Assignment No.1

Suyog Tangade
MD/2020/710

Download all python codes from

https://github.com/suyogtangade/AI.git

and latex-tikz codes from

https://github.com/suyogtangade/AI.git

1 Question No.16(b) (cbse/2006/set-2)

Find the co-ordinates of the point equidistant from three given points $\mathbf{A}\begin{pmatrix}5\\3\end{pmatrix}$, $\mathbf{B}\begin{pmatrix}5\\-5\end{pmatrix}$ and $\mathbf{C}\begin{pmatrix}1\\-5\end{pmatrix}$

Solution:

Let the point equidistant from **A** & **B** & **C** be

$$\mathbf{P} = \begin{pmatrix}x\\y\end{pmatrix} \tag{1.0.1}$$

$$\|\mathbf{P}-\mathbf{A}\|^2 = \|\mathbf{P}-\mathbf{B}\|^2 \tag{1.0.2}$$

$$\mathbf{P} = \begin{pmatrix}x\\y\end{pmatrix} \tag{1.0.3}$$

$$\|\mathbf{P}\|^2 + \|\mathbf{A}\|^2 - 2\mathbf{A}^T\mathbf{P} \tag{1.0.4}$$

$$= \|\mathbf{P}\|^2 + \|\mathbf{B}\|^2 - 2\mathbf{B}^T\mathbf{P} \tag{1.0.5}$$

$$\left(A-B\right)^T\mathbf{P} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2} \tag{1.0.6}$$

$$\left(B-C\right)^T\mathbf{P} = \frac{\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2}{2} \tag{1.0.7}$$

$$\begin{bmatrix}(A-B)^T\\(B-C)^T\end{bmatrix}\mathbf{P} = \frac{1}{2}\frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2} \tag{1.0.8}$$

$$\mathbf{A}\begin{pmatrix}5\\3\end{pmatrix}, \mathbf{B}\begin{pmatrix}5\\-5\end{pmatrix}, \mathbf{C}\begin{pmatrix}1\\-5\end{pmatrix} \tag{1.0.9}$$

$$\left(A-B\right)^T\mathbf{P} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2} \tag{1.0.10}$$

$$\begin{bmatrix}5 & 3\end{bmatrix} - \begin{bmatrix}5 & -5\end{bmatrix}\mathbf{P} = \frac{\left\|\begin{pmatrix}5\\3\end{pmatrix}\right\|^2 - \left\|\begin{pmatrix}5\\-5\end{pmatrix}\right\|^2}{2} \tag{1.0.11}$$

$$\begin{pmatrix}0 & 8\end{pmatrix}\mathbf{P} = \left[\frac{\left(\sqrt{5^2}+\sqrt{3^2}\right)^2 - \left(\sqrt{5^2}+\sqrt{-5^2}\right)^2}{2}\right] \tag{1.0.12}$$

$$\begin{pmatrix}0 & 8\end{pmatrix}\mathbf{P} = \left[\frac{\left(\sqrt{25}+\sqrt{9}\right)^2 - \left(\sqrt{25}+\sqrt{25}\right)^2}{2}\right] \tag{1.0.13}$$

$$\begin{pmatrix}0 & 8\end{pmatrix}\mathbf{P} = \left[\frac{\left(\sqrt{34}\right)^2 - \left(\sqrt{50}\right)^2}{2}\right] \tag{1.0.14}$$

$$\begin{pmatrix}0 & 8\end{pmatrix}\mathbf{P} = \frac{-16}{2} \tag{1.0.15}$$

$$\begin{pmatrix}0 & 8\end{pmatrix}\mathbf{P} = -\mathbf{8} \tag{1.0.16}$$

$$\implies \mathbf{P} = \mathbf{y} = -1 \tag{1.0.17}$$

$$\left(B-C\right)^T\mathbf{P} = \frac{\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2}{2} \tag{1.0.18}$$

$$\begin{bmatrix}5 & -5\end{bmatrix} - \begin{bmatrix}1 & -5\end{bmatrix}\mathbf{P} = \frac{\left\|\begin{pmatrix}5\\-5\end{pmatrix}\right\|^2 - \left\|\begin{pmatrix}1\\-5\end{pmatrix}\right\|^2}{2} \tag{1.0.19}$$

$$\begin{pmatrix}4 & 0\end{pmatrix}\mathbf{P} = \left[\frac{\left(\sqrt{5^2}+\sqrt{-5^2}\right)^2 - \left(\sqrt{1^2}+\sqrt{-5^2}\right)^2}{2}\right] \tag{1.0.20}$$

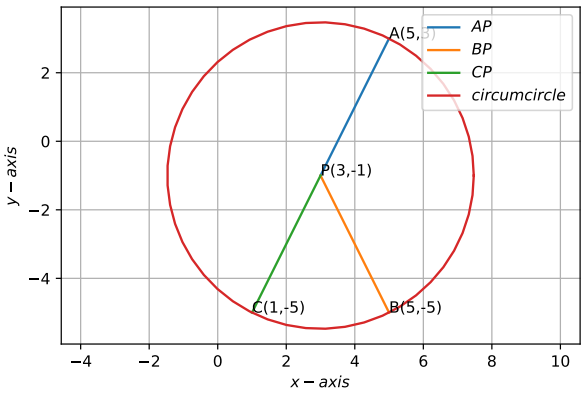

Fig. 1.1: Graphical Solution

$$\begin{pmatrix}4 & 0\end{pmatrix}\mathbf{P} = \left[\frac{\left(\sqrt{25}+\sqrt{25}\right)^2-\left(\sqrt{1}+\sqrt{25}\right)^2}{2}\right] \tag{1.0.21}$$

$$\begin{pmatrix}4 & 0\end{pmatrix}\mathbf{P} = \left[\frac{\left(\sqrt{50}\right)^2-\left(\sqrt{26}\right)^2}{2}\right] \tag{1.0.22}$$

$$\begin{pmatrix}4 & 0\end{pmatrix}\mathbf{P} = \frac{24}{2} \tag{1.0.23}$$

$$\begin{pmatrix}4 & 0\end{pmatrix}\mathbf{P} = \mathbf{12} \tag{1.0.24}$$

$$\implies \mathbf{P} = \mathbf{x} = 3 \tag{1.0.25}$$

$$\mathbf{P} = \begin{pmatrix}x\\y\end{pmatrix} \tag{1.0.26}$$

$$\mathbf{P} = \begin{pmatrix}3\\-1\end{pmatrix}. \tag{1.0.27}$$