal{I}_{\text{cb}}^c$ be the set of the remaining indices corresponding to RTOs. Let $\mathcal{I}_{\text{ck}} \subset \{1, 2, \ldots, N\}$ be the set of indices corresponding to coinbase kernels and $\mathcal{I}_{\text{ck}}^c$ be the set of indices corresponding to regular transaction kernels. A valid block has to satisfy the following equations.

$$\sum_{i \in \mathcal{I}_{\text{cb}}} O_i - \left(\sum_{i=1}^{N} f_i \right) H - rH = \sum_{i \in \mathcal{I}_{\text{ck}}} X_i, \tag{3.9}$$

$$\sum_{i \in \mathcal{I}_{\text{cb}}^c} O_i + \left(\sum_{i=1}^{N} f_i \right) H - \sum_{i=1}^{L} I_i = \sum_{i \in \mathcal{I}_{\text{ck}}^c} X_i + k_{\text{off}} G. \tag{3.10}$$

Here $r = 60 \times 10^9$ is the block subsidy in nanogrin units. As the right hand sides of both equations are commitments to the zero amount, the binding property of Pedersen commitments and the range proofs imply that

(i) the total amount in the coinbase outputs of a block is $r + \sum_{i=1}^{N} f_i$ and

(ii) the sum of the input amounts is equal to the sum of the transaction fees and RTO amounts.

3.5 Revelio - A MimbleWimble Proof of Reserves Protocol

Revelio [1] was the first proof of reserves protocol for MimbleWimble backed cryptocurrencies. It uses non-interactive zero-knowledge (NIZK) proofs of knowledge for proving statements involving discrete logarithms which were originally introduced by [32].

3.5.1 Proving Statements About Discrete Logarithms

Let $\mathbb{G}$ be a cyclic group of prime order q. Let G, G', H be the generators of the group such that the discrete log relation between any two of them is assumed to be unknown. We will use additive notation to be consistent with the notation used in Revelio paper. Revelio uses three particular types of NIZK proofs of knowledge as defined below. Let $\mathcal{H}\{0,1\}^* \rightarrow \mathbb{Z}_q$ be a cryptographic hash function modelled as a random oracle. The scalar pair $(\alpha, \beta) \in \mathbb{Z}_q^2$ is called as the representation of $X \in \mathbb{G}$ with respect to generators G, H such that $X = \alpha G + \beta H$.

Definition 3.1 *A pair of scalars $(c, s) \in \mathbb{Z}_q^2$ is a NIZK proof of knowledge of the discrete log of an element $X \in \mathbb{G}$ with respect to a generator G if they satisfy*

$$c = \mathcal{H}(G, X, sG + cX).$$

We will denote such a pair by $PoK\{\alpha \mid X = \alpha G\}$.

Definition 3.2 *A triple of scalars $(c, s_1, s_2) \in \mathbb{Z}_q^3$ is a NIZK proof of knowledge and equality of the representations of $X, Y \in \mathbb{G}$ with respect to generator pairs (G, H) and (G', H) respectively if they satisfy*

$$c = \mathcal{H}(S, s_1 G + s_2 H + cX, s_1 G' + s_2 H + cX).$$

where $S = (G \| G' \| H \| X \| Y)$. We will denote such a triple by

$$PoK\{\underbrace{(\alpha, \beta)}_{wit} \mid \underbrace{X = \alpha G + \beta H \ \wedge \ Y = \alpha G' + \beta H}_{stmt}\}.$$

Definition 3.3 *A 5-tuple of scalars $(c_1, c_2, s_1, s_2, s_3) \in \mathbb{Z}_q^5$ is a NIZK proof of either*

(i) the knowledge and equality of the representations of $X, Y \in \mathbb{G}$ with respect to generator pairs (G, H) and (G', H) respectively OR

(ii) knowledge of the discrete logarithm of the element $Y \in \mathbb{G}$ with respect to generator G',

if they satisfy

$$c_1 + c_2 = \mathcal{H}(S, V_1, V_2, V_3).$$

where $S = (G \| G' \| H \| X \| Y)$ *and*

$$\begin{aligned} V_1 &= s_1 G + s_2 H + c_1 X, \\ V_2 &= s_1 G' + s_2 H + c_1 X, \\ V_3 &= s_3 G' + c_2 Y \end{aligned}$$

We will denote such a triple by

$$PoK\{\underbrace{(\alpha, \beta, \gamma)}_{wit} \mid \underbrace{(X = \alpha G + \beta H \ \wedge \ Y = \alpha G' + \beta H) \vee (Y = \gamma G')}_{OR\ stmt}\}.$$

3.5.2 Main Idea of Revelio

To generate a Revelio proof, an exchange chooses an anonymity set $\mathcal{C}_{\text{anon}} = (C_1, C_2, \ldots, C_n)$ from the Grin blockchain. Let the set of outputs owned by the exchange be $\mathcal{C}_{\text{own}} \subset \mathcal{C}_{\text{anon}}$. For $C_i \in C_{\text{own}}$, the exchange knows the blinding factor $k_i \in \mathbb{Z}_q$ such that $C_i = k_i G + a_i H$. For each $C_i \in C_{\text{anon}}$, the exchange also publishes a tag I_i defined as

$$I_i = \begin{cases} k_i G' + v_i H & \text{if } C_i \in C_{\text{own}} \\ y_i G' & \text{if } C_i \notin C_{\text{own}} \end{cases} \tag{3.11}$$

where $y_i = \mathcal{H}(k_{\text{exch}}, C_i)$ and k_{exch} is a long-term secret key of the exchange. The reason for such a definition of y_i is that it needs to be a determinitic function of the chosen cover output C_i. This is because we need to have consistent tag for a particular cover output C_i appearing in $\mathcal{C}_{\text{anon}}$ over multiple Revelio proofs. If a randomly generated y_i was used in different Revelio proofs of the same exchange, the tags corresponding to exchange-owned outputs would remain the same while the tags of cover outputs would change. In such a case, it would be trivial to point out the outputs owned by the exchange. Therefore, we need tags to be a determinitic function of the outputs in the anonymity set. Additionally, the exchange publishes NIZK PoK $\sigma_i = (c_1^i, c_2^i, s_1^i, s_2^i, s_3^i)$ of the form

$$PoK\{(\alpha, \beta, \gamma) \mid (C_i = \alpha G + \beta H \ \wedge \ I_i = \alpha G' + \beta H) \vee (I_i = \gamma G')\}.$$

Lastly, the exchange claims that the commitment $C_{\text{assets}} = \sum_{i \in [n]} I_i$ is a Pedersen commitment to the total assets owned by the exchange. Note that the tag list $(I_1, I_2, \ldots, I_n)$ is used to check collusion among exchanges. A Revelio proof verification involves checking of any matches in tags of different exchanges and verification of the NIZK PoKs $\sigma_i \ \forall i \in [n]$.

3.5.3 Drawbacks of Revelio

The size of a Revelio proof is $5n$ elements in $\mathbb{Z}_q$ and $n + 1$ elements in $\mathbb{G}$. Privacy provided by Revelio for exchange-owned outputs directly depends on the number of cover outputs used. Larger is the size of the anonymity set, greater is the privacy of the exchange-owned outputs. An exchange would ideally like to have the entire UTXO set as the anonymity set. Since the proof sizes in Revelio increase linearly with the size of C_{anon}, setting C_{anon} to be equal to the whole UTXO set is not a scalable strategy.

The collusion-resistance property of Revelio works only if all the exchanges generate their proofs using the same blockchain state. Enabling simultaneous proof generation is necessary to avoid cheating by exchanges. By simultaneous proof generation, we mean that the exchanges must have the same anonymity sets (ideally, the whole of UTXO set at a give point in time) and different exchanges could generate proofs in synchronisation with the blockchain state. Both of the above drawbacks led us to the development of RevelioBP which alleviates the above drawbacks with a higher computational cost. We describe RevelioBP in the next chapter.

Chapter 4

RevelioBP - MimbleWimble Proof of Reserves Protocol with Short Proofs

4.1 Introduction

MimbleWimble is a design for a scalable cryptocurrency which was proposed in 2016 [33]. Beam and Grin are two implementations of the MimbleWimble protocol which are available on several exchanges [21]. Revelio [1] was the first proof of reserves protocol for MimbleWimble coins which provided some privacy to exchanges by hiding the exchange-owned outputs inside an anonymity set of outputs. As the anonymity set is revealed as part of the proof of reserves, a larger anonymity set results in better privacy for the exchange. Since the Revelio proof size scales linearly with the anonymity set, it becomes an impediment in scaling the anonymity set to the set of all unspent transaction outputs (UTXOs). To solve the scalability issue of Revelio, we designed RevelioBP leveraging the Bulletproofs [2] framework, resulting in the proof size being logarithmic in the anonymity set size.

4.2 Our Contribution

In this chapter, we present RevelioBP, a proof of reserves protocol for MimbleWimble with proof sizes scaling *logarithmcally* in the size of the anonymity set and *linearly* in the size of the exchange-owned output set. This makes it feasible to choose the anonymity set to be the set of all UTXOs on the blockchain. To make quantitative comparisons, we have implemented RevelioBP in Rust. At the time of writing this

paper, the number of UTXOs on the Grin blockchain is approximately 161,000 [34]. A Revelio proof of reserves for this anonymity set will have size 32 MB as against 0.27 MB using RevelioBP instead.* This reduction in proof size, however, comes at the cost of larger proof generation and verification times. If an exchange is willing to compromise on the size of the proof and is required to give frequent proofs of reserves, Revelio serves as a better choice. If proof sizes are critical for an exchange and it is willing to spend more time generating the proof, RevelioBP clearly outperforms Revelio. In conclusion, we quantitatively highlight the trade-off between proof size and performance in using Revelio and RevelioBP, both of which are based on the discrete log assumption.

4.3 Outputs in MimbleWimble

In MimbleWimble, coins are stored in outputs which consist of Pedersen commitments of the form $C = g^r h^a \in \mathbb{G}$ where $g, h \in \mathbb{G}$ and $r, a \in \mathbb{Z}_q$. Here a represents the amount of coins stored in the output and r is a blinding factor. Each commitment is accompanied by a range proof which proves that the amount a lies in the range $\{0, 1, 2, \ldots, 2^{64} - 1\}$.

The group elements g and h are assumed to have an unknown discrete logarithm relationship. For example, in Grin $\mathbb{G}$ is the secp256k1 elliptic curve group, g is the base point of the secp256k1 curve, and h is obtained by hashing g with the SHA256 hash function [36]. The unknown discrete logarithm relationship makes the commitment computationally binding, i.e. a polynomial-time adversary cannot find $r' \neq r$ and $a' \neq a$ such that $C = g^r h^a = g^{r'} h^{a'}$.

To spend an output having the commitment $C = g^r h^a$, knowledge of the blinding factor r is required [31]. As spending ability is equivalent to ownership, a proof of reserves protocol for MimbleWimble involves a proof of knowledge of blinding factors of several outputs.

4.4 From Omniring to RevelioBP

In Monero, source addresses in a transaction are obfuscated using ring signatures and the amounts are hidden in Pedersen commitments [37]. The current transaction structure in Monero, called *ring confidential transaction (RingCT)*, has proof sizes

*Under the assumption that the exchange owns 5% of all UTXOs.
Some sections of this chapter originally appeared in [35].

which scale linearly in the ring size. Omniring [28] is a recent proposal for RingCTs with proof sizes which scale logarithmically in the ring size. It relies on Bulletproofs [2] to achieve this size reduction. Given a ring $\mathcal{R} = (R_1, R_2, \ldots, R_n)$ of public keys where $R_i = h^{x_i}$ for $h \in \mathbb{G}, x_i \in \mathbb{Z}_q$, the Omniring construction enables a prover to prove knowledge of the private keys $x_{i_1}, x_{i_2}, \ldots, x_{i_m}$ corresponding to a subset $\mathcal{R}_\mathcal{S}$ of $\mathcal{R}$ without revealing $\mathcal{R}_\mathcal{S}$. For each public key R_j in this subset $\mathcal{R}_\mathcal{S}$, the prover also outputs a *tag* given by $tag_j = g^{x_j^{-1}}$ for $g \in \mathbb{G}$. This tag is used to detect double spending from a source address.

The design of RevelioBP is inspired by the Omniring construction. Given the set of UTXOs $\mathcal{C}_{\text{utxo}} = (C_1, C_2, \ldots, C_n)$ on the blockchain where $C_i = g^{r_i} h^{a_i}$ for some $r_i, a_i \in \mathbb{Z}_q$, the prover in RevelioBP proves knowledge of blinding factors r_i and amounts a_i for all C_i in a subset $\mathcal{C}_{\text{own}}$ of $\mathcal{C}_{\text{utxo}}$ without revealing $\mathcal{C}_{\text{own}}$. For each output $C_j \in \mathcal{C}_{\text{own}}$, the prover outputs a tag $tag_j = g_t^{r_j} h^{a_j}$ where $g_t \in \mathbb{G}$ is a randomly chosen group element. In RevelioBP, the tag has a dual role. Firstly, it is used to detect output sharing between exchanges. Secondly, the product of the tags is a Pedersen commitment to the total reserves of the exchange.

4.5 RevelioBP Proof of Reserves Protocol

To spend a MimbleWimble output having the commitment $C = g^r h^a$, knowledge of the blinding factor r is required [31]. Technically, the ability to spend an output also requires knowledge of the amount a. But the amount can be at most $2^{64} - 1$, and hence can be found by brute force search given C and r.

Let $\mathcal{C}^t_{\text{utxo}}$ be the set of UTXOs on a MimbleWimble blockchain after the block with height t has been mined. An exchange will own a subset $\mathcal{C}^t_{\text{own}} \subset \mathcal{C}^t_{\text{utxo}}$, where ownership implies knowledge of the blinding factor for each output $C \in \mathcal{C}^t_{\text{own}}$. Using the RevelioBP protocol, the exchange can construct a Pedersen commitment C_{res} to an amount which is equal to the sum of the amounts commited to by each of the outputs in $\mathcal{C}^t_{\text{own}}$. Given a Pedersen commitment C_{liab} to the total liabilities of the exchange, it can give a proof of solvency via a range proof which shows that the amount committed to in $C_{\text{res}} C_{\text{liab}}^{-1}$ is non-negative. If there is no suitable method to construct $\mathcal{C}_{\text{liab}}$, then the exchange can reveal a base amount a_{base} and prove that $\mathcal{C}_{\text{res}} h^{-a_{\text{base}}}$ is a commitment to a non-negative amount.

While RevelioBP does not reveal $\mathcal{C}^t_{\text{own}}$, it does reveal its cardinality $s_t = |\mathcal{C}^t_{\text{own}}|$. We give a reasonable workaround for this issue in Section 4.5.1.

If the Decisional Diffie-Hellman (DDH) assumption holds in the group $\mathbb{G}$, the RevelioBP proof of reserves protocol satisfies the following properties:

- *Inflation resistance*: Using RevelioBP, a probabilistic polynomial time (PPT) exchange will not be able to generate a commitment to an amount which exceeds the reserves it actually owns.
- *Collusion detection*: Situations where two exchanges share an output while generating their respective RevelioBP proofs will be detected.
- *Output privacy*: A PPT adversary who observes RevelioBP proofs from an exchange cannot do any better than random guessing while identifying members of C^t_{own}.

The security proofs are given in Section 4.7.

4.5.1 Proof Generation

The RevelioBP protocol requires one randomly chosen group element $g_t \in \mathbb{G}$ per block such that the discrete log relation between g_t and g, h is unknown. All the exchanges need to agree upon the procedure used to generate the sequence of g_ts. For example, g_t could be generated by hashing the contents of the block at height t. An exchange giving a RevelioBP proof of its reserves at the block with height t performs the following procedure:

1. From the UTXO set C^t_{utxo} at block t, the exchange constructs the vector $\mathbf{C} = (C_1, C_2, \ldots, C_n)$ where the C_is are all the UTXOs arranged in the order of their appearance on the blockchain. So $n = |C^t_{\text{utxo}}|$. To keep the notation simple, we do not make the dependence of $\mathbf{C}$ and n on t explicit.
2. The exchange owns a subset $C^t_{\text{own}} = \{C_{i_1}, C_{i_2}, \ldots, C_{i_s}\}$ of C^t_{utxo} where $1 \leq i_1 < \cdots < i_s \leq n$. For each $C_{i_j} \in C^t_{\text{own}}$, the exchange knows r_j and a_j such that $C_{i_j} = g^{r_j} h^{a_j}$. Using this information, the exchange constructs the *tag vector* $\mathbf{I} = (I_1, I_2, \ldots, I_s)$ where $I_j = g_t^{r_j} h^{a_j}$. Note that I_j is a Pedersen commitment to the amount a_j with blinding factor r_j using bases g_t, h. So the only difference between C_{i_j} and I_j is that the base g in the former is replaced with g_t in the latter.
3. Let $\mathbf{a} = (a_1, a_2, \ldots, a_s)$ and $\mathbf{r} = (r_1, r_2, \ldots, r_s)$ be the amount and blinding factor vectors corresponding to the exchange-owned outputs. Let $\mathbf{e}_{i_j} \in \{0,1\}^n$

be the unit vector with a 1 in position i_j and 0s everywhere else. Let $\mathcal{E} \in \{0,1\}^{s \times n}$ be the matrix with $\mathbf{e}_{i_1}, \mathbf{e}_{i_2}, \ldots, \mathbf{e}_{i_s}$ as rows.

The exchange publishes $(t, \mathbf{I})$ and generates a zero-knowledge argument of knowledge Π_{RevBP} of quantities $(\mathcal{E}, \mathbf{a}, \mathbf{r})$ such that for all $j = 1, 2, \ldots, s$

$$\mathbf{C}^{\mathbf{e}_{i_j}} = g^{r_j} h^{a_j}, \; I_j = g_t^{r_j} h^{a_j}. \tag{4.1}$$

4. The exchange publishes its RevelioBP proof as $(t, \mathbf{I}, \Pi_{\mathsf{RevBP}})$ and claims that $C_{\text{res}} = \prod_{j=1}^{s} I_j$ is a Pedersen commitment to its reserves $\sum_{j=1}^{s} a_j$.

Note that the tag I_j is a deterministic function of the output C_{i_j} at a given block height t. So if two exchanges try to use the same output in their respective RevelioBP proofs, the same tag I_j will appear in both their proofs, revealing the collusion.

The reason for changing the base g_t with the block height is to change the tag of the same output across RevelioBP proofs at different block heights. If g_t were unchanged (as in Revelio [1]), then the appearance of the same tag in two RevelioBP proofs at different block heights will reveal that some exchange-owned output has remained unspent between these two block heights.

