# Assignment No.1

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## 1 QUESTION No.16(B) (CBSE/2006/SET-2)

Find the co-ordinates of the point equidistant from three given points $\mathbf{A}\begin{pmatrix}5\\3\end{pmatrix}$, $\mathbf{B}\begin{pmatrix}5\\-5\end{pmatrix}$ and $\mathbf{C}\begin{pmatrix}1\\-5\end{pmatrix}$

**Solution:**

Let the point equidistant from $\mathbf{A}$ & $\mathbf{B}$ & $\mathbf{C}$ be

$$\mathbf{P} = \begin{pmatrix}x\\y\end{pmatrix} \tag{1.0.1}$$

$$\|\mathbf{P} - \mathbf{A}\|^2 = \|\mathbf{P} - \mathbf{B}\|^2 \tag{1.0.2}$$

$$\mathbf{P} = \begin{pmatrix}x\\y\end{pmatrix} \tag{1.0.3}$$

$$\|\mathbf{P}\|^2 + \|\mathbf{A}\|^2 - 2\mathbf{A}^T\mathbf{P} \tag{1.0.4}$$
$$= \|\mathbf{P}\|^2 + \|\mathbf{B}\|^2 - 2\mathbf{B}^T\mathbf{P} \tag{1.0.5}$$

$$(\mathbf{A} - \mathbf{B})^T\mathbf{P} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2} \tag{1.0.6}$$

$$(\mathbf{B} - \mathbf{C})^T\mathbf{P} = \frac{\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2}{2} \tag{1.0.7}$$

$$\begin{pmatrix}(A - B)^T\\(B - C)^T\end{pmatrix}\mathbf{P} = \frac{1}{2}\frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2} \tag{1.0.8}$$

$$\mathbf{A}\begin{pmatrix}5\\3\end{pmatrix}, \mathbf{B}\begin{pmatrix}5\\-5\end{pmatrix}, \mathbf{C}\begin{pmatrix}1\\-5\end{pmatrix} \tag{1.0.9}$$

$$(\mathbf{A} - \mathbf{B})^T\mathbf{P} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2} \tag{1.0.10}$$

$$\left[\begin{pmatrix}5 & 3\end{pmatrix} - \begin{pmatrix}5 & -5\end{pmatrix}\right]\mathbf{P} = \frac{\left\|\begin{pmatrix}5\\3\end{pmatrix}\right\|^2 - \left\|\begin{pmatrix}5\\-5\end{pmatrix}\right\|^2}{2} \tag{1.0.11}$$

$$\begin{pmatrix}0 & 8\end{pmatrix}\mathbf{P} = \left[\frac{\left(\sqrt{5^2} + \sqrt{3^2}\right)^2 - \left(\sqrt{5^2} + \sqrt{-5^2}\right)^2}{2}\right] \tag{1.0.12}$$

$$\begin{pmatrix}0 & 8\end{pmatrix}\mathbf{P} = \left[\frac{\left(\sqrt{25} + \sqrt{9}\right)^2 - \left(\sqrt{25} + \sqrt{25}\right)^2}{2}\right] \tag{1.0.13}$$

$$\begin{pmatrix}0 & 8\end{pmatrix}\mathbf{P} = \left[\frac{\left(\sqrt{34}\right)^2 - \left(\sqrt{50}\right)^2}{2}\right] \tag{1.0.14}$$

$$\begin{pmatrix}0 & 8\end{pmatrix}\mathbf{P} = \frac{-16}{2} \tag{1.0.15}$$

$$\begin{pmatrix}0 & 8\end{pmatrix}\mathbf{P} = -\mathbf{8} \tag{1.0.16}$$

$$\implies \mathbf{P} = \mathbf{y} = -1 \tag{1.0.17}$$

$$(\mathbf{B} - \mathbf{C})^T\mathbf{P} = \frac{\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2}{2} \tag{1.0.18}$$

$$\left[\begin{pmatrix}5 & -5\end{pmatrix} - \begin{pmatrix}1 & -5\end{pmatrix}\right]\mathbf{P} = \frac{\left\|\begin{pmatrix}5\\-5\end{pmatrix}\right\|^2 - \left\|\begin{pmatrix}1\\-5\end{pmatrix}\right\|^2}{2} \tag{1.0.19}$$

$$\begin{pmatrix}4 & 0\end{pmatrix}\mathbf{P} = \left[\frac{\left(\sqrt{5^2} + \sqrt{-5^2}\right)^2 - \left(\sqrt{1^2} + \sqrt{-5^2}\right)^2}{2}\right] \tag{1.0.20}$$
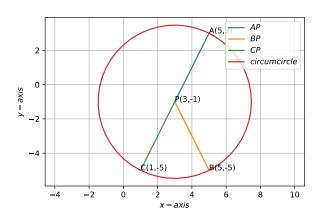
Fig. 1.1: Graphical Solution

$$\begin{pmatrix} 4 & 0 \end{pmatrix} \mathbf{P} = \left[ \frac{\left( \sqrt{25} + \sqrt{25} \right)^2 - \left( \sqrt{1} + \sqrt{25} \right)^2}{2} \right]$$

(1.0.21)

$$\begin{pmatrix} 4 & 0 \end{pmatrix} \mathbf{P} = \left[ \frac{\left( \sqrt{50} \right)^2 - \left( \sqrt{26} \right)^2}{2} \right]$$

(1.0.22)

$$\begin{pmatrix} 4 & 0 \end{pmatrix} \mathbf{P} = \frac{24}{2}$$

(1.0.23)

$$\begin{pmatrix} 4 & 0 \end{pmatrix} \mathbf{P} = 12$$

(1.0.24)

$$\implies \mathbf{P} = \mathbf{x} = 3$$

(1.0.25)

$$\mathbf{P} = \begin{pmatrix} x \\ y \end{pmatrix}$$

(1.0.26)

$$\mathbf{P} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}.$$

(1.0.27)