EEL4930/EEL5840 Fall 2016 - Homework 1
Linear Algebra and PCA

September 6, 2016

Due: September 13, 2016, 11:59 PM

Instructions

Your homework submission must cite any references used (including articles, books, code, websites, and personal communications). All solutions must be written in your own words, and you must program the algorithms yourself. If you do work with others, you must list the people you worked with. Submit your solutions as a PDF to the E-Learning at UF (http://elearning.ufl.edu/).

Your programs must be written in either MATLAB or Python. The relevant code to the problem should be in the PDF you turn in. If a problem involves programming, then the code should be shown as part of the solution to that problem. If you solve any problems by hand just digitize that page and submit it (make sure the problem is labeled).

If you have any questions address them to:

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Problems

1. (1 point) Let X be a 4×2 matrix, Y be a 2×2 matrix, and Z be a matrix such that $X = ZY$. What is the size of Z?

2. (2 points) Compute a formula for A^k, i.e., the matrix power operation defined as the matrix product of k copies of A, using **eigenvalue decomposition**. Show the necessary steps of your derivation.

3. (3 points) Consider the following equation $\phi = \left(X^T w - y\right)^T \left(X^T w - y\right)$, where w is a 3×1 vector of elements of w_i, X^T is an 5×3 matrix of elements of x_{ij} and y is a 5×1 vector of elements of y_i. You can think of each row of X^T as a sample of 3 dimensions. Solve the equation

$$\frac{\partial \phi}{\partial w} = 0$$

to determine the value of w that minimizes ϕ.

Use the equation you derived to solve for these values of $X^T = \begin{pmatrix} -3 & 2 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 4 & 1 \\ 0 & 1 & -1 \end{pmatrix}$ and $y = \begin{pmatrix} 1 \\ -1 \\ 1 \\ 2 \\ 10 \end{pmatrix}$.

4. (5 points) Principal component analysis (PCA) reduces the dimensionality of the data by finding projection directions(s) that *minimizes the squared errors in reconstructing the original data* or equivalently *maximizes the variance of the projected data.*

4.1. (3 points) Consider 3 data points in the 2-d space: $x_1 = [-1, 2]^T$, $x_2 = [0, 0]^T$ and $x_3 = [2, 3]^T$. What is the first principal component (write down the vector values)?

4.2. (1 points) If you project the original data points into the 1-d subspace by the principal component (a line in this case), what are their coordinates in the 1-d subspace? And what is the variance of the projected data?

4.3. (1 points) For the projected data you just obtained above, if you consider them as an approximation of the original data points, what is the error?