As C_{res} is a Pedersen commitment with respect to bases g_t and h, the Pedersen commitment C_{liab} to the exchange's liabilities should also be generated using these bases. Otherwise, it will be not be possible to generate a range proof on $C_{\text{res}} C_{\text{liab}}^{-1}$.

The proof reveals the cardinality s of $\mathcal{C}_{\text{own}}^{t}$. An exchange which wants to hide the number of outputs it owns can create some outputs which commit to the zero amount and use these to pad the outputs with non-zero amounts. For example, suppose that the number of outputs owned by the exchange is expected to be in the range 600 to 1000. It can create 400 outputs which commit to the zero amount and use these to always pad the number s revealed in the proof to be always 1000. Of course, the exchange would need to spend a nominal amount as transaction fees in creation of such outputs.

Finally, note that an exchange can under-report its reserves by excluding an output it owns from the subset $\mathcal{C}_{\text{own}}^{t}$ used to generate the RevelioBP proof. An exchange may choose to do this if its liabilities are much lower than its reserves.

4.5.2 Proof Verification

Given a RevelioBP proof of reserves $(t, \mathbf{I}, \Pi_{\mathsf{RevBP}})$ from an exchange referring to the block height t, the verifier performs the following procedure:

1. First, it reads the set of all UTXOs at block height t and forms the vector $\mathbf{C} = (C_1, \ldots, C_n)$ such that C_is are listed in the order of their appearance on the blockchain.

2. It verifies the argument of knowledge Π_{RevBP} by checking that the verification equations described in Figure 4.6 hold.

3. Finally, the verifier checks if any of the tags in the $\mathbf{I}$ vector appear in another exchange's RevelioBP proof for the same block height t. If the same tag I_j appears in the RevelioBP proofs of two different exchanges, then collusion is declared and the proofs is considered invalid.

4.6 ZK Argument of Knowledge Π_{RevBP}

Let $\mathbf{C} = (C_1, \ldots, C_n)$ be the vector representation of the UTXO set $\mathcal{C}^t_{\text{utxo}}$ at block height t. Let $\mathbf{I} = (I_1, \ldots, I_s)$ be the tag vector published by the exchange as part of the RevelioBP proof. The exchange constructs an argument of knowledge Π_{RevBP} to convince a verifier of the following:

(i) It knows r_j and a_j such that $I_j = g_t^{r_j} h^{a_j}\ \forall j \in [s]$.

(ii) $\exists i_j \in [n]$ (index) such that $C_{i_j} = g^{r_j} h^{a_j}\ \forall j \in [s]$.

Note that while the existence of the indices i_j will be proved by Π_{RevBP}, the indices themselves are not revealed. Since the term h^{a_j} is common in both equations in the above statements, combining the two equations, we can equivalently state that the exchange knows r_j and i_j such the following holds for all $j \in [s]$

$$I_j g_t^{-r_j} = C_{i_j} g^{-r_j}. \tag{4.2}$$

In other words, knowledge of r_j and i_j for all $j \in [s]$ suffices for an honest exchange to construct Π_{RevBP}.

Consider the language $\mathcal{L}_{\mathsf{RevBP}}$ given in (4.3) where $\mathbf{r} = (r_1, r_2, \ldots, r_s) \in \mathbb{Z}_q^s$ and $\mathbf{e}_{i_j} \in \{0,1\}^n$ is a unit vector with a 1 at index i_j and zeros everywhere else. The language depends on the common reference string $crs = \{\mathbb{G}, q, g, h, g_t\}$.

$$\mathcal{L}_{\mathsf{RevBP}} = \left\{ (\mathbf{C}, \mathbf{I}) \;\middle|\; \begin{array}{l} \exists (\mathbf{r}, \mathbf{e}_{i_1}, \ldots, \mathbf{e}_{i_s}) \text{ such that} \\ I_j g_t^{-r_j} = \mathbf{C}^{\mathbf{e}_{i_j}} g^{-r_j}\ \forall j \in [s] \end{array} \right\} \tag{4.3}$$

To leverage the Bulletproofs framework for the construction of a *log*-sized argument of knowledge for the language $\mathcal{L}_{\mathsf{RevBP}}$, we need to do the following:

(i) Embed the secrets $(\mathbf{r}, \mathbf{e}_{i_1}, \ldots, \mathbf{e}_{i_s})$ as the exponents in a Pedersen vector commitment satisfying some inner product relation.

(ii) Using the public information $(\mathbf{C}, \mathbf{I})$, construct the base vectors of the Pedersen vector commitment in such a way that the prover would not know the discrete logarithm relation between elements of the base vectors.

The first requirement seems natural since the Bulletproofs technique helps us prove the knowledge of exponents in a Pedersen vector commitment satisfying some inner product relations. The second one is a more technical requirement. In Bulletproofs, the elements in the base vectors $\mathbf{g}, \mathbf{h} \in \mathbb{G}^n$ are uniformly chosen from the group $\mathbb{G}$ to ensure that a discrete logarithm relation between them is not known to a PPT prover. The soundness of the Bulletproofs protocol relies on this assumption. Lai *et al* [28] noted that if base vector components are chosen from a blockchain a prover might know the discrete logarithm relation between them. To solve this problem, they proposed using a base vector which is the Hadamard product of the vectors taken from the blockchain (with a random exponent) and a randomly chosen base vector.

To construct a base vector satisfying the above requirements, we write the statement of $\mathcal{L}_{\mathsf{RevBP}}$ from (4.3) as

$$g^{-r_j} g_t^{r_j} \mathbf{C}^{\mathbf{e}_{i_j}} I_j^{-1} = 1 \ \forall j \in [s]. \tag{4.4}$$

For $u \xleftarrow{\$} \mathbb{Z}_q$, combining the above constraints, we have

$$\begin{aligned} &\prod_{j \in [s]} \left(g^{-r_j} g_t^{r_j} \mathbf{C}^{\mathbf{e}_{i_j}} I_j^{-1} \right)^{u^{j-1}} = 1, \\ \implies \quad & g^{-\langle \boldsymbol{u}^s, \boldsymbol{r} \rangle} g_t^{\langle \boldsymbol{u}^s, \boldsymbol{r} \rangle} \mathbf{C}^{\boldsymbol{u}^s \boldsymbol{\mathcal{E}}} \mathbf{I}^{-\boldsymbol{u}^s} = 1, \end{aligned} \tag{4.5}$$

where $\boldsymbol{\mathcal{E}}$ is a $s \times n$ matrix having the $\mathbf{e}_{i_j}$ vectors as rows. We write the exponents in (4.5) as *compressed secrets*, namely $\xi = -\langle \boldsymbol{u}^s, \boldsymbol{r} \rangle$, $\xi' = \langle \boldsymbol{u}^s, \boldsymbol{r} \rangle$, $\hat{\mathbf{e}} = \boldsymbol{u}^s \boldsymbol{\mathcal{E}}$ and let $\hat{I} = \mathbf{I}^{-\boldsymbol{u}^s}$. Given a vector $\mathbf{p} \in \mathbb{G}^{n+3}$ and a scalar $w \in \mathbb{Z}_q$, we construct the base and exponent vectors as follows

$$\mathbf{g}'_w \coloneqq \left((g \| g_t \| \mathbf{C} \| \hat{I})^{\circ w} \circ \mathbf{p} \right), \tag{4.6}$$

$$\mathbf{a}' \coloneqq (\xi \| \xi' \| \hat{\mathbf{e}} \| 1). \tag{4.7}$$

Note that the compressed secrets are a linear combination of the actual secrets. We need to append the actual secrets $(\mathbf{r}, \mathbf{e}_{i_1}, \ldots, \mathbf{e}_{i_s})$ for completeness to (4.7). Thus,

we have

$$\mathbf{g}_w \coloneqq \Big[\Big((g\|g_t\|\mathbf{C}\|\hat{I})^{\circ w} \circ \mathbf{p}\Big)\|\mathbf{g}'\Big], \tag{4.8}$$

$$\mathbf{a} \coloneqq \Big[(\xi\|\xi'\|\hat{\mathbf{e}}\|1)\|(\mathbf{e}_{i_1}\|\ldots\|\mathbf{e}_{i_s}\|\mathbf{r})\Big]. \tag{4.9}$$

where $\mathbf{g}' \xleftarrow{\$} \mathbb{G}^{sn+s}$. We now state a lemma in regards to the non-trivial discrete-log relation between the components of the base vector $\mathbf{g}_w$.

Lemma 4.1 *If the components of $\boldsymbol{p}$ are chosen uniformly from $\mathbb{G}$ and independent of $(\boldsymbol{C}, \boldsymbol{I})$, then a PPT adversary cannot find a non-trival discrete logarithm relation between components of $\boldsymbol{g}_w$.*

Proof: As the components of $\mathbf{p}$ and $\mathbf{g}'$ are uniformly chosen from $\mathbb{G}$, the components of $\mathbf{g}_w$ are iid with a uniform distribution in $\mathbb{G}$. Hence a PPT adversary cannot find a non-trivial discrete logarithm relation between these components. ■

We can now construct a Pedersen vector commitment as $A = (h')^r\mathbf{g}_w^{\mathbf{a}}\mathbf{h}^{\mathbf{b}}$ for appropriately chosen $\mathbf{b} \in \mathbb{Z}_q^N, \mathbf{h} \xleftarrow{\$} \mathbb{G}^N$ where $N = |\mathbf{a}|$, satisfying the first requirement in using the Bulletproofs framework. Owing to (4.5), (4.8), (4.9), we have $\mathbf{g}_w^{\mathbf{a}} = \mathbf{g}_{w'}^{\mathbf{a}}$ for any $w, w' \in \mathbb{Z}_q$. Thus, the above vector commitment to $\mathbf{a}$ remains the same for any $w \in \mathbb{Z}_q$. By successfully running the Bulletproofs protocol twice on A with respect to two different bases $(h'\|\mathbf{g}_w\|\mathbf{h})$ and $(h'\|\mathbf{g}_{w'}\|\mathbf{h})$ we can extract the secret vector $\mathbf{a}$ such that $A = (h')^r\mathbf{g}_w^{\mathbf{a}}\mathbf{h}^{\mathbf{b}} = (h')^r\mathbf{g}_{w'}^{\mathbf{a}}\mathbf{h}^{\mathbf{b}}$. This solves the problem of extractability (soundness) of the protocol. In summary, we are now ready to construct a Bulletproofs-based argument of knowledge proving that the exchange owns some outputs from the entire set of UTXOs. The next step is to develop an inner product relation between the secret vectors $\mathbf{a}$ and $\mathbf{b}$ to enable us in designing a Bulletproofs-like protocol.

4.6.1 Building Inner Product Relation

We have computed the secret vector $\mathbf{a}$ and the base vector $\mathbf{g}_w$ given by equations (4.6), (4.7) overcoming the difficulties in using a Bulletproofs-like framework. We rename secret vectors $\mathbf{a}, \mathbf{b}$ as $\mathbf{c}_L, \mathbf{c}_R \in \mathbb{Z}_q^N$ respectively and define them in Figure 4.2. Note that $\mathbf{c}_R$ is defined based on $\mathbf{c}_L$. We now wish to construct an inner product relationship between $\mathbf{c}_L$ and $\mathbf{c}_R$. We use vector indexing starting from 1, i.e. $\mathbf{c}_L[k_1 : k_2] = (\mathbf{c}_L[k_1], \mathbf{c}_L[k_1+1], \ldots, \mathbf{c}_L[k_2-1])$ where $k_1, k_2 \in \mathbb{N}, k_1 < k_2$.

(i) $\mathbf{e}_L = \mathbf{c}_L[n+4 : n+4+sn] = vec(\mathfrak{E}) \in \{0,1\}^{sn}$ and $\mathbf{e}_R = \mathbf{c}_R[n+4 : n+4+sn] = \mathbf{1}^{sn} - vec(\mathfrak{E}) \in \{0,1\}^{sn}$, i.e. those slices are element-wise binary complements.

Therefore, for a scalar vector $\boldsymbol{y}^{sn} \in \mathbb{Z}_q^{sn}$, we should have $\mathbf{e}_L \circ (\mathbf{e}_R \circ \boldsymbol{y}^{sn}) = \mathbf{0}^{sn}$, leading to inner-product constraint (4.10).

(ii) Next, compressed secrets ξ, ξ' are related such that $\xi + \xi' = 0$ and $\xi = -\langle \boldsymbol{u}^s, \mathbf{r} \rangle$. This means that for a scalar $v \in \mathbb{Z}_q$, we must have

$$\begin{aligned} v\xi + \xi' = (v-1)\xi &= -(v-1)\langle \boldsymbol{u}^s, \mathbf{r} \rangle, \\ \implies \xi + \xi' + \langle (v-1)\boldsymbol{u}^s, \mathbf{r} \rangle &= 0. \end{aligned}$$

This leads us to the constraint (4.11).

(iii) We have the compressed secret vector for indices as

$$\hat{\mathbf{e}} = \boldsymbol{u}^s \boldsymbol{\mathcal{E}} = (0, 0, \underbrace{1}_{i_1}, \ldots, \underbrace{u}_{i_2}, \ldots, \underbrace{u^2}_{i_3}, \ldots, \underbrace{u^{s-1}}_{i_s}, \ldots, 0, 0) \in \mathbb{Z}_q^n.$$

To prove that $\hat{\mathbf{e}}$ actually is of this form, we multiply it with a scalar vector $\boldsymbol{y}^n$.

$$\boldsymbol{y}^n \circ \hat{\mathbf{e}} = (0, 0, \underbrace{y^{i_1} \cdot 1}_{i_1}, \ldots, \underbrace{y^{i_2} \cdot u}_{i_2}, \ldots, \underbrace{y^{i_3} \cdot u^2}_{i_3}, \ldots, \underbrace{y^{i_s} \cdot u^{s-1}}_{i_s}, \ldots, 0, 0) \in \mathbb{Z}_q^n.$$

Now we get this same structure if we do the following: $\mathbf{e}_L \circ (\boldsymbol{u}^s \otimes \boldsymbol{y}^n)$ leading us to constraint (4.12).

(iv) We know that if we add up all the elements of $\mathbf{e}_L \circ (\boldsymbol{y}^s \otimes \mathbf{1}^n)$, we will get the sum as $\sum_{j=1}^{s} y^{j-1}$. To prove that $\mathbf{c}_L[n+3] = 1$, if we multiply $\mathbf{c}_L[n+3]$ with y^s and add it to the sum computed above, we must get $\langle \mathbf{1}^{s+1}, \boldsymbol{y}^{s+1} \rangle$, leading us to constraint (4.13).

(v) Lastly, to ensure that elements of $\mathbf{e}_L$ and $\mathbf{e}_R$ are actually $\{0, 1\}$, checking if $\langle \mathbf{e}_L + \mathbf{e}_R - \mathbf{1}^{sn}, \boldsymbol{y}^{sn} \rangle = 0$ should suffice. This forms the last constraint (4.14).

Now, we have all the required relations between the secret vectors in Figure 4.5 to ensure that an honest prover's witness encoding is correct. We now combine all these constraints in a single inner product relation by multiplying each and constraint by a power of a scalar and adding them up.

$$\begin{aligned} &\langle \mathbf{c}_L, \mathbf{c}_R \circ \mathbf{v}_0 \rangle + z \cdot \langle \mathbf{c}_L, \mathbf{v}_1 \rangle + z^2 \cdot \langle \mathbf{c}_L, \mathbf{v}_2 \rangle + \\ &\qquad z^3 \cdot \langle \mathbf{c}_L, \mathbf{v}_3 \rangle + z^4 \cdot \langle \mathbf{c}_L + \mathbf{c}_R - \mathbf{1}^t, \mathbf{v}_4 \rangle = z^3 \cdot \langle \mathbf{1}^{s+1}, \boldsymbol{y}^{s+1} \rangle, \end{aligned}$$

$$\implies \langle \mathbf{c}_L, \mathbf{c}_R \circ \mathbf{v}_0 \rangle + \langle \mathbf{c}_L, \sum_{i=1}^{4} z^i \cdot \mathbf{v}_i \rangle + \langle \mathbf{c}_R, z^4 \cdot \mathbf{v}_4 \rangle = z^3 \cdot \langle \mathbf{1}^{s+1}, \boldsymbol{y}^{s+1} \rangle + \langle \mathbf{1}^t, z^4 \cdot \mathbf{v}_4 \rangle$$

Substituting the compressed constraint vectors $\boldsymbol{\theta}, \boldsymbol{\mu}, \boldsymbol{\nu}$ as defined in Figure 4.4,

$$\implies \langle \mathbf{c}_L, \mathbf{c}_R \circ \boldsymbol{\theta} \rangle + \langle \mathbf{c}_L, \boldsymbol{\mu} \rangle + \langle \mathbf{c}_R, \boldsymbol{\nu} \rangle = z^3 \cdot \langle \mathbf{1}^{s+1}, \boldsymbol{y}^{s+1} \rangle + \langle \mathbf{1}^t, \boldsymbol{\nu} \rangle,$$

Combining the first and third terms on the lhs using constraint vector $\boldsymbol{\theta}^{\circ -1}$,

$$\implies \langle \mathbf{c}_L, \mathbf{c}_R \circ \boldsymbol{\theta} \rangle + \langle \mathbf{c}_L, \boldsymbol{\mu} \rangle + \langle \mathbf{c}_R \circ \boldsymbol{\theta}, \boldsymbol{\nu} \circ \boldsymbol{\theta}^{\circ -1} \rangle = z^3 \cdot \langle \mathbf{1}^{s+1}, \boldsymbol{y}^{s+1} \rangle + \langle \mathbf{1}^t, \boldsymbol{\nu} \rangle,$$
$$\implies \langle \mathbf{c}_L + \boldsymbol{\nu} \circ \boldsymbol{\theta}^{\circ -1}, \mathbf{c}_R \circ \boldsymbol{\theta} \rangle + \langle \mathbf{c}_L, \boldsymbol{\mu} \rangle = z^3 \cdot \langle \mathbf{1}^{s+1}, \boldsymbol{y}^{s+1} \rangle + \langle \mathbf{1}^t, \boldsymbol{\nu} \rangle,$$

Substituting compressed constraint vector $\boldsymbol{\alpha}$ and scalar δ, we get

$$\implies \langle \mathbf{c}_L + \boldsymbol{\alpha}, \mathbf{c}_R \circ \boldsymbol{\theta} \rangle + \langle \mathbf{c}_L, \boldsymbol{\mu} \rangle = \delta,$$
$$\implies \langle \mathbf{c}_L + \boldsymbol{\alpha}, \mathbf{c}_R \circ \boldsymbol{\theta} \rangle + \boldsymbol{\mu} = \delta + \langle \boldsymbol{\mu}, \boldsymbol{\alpha} \rangle$$

The final equation is the required single inner product relation. We can now proceed in constructing a Bulletproofs-like protocol for this inner product relation.

The interactive protocol $\Pi_{\mathsf{RevBP}} = (\mathit{Setup}, \langle \mathscr{P}, \mathscr{V} \rangle)$ for the language $\mathscr{L}_{\mathsf{RevBP}}$ is shown in Figure 4.6. Note that *Setup*, prover $\mathscr{P}$ and verifier $\mathscr{V}$ are PPT algorithms. The notation used in the protocol is given in Fig. 4.1, 4.2, 4.3, 4.4, 4.5.

Notation	*Description*
$\hat{I} = \hat{I}(u) \coloneqq \mathbf{I}^{-\boldsymbol{u}^s}$	Compressed key-images $I_1, I_2, \ldots, I_s$
$\boldsymbol{\mathcal{E}} \in \mathbb{Z}_2^{s \times n}$	Secret indices matrix, $vec(\boldsymbol{\mathcal{E}}) = (\mathbf{e}_{i_1}, \ldots, \mathbf{e}_{i_s})$
$\hat{\mathbf{e}} = \boldsymbol{u}^s \boldsymbol{\mathcal{E}}$ $\xi \coloneqq -\langle \boldsymbol{u}^s, \mathbf{r} \rangle$ $\xi' \coloneqq \langle \boldsymbol{u}^s, \mathbf{r} \rangle$	Compressed secrets $(\hat{\mathbf{e}}, \xi, \xi')$ satisfying $g^{\xi} g_t^{\xi'} \mathbf{C}^{\hat{\mathbf{e}}} \hat{I} = 1$
$\mathbf{c}_L, \mathbf{c}_R$	Honest encoding of witness (Fig. 4.2)
$N = sn + n + s + 3$	Size of the vectors $\mathbf{c}_L, \mathbf{c}_R$
$(\mathbf{v}_0, \ldots, \mathbf{v}_4)(u, v, y)$	Constraint vectors (Fig. 4.3, 4.4)
$\boldsymbol{\alpha}, \boldsymbol{\beta}, \delta, \boldsymbol{\mu}, \boldsymbol{\nu}, \boldsymbol{\theta}(u, v, y, z)$	Compressed constraint vectors (Fig. 4.3, 4.4, 4.5)
$\mathsf{EQ}(\boldsymbol{\gamma}_L, \boldsymbol{\gamma}_R)$	Relationship between valid witnesses (Fig. 4.5)

Figure 4.1: Notation used in the argument of knowledge Π_{RevBP}

$$\mathbf{c}_L \coloneqq (\ \xi \parallel \xi' \parallel\ \hat{\mathbf{e}} \parallel 1 \parallel \quad vec(\mathfrak{E}) \quad \parallel\ \mathbf{r}\ )$$
$$\mathbf{c}_R \coloneqq (\quad \mathbf{0}^{n+3} \quad \parallel \mathbf{1}^{sn} - vec(\mathfrak{E}) \parallel \mathbf{0}^{s}\ )$$

Figure 4.2: Honest encoding of witness vectors

$$\begin{bmatrix}\mathbf{v}_0\\ \mathbf{v}_1\\ \mathbf{v}_2\\ \mathbf{v}_3\\ \mathbf{v}_4\end{bmatrix} \coloneqq \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \boldsymbol{y}^{sn} & \cdot \\ v & 1 & \cdot & \cdot & \cdot & (v-1)\boldsymbol{u}^s \\ \cdot & \cdot & -\boldsymbol{y}^n & \cdot & \boldsymbol{u}^s \otimes \boldsymbol{y}^n & \cdot \\ \cdot & \cdot & \cdot & y^s & \boldsymbol{y}^s \otimes \mathbf{1}^n & \cdot \\ \cdot & \cdot & \cdot & \cdot & \boldsymbol{y}^{sn} & \cdot \end{bmatrix}$$

Figure 4.3: Definitions of constraint vectors where dots mean zero scalars or vectors

$$\boldsymbol{\theta} \coloneqq \mathbf{v}_0, \quad \boldsymbol{\mu} \coloneqq \sum_{i=1}^{4} z^i \mathbf{v}_i, \quad \boldsymbol{\nu} \coloneqq z^4 \mathbf{v}_4,$$
$$\boldsymbol{\theta}^{\circ -1}[j] = \begin{cases} (\boldsymbol{\theta}[j])^{-1} & \text{if } \boldsymbol{\theta}[j] \neq 0, \\ 0 & \text{otherwise,} \end{cases}$$
$$\boldsymbol{\alpha} \coloneqq \boldsymbol{\theta}^{\circ -1} \circ \boldsymbol{\nu}, \qquad \boldsymbol{\beta} \coloneqq \boldsymbol{\theta}^{\circ -1} \circ \boldsymbol{\mu},$$
$$\delta \coloneqq z^3 \cdot \langle \mathbf{1}^{s+1}, \boldsymbol{y}^{s+1} \rangle + \langle \boldsymbol{\alpha}, \boldsymbol{\mu} \rangle + \langle \mathbf{1}^N, \boldsymbol{\nu} \rangle.$$

Figure 4.4: Definitions of compressed constraint vectors

$$\mathsf{EQ}(\boldsymbol{\gamma}_L, \boldsymbol{\gamma}_R) = 0 \iff$$
$$\langle \boldsymbol{\gamma}_L, \boldsymbol{\gamma}_R \circ \mathbf{v}_0 \rangle = 0, \tag{4.10}$$
$$\langle \boldsymbol{\gamma}_L, \mathbf{v}_1 \rangle = 0, \tag{4.11}$$
$$\langle \boldsymbol{\gamma}_L, \mathbf{v}_2 \rangle = 0, \tag{4.12}$$
$$\langle \boldsymbol{\gamma}_L, \mathbf{v}_3 \rangle = \langle \mathbf{1}^{s+1}, \boldsymbol{y}^{s+1} \rangle, \tag{4.13}$$
$$\langle \boldsymbol{\gamma}_L + \boldsymbol{\gamma}_R - \mathbf{1}^t, \mathbf{v}_4 \rangle = 0. \tag{4.14}$$

Figure 4.5: A system of equations guaranteeing the integrity of the encoding of the witnesses

Figure 4.6: Argument of knowledge for $\mathcal{L}_{\mathsf{RevBP}}$

Setup$(\lambda, \mathcal{L})$:

$\mathcal{L}(\mathbb{G}, q, g, h, g_t)$ as defined in (4.3),

Generate following elements randomly from $\mathbb{G}$

$h' \xleftarrow{\$} \mathbb{G}, \mathbf{p} \xleftarrow{\$} \mathbb{G}^{n+3}, \mathbf{g}' \xleftarrow{\$} \mathbb{G}^{sn+s}, \mathbf{h} \xleftarrow{\$} \mathbb{G}^{N}$

Output: $crs = (\mathbb{G}, q, g, h, g_t, h', \mathbf{p}, \mathbf{g}', \mathbf{h})$

$\langle \mathcal{P}(crs,\ stmt,\ wit), \mathcal{V}(crs,\ stmt) \rangle$:

$\mathcal{P}$:

(i) $r_A \xleftarrow{\$} \mathbb{Z}_q$

(ii) $\mathbf{g}_0 = (\mathbf{p} \parallel \mathbf{g}')$

(iii) $A \coloneqq (h')^{r_A} \mathbf{g}_0^{\mathbf{c}_L} \mathbf{h}^{\mathbf{c}_R}$

$\mathcal{P} \longrightarrow \mathcal{V}$: A

$\mathcal{V}$: $u, v, w \xleftarrow{\$} \mathbb{Z}_q$, $\mathcal{V} \longrightarrow \mathcal{P}$: u, v, w

$\mathcal{P},\ \mathcal{V}$:

(i) $\hat{I} \coloneqq \mathbf{I}^{-\boldsymbol{u}^s}$

(ii) $\mathbf{g}_w \coloneqq \left[\left((g \| g_t \| \mathbf{C} \| \hat{I})^{\circ w} \circ \mathbf{p}\right) \| \mathbf{g}'\right]$

$\mathcal{P}$:

(i) $r_S \xleftarrow{\$} \mathbb{Z}_q,\ \mathbf{s}_L \xleftarrow{\$} \mathbb{Z}_q^N,\ \mathbf{s}_R \in \mathbb{Z}_q^N$ s.t. $\forall j \in [N]$

$$\mathbf{s}_R[j] = \begin{cases} s_j \xleftarrow{\$} \mathbb{Z}_q & \text{if } n+4 \leq j \leq N-s, \\ 0 & \text{otherwise} \end{cases}$$

(ii) $S = (h')^{r_S} \mathbf{g}_w^{\mathbf{s}_L} \mathbf{h}^{\mathbf{s}_R}$

$\mathcal{P} \longrightarrow \mathcal{V}$: S

$\mathcal{V}$: $y, z \xleftarrow{\$} \mathbb{Z}_q$, $\mathcal{V} \longrightarrow \mathcal{P}$: y, z

$\mathcal{P}$:

(i) Define the following polynomials in $\mathbb{Z}_q^N[X]$

$$\begin{aligned} l(X) &\coloneqq \mathbf{c}_L + \boldsymbol{\alpha} + \mathbf{s}_L \cdot X \\ r(X) &\coloneqq \boldsymbol{\theta} \circ (\mathbf{c}_R + \mathbf{s}_R \cdot X) + \boldsymbol{\mu} \\ t(X) &\coloneqq \langle l(X), r(X) \rangle = t_2 X^2 + t_1 X + t_0 \end{aligned}$$

for $t_2, t_1, t_0 \in \mathbb{Z}_q$. Also, $t_0 = \delta$.

(ii) $\tau_1, \tau_2 \xleftarrow{\$} \mathbb{Z}_q$

(iii) $T_1 = g^{t_1} h^{\tau_1}, T_2 = g^{t_2} h^{\tau_2}$

$\mathcal{P} \longrightarrow \mathcal{V}$: T_1, T_2

$\mathcal{V}$: $x \xleftarrow{\$} \mathbb{Z}_q$, $\mathcal{V} \longrightarrow \mathcal{P}$: x

$\mathcal{P}$:

(i) $\boldsymbol{\ell} \coloneqq l(x) = \mathbf{c}_L + \boldsymbol{\alpha} + \mathbf{s}_L \cdot x \in \mathbb{Z}_q^N$

(ii) $\boldsymbol{\mathfrak{r}} \coloneqq r(x) = \boldsymbol{\theta} \circ (\mathbf{c}_R + \mathbf{s}_R \cdot x) + \boldsymbol{\mu} \in \mathbb{Z}_q^N$

(iii) $\hat{t} \coloneqq \langle \boldsymbol{\ell}, \boldsymbol{\mathfrak{r}} \rangle \in \mathbb{Z}_q$

(iv) $\tau_x \coloneqq \tau_2 x^2 + \tau_1 x$

(v) $r \coloneqq r_A + r_S x$

$\mathcal{P} \longrightarrow \mathcal{V}$: $\boldsymbol{\ell}, \boldsymbol{\mathfrak{r}}, \hat{t}, \tau_x, r$

$\mathcal{V}$:

(i) $\hat{t} \stackrel{?}{=} \langle \boldsymbol{\ell}, \boldsymbol{\mathfrak{r}} \rangle$ *// $\hat{t}$ was computed correctly*

(ii) $g^{\hat{t}} h^{\tau_x} \stackrel{?}{=} g^{\delta} T_1^x T_2^{x^2}$ *// $\hat{t}$ satisfies $t_0 + t_1 x + t_2 x^2$*

(iii) $(h')^r \mathbf{g}_w^{\boldsymbol{\ell}} \mathbf{h}^{\boldsymbol{\theta}^{\circ -1} \circ \boldsymbol{\mathfrak{r}}} \stackrel{?}{=} A S^x \mathbf{g}_w^{\boldsymbol{\alpha}} \mathbf{h}^{\boldsymbol{\beta}}$ *// Check if $\boldsymbol{\ell} = l(x)$ and $\boldsymbol{\mathfrak{r}} = r(x)$*

As the prover has to send vectors $\boldsymbol{\ell}, \boldsymbol{\mathfrak{r}} \in \mathbb{Z}_q^N$ in the last round, the Π_{RevBP} protocol results in a communication cost of $\mathcal{O}(N)$ for the prover, where N is the length of the secret vectors. We reduce this to $\mathcal{O}(\log_2(N))$ using the inner-product argument in [2]. Concretely, the language for the inner-product argument is expressed as

$$\mathcal{L}_{\mathsf{IP}} = \left\{ P \in \mathbb{G}, c \in \mathbb{Z}_q \;\middle|\; \begin{array}{c} \exists (\mathbf{a}, \mathbf{b}) \text{ such that} \\ P = u^c \mathbf{g}^{\mathbf{a}} \mathbf{h}^{\mathbf{b}} \wedge c = \langle \mathbf{a}, \mathbf{b} \rangle \end{array} \right\} \tag{4.15}$$

where $\mathbf{a}, \mathbf{b} \in \mathbb{Z}_q^{|\mathbf{a}|}$, $\mathbf{g}, \mathbf{h} \xleftarrow{\$} \mathbb{G}^{|\mathbf{a}|}$, $u \xleftarrow{\$} \mathbb{G}$. Thus, in our case, we construct a Pedersen vector commitment to $(\boldsymbol{\ell}, \boldsymbol{\imath})$

$$P = u^{\hat{t}}\mathbf{g}_w^{\boldsymbol{\ell}}(\mathbf{h}')^{\boldsymbol{\imath}} = u^{\hat{t}}(h')^{-r}AS^x\mathbf{g}_w^{\boldsymbol{\alpha}}\mathbf{h}^{\boldsymbol{\beta}},$$

where $\mathbf{h}' = \mathbf{h}^{\boldsymbol{\theta}^{\circ -1}}$. Note that P, as shown above, could be computed by verifier. Thus, running the inner-product argument with input $(P, \hat{t})$ proves the knowledge of $(\boldsymbol{\ell}, \boldsymbol{\imath})$ in $\mathcal{O}(\log_2 N)$ communication. Furthermore, since the Π_{RevBP} protocol is public-coin, we can make it non-interactive using the Fiat-Shamir heuristic [38].

Theorem 4.1 *The argument presented in Figure 4.6 is public-coin, constant-round, perfectly complete and perfect special honest-verifier zero-knowledge.*

Proof sketch: The Π_{RevBP} protocol is public-coin since all of the challenges from $\mathcal{V}$ are generated uniformly randomly from $\mathbb{Z}_q$. Given a $crs = (\mathbb{G}, q, g, h, g_t)$ and an honest prover with knowledge of witness $wit = (\mathbf{r}, \mathbf{e}_{i_1}, \ldots, \mathbf{e}_{i_s})$ for a $stmt = (\mathbf{C}, \mathbf{I}) \in \mathcal{L}_{\mathsf{RevBP}}$, it is easy to see that the three verification conditions at the end of Figure 4.6 hold. Thus, the protocol is perfectly complete. Next, we need show that Π_{RevBP} is perfect special honest-verifier zero-knowledge (SHVZK) by constructing an efficient simulator $\mathcal{S}$, which can simulate the transcript of Π_{RevBP} without knowing the witness. Note the difference between a Special HVZK and HVZK is that in the former, the simulator $\mathcal{S}$ is given the challenges chosen by the verifier while in the latter, simulator $\mathcal{S}$ is allowed to choose the challenges by itself. Perfect SHVZK implies that no adversary, even if it is computationally unbounded, would be able to distinguish between the simulated and the honestly generated transcripts for valid statements and witnesses. For the protocol to be perfect SHVZK, the simulated transcript needs to be identically distributed to the transcript of Π_{RevBP}. Detailed construction of $\mathcal{S}$ is shown in Appendix A.2.

Theorem 4.2 *Assuming the discrete logarithm assumption holds over $\mathbb{G}$, Π_{RevBP} has computational witness-extended-emulation for extracting a valid witness wit.*

Proof sketch: Proving that Π_{RevBP} has computational witness-extended emulation implies showing that it is *computationally sound*. To do so, we need to construct a PPT extractor $\mathscr{E}$, which when given enough number of transcripts of Π_{RevBP} via rewinding, succeeds in construction of a valid witness as defined in the language $\mathcal{L}_{\mathsf{RevBP}}$. We show the detailed construction of $\mathscr{E}$ in Appendix A.3.

4.7 Security Properties of RevelioBP

In this section, we discuss the security properties of the Π_{RevBP} protocol, namely inflation resistance, collusion detection, and output privacy (as defined in Section 4.5). We defer the rigorous treatment of the security properties to an extended version of this paper.

4.7.1 Inflation Resistance

Theorem 4.2 proves that a PPT exchange can include the tag $I_j = g_t^{r_j} h^{a_j}$ in the vector $\mathbf{I}$ *only if* it knows the blinding factor r_j and amount a_j corresponding to the output $C_{i_j} = g^{r_j} h^{a_j}$. Thus an exchange can only create tags corresponding to outputs it owns. Furthermore, since each tag I_j is forced to be a Pedersen commitment to the *same amount* as the output C_{i_j}, the exchange cannot inflate the amount being contributed by I_j to C_{res}. Thus C_{res} is a Pedersen commitment to the actual reserves $\sum_{j=1}^{s} a_j$.

4.7.2 Collusion Resistance

Suppose two exchanges generate RevelioBP proofs at the same block height t. Ideally, each $C_i \in C_{\text{utxo}}$ can contribute to the reserves of at most one of the exchanges. If exchange 1 who owns an output $C_i = g^{r_i} h^{a_i}$ reveals r_i and a_i to exchange 2, both of them can try using C_i as a contributing output in their proofs of reserves. Then while creating their respective arguments Π_{RevBP} both exchanges will be forced to include the tag $I_j = g_t^{r_i} h^{a_i}$ in their tag vectors, revealing the collusion. This technique will work only if all exchanges agree to use the same sequence of g_ts in their RevelioBP proofs. If exchanges 1 and 2 were to use different bases g_t and g'_t to generate their proofs, then collusion cannot be detected. As of now, pressure from customers and regulators seems to be the only way to ensure that all exchanges use the same g_t.

4.7.3 Output Privacy

Let λ be the security parameter such that $\mathit{Setup}(1^\lambda)$ generates the group $(\mathbb{G}, q, g)$ with $\log_2 q \approx \lambda$. Suppose an exchange publishes a polynomial number of proofs of reserves at block heights $t_1, t_2, \ldots, t_{f(\lambda)}$ where f is a polynomial. Output privacy requires that a PPT distinguisher $\mathcal{D}$, which is given the $f(\lambda)$ proofs of reserves as input, cannot do better than random guessing while classifying an output as owned by the exchange. Note that the Π_{RevBP} protocol itself does not reveal

anything about the secrets. Here, we intend to analyse privacy of outputs due to the revelation of the tag vector.

The RevelioBP protocol provides output privacy under the following assumptions:

(i) The blinding factors of the exchange-owned outputs are chosen independently, uniformly from $\mathbb{Z}_q$.

(ii) The DDH problem is hard in the group $\mathbb{G}$, i.e. there is no algorithm which can solve the DDH problem in $\mathbb{G}$ with a running time polynomial in λ.

If the first assumption does not hold, a PPT adversary could identify exchange-owned outputs given a RevelioBP proof. For example, consider the case when two exchange-owned outputs C_i and C_j have the same blinding factor r but different amounts a_i and a_j, i.e. $C_i = g^r h^{a_i}$ and $C_j = g^r h^{a_j}$. If the exchange uses both outputs in a RevelioBP proof at block height t, then the tags corresponding to the outputs will be $I_j = g_t^r h^{a_i}$ and $I_j = g_t^r h^{a_j}$. An adversary can figure out that these two tags have the same blinding factor by checking if the equality $I_j h^{-a_1} = I_j h^{-a_2}$ holds for some $(a_1, a_2) \in V^2$ where V is the range of possible amounts. As the size of the set V is usually small, such a search is feasible. Once the amounts a_i and a_j have been found, the adversary could iterate through all possible output pairs (C, C') in $\mathcal{C}^t_{\text{utxo}} \times \mathcal{C}^t_{\text{utxo}}$ and check if $Ch^{-a_i} = C'h^{a_j}$. If a pair of outputs satisfying this equality is found, then the adversary concludes that both of them belong to the exchange. By requiring that the blinding factors are randomly chosen, such attacks become infeasible.

To precisely define output privacy, we use an experiment called `OutputPriv` which proceeds as follows:

1. For security parameter λ, the generate group parameters $(\mathbb{G}, q, g) \leftarrow \mathit{Setup}(1^\lambda)$ where $q \approx 2^\lambda$. A sequence of generators $g_1, g_2, \ldots, g_{f(\lambda)}$ are chosen uniformly from $\mathbb{G}$. These generators will be used to instantiate g_t in each of the $f(\lambda)$ RevelioBP proofs.

2. The exchange creates two outputs C_1, C_2 with amounts a_1, a_2 and blinding factors $r_1, r_2 \xleftarrow{\$} \mathbb{Z}_q$, i.e. $C_1 = g^{r_1} h^{a_1}$ and $C_2 = g^{r_2} h^{a_2}$.

3. The exchange chooses an integer $\mathfrak{b} \xleftarrow{\$} \{1, 2\}$. Note that $C_\mathfrak{b} = g^{r_\mathfrak{b}} h^{a_\mathfrak{b}}$.

4. The exchange now generates $f(\lambda)$ RevelioBP proofs where the UTXO vector is $\mathbf{C} = (C_1, C_2)$ and the number of exchange-owned outputs is 1 in all of them.

The lth proof reveals a single tag $I_l = g_l^{r_{\mathfrak{b}}} h^{a_{\mathfrak{b}}}$ for $l = 1, 2, \ldots, f(\lambda)$, i.e. the same exchange-owned output is used to generate each of the l proofs. Let the argument of knowledge corresponding to the lth RevelioBP proof be Π^l_{RevBP}.

5. The $f(\lambda)$ RevelioBP proofs consisting of tags $\bar{I} = (I_1, \ldots, I_{f(\lambda)})$, arguments $\bar{\Pi} = (\Pi^1_{\mathsf{RevBP}}, \ldots, \Pi^{f(\lambda)}_{\mathsf{RevBP}})$ along with the generators $\bar{g} = (g_1, g_2, \ldots, g_{f(\lambda)})$, outputs C_1, C_2, and amounts a_1, a_2 are given as input to a distinguisher $\mathcal{D}$ which outputs $\mathfrak{b}' \in \{1, 2\}$, i.e.

$$\mathfrak{b}' = \mathcal{D}\left(\bar{I}, \bar{\Pi}, \bar{g}, C_1, C_2, a_1, a_2\right) \tag{4.16}$$

6. $\mathcal{D}$ succeeds if $\mathfrak{b}' = \mathfrak{b}$. Otherwise it fails.

Definition 4.1 *The* RevelioBP *proof of reserves protocol provides output privacy if every PPT distinguisher $\mathcal{D}$ in the* `OutputPriv` *experiment succeeds with a probability which is negligibly close to $\frac{1}{2}$.*

To motivate the above definition, consider an adversary who observes a RevelioBP proof generated by an exchange for UTXO set $\mathcal{C}^t_{\text{utxo}}$ having size n. The length of the tag vector $\mathbf{I}$ reveals the number of outputs s owned by the exchange. Suppose the adversary is asked to identify an exchange-owned output from $\mathcal{C}^t_{\text{utxo}}$. If it chooses an output uniformly from $\mathcal{C}^t_{\text{utxo}}$, then it succeeds with probability $\frac{s}{n}$. But the adversary may itself own some outputs (say, n_a) in $\mathcal{C}^t_{\text{utxo}}$. Then, the success probability can be increased to $\frac{s}{n-n_a}$. The above definition models the extreme case when $n - n_a = 2$ and $s = 1$. The definition states that a PPT adversary can only do negligibly better than a random guessing strategy.

The justification for revealing the amounts a_1, a_2 to the distinguisher $\mathcal{D}$ is that the amount in a MimbleWimble output may be known to an entity other than the output owner. For instance, a MimbleWimble transaction where Alice sends some coins to Bob will result in a new output whose blinding factor is known only to Bob but the amount in this output is known to Alice. The above definition captures the requirement that even entities with knowledge of the amounts in outputs should not be able to identify exchange-owned outputs from the RevelioBP proofs. We have the following theorem whose proof is given in Appendix A.4.

Theorem 4.3 *The* RevelioBP *proof of reserves protocol provides output privacy in the random oracle model under the DDH assumption provided that the exchange chooses the blinding factors in its outputs uniformly and independently of each other.*

4.8 Performance

We compare the performance of our proof of reserves protocol with Revelio which was the first MimbleWimble proof of reserves protocol [1]. In Revelio, an exchange publishes an anonymity set $\mathcal{C}_{\text{anon}} \subseteq \mathcal{C}_{\text{utxo}}$ such that $\mathcal{C}_{\text{own}} \subseteq \mathcal{C}_{\text{anon}}$. In RevelioBP, the anonymity set is always equal to $\mathcal{C}_{\text{utxo}}$. For a fair comparison, we set $\mathcal{C}_{\text{anon}} = \mathcal{C}_{\text{utxo}}$ in Revelio. Let $n = |\mathcal{C}_{\text{utxo}}|$ and $s = |\mathcal{C}_{\text{own}}|$. Revelio proof sizes are $\mathcal{O}(n)$ while RevelioBP proof sizes are $\mathcal{O}(s + \log_2(sn))$. The exact proof sizes are:

	#Elements in $\mathbb{G}$	#Elements in $\mathbb{Z}_q$
Revelio	$n + 1$	$5n$
RevelioBP	$s + 2\lceil \log_2 N \rceil + 4$	5

Table 4.1: Proof sizes of Revelio and RevelioBP protocols

where $N = sn + n + s + 3$. The logarithmic dependence of the RevelioBP proof size on the UTXO set size n is the main advantage of RevelioBP over Revelio. This is illustrated in Figure 4.7a where we compare the proof sizes of the Revelio and RevelioBP protocols as a function of n for $s = 20$. For a UTXO set size of 2×10^5, the RevelioBP proof is a mere 2.5 KB compared to a 41 MB Revelio proof.

We have implemented RevelioBP in Rust over the secp256k1 elliptic curve. The Revelio running times were estimated using the simulation code made available by Dutta *et al* [39]. All the experiments were run on an Intel Core i7 2.6 GHz CPU. Our simulation code is open-sourced on GitHub [40]. Figure 4.7c shows the RevelioBP and Revelio proof generation and verification times as a function of the UTXO set size for $s = 20$. For a constant own set size $s = 10$, we observe the following:

(i) A linear growth in the running times of both RevelioBP and Revelio as a function of n.

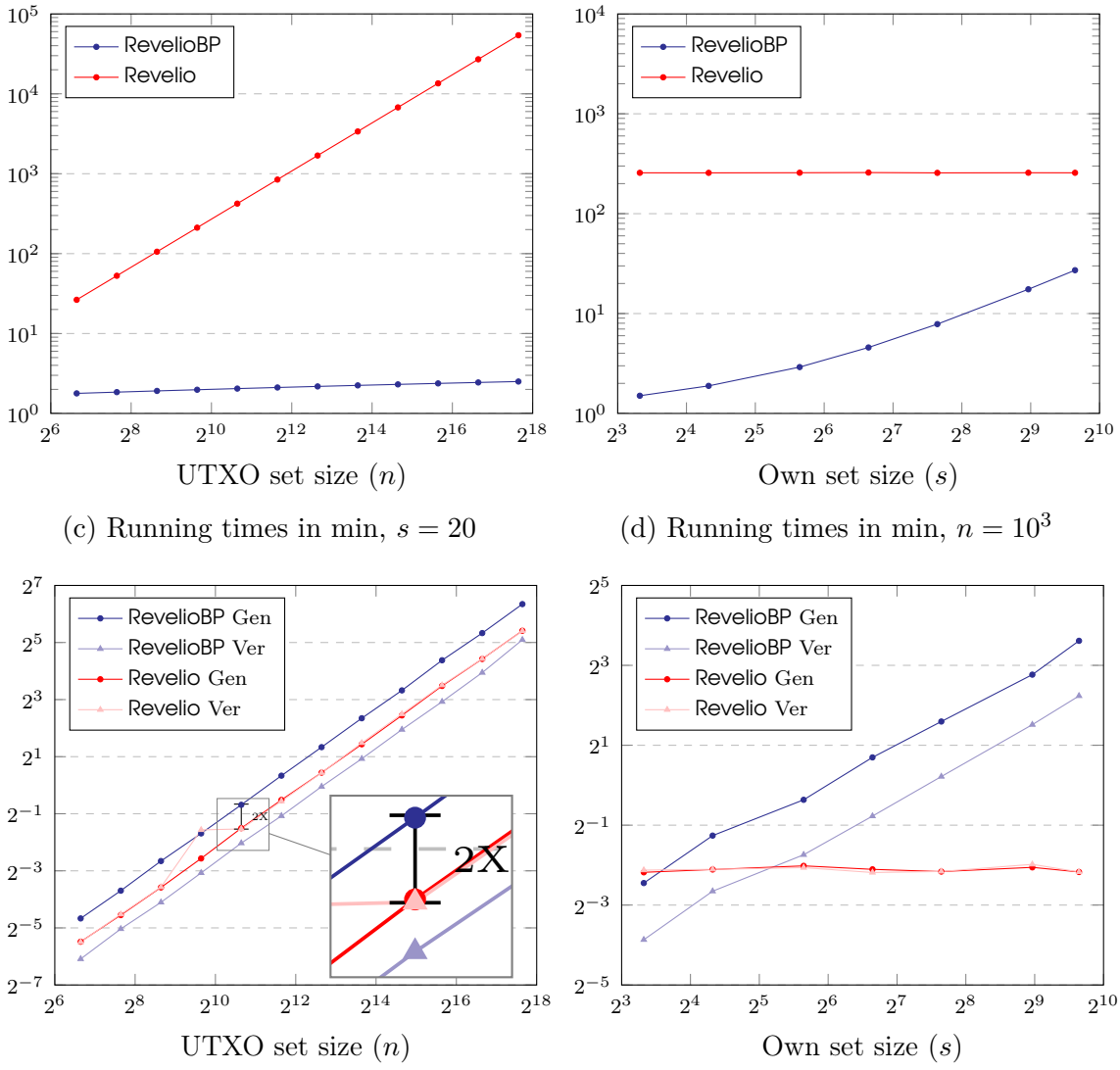
(ii) The RevelioBP proof generation is typically 2X slower than that of Revelio proof generation.

(iii) Although both the generation and verification of RevelioBP proofs if linear in n for a constant s, the verification is around 2.5X faster than its generation because of a single multi-exponentiation check in the verification of inner product protocol.

(iv) The RevelioBP verification is 20% faster than the verification of a Revelio proof.

(a) Proof size in KB for $s = 20$

(b) Proof size in KB for $n = 10^3$



(c) Running times in min, $s = 20$

(d) Running times in min, $n = 10^3$

Figure 4.7: Performance comparison of RevelioBP and Revelio for $\mathbb{G} =$ `secp256k1` elliptic curve. All the plots are in log-log scale.

Furthermore, while the running time of Revelio is independent of s, it scales as $\mathcal{O}(sn)$ for RevelioBP. Figure 4.7d shows the generation and verification times respectively as a function of s for $n = 10^3$. Also, the proof size of RevelioBP proofs grow linearly with s while that of Revelio remains constant. This is shown in Figure 4.7b for a constant anonymity set size $n = 10^3$. This implies that for a constant UTXO set n, the verification in RevelioBP is faster than Revelio only upto a particular s. Similarly, the difference between RevelioBP and Revelio proof generation widens as s increases. To illustrate this in practical terms, the size of the current UTXO set in the Grin blockchain as of June 7, 2020 is 161,000 [34]. For this UTXO set size and own output set size equal to 100, an exchange could take

upto 160 minutes to generate a RevelioBP proof while taking less than 34 minutes to generate a Revelio proof. The customers of the exchange can verify RevelioBP and Revelio proofs in 68 and 34 minutes respectively. For an exchange which owns 200 outputs in the current UTXO set, the RevelioBP generation and verification times would rise to 320 minutes and 130 minutes respectively, while the timings of Revelio remain unchanged.

4.8.1 Scalability and Performance Trade-off

The smaller proof sizes of RevelioBP would enable exchanges to store several historical proofs for audit purposes. Further, if proofs of reserves are to be uploaded on the blockchain for public verifiability, smaller proof size becomes crucial. The benefits in scalability due to RevelioBP comes at the price of performance. From the customer point of view too, larger verification times are undesirable given their limited computational resources.

The simpler formulation of Revelio, on the other hand, allows faster generation and verification. Therefore, if the proof size is not a concern for an exchange, using Revelio can reduce computational cost for the exchanges as well as the customers. Moreover, the design of Revelio allows parallelization of its proof generation as well as verification. On the contrary, RevelioBP relies on the recursive inner product protocol, preventing parallelization of proof generation. Also, since the base vector used in RevelioBP depends on the tag vector published by an exchange, RevelioBP proofs of multiple exchanges on the same blockchain state cannot be aggregated. Lastly, if exchanges are required to generate proofs of reserves after every K blocks are mined, the proof generation time needs to be less than K minutes[†]. In such cases, proofs with smaller running times would be preferred. We conclude by summarizing the trade-offs in using Revelio and RevelioBP in Table 4.2.

4.9 Conclusion

To avoid proof sizes which are linear in the size of anonymity set, we have presented Bulletproofs-based proof of reserves protocol RevelioBP with proof size scaling logarithmically in the size of the anonymity set. Having implemented RevelioBP, we conclude that the smaller proof sizes it offers comes with the cost of larger proof generation and verification times. Revelio, the first proof of reserves for MimbleWimble,

[†]The average inter-block time is one minute in both Grin and Beam (the two most popular MimbleWimble-based cryptocurrencies).

	RevelioBP	Revelio
Proof size	$\mathcal{O}(\log(sn) + s)$	$\mathcal{O}(n)$
Gen/Ver times	$\mathcal{O}(sn)$	$\mathcal{O}(n)$
Scalability	✓	✗
Blockchain state	✓	✗
Output privacy	✓	✓
Non-collusion	✓	✓
Inflation resistance	✓	✓
Parallelizable	✗	✓
Hiding own set size	✗	✓

Table 4.2: Summary of performance comparison between Revelio and RevelioBP

does better in terms of generation and verification times. An exchange has to make a trade-off between scalability and performance and choose which protocol suits their needs better: RevelioBP with smaller proof size and large generation times or Revelio with larger proof sizes and faster generation times.

Chapter 5

Confidentiality of Amounts in Grin

MimbleWimble [41] is a scalable cryptocurrency design where coins are stored in Pedersen commitments [42]. The blinding factor of the Pedersen commitment which obscures the amount of coins also serves as the spending key. Like many other cryptocurrency designs, transactions in MimbleWimble are of two types: *regular transactions* and *coinbase transactions*. Regular transactions involve a transfer of coins from some input commitments already present on the blockchain to new output commitments. A combination of digital signatures and range proofs are used to prove that the total coins in the input commitments equals the total coins in the output commitments plus transaction fees, without revealing the amounts in the commitments [31]. Coinbase transactions reward miners for adding blocks to the blockchain. They only consist of output commitments and have no input commitments. The total amount of coins in the coinbase output commitments of a block is *public*, being equal to the sum of the block subsidy and the transaction fees paid by the regular transactions in the block.

Every regular transaction output commitment can be traced back to a set of *donor* coinbase output commitments with public amounts which could have possibly contributed to it. The key observation is that *the amount of coins in a regular transaction output is bounded from above by the sum of the public amounts in its donor coinbase outputs minus the total transaction fees paid on the paths from these donor coinbase outputs to the regular transaction output*. While this observation is probably well-known in the MimbleWimble community, to the best of our knowledge there has been no effort to quantitatively compute such upper bounds for

Some sections of this chapter originally appeared in [43].

a MimbleWimble-based cryptocurrency. In this paper, we compute these upper bounds for the Grin implementation [7] of MimbleWimble. Our method can be applied to the other implementations like Beam [8]. We chose Grin because we were able to obtain its blockchain data from the administrator of the GrinExplorer site [44]. Note that, unlike other cryptocurrencies, it is not possible for a new node in Grin to download all the historical blocks starting with the genesis block [45]. This is a deliberate design choice as the MimbleWimble protocol does not require all the blocks to check the validity of the current blockchain state. The network load on existing nodes in the Grin P2P network is reduced by not requiring them to send historical blocks to new nodes.*

The first Grin block was mined on January 15, 2019. We used a snapshot of the blockchain from March 17, 2020 in our analysis which had 612,102 blocks. Grin has a block subsidy of 60 grins per block and a target inter-block time of one minute. Coinbase outputs cannot be spent until they receive 1440 confirmations which corresponds to 24 hours worth of blocks [46]. For a regular transaction output (RTO) in a block at height h (with genesis block having height 0), a trivial upper bound on the amount of coins in the output is $60 \times \max(0, h - 1439)$ grins. This corresponds to the cumulative block subsidy in the blocks from height 0 to height $\max(0, h - 1440)$. We define the *flow upper bound* for an RTO to be the sum of the amounts in its donor coinbase outputs minus the total transaction fees paid in the paths from these donor coinbase outputs to the RTO (see Section 5.3 for an illustration). The effectiveness of the flow upper bound can be quantified using the *flow ratio* of an RTO which is defined as

$$\text{Flow ratio of RTO} = \frac{\text{Flow upper bound of RTO}}{\text{Trivial upper bound of RTO}}.$$

A value of flow ratio close to 1 implies that the flow upper bound does not reveal much information about the amount in the RTO. But a flow ratio value close to 0 implies that the flow upper bound is effective in constraining the amount in the RTO to a narrow range in the relative sense.

5.1 Our Contribution

Our main contribution is an empirical analysis of the confidentiality of amounts in the Grin blockchain which takes the transaction graph into account. To enable

*Beam does allow the download of all historical blocks from its P2P network, in addition to allowing the download of a compressed blockchain state like Grin.

efficient computation of the flow upper bound, we define a graph with vertex set equal to the union of the set of coinbase outputs and the set of blocks. Note that regular transaction inputs or outputs are not represented as vertices in this graph. The graph edges are defined to reflect all possible flows of coins between transaction inputs and outputs. Using this graph, we calculate the flow ratio as a function of the block height for the Grin blockchain (see Figure 5.3). For the blocks in our snapshot, we find that the flow ratio is less than 0.5 for RTOs in 6.6% of the blocks and more than 0.9 for 88% of the blocks. As these statistics may be biased by early blocks mined during periods of low transaction activity, we consider the distribution of flow ratio for only unspent regular transaction outputs (URTOs) in our snapshot (see Figure 5.4). We find that while 95% of the 110,149 URTOs have a flow ratio greater than 0.9, about 0.8% of them have a flow ratio less than 0.01. We conclude that while the flow upper bound does not violate the confidentiality of most of the URTOs, it can constrain the amounts in some URTOs to a narrow range.

5.2 Related Work

Inputs and outputs from disparate transactions are aggregated in a MimbleWimble block which hides the link between those involved in the same transaction. Ivan Bogatty [47] demonstrated a practical attack to uncover links between inputs and outputs in a Grin block by listening to transactions broadcast in the peer-to-peer network. We can obtain tighter flow upper bounds by incorporating such link information. Due to unavailability of such link information for historical blocks, we do not consider this information in the flow upper bound calculations described in this paper. So our results represent a conservative estimate of the upper bound and could be improved upon by incorporating links between inputs and outputs.

Apart from Bogatty's attack [47], we are not aware of any other work addressing the privacy of MimbleWimble-based cryptocurrencies. To the best of our knowledge, our work is the first to address the confidentiality of amounts in MimbleWimble. Bogatty's attack is only concerned with linking inputs and outputs in a Grin block and does not consider privacy of amounts. While Monero also has amounts hidden by Pedersen commitments, previous work addressing privacy in Monero by Kumar et al. [48] and Möser et al. [49] has been primarily concerned with identifying the actual source address in the ring of addresses present in a Monero transaction. These papers do not address the privacy of amounts in Monero, which is an interesting

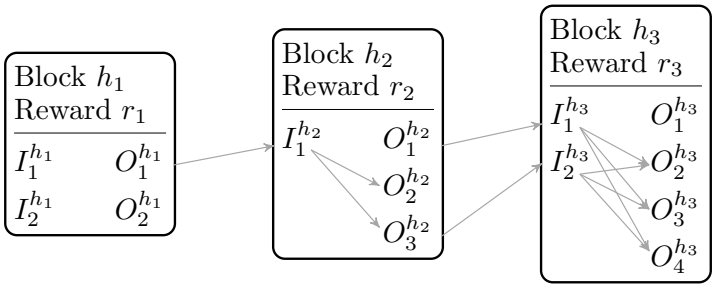


Figure 5.1: llustration of flow upper bound calculation for blocks at height h_1, h_2, h_3.

direction for future work (our approach cannot be used directly due to the source address obfuscation in Monero).

5.3 Illustration of Flow Upper Bound Calculation

In this section, we illustrate the flow upper bound calculation with an example. Before proceeding, we assume that the reader is familiar with the structure of blocks and transactions in Grin, which is also described in detail in Section 3.4.4 of Chapter 3. Consider blocks at heights h_1, h_2, and h_3 on the Grin blockchain as shown in Figure 5.1. Let the total fees for the block at height h_i be f_i^{tot}. The block reward in block h_i is then $r_i = r + f_i^{\text{tot}}$ where $r = 60$ grins is the block subsidy. An inter-block arrow from an output $O_l^{h_i}$ to an input $I_j^{h_{i+1}}$ denotes that the input is spending the output. An intra-block arrow from an input $I_j^{h_i}$ to an output $O_l^{h_i}$ indicates that the input could be contributing coins to the output; however such an arrow does not indicate that the input is definitely contributing to the output. In the absence of linkability information, we assume any input in a block can contribute to any RTO in the same block.

Let $\mathfrak{a}(C)$ be the amount hidden in a commitment C. We will assume that the first output $O_1^{h_i}$ is the only coinbase output in all three blocks. Then $\mathfrak{a}(O_1^{h_i}) = r_i$ for $i = 1, 2, 3$. The coinbase output $O_1^{h_1}$ is spent in block h_2 via the input $I_1^{h_2}$. So we have $I_1^{h_2} = O_1^{h_1} \implies \mathfrak{a}(I_1^{h_2}) = r_1$. The coinbase output $O_1^{h_2}$ and regular transaction output $O_3^{h_2}$ are spent in block h_3 via the inputs $I_1^{h_3}$ and $I_2^{h_3}$ respectively. So we have $I_1^{h_3} = O_1^{h_2}$ and $I_2^{h_3} = O_3^{h_2}$ which implies $\mathfrak{a}(I_1^{h_3}) = r_2$ and $\mathfrak{a}(I_2^{h_3}) = \mathfrak{a}(O_3^{h_2})$.

The consequence of equation (3.10) in block h_2 is

$$\mathfrak{a}(O_2^{h_2}) + \mathfrak{a}(O_3^{h_2}) = r_1 - f_2^{\text{tot}}. \tag{5.1}$$

While the sum of the amounts in $O_2^{h_2}$ and $O_3^{h_2}$ is known, the allocation of the sum to each output is hidden. As the sum represents an upper bound on the individual amounts in each of the outputs, we have

$$\mathfrak{a}(O_l^{h_2}) \leq r_1 - f_2^{\text{tot}} \quad \text{for } l = 2, 3. \tag{5.2}$$

The term on the right hand side is the flow upper bound for the RTOs in block h_2. The set of donor coinbase outputs for $O_2^{h_2}, O_3^{h_2}$ contains only $O_1^{h_1}$ which contributes r_1 to the upper bound. The f_2^{tot} term corresponds to the total fees paid on the path from the donor coinbase output $O_1^{h_1}$ to the RTOs $O_2^{h_2}, O_3^{h_2}$. In general, the flow upper bound is the same for all the RTOs in the same block.

The consequence of equation (3.10) in block h_3 is

$$\begin{aligned}
&\mathfrak{a}(O_2^{h_3}) + \mathfrak{a}(O_3^{h_3}) + \mathfrak{a}(O_4^{h_3}) \\
&= \mathfrak{a}(I_1^{h_3}) + \mathfrak{a}(I_2^{h_3}) - f_3^{\text{tot}} \\
&= \mathfrak{a}(O_1^{h_2}) + \mathfrak{a}(O_3^{h_2}) - f_3^{\text{tot}} \\
&= r_2 + \mathfrak{a}(O_3^{h_2}) - f_3^{\text{tot}} \\
&\leq r_2 + r_1 - f_2^{\text{tot}} - f_3^{\text{tot}},
\end{aligned}$$

where the last inequality follows from equation (5.2). Once again the upper bound on the sum of the amounts in the RTOs, yields the flow upper bound on the individual amounts given by

$$\mathfrak{a}(O_l^{h_3}) \leq r_2 + r_1 - f_2^{\text{tot}} - f_3^{\text{tot}} \quad \text{for } l = 2, 3, 4. \tag{5.3}$$

The $r_2 + r_1$ term in the flow upper bound is due to the donor coinbase outputs $O_1^{h_1}, O_2^{h_2}$ and the $f_2^{\text{tot}} + f_3^{\text{tot}}$ term corresponds to the total fees paid on the paths from these donor coinbase outputs to the RTOs in block h_3.

5.4 The Flow Graph

To automate the calculation of flow upper bounds, we construct a directed graph $G = (V, E)$ from the information available on the Grin blockchain, which we call the *flow graph*. In the Grin blockchain snapshot we considered, every block had exactly one coinbase output even though this is not mandatory.[†] We assume that this condition holds in our discussion below.

Let V_{bl} be the set of blocks in the blockchain and let V_{cb} be the set of all coinbase outputs. The set of vertices is defined as $V = V_{\text{bl}} \cup V_{\text{cb}}$. The set of directed edges E will capture two types of flows of coins.

[†] `https://github.com/mimblewimble/grin/issues/689`

(i) For $c \in V_{\text{cb}}$ and $b \in V_{\text{bl}}$, the directed edge (c, b) belongs to the edge set E if the coinbase output c is spent by a regular transaction input in block b.

(ii) For $b_1, b_2 \in V_{\text{bl}}$, the directed edge (b_1, b_2) belongs to the edge set E if *at least* one regular transaction output in block b_1 is spent by a regular transaction input in block b_2.[‡]

The edge set E has no other edges. Note that the intra-block flow of coins from inputs to outputs in the same block has not been explicitly represented in the flow graph. This is because we are not taking linkability information about inputs and outputs into account. For block $b \in V_{\text{bl}}$, Algorithm 1 generates the subgraph $G_s^{(b)} = (V_s^{(b)}, E_s^{(b)})$, $V_s^{(b)} \subset V_{\text{bl}}$, $E_s^{(b)} \subset E$ and computes the set $R^{(b)} \subset V_{\text{cb}}$ of donor coinbase vertices. Let $\texttt{pred}(b)$ denote the set of predecessors to node b.

Algorithm 1 Create subgraph for node $b \in V_{\text{bl}}$

```
1: procedure subG(b)
2:     for p in pred(b) do
3:         V_s^(b) ← V_s^(b) ∪ {p},  E_s^(b) ← E_s^(b) ∪ {(p, b)}    // Predecessors added to subgraph
4:         if p ∈ V_cb and p ∉ R^(b) then                            // Stop if coinbase node is found
5:             R^(b) ← R^(b) ∪ {p}
6:         else                                                      // Else continue recursively
7:             subG(p)
```

Definition 5.1 *A coinbase output vertex c in G is called a donor of a block b if there is a directed path from c to b in G. A donor of a block b is also referred to as the donor of the regular transaction outputs in the block b.*

For example, Figure 5.2 shows the subgraph generated by the donor coinbase outputs of the block at height 1499 and the blocks which lie on paths from these coinbase outputs to it. The square vertices represent blocks and the circular vertices represent coinbase outputs labelled with the block height in which they were generated. Edges denote the flow of coins. Block 1499 has 7 donors at block heights 5, 7, 9, 16, 18, 33, and 38. The flow upper bound for the RTOs in block 1499 is equal to the sum of the block rewards of these 7 coinbase outputs minus the transaction fees paid in the 9 blocks which lie on the paths from the 7 donors to 1499 (inclusive of 1499).

[‡]Note that output being spent in block b_1 has to be an RTO and not a coinbase transaction output. The flow of coins out of coinbase outputs is captured by the previous type of edge. Also note that there can exist at most one edge between two blocks b_1 and b_2 as the edge existence condition requires only that at least one RTO in block b_1 be spent in block b_2.

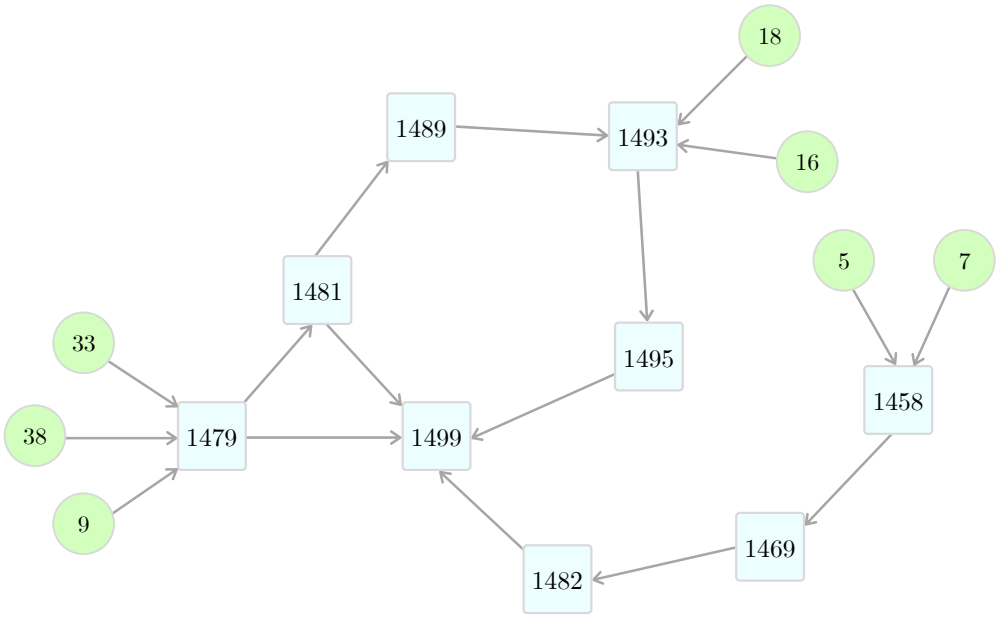


Figure 5.2: Subgraph generated for block at height 1499.

Let $V_{\text{cb}}^{(h)}$ be the set of donor coinbase outputs of the block at height h. Let $V_{\text{bl}}^{(h)}$ be the set of blocks which lie on a directed path in the flow graph G from any vertex in $V_{\text{cb}}^{(h)}$ to the block at height h. The block at height h is also included in $V_{\text{bl}}^{(h)}$. For a coinbase output vertex $c \in V_{\text{cb}}$, let $\mathit{reward}(c)$ denote the block reward (subsidy plus fees) of the block in which c appeared. For a block vertex $b \in V_{\text{bl}}$, let $\mathit{fees}(b)$ denote the total transaction fees paid by all the regular transactions in block b. For a regular transaction output O in a block at height h, the flow upper bound is given by

$$\mathfrak{a}(O) \leq \sum_{c \in V_{\text{cb}}^{(h)}} \mathit{reward}(c) - \sum_{b \in V_{\text{bl}}^{(h)}} \mathit{fees}(b). \tag{5.4}$$

The correctness of this upper bound can be argued as follows. It is clear that

$$\mathfrak{a}(O) \leq \sum_{c \in V_{\text{cb}}^{(h)}} \mathit{reward}(c) \tag{5.5}$$

as the total coins in O cannot exceed the sum of all possible sources of coins for block h. Let $f_{\text{tot}}^{(h)} = \sum_{b \in V_{\text{bl}}^{(h)}} \mathit{fees}(b)$. We claim that the fees in $f_{\text{tot}}^{(h)}$ must be paid *only* from the coins minted in coinbase outputs belonging to $V_{\text{cb}}^{(h)}$. If our claim is true, $f_{\text{tot}}^{(h)}$ must be deducted from the upper bound as these fees are paid before the coins reach the output O. To verify our claim, suppose coins minted in a coinbase output $c' \notin V_{\text{cb}}^{(h)}$ contributed ϵ coins to the fees in $f_{\text{tot}}^{(h)}$. Then there is a sequence of transactions which resulted in the ϵ coins being deposited in a block $b \in V_{\text{bl}}^{(h)}$.

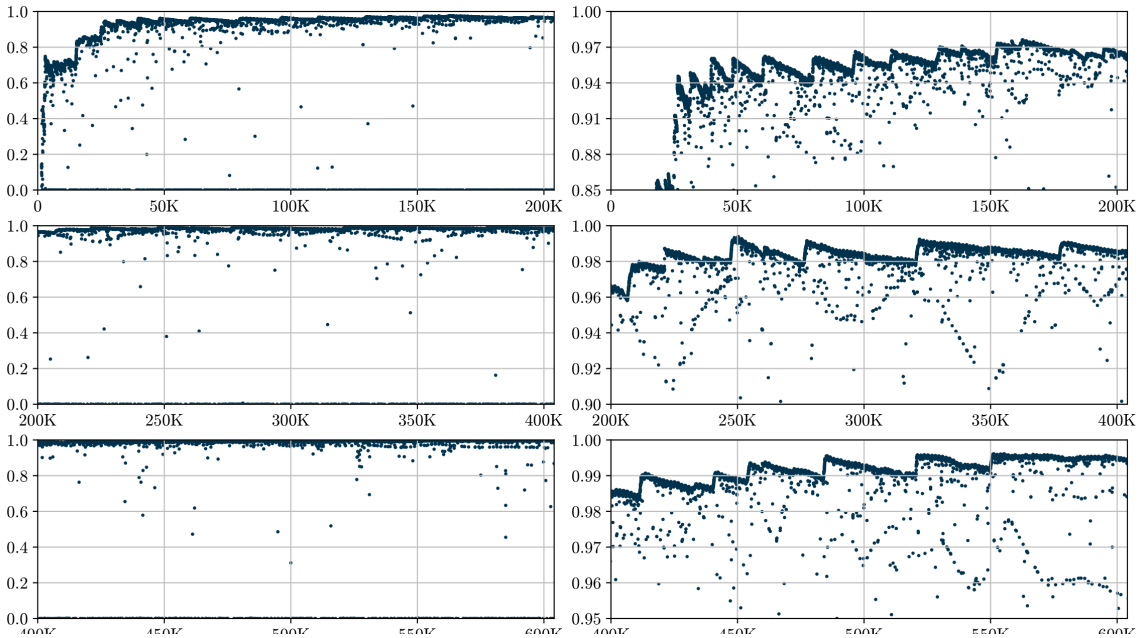


Figure 5.3: Plot of flow ratio vs block height. Plot of flow ratio close to 1 shown magnified on right.

This implies c' is a donor of this block b. As block b itself lies on a directed path from a donor coinbase output to the block at height h, there is a directed path from c' to the block at height h, i.e. c' is a donor of the block at height h. This is a contradiction as $c' \notin V_{\text{cb}}^{(h)}$.

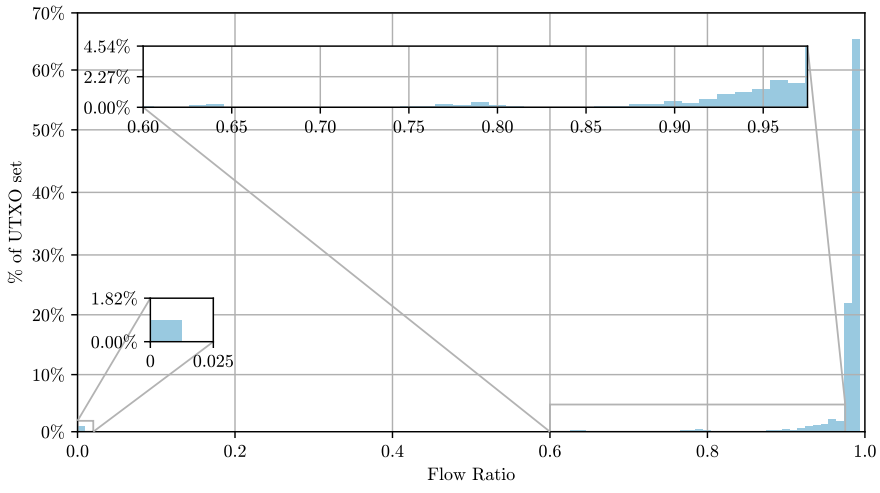


Figure 5.4: The distribution of flow ratio for outputs in the URTO set.

5.5 Results

For our analysis, we used a snapshot of the Grin blockchain from March 17, 2020 containing 612,102 blocks. As historical blocks are not available for download from the Grin P2P network, we were fortunate to obtain a PostgreSQL database containing the blockchain data from the administrator of the GrinExplorer website. We used the community edition of the Neo4j graph database for the construction of the flow graph and flow upper bound calculations.

Figure 5.3 shows scatter plots of the flow ratio as a function of the block height, *only for blocks with at least one RTO*. Note that the flow ratio corresponding to a block height is the flow ratio for all RTOs in the block at that height. We used a step size of 10 in the block height to reduce calculation time (which still took 13.5 hours). The subplots in the left column plot the flow ratio for three block height range of approximately 612,000. The subplots in the right column plot the flow ratio for the same block height ranges but with y-axis ranges 0.85 to 1, 0.9 to 1, and 0.95 to 1. The left column subplots show that while the flow ratio was initially small it had exceeded 0.9 for most blocks by block height 50,000. We found that 88% of the blocks we considered had a flow ratio above 0.9. However, there were still block heights with small flow ratios even at heights larger than 100,000. For instance, about 6.6% of the blocks we considered with heights larger than 100,000 had a flow ratio less than 0.5. The column subplots on the right reveal a jagged structure in the scatter plots for flow ratios close to 1. While a precise explanation eludes us at this point, we suspect that the rising edges are due to the accumulation of coins by miners while the falling edges are due to the increase in the trivial upper bound in the flow ratio denominator.

To get an idea of the current state of the Grin blockchain, we plot the distribution of flow ratio for only unspent regular transaction outputs (URTOs) in our snapshot in Figure 5.4. We find that while 95% of the 110,149 URTOs have a flow ratio greater than 0.9, about 0.8% of them have a flow ratio less than 0.01. In terms of upper bounds, we found that 983 of the URTOs have an upper bound of 1800 grins. We conclude that while the flow upper bound does not violate the confidentiality of most of the URTOs, it can constrain the amounts in some URTOs to a narrow range.

As noted earlier, we used a step size of 10 in the block height while computing flow ratio for RTOs in blocks. We did so for two reasons: (i) Computing flow upper bounds per block is a computationally heavy process as the block height increases, (ii) The trend observed in flow ratio for step size 10 and step size 1 is similar. For

example, Figure 5.5 shows flow ratio for blocks in range [39000, 41000]. The top subplot plots flow ratio for each block in the range [39000, 41000] while the bottom subplot plots for blocks in steps of 10. Clearly, the trend is very similar in both the subplots.

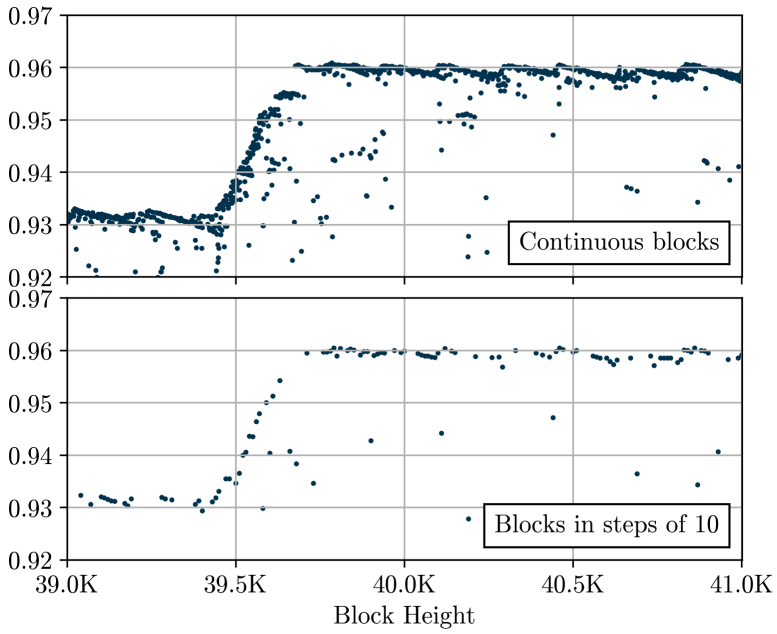


Figure 5.5: Flow ratio plot for blocks in range [39000,41000], the top and bottom plots have steps of size 1 and 10 respectively.

5.6 Conclusion

In this chapter, we explore the question of whether the transaction graph in Grin violates the confidentiality of amounts hidden in the Pedersen commitments corresponding to regular transaction outputs. We find that the confidentiality for most outputs is preserved if the linkability information between inputs and outputs is ignored. As incorporating linkability information will lead to tighter upper bounds, it is a direction worthy of more investigation. The presence of regular transaction outputs with small upper bounds shows that the perfectly hiding property of Pedersen commitments in MimbleWimble-based cryptocurrencies should be taken with a grain of salt. We also intend to examine in future if similar analysis if possible for other cryptocurrency systems like Beam and Monero which use Pedersen Commitments.

Appendix A

Security Proofs of RevelioBP

We present the complete security analysis of the argument of knowledge Π_{RevBP} protocol in the following sections. Although the soundness and zero-knowledge proofs for Π_{RevBP} have some similarities to the security proofs of the ring signature construction in Omniring [28], they are not trivial and necessary to describe the RevelioBP protocol completely.

A.1 Proof of Lemma 4.1

Since $\mathbf{g}_w = (\mathbf{g}'_w \| \mathbf{g}')$ and $\mathbf{g}'$ is generated uniformly, we need only to prove that there is no non-trivial discrete logarithm relation between elements of $\mathbf{g}'_w$. We show that it is difficult to find $\mathbf{l}, \mathbf{l}' \in \mathbb{Z}^{n+3}$ such that $(\mathbf{g}'_w)^{\mathbf{l}} = (\mathbf{g}'_w)^{\mathbf{l}'}$. Let us denote $\mathbf{p} = (p_1, p_2, \ldots, p_{n+3})$, $\mathbf{l} = (l_1, l_2, \ldots, l_{n+3})$ and $\mathbf{l}' = (l'_1, l'_2, \ldots, l'_{n+3})$.

Further, suppose the simulator is given a discrete log problem query (g_0, X) where it wants to compute the discrete log of X with respect to g_0. The simulator picks randomly scalars $\lambda_i, \rho_1, \rho_2 \xleftarrow{\$} \mathbb{Z}_q$ for all $i \in [n+3]$ and sets $g = g_0, h = g^{\rho_1}, g_t = g^{\rho_2}$, and $p_i = X^{\lambda_i}$. Thus, $C_i = g^{r_i} h^{a_i} = g_0^{(r_i + \rho_1 a_i)}$, and $I_j = g_t^{r_j} h^{a_j} = g_0^{(\rho_2 r_j + a_j)}$.

$$
\begin{aligned}
(\mathbf{g}'_w)^{\mathbf{l}} &= ((g \| g_t \| \mathbf{C} \| I_j^{-u})^w \circ \mathbf{p})^{\mathbf{l}} \\
&= (g^w p_1 \| g_t^w p_2 \| C_1^w p_3 \| \ldots \| C_n^w p_{n+2} \| I_j^{-uw} p_{n+3})^{\mathbf{l}} \\
&= (g^w p_1)^{l_1} (g_t^w p_2)^{l_2} \prod_{i=3}^{n+2} (C_{i-2}^w p_i)^{l_i} (I_j^{-uw} p_{n+3})^{l_{n+3}} \\
&= g^{w l_1} g_t^{w l_2} I_j^{-uw l_{n+3}} \prod_{i=1}^{n} C_i^{w l_{i+2}} \prod_{i=1}^{n+3} p_i^{l_i} \\
&= g_0^{w(l_1 + l_2\rho_2 - u(\rho_2 r_j + a_j) l_{n+3} + \sum_{i=1}^{n} ((r_i + \rho_1 a_i) l_{i+2}))} \cdot X^{\sum_{i=1}^{3} \lambda_i l_i}
\end{aligned}
$$

Thus, we can write $(\mathbf{g}'_w)^{\mathbf{l}} = (\mathbf{g}'_w)^{\mathbf{l}'}$ as $(\mathbf{g}'_w)^{(\mathbf{l}-\mathbf{l}')} = 1$.

$$\implies g_0^{w((l_1-l'_1)+(l_2-l'_2)\rho_2 - u(\rho_2 r_j + a_j)(l_{n+3}-l'_{n+3}))} \\ g_0^{w\sum_{i=1}^{n}((r_i+\rho_1 a_i)(l_{i+2}-l'_{i+2}))} \cdot X^{\sum_{i=1}^{3}\lambda_i(l_i-l'_i)} = 1.$$

Since $\mathbf{l}, \mathbf{l}'$ are distinct, $l_i \neq l'_i$ for atleast one $i \in [n+3]$, the simulator is able to find a non-trivial discrete log representation of 1 with respect to (g_0, X) given as

$$\frac{w}{\sum_{i=1}^{n+3}\lambda_i(l_i - l'_i)} \cdot \Big[\Big((l_1 - l'_1) + (l_2 - l'_2)\rho_2 - u(\rho_2 r_j + a_j)(l_{n+3} - l'_{n+3})\Big) \\ + \sum_{i=1}^{n}\Big((r_i + \rho_1 a_i)(l_{i+2} - l'_{i+2})\Big)\Big].$$

■

A.2 Proof of Theorem 1 (Perfect SHVZK)

Let the verifier challenges be (u, v, w, y, z, x). Simulator $\mathcal{S}$ computes $\hat{I}(u), \mathbf{g}_w(u, w)$ as described in Π_{RevBP}. We will use $\delta(u, v, y, z)$ to make the dependence of δ (as defined in Fig. 4.4) on the challenges explicit. Now, $\mathcal{S}$ samples the following quantities uniformly from respective groups: $S, T_2 \xleftarrow{\$} \mathbb{G}$, $\boldsymbol{\ell}, \boldsymbol{\mathfrak{r}} \xleftarrow{\$} \mathbb{Z}_q^N$ $\tau_x, r \xleftarrow{\$} \mathbb{Z}_q$. It then computes the remaining quantities corresponding to the ones which were sent by $\mathcal{P}$ to $\mathcal{V}$ in Π_{RevBP} as follows:

$$\hat{t} = \langle \boldsymbol{\ell}, \boldsymbol{\mathfrak{r}} \rangle, \tag{A.1}$$

$$T_1 = (g^{\hat{t}-\delta} h^{\tau_x} T_2^{-x^2})^{x^{-1}}, \tag{A.2}$$

$$A = (h')^r S^{-x} \mathbf{g}_w^{\boldsymbol{\ell}-\boldsymbol{\alpha}} \mathbf{h}^{\boldsymbol{\theta}^{\circ -1} \circ \boldsymbol{\mathfrak{r}} - \boldsymbol{\beta}}. \tag{A.3}$$

Finally, $\mathcal{S}$ outputs the simulated transcript $(A, S, T_1, T_2, \boldsymbol{\ell}, \boldsymbol{\mathfrak{r}}, \hat{t}, \tau_x, r)$. Note that since $\boldsymbol{\ell}, \boldsymbol{\mathfrak{r}}$ are uniformly sampled from $\mathbb{Z}_q^N$, $\hat{t}$ is uniformly distributed in $\mathbb{Z}_q$. T_1 is also uniformly distributed in $\mathbb{G}$ as g, h, T_2 are uniformly sampled from $\mathbb{G}$ and the corresponding exponents are also uniformly distributed in $\mathbb{Z}_q$. From Lemma 4.1, recall that $\mathbf{g}_w$ can also be considered as uniformly distributed in $\mathbb{G}^{n+3}$. Thus, A is also uniformly distributed in $\mathbb{G}$ since the generators as well as exponents in the equation for computing A are uniformly distributed in the respective groups. This implies that all the elements produced by $\mathcal{S}$ and those produced by Π_{RevBP} are identically distributed and also satisfy the verification equations at the end of Figure 4.6. Therefore, the protocol is perfect special honest-verifier zero knowledge. ■

A.3 Proof of Theorem 4.2 (Soundness)

To prove that Π_{RevBP} has witness-extended emulation, first we state a couple of useful lemmas and a corollary. Before we proceed, we define some notation and a new system of constraint equations CS. Let $\boldsymbol{\gamma}_L, \boldsymbol{\gamma}_R \in \mathbb{Z}_q^N$ be

$$\boldsymbol{\gamma}_L \coloneqq (\gamma_{L,1} \| \gamma_{L,2} \| \boldsymbol{\gamma}_{L,3} \| \gamma_{L,4} \| \quad \boldsymbol{\gamma}_{L,5} \quad \| \boldsymbol{\gamma}_{L,6}),$$
$$\boldsymbol{\gamma}_R \coloneqq (\underbrace{\gamma_{R,1}}_{1} \| \underbrace{\gamma_{R,2}}_{1} \| \underbrace{\boldsymbol{\gamma}_{R,3}}_{n} \| \underbrace{\gamma_{R,4}}_{1} \| \underbrace{\boldsymbol{\gamma}_{R,5}}_{sn} \| \underbrace{\boldsymbol{\gamma}_{R,6}}_{s}),$$

where for $i \in \{L, R\}$, $\gamma_{i,j} \in \mathbb{Z}_q$ for $j \in \{1, 2, 4\}$, $\boldsymbol{\gamma}_{i,3} \in \mathbb{Z}_q^n$, $\boldsymbol{\gamma}_{i,6} \in \mathbb{Z}_q^s$ and for some matrices $\Gamma_L, \Gamma_R \in \mathbb{Z}_q^{s \times n}$, $\boldsymbol{\gamma}_{i,5} = vec(\Gamma_i) \in \mathbb{Z}_q^{sn}$. Define the constraint system CS with parameter u such that $\mathsf{CS}(\boldsymbol{\gamma}_L, \boldsymbol{\gamma}_R) = 0 \iff$

$$\begin{cases} \boldsymbol{\gamma}_{L,5} \circ \boldsymbol{\gamma}_{R,5} &= \mathbf{0}^{sn}, & \text{(A.4)} \\ \gamma_{L,1} &= -\langle \boldsymbol{u}^s, \boldsymbol{\gamma}_{L,6} \rangle, & \text{(A.5)} \\ \gamma_{L,2} &= \langle \boldsymbol{u}^s, \boldsymbol{\gamma}_{L,6} \rangle, & \text{(A.6)} \\ \boldsymbol{\gamma}_{L,3} &= \boldsymbol{u}^s \Gamma_L, & \text{(A.7)} \\ \Gamma_L \mathbf{1}^n &= \mathbf{1}^s, & \text{(A.8)} \\ \gamma_{L,4} &= 1, & \text{(A.9)} \\ \boldsymbol{\gamma}_{L,5} &= -\boldsymbol{\gamma}_{R,5} + \mathbf{1}^{sn}. & \text{(A.10)} \end{cases}$$

Lemma A.1 *For a fixed $q > 2^\lambda$ and $u, v \in \mathbb{Z}_q$, suppose there exist $\boldsymbol{\gamma}_L, \boldsymbol{\gamma}_R \in \mathbb{Z}_q^N$ such that we have $\mathsf{EQ}(\boldsymbol{\gamma}_L, \boldsymbol{\gamma}_R) = 0$ for sn different values of y and two different values of v, then $\mathsf{CS}(\boldsymbol{\gamma}_L, \boldsymbol{\gamma}_R) = 0$.*

Proof: Since $\mathsf{EQ}(\boldsymbol{\gamma}_L, \boldsymbol{\gamma}_R) = 0$ for sn different values of y, the following polynomials in y of degree at most $sn - 1$ have sn different roots. Thus, all of them must be equal to *zero* polynomials.

$$\begin{aligned} \langle \boldsymbol{\gamma}_{L,5} \circ \boldsymbol{\gamma}_{R,5}, \boldsymbol{y}^{sn} \rangle = 0 \qquad & \text{by (4.10)}, \\ v \cdot \gamma_{L,1} + \gamma_{L,2} + (v-1)\langle \boldsymbol{\gamma}_{L,6}, \boldsymbol{u}^s \rangle = 0 \qquad & \text{by (4.11)}, \\ \langle \boldsymbol{\gamma}_{L,3} - \boldsymbol{u}^s \Gamma_L, \boldsymbol{y}^n \rangle = 0 \qquad & \text{by (4.12)}, \\ (\gamma_{L,4} - 1) y^s + \langle \Gamma_L \mathbf{1}^n - \mathbf{1}^s, \boldsymbol{y}^s \rangle = 0 \qquad & \text{by (4.13)}, \\ \langle \boldsymbol{\gamma}_{L,5} + \boldsymbol{\gamma}_{R,5} - \mathbf{1}^{sn}, \boldsymbol{y}^{sn} \rangle = 0 \qquad & \text{by (4.14)}. \end{aligned}$$

Furthermore, since $\mathsf{EQ}(\boldsymbol{\gamma}_L, \boldsymbol{\gamma}_R) = 0$ for two different values of v too, the coefficient of v and the constant term in the second equation both must be *zero*. By comparing coefficients, we get $\mathsf{CS}(\boldsymbol{\gamma}_L, \boldsymbol{\gamma}_R) = 0$. ■

Lemma A.2 *If* $\mathsf{CS}(\boldsymbol{\gamma}_L, \boldsymbol{\gamma}_R) = 0$ *then each row of* Γ_L *is a unit vector of length* n.

Proof: This follows from equations (A.4), (A.8) and (A.10). ■

Corollary 1 *Assuming the discrete logarithm assumption holds over* $\mathbb{G}$*, a* PPT *adversary cannot find a non-trivial discrete logarithm relation between the components of the base* $(h' \| \boldsymbol{g}_w \| \boldsymbol{h})$

Proof: Since $(h', \mathbf{h})$ are generated uniformly from $\mathbb{G}$, it is infeasible for a PPT adversary to compute its discrete log relation with base vector $\mathbf{g}_w$. Proving that a PPT adversary cannot find a non-trivial discrete log relation between components of $\mathbf{g}_w$ follows from Lemma 4.1. ■

With the above lemmas and the corollary, we proceed to construct an extractor $\mathscr{E}$. Let $pp \leftarrow \mathit{Setup}(\lambda)$ and $\mathit{stmt}, \mathit{wit} \leftarrow \mathscr{A}(pp)$. The aim of $\mathscr{E}$ is to produce a *valid* transcript and consequently the witness wit' corresponding to that transcript. Since $\mathscr{E}$ has oracle access to $\langle \mathscr{P}^\star(pp, \mathit{stmt}; \mathit{wit}), \mathcal{V}(pp, \mathit{stmt}) \rangle$ for any prover $\mathscr{P}^\star$, producing a valid transcript is trivial for $\mathscr{E}$. We hence focus on how $\mathscr{E}$ could extract a valid witness.

Extractor $\mathscr{E}$ runs $\mathscr{P}^\star$ on one value each of u, v, 2 different values of w, sn different values of y, 5 different values of z and 3 different values of x. This results in $30 \times sn$ transcripts. $\mathscr{E}$ fixes the values of (w, y, z) and runs $\mathscr{P}^\star$ for $x = (x_1, x_2, x_3)$. Let the transcripts for the respective x be $(A, S, T_1, T_2, \tau_{x_i}, r_{x_i}, \boldsymbol{\ell}_{x_i}, \boldsymbol{\imath}_{x_i}, \hat{t}_{x_i})$ for $i = 1, 2, 3$. Now $\mathscr{E}$ will extract the discrete logarithm representations of A, S, T_1, T_2 using the above transcripts.

Extracting A: Choose $k_i \in \mathbb{Z}_q$ for $i = 1, 2$ such that $\sum_{i=1}^{2} k_i = 1$ and $\sum_{i=1}^{2} k_i x_i = 0$. From (A.3), we have

$$A^{k_i} = h^{r_{x_i} k_i} S^{-x k_i} \mathbf{g}_w^{k_i \cdot (\boldsymbol{\ell}_{x_i} - \boldsymbol{\alpha})} \mathbf{h}^{k_i \cdot (\boldsymbol{\theta}^{\circ -1} \circ \boldsymbol{\imath}_{x_i} - \boldsymbol{\beta})} \ \forall i \in \{1, 2\}$$

$$\prod_{i=1}^{2} A^{k_i} = h^{\sum_i r_{x_i} k_i} S^{-\sum_i k_i x_i} \mathbf{g}_w^{(\sum_i k_i \cdot \boldsymbol{\ell}_{x_i}) - \boldsymbol{\alpha}(\sum_i k_i)} \mathbf{h}^{(\sum_i k_i \boldsymbol{\theta}^{\circ -1} \circ \boldsymbol{\imath}_{x_i}) - \boldsymbol{\beta}(\sum_i k_i)}$$

$$\implies A = h^{\sum_i r_{x_i} k_i} \mathbf{g}_w^{(\sum_i k_i \cdot \boldsymbol{\ell}_{x_i}) - \boldsymbol{\alpha}} \mathbf{h}^{(\sum_i k_i \boldsymbol{\theta}^{\circ -1} \circ \boldsymbol{\imath}_{x_i}) - \boldsymbol{\beta}}$$

$$\implies A = h^{r'_A} \mathbf{g}_w^{\mathbf{c}'_L} \mathbf{h}^{\mathbf{c}'_R}.$$

where $r'_A = \sum_i r_{x_i} k_i$, $\mathbf{c}'_L = (\sum_i k_i \cdot \boldsymbol{\ell}_{x_i}) - \boldsymbol{\alpha}$ and $\mathbf{c}'_R = \left(\sum_i k_i \cdot (\boldsymbol{\theta}^{\circ -1} \circ \boldsymbol{\imath}_{x_i})\right) - \boldsymbol{\beta}$. Since we have considered the above extraction for a particular w out of the 2 of its values,

$r'_A, \mathbf{c}'_L, \mathbf{c}'_R$ depend on w. To show that the discrete logarithm representation of A is independent of w, $\mathscr{E}$ repeats the above for a different w'. In particular, we have $A = h^{r''_A}\mathbf{g}_{w'}^{\mathbf{c}''_L}\mathbf{h}^{\mathbf{c}''_R}$. Now we have two possibly different representations of A. Write $\mathbf{c}'_L = (\mathbf{c}'_{L,1}\|\mathbf{c}'_{L,2})$ and $\mathbf{c}''_L = (\mathbf{c}''_{L,1}\|\mathbf{c}''_{L,2})$ of appropriate dimensions. We have

$$h^{r'_A}\mathbf{g}_w^{\mathbf{c}'_L}\mathbf{h}^{\mathbf{c}'_R} = h^{r''_A}\mathbf{g}_{w'}^{\mathbf{c}''_L}\mathbf{h}^{\mathbf{c}''_R} \implies$$
$$1 = h^{r'_A - r''_A}(g\|g_t\|\mathbf{C}\|\hat{I})^{w\cdot\mathbf{c}'_{L,1} - w'\cdot\mathbf{c}''_{L,1}}(\mathbf{p}\|\mathbf{g}')^{\mathbf{c}'_L - \mathbf{c}''_L}\mathbf{h}^{\mathbf{c}'_R - \mathbf{c}''_R}.$$

Now, if $r'_A \neq r''_A,\ \mathbf{c}'_L \neq \mathbf{c}''_L,\ \mathbf{c}'_R \neq \mathbf{c}''_R$, since $(\mathbf{p}\|\mathbf{g}'\|\mathbf{h})$ is uniformly chosen after fixing $(g\|g_t\|\mathbf{C}\|\hat{I})$, we would have violated the discrete logarithm assumption. Thus, $r'_A = r''_A,\ \mathbf{c}'_L = \mathbf{c}''_L,\ \mathbf{c}'_R = \mathbf{c}''_R$ and letting $\mathbf{c}'_L = (\xi'\|\xi''\|\hat{\mathbf{e}}'\|\psi')$, we get

$$\begin{aligned} & 1 = (g\|g_t\|\mathbf{C}\|\hat{I})^{(w-w')\cdot\mathbf{c}'_L} \\ \implies\ & 1 = (g\|g_t\|\mathbf{C}\|\hat{I})^{\mathbf{c}'_L} \text{ since } w \neq w' \\ \implies\ & 1 = g^{\xi'}\cdot g_t^{\xi''}\cdot\mathbf{C}^{\hat{\mathbf{e}}'}\cdot\hat{I}^{\psi'}. \end{aligned} \tag{A.11}$$

We will use equation (A.11) in the last part of the proof.

Extracting S: $\mathscr{E}$ samples some $k_1, k_2 \in \mathbb{Z}_q$ such that $\sum_i k_i = 0$ and $\sum_i k_i x_i = 1$. From (A.3), we have

$$S^{x_i} = h^{r_{x_i}}A^{-1}\mathbf{g}_w^{\boldsymbol{\ell}_{x_i} - \boldsymbol{\alpha}}\mathbf{h}^{\boldsymbol{\theta}^{\circ -1}\circ\boldsymbol{\tau}_{x_i} - \boldsymbol{\beta}}\ \forall i \in \{1,2\}$$

$$\prod_{i=1}^{2} S^{k_i x_i} = h^{\sum_i k_i r_{x_i}}A^{-\sum_i k_i}\mathbf{g}_w^{(\sum_i k_i\cdot\boldsymbol{\ell}_{x_i}) - (\sum_i k_i)\boldsymbol{\alpha}}\mathbf{h}^{(\sum_i k_i\cdot\boldsymbol{\theta}^{\circ -1}\circ\boldsymbol{\tau}_{x_i}) - (\sum_i k_i)\boldsymbol{\beta}}$$

$$S = h^{r'_S}\mathbf{g}_w^{\sum_i k_i\cdot\boldsymbol{\ell}_{x_i}}\mathbf{h}^{\sum_i k_i\cdot\boldsymbol{\theta}^{\circ -1}\circ\boldsymbol{\tau}_{x_i}} \tag{A.12}$$

$$\implies S = h^{r'_S}\mathbf{g}_w^{\mathbf{s}'_L}\mathbf{h}^{\mathbf{s}'_R} \tag{A.13}$$

For a fixed w, the extracted A, S hold for all possible (x, y, z) because otherwise, the discrete log assumption would be violated owing to Corollary 1 as a non-trivial discrete logarithm representation of 1 with respect to the base $(h'\|\mathbf{g}_w\|\mathbf{h})$ would be known.

Substituting these expressions of A, S in the expressions for $\boldsymbol{\ell}, \boldsymbol{\tau}$ from the protocol, we get

$$\begin{aligned} \boldsymbol{\ell}'_x &= \mathbf{c}'_L + \boldsymbol{\alpha} + \mathbf{s}'_L\cdot x \ \in \mathbb{Z}_q^N, \\ \boldsymbol{\tau}'_x &= \boldsymbol{\theta}\circ\ (\mathbf{c}'_R + \mathbf{s}'_R\cdot x) + \boldsymbol{\mu}\ \in \mathbb{Z}_q^N. \end{aligned}$$

These vectors must also hold for all (x, y, z) because if that was not the case, then we would know a non-trivial discrete logarithm representation of 1 with respect to the base $(h'\|\mathbf{g}_w\|\mathbf{h})$ due to Corollary 1.

Extracting T_1, T_2: $\mathscr{E}$ chooses $k_i \in \mathbb{Z}_q$ for $i \in \{1, 2, 3\}$ such that $\sum_{i=1}^{3} k_i = 0$, $\sum_{i=1}^{3} k_i x_i = 1$ and $\sum_{i=1}^{3} k_i x_i^2 = 0$. Thus, we have

$$T_1 = \prod_{i=1}^{3} T_1^{k_i x_i} = g^{\sum_{i=1}^{3} k_i \hat{t}_{x_i}} h^{\sum_{i=1}^{3} k_i \tau_{x_i}} = g^{t'_1} h^{r'_1}.$$

Similarly, to extract T_2, $\mathscr{E}$ chooses $k'_i \in \mathbb{Z}_q$ for $i \in \{1, 2, 3\}$ such that $\sum_{i=1}^{3} k'_i = 0$, $\sum_{i=1}^{3} k'_i x_i = 0$ and $\sum_{i=1}^{3} k'_i x_i^2 = 1$. Thus, we have

$$T_2 = \prod_{i=1}^{3} T_2^{k'_i x_i^2} = g^{\sum_{i=1}^{3} k'_i \hat{t}_{x_i}} h^{\sum_{i=1}^{3} k'_i \tau_{x_i}} = g^{t'_2} h^{r'_2}.$$

Again, the above expressions for T_1, T_2 hold for all x, or otherwise we would have obtained a non-trivial discrete logarithm representation of 1 base $(g\|h)$ violating the discrete logarithm assumption. Therefore, we have obtained $t'_1, t'_2 \in \mathbb{Z}_q$ as the exponents of g in the extracted T_1 and T_2 respectively.

Extracting witness: $\mathscr{E}$ parses $\mathbf{c}'_L$ as below and outputs the witness wit'

$$\begin{aligned} \mathbf{c}'_L &= (\xi'\|\xi''\|\hat{\mathbf{e}}'\|\psi'\|vec(\mathfrak{E}')\|\mathbf{r}'), \\ wit' &= (\mathbf{r}', \mathbf{e}'_{i_1}, \ldots \mathbf{e}'_{i_s}). \end{aligned}$$

Finally, what remains to show is that the extracted witness is a valid witness to the statement *stmt*. Using the extracted t'_1, t'_2 we have

$$t'_x = \delta(u, v, y, z) + t'_1 x + t'_2 x^2.$$

for all (x, y, z) or else we would violate the DL assumption by having a DL relation between g, h. Let

$$\begin{aligned} t'_0 &\coloneqq \delta(u, v, y, z), \\ l'(X) &\coloneqq \mathbf{c}'_L + \boldsymbol{\alpha} + \mathbf{s}'_L \cdot X, \\ r'(X) &\coloneqq \boldsymbol{\theta} \circ (\mathbf{c}'_R + \mathbf{s}'_R \cdot X) + \boldsymbol{\mu}, \\ t'(X) &\coloneqq \langle l'(X), r'(X) \rangle. \end{aligned}$$

Now, the following polynomial, for all (y, z), has at least 3 roots and hence must be a zero polynomial.

$$t'(X) - (t'_0 + t'_1 X + t'_2 X^2).$$

We have $t'(X) = t'_0 + t'_1 X + t'_2 X^2$ and particularly, $t'(0) = t'_0$. The latter two quantities are given by

$$t'_0 = z^3 \cdot \langle \mathbf{1}^{s+1}, \boldsymbol{y}^{s+1} \rangle + \langle \boldsymbol{\alpha}, \boldsymbol{\mu} \rangle + \langle \mathbf{1}^N, \boldsymbol{\nu} \rangle, \tag{A.14}$$

$$\begin{aligned} t'(0) &= \langle \mathbf{c}'_L, \boldsymbol{\theta} \circ \mathbf{c}'_R \rangle + \langle \mathbf{c}'_L, \boldsymbol{\mu} \rangle + \langle \mathbf{c}'_R, \boldsymbol{\theta} \circ \boldsymbol{\alpha} \rangle + \langle \boldsymbol{\alpha}, \boldsymbol{\mu} \rangle \\ &= \langle \mathbf{c}'_L, \boldsymbol{\theta} \circ \mathbf{c}'_R \rangle + \langle \mathbf{c}'_L, \boldsymbol{\zeta} \rangle + \langle \mathbf{c}'_L, \boldsymbol{\nu} \rangle + \langle \mathbf{c}'_R, \boldsymbol{\nu} \rangle + \langle \boldsymbol{\alpha}, \boldsymbol{\mu} \rangle \\ &= \langle \mathbf{c}'_L, \boldsymbol{\theta} \circ \mathbf{c}'_R \rangle + \langle \mathbf{c}'_L, \boldsymbol{\zeta} \rangle \ + \langle \mathbf{c}'_L + \mathbf{c}'_R, \boldsymbol{\nu} \rangle + \langle \boldsymbol{\alpha}, \boldsymbol{\mu} \rangle, \end{aligned} \tag{A.15}$$

where $\boldsymbol{\zeta} = \sum_{i=1}^{3} z^i \mathbf{v}_i$. Equations (A.14) and (A.15) imply

$$\begin{aligned} z^3 \langle \mathbf{1}^{s+1}, \boldsymbol{y}^{s+1} \rangle &= \langle \mathbf{c}'_L, \boldsymbol{\theta} \circ \mathbf{c}'_R \rangle + \langle \mathbf{c}'_L, \boldsymbol{\zeta} \rangle + \langle \mathbf{c}'_L - \mathbf{c}'_R - \mathbf{1}^t, \boldsymbol{\nu} \rangle + \langle \boldsymbol{\alpha}, \boldsymbol{\mu} \rangle \\ &= \langle \mathbf{c}'_L, \mathbf{v}_0 \circ \mathbf{c}'_R \rangle + \sum_{i=1}^{3} z^i \langle \mathbf{c}'_L, \mathbf{v}_i \rangle + z^4 \langle \mathbf{c}'_L + \mathbf{c}'_R - \mathbf{1}^N, \mathbf{v}_4 \rangle. \end{aligned}$$

The above equation holds for 5 different values of z. As the equation involves a degree 4 polynomial, the coefficients on both sides must be equal. This implies that $\mathsf{EQ}(\mathbf{c}'_L, \mathbf{c}'_R) = 0$ for sn different values of y and 2 values of v. By Lemma A.1, we have $\mathsf{CS}(\mathbf{c}'_L, \mathbf{c}'_R) = 0$. Further, Lemma A.2 implies that each row vector of $\boldsymbol{\mathcal{E}}'$ is a unit vector of length n. Let $vec(\boldsymbol{\mathcal{E}}') = (\mathbf{e}'_{i_1}, \ldots, \mathbf{e}'_{i_s})$ and write

$$\xi' = -\langle \boldsymbol{u}^s, \mathbf{r}' \rangle, \ \xi'' = \langle \boldsymbol{u}^s, \mathbf{r}' \rangle, \ \psi' = 1, \ \hat{\mathbf{e}}' = \boldsymbol{v}^s \boldsymbol{\mathcal{E}}'.$$

Also, let $i'_1, i'_2, \ldots, i'_s$ be the indices of the non-zero numbers in vector $\hat{\mathbf{e}}'$. We now show that these exponents computed from the extracted witness $(\mathbf{r}', \mathbf{e}'_{i_1}, \mathbf{e}'_{i_2}, \ldots, \mathbf{e}'_{i_s})$ is correct. From (A.11), we have

$$\begin{aligned} 1 &= g^{\xi'} \cdot g_t^{\xi''} \cdot \mathbf{C}^{\hat{\mathbf{e}}'} \cdot \hat{I}^{\psi'} \\ &= g^{-\langle \boldsymbol{u}^s, \mathbf{r}' \rangle} \cdot g_t^{\langle \boldsymbol{u}^s, \mathbf{r}' \rangle} \cdot \left(\prod_{j=1}^{s} \mathbf{C}^{u^{j-1} \cdot \mathbf{e}'_{i_j}} \right) \cdot \left(\prod_{j=1}^{s} I_j^{-u^{j-1}} \right) \\ &= \prod_{j=1}^{s} \left(g^{-r'_j} g_t^{r'_j} \mathbf{C}^{\mathbf{e}'_{i_j}} I_j^{-1} \right)^{u^{j-1}}. \end{aligned}$$

The final equality can be interpreted as an evaluation of an s-degree polynomial in the exponent at a random point u. The probability of such an evaluation being zero for a non-zero polynomial is bounded by $\frac{s+1}{q}$, which is negligible since $q > 2^\lambda$ by Schwartz-Zippel lemma. Thus, we assume that the polynomial is always *zero*. This implies that for all $j \in [s]$, $g^{-r'_j} g_t^{r'_j} \mathbf{C}^{\mathbf{e}'_{i_j}} I_j^{-1} = 1$. Now the amount a'_j can be calculated (after extracting $(r'_j, \mathbf{e}'_{i_j})$) by an honest PPT prover (or extractor) since the amount lies in the finite range $\{0, 1, \ldots, 2^{64} - 1\}$. Therefore, *wit*′ is a valid witness corresponding to the statement *stmt* for the language $\mathcal{L}_{\mathsf{RevBP}}$. ■

A.4 Proof of Theorem 4.3 (Output Privacy)

We will prove Theorem 4.3 by contradiction. We will prove that if there is a PPT distinguisher $\mathscr{D}$ who can succeed in the `OutputPriv` experiment with probability at least $\frac{1}{2} + \frac{1}{\mathfrak{p}(\lambda)}$ for a polynomial $\mathfrak{p}$, then we can construct a PPT adversary $\mathscr{E}$ who can solve the generalized DDH problem [50] with success probability at least $\frac{1}{2} + \frac{1}{2\mathfrak{p}(\lambda)}$. This is a contradiction as the generalized DDH problem is equivalent to the DDH problem and the latter is assumed to be hard in the group $\mathbb{G}$.

Let $\mathscr{E}$ be an adversary who is tasked with solving the generalized DDH problem given a tuple $(g_0, g_1, \ldots, g_{f(\lambda)}, u_0, u_1, \ldots, u_{f(\lambda)}) \in \mathbb{G}^{2f(\lambda)+2}$. $\mathscr{E}$ wants to distinguish between the following two cases:

- In the tuple $(g_0, g_1, \ldots, g_{f(\lambda)}, u_0, u_1, \ldots, u_{f(\lambda)}) \in \mathbb{G}^{2f(\lambda)+2}$, $g_l \xleftarrow{\$} \mathbb{G}$, $u_l \xleftarrow{\$} \mathbb{G}$ $\forall l = 0, 1, 2, \ldots, f(\lambda)$.
- In the tuple $(g_0, g_1, \ldots, g_{f(\lambda)}, u_0, u_1, \ldots, u_{f(\lambda)}) \in \mathbb{G}^{2f(\lambda)+2}$, $g_l \xleftarrow{\$} \mathbb{G}$, $u_l = g_l^r$ $\forall l = 0, 1, 2, \ldots, f(\lambda)$ where $r \xleftarrow{\$} \mathbb{Z}_q$.

Let $\mathfrak{d} = 1$ and $\mathfrak{d} = 2$ denote the above two cases, which are assumed to be equally likely. $\mathscr{E}$ needs to output its estimate $\mathfrak{d}'$ of $\mathfrak{d}$. To estimate $\mathfrak{d}$ correctly, $\mathscr{E}$ constructs a valid input to the `OutputPriv` distinguisher $\mathscr{D}$ as follows:

1. $\mathscr{E}$ sets $g = g_0$ and chooses $h \xleftarrow{\$} \mathbb{G}$. It also chooses amounts $a_1, a_2 \xleftarrow{\$} V$.
2. $\mathscr{E}$ chooses an integer $\mathfrak{b}$ uniformly from $\{1, 2\}$. It sets $C_{\mathfrak{b}} = u_0 h^{a_{\mathfrak{b}}}$ and chooses the other output uniformly from $\mathbb{G}$. For $l = 1, 2, \ldots, f(\lambda)$, $\mathscr{E}$ sets the tags $I_l = u_l h^{a_{\mathfrak{b}}}$.
3. For $l = 1, 2, \ldots, f(\lambda)$, $\mathscr{E}$ creates the lth argument Π_{RevBP} using the PPT simulator $\mathcal{S}$ in Appendix A.3 with $g_t = g_l$, $\mathbf{C} = (C_1, C_2)$, and $\mathbf{I} = (I_l)$.
4. $\mathscr{E}$ feeds the computed quantities to $\mathscr{D}$ and gets
$$\mathfrak{b}' = \mathscr{D}\left(\left\{I_j, \Pi^j_{\mathsf{RevBP}}, g_j\right\}_{j=1}^{f(\lambda)}, C_1, C_2, a_1, a_2\right).$$
5. If $\mathfrak{b}' = \mathfrak{b}$, then $\mathscr{E}$ sets $\mathfrak{d}' = 2$. Otherwise, $\mathfrak{d}' = 1$.

The motivation behind this construction is that when $\mathscr{D}$ estimates $\mathfrak{b}$ correctly it could be exploiting some structure in the inputs given to it.

- When $\mathfrak{d} = 1$, the $u_0, u_1, \ldots, u_{f(\lambda)}$ components of the tuple given to $\mathscr{E}$ are uniformly distributed. This makes the distribution of $(I_1, I_2, \ldots, I_{f(\lambda)}, C_1, C_2)$ identical for both $\mathfrak{b} = 1$ and $\mathfrak{b} = 2$. This in turn makes the distributions of the simulated arguments $\Pi^1_{\mathsf{RevBP}}, \ldots, \Pi^{f(\lambda)}_{\mathsf{RevBP}}$ identical for both values of $\mathfrak{b}$. Thus $\mathscr{D}$ can only estimate $\mathfrak{b}$ with a success probability of $\frac{1}{2}$. Thus $\Pr[\mathfrak{b}' = \mathfrak{b} \mid \mathfrak{d} = 1] = \frac{1}{2}$.

- When $\mathfrak{d} = 2$, the $u_l = g_l^r$ for all $l = 0, 1, \ldots, f(\lambda)$. By construction, the vector $(I_1, I_2, \ldots, I_{f(\lambda)}, C_1, C_2)$ has a distribution which is different for $\mathfrak{b} = 1$ and $\mathfrak{b} = 2$. More importantly, the input $\mathscr{E}$ feeds to $\mathscr{D}$ is identically distributed to the input $\mathscr{D}$ receives in the `OutputPriv` experiment. If $\mathscr{D}$ can estimate $\mathfrak{b}$ correctly, then $\mathscr{E}$ bets on the distinguisher $\mathscr{D}$'s ability to win in the `OutputPriv` experiment and concludes that the tuple it received is a generalized DDH tuple.

Clearly, if adversary $\mathscr{D}$ is PPT then so is $\mathscr{E}$. Suppose there is a PPT distinguisher $\mathscr{D}$ which succeeds in the `OutputPriv` experiment with probability of success which is lower bounded by $\frac{1}{2} + \frac{1}{\mathfrak{p}(\lambda)}$ where $\mathfrak{p}$ is a polynomial. Thus we have $\Pr[\mathfrak{b}' = \mathfrak{b} \mid \mathfrak{d} = 2] \geq \frac{1}{2} + \frac{1}{\mathfrak{p}(\lambda)}$.

The success probability of $\mathscr{E}$ is given by

$$\begin{aligned}
\Pr[\mathfrak{d}' = \mathfrak{d}] &= \frac{1}{2}\Pr[\mathfrak{d}' = 1 | \mathfrak{d} = 1] + \frac{1}{2}\Pr[\mathfrak{d}' = 2 | \mathfrak{d} = 2], \\
&= \frac{1}{2}\Pr[\mathfrak{b}' \neq \mathfrak{b} \mid \mathfrak{d} = 1] + \frac{1}{2}\Pr[\mathfrak{b}' = \mathfrak{b} \mid \mathfrak{d} = 2], \\
&\geq \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \left(\frac{1}{2} + \frac{1}{\mathfrak{p}(\lambda)}\right) = \frac{1}{2} + \frac{1}{2\mathfrak{p}(\lambda)}.
\end{aligned}$$

Thus, $\mathscr{E}$ succeeds in solving the generalized DDH problem with a probability non-negligibly larger than $\frac{1}{2}$. As a PPT adversary who can solve the generalized DDH problem is equivalent to a PPT adversary solving the classical DDH problem [50], we get a contradiction. ■

References

[1] A. Dutta and S. Vijayakumaran, "Revelio: A MimbleWimble proof of reserves protocol," in *2019 Crypto Valley Conference on Blockchain Technology (CVCBT)*, June 2019, pp. 7–11.

[2] B. Bünz, J. Bootle, D. Boneh, A. Poelstra, P. Wuille, and G. Maxwell, "Bulletproofs: Short proofs for confidential transactions and more," in *2018 IEEE Symposium on Security and Privacy (SP)*, May 2018, pp. 315–334.

[3] S. Nakamoto. (2008) Bitcoin: A peer-to-peer electronic cash system. [Online]. Available: http://bitcoin.org/bitcoin.pdf

[4] M. Conti, E. S. Kumar, C. Lal, and S. Ruj, "A survey on security and privacy issues of bitcoin," *IEEE Communications Surveys & Tutorials*, vol. 20, pp. 3416–3452, 2018.

[5] N. v. Saberhagen, "CryptoNote v 2.0," White paper, 2013. [Online]. Available: https://cryptonote.org/whitepaper.pdf

[6] E. B. Sasson, A. Chiesa, C. Garman, M. Green, I. Miers, E. Tromer, and M. Virza, "Zerocash: Decentralized anonymous payments from bitcoin," in *2014 IEEE Symposium on Security and Privacy*, 2014, pp. 459–474.

[7] "Grin project website." [Online]. Available: https://grin-tech.org/

[8] "Beam project website." [Online]. Available: https://www.beam.mw/

[9] A. Poelstra, "Mimblewimble," 2016. [Online]. Available: https://download.wpsoftware.net/bitcoin/wizardry/mimblewimble.pdf

[10] H. Natarajan, S. K. Krause, and H. L. Gradstein, "Distributed Ledger Technology (DLT) and blockchain (English)," *FinTech note; no. 1. Washington, D.C. : World Bank Group*, 2017. [Online].

Available: http://documents.worldbank.org/curated/en/177911513714062215/Distributed-Ledger-Technology-DLT-and-blockchain

[11] K. Gary C., "An Overview of Cryptography," *Handbook on Local Area Networks*, September 1998. [Online]. Available: https://www.garykessler.net/library/crypto.html

[12] C. David, "Blind Signatures for Payments," *Springer-Verlag*, 1982. [Online]. Available: http://www.hit.bme.hu/~buttyan/courses/BMEVIHIM219/2009/Chaum.BlindSigForPayment.1982.PDF

[13] IDEX blog. A complete list of cryptocurrency exchange hacks [updated]. [Accessed 27-MAY-2020]. [Online]. Available: https://blog.idex.io/all-posts/a-complete-list-of-cryptocurrency-exchange-hacks-updated

[14] R. A. Musiala, T. M. Goody, V. Reynolds, L. Tenery, M. McGrath, C. Rowland, and S. Sekhri, "Cryptocurrencies: Forensic techniques to meet the challenge of new fraud and corruption risks," *Springer-Verlag*, 2020. [Online]. Available: https://www.aicpa.org/content/dam/aicpa/interestareas/forensicandvaluation/newsandpublications/downloadabledocuments/eye-on-fraud-cryptocurrency-202003.pdf

[15] G. G. Dagher, B. Bünz, J. Bonneau, J. Clark, and D. Boneh, "Provisions: Privacy-preserving proofs of solvency for Bitcoin exchanges," in *Proceedings of the 22nd ACM SIGSAC Conference on Computer and Communications Security (ACM CCS)*, New York, NY, USA, 2015, pp. 720–731.

[16] A. Dutta and S. Vijayakumaran, "MProve: A proof of reserves protocol for Monero exchanges," in *2019 IEEE European Symposium on Security and Privacy Workshops (EuroS&PW)*, June 2019, pp. 330–339.

[17] K. Chalkias, K. Lewi, P. Mohassel, and V. Nikolaenko, "Distributed auditing proofs of liabilities," Cryptology ePrint Archive, Report 2020/468, 2020, https://eprint.iacr.org/2020/468.

[18] S. Roose, "Standardizing Bitcoin Proof of Reserves," Blockstream Blog Post, Feb. 2018. [Online]. Available: https://blockstream.com/2019/02/04/standardizing-bitcoin-proof-of-reserves/

[19] Wikipedia contributors. Mt. Gox — Wikipedia, the free encyclopedia. [Accessed 10-JUNE-2020]. [Online]. Available: https://en.bitcoin.it/wiki/Mt._Gox

[20] C. Decker, J. Guthrie, J. Seidel, and R. Wattenhofer, "Making bitcoin exchanges transparent," in *20th European Symposium on Research in Computer Security (ESORICS)*, 2015, pp. 561–576.

[21] "CoinMarketCap Markets." [Online]. Available: https://coinmarketcap.com/#markets

[22] A. Wood, "QuadrigaCX Wallets Have Been Empty, Unused Since April 2018, Ernst and Young Finds," Mar. 2019. [Online]. Available: https://cointelegraph.com/news/report-quadrigacx-wallets-have-been-empty-unused-since-april

[23] "Ernst & Young Inc — Third Report of the Monitor," Mar. 2019. [Online]. Available: https://documentcentre.eycan.com/eycm_library/Quadriga%20Fintech%20Solutions%20Corp/English/CCAA/1.%20Monitor%27s%20Reports/4.%20Third%20Report%20of%20the%20Monitor/Third%20Report%20of%20the%20Monitor%20dated%20March%201,%202019.pdf

[24] A. H. Koblitz, N. Koblitz, and A. Menezes, "Elliptic curve cryptography: The serpentine course of a paradigm shift," *Journal of Number Theory*, vol. 131, no. 5, pp. 781 – 814, 2011, elliptic Curve Cryptography. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0022314X09000481

[25] S. Vijayakumaran. (2017) An introduction to Bitcoin, 2017. [Online]. Available: https://www.ee.iitb.ac.in/~sarva/bitcoin/bitcoin-notes-v0.1.pdf

[26] P. Hess, "Sec 2: Recommended elliptic curve domain parameters," 2000.

[27] R. L. Rivest, A. Shamir, and L. Adleman, "A method for obtaining digital signatures and public-key cryptosystems," *Commun. ACM*, vol. 21, no. 2, pp. 120–126, Feb. 1978. [Online]. Available: https://doi.org/10.1145/359340.359342

[28] R. W. F. Lai, V. Ronge, T. Ruffing, D. Schröder, S. A. K. Thyagarajan, and J. Wang, "Omniring: Scaling private payments without trusted setup," in *Proceedings of the 2019 ACM SIGSAC Conference on Computer and Communications Security*, November 2019, pp. 31–48.

[29] J.-J. Quisquater, M. Quisquater, M. Quisquater, M. Quisquater, L. Guillou, M. A. Guillou, G. Guillou, A. Guillou, G. Guillou, and S. Guillou, "How to explain zero-knowledge protocols to your children," in *Advances in Cryptology — CRYPTO' 89 Proceedings*, G. Brassard, Ed. New York, NY: Springer New York, 1990, pp. 628–631.

[30] L. Mathieson. (2013) The difference between soundness and completeness . [Online]. Available: https://philosophy.stackexchange.com/questions/6992/the-difference-between-soundness-and-completeness

[31] "Introduction to MimbleWimble and Grin." [Online]. Available: https://github.com/mimblewimble/grin/blob/master/doc/intro.md

[32] J. Camenisch and M. Stadler, "Proof systems for general statements about discrete logarithms," Dept. of Computer Science, ETH Zürich, Tech. Rep. 260, Mar 1997.

[33] T. E. Jedusor, "Mimblewimble," 2016. [Online]. Available: https://download.wpsoftware.net/bitcoin/wizardry/mimblewimble.txt

[34] Grinscan website. Date accessed: June 7, 2020. [Online]. Available: https://grinscan.net/charts

[35] S. Bagad and S. Vijayakumaran, "Performance trade-offs in design of mimblewimble proofs of reserves," in *IEEE European Symposium on Security and Privacy Workshops (EuroS PW)*, 2020.

[36] "constants.rs — Grin rust-secp256k1-zkp GitHub repository." [Online]. Available: https://github.com/mimblewimble/rust-secp256k1-zkp/blob/master/src/constants.rs

[37] S. Noether and A. Mackenzie, "Ring confidential transactions," *Ledger*, vol. 1, pp. 1–18, 2016.

[38] A. Fiat and A. Shamir, "How to prove yourself: Practical solutions to identification and signature problems," in *Advances in Cryptology — CRYPTO' 86*, A. M. Odlyzko, Ed. Berlin, Heidelberg: Springer Berlin Heidelberg, 1987, pp. 186–194.

[39] Revelio simulation code. [Online]. Available: https://github.com/avras/revelio

[40] RevelioBP simulation code. [Online]. Available: https://github.com/suyash67/RevelioBP

[41] "MimbleWimble." [Online]. Available: https://scalingbitcoin.org/papers/mimblewimble.txt

[42] T. P. Pedersen, "Non-interactive and information-theoretic secure verifiable secret sharing," in *Advances in Cryptology — CRYPTO '91*. Springer, 1992, pp. 129–140.

[43] S. Bagad and S. Vijayakumaran, "On the confidentiality of amounts in grin," in *Crypto Valley Conference on Blockchain Technology (CVCBT)*, 2020.

[44] "GrinExplorer website." [Online]. Available: https://grinexplorer.net/

[45] "Grin Forum Thread on Grinnode.live." [Online]. Available: https://forum.grin.mw/t/https-grinnode-live-public-grin-api-and-sync-service/6671/11

[46] "The Coinbase Maturity Rule." [Online]. Available: https://github.com/mimblewimble/grin/blob/master/doc/coinbase_maturity.md

[47] Linking 96% of Grin Transactions. [Online]. Available: https://github.com/bogatyy/grin-linkability

[48] A. Kumar, C. Fischer, S. Tople, and P. Saxena, "A traceability analysis of Monero's blockchain," in *European Symposium on Research in Computer Security – ESORICS 2017*, 2017, pp. 153–173.

[49] M. Möser, K. Soska, E. Heilman, K. Lee, H. Heffan, S. Srivastava, K. Hogan, J. Hennessey, A. Miller, A. Narayanan, and N. Christin, "An empirical analysis of traceability in the Monero blockchain," *Proceedings on Privacy Enhancing Technologies*, vol. 2018, no. 3, pp. 143–163, 2018.

[50] F. Bao, R. H. Deng, and H. Zhu, "Variations of Diffie-Hellman problem," in *Information and Communications Security*, 2003, pp. 301–312.