

Assignment No.1

Suyog Tangade
MD/2020/710

Download all python codes from

<https://github.com/suyogtangade/AI.git>

and latex-tikz codes from

<https://github.com/suyogtangade/AI.git>

1 QUESTION No.16(B) (CBSE/2006/SET-2)

Find the co-ordinates of the point equidistant from three given points $\mathbf{A}\begin{pmatrix} 5 \\ 3 \end{pmatrix}$, $\mathbf{B}\begin{pmatrix} 5 \\ -5 \end{pmatrix}$ and $\mathbf{C}\begin{pmatrix} 1 \\ -5 \end{pmatrix}$

Solution:

Let the point equidistant from \mathbf{A} & \mathbf{B} & \mathbf{C} be

$$\mathbf{P} = \begin{pmatrix} x \\ y \end{pmatrix} \quad (1.0.1)$$

$$\|\mathbf{P} - \mathbf{A}\|^2 = \|\mathbf{P} - \mathbf{B}\|^2 \quad (1.0.2)$$

$$\mathbf{P} = \begin{pmatrix} x \\ y \end{pmatrix} \quad (1.0.3)$$

$$\|\mathbf{P}\|^2 + \|\mathbf{A}\|^2 - 2\mathbf{A}^T \mathbf{P} \quad (1.0.4)$$

$$= \|\mathbf{P}\|^2 + \|\mathbf{B}\|^2 - 2\mathbf{B}^T \mathbf{P} \quad (1.0.5)$$

$$(\mathbf{A} - \mathbf{B})^T \mathbf{P} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2} \quad (1.0.6)$$

$$(\mathbf{B} - \mathbf{C})^T \mathbf{P} = \frac{\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2}{2} \quad (1.0.7)$$

$$\begin{pmatrix} (\mathbf{A} - \mathbf{B})^T \\ (\mathbf{B} - \mathbf{C})^T \end{pmatrix} \mathbf{P} = \frac{1}{2} \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2} \quad (1.0.8)$$

$$\mathbf{A}\begin{pmatrix} 5 \\ 3 \end{pmatrix}, \mathbf{B}\begin{pmatrix} 5 \\ -5 \end{pmatrix}, \mathbf{C}\begin{pmatrix} 1 \\ -5 \end{pmatrix} \quad (1.0.9)$$

$$(\mathbf{A} - \mathbf{B})^T \mathbf{P} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2} \quad (1.0.10)$$

$$\left[\begin{pmatrix} 5 & 3 \end{pmatrix} - \begin{pmatrix} 5 & -5 \end{pmatrix} \right] \mathbf{P} = \frac{\left\| \begin{pmatrix} 5 \\ 3 \end{pmatrix} \right\|^2 - \left\| \begin{pmatrix} 5 \\ -5 \end{pmatrix} \right\|^2}{2} \quad (1.0.11)$$

$$\begin{pmatrix} 0 & 8 \end{pmatrix} \mathbf{P} = \left[\frac{(\sqrt{5^2 + 3^2})^2 - (\sqrt{5^2 + (-5)^2})^2}{2} \right] \quad (1.0.12)$$

$$\begin{pmatrix} 0 & 8 \end{pmatrix} \mathbf{P} = \left[\frac{(\sqrt{25 + 9})^2 - (\sqrt{25 + 25})^2}{2} \right] \quad (1.0.13)$$

$$\begin{pmatrix} 0 & 8 \end{pmatrix} \mathbf{P} = \left[\frac{(\sqrt{34})^2 - (\sqrt{50})^2}{2} \right] \quad (1.0.14)$$

$$\begin{pmatrix} 0 & 8 \end{pmatrix} \mathbf{P} = \frac{-16}{2} \quad (1.0.15)$$

$$\begin{pmatrix} 0 & 8 \end{pmatrix} \mathbf{P} = -8 \quad (1.0.16)$$

$$\Rightarrow \mathbf{P} = \mathbf{y} = -1 \quad (1.0.17)$$

$$(\mathbf{B} - \mathbf{C})^T \mathbf{P} = \frac{\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2}{2} \quad (1.0.18)$$

$$\left[\begin{pmatrix} 5 & -5 \end{pmatrix} - \begin{pmatrix} 1 & -5 \end{pmatrix} \right] \mathbf{P} = \frac{\left\| \begin{pmatrix} 5 \\ -5 \end{pmatrix} \right\|^2 - \left\| \begin{pmatrix} 1 \\ -5 \end{pmatrix} \right\|^2}{2} \quad (1.0.19)$$

$$\begin{pmatrix} 4 & 0 \end{pmatrix} \mathbf{P} = \left[\frac{(\sqrt{5^2 + (-5)^2})^2 - (\sqrt{1^2 + (-5)^2})^2}{2} \right] \quad (1.0.20)$$

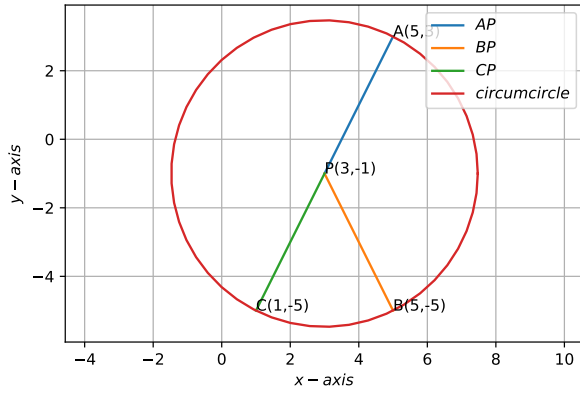


Fig. 1.1: Graphical Solution

$$(4 \ 0) \mathbf{P} = \left[\frac{(\sqrt{25} + \sqrt{25})^2 - (\sqrt{1} + \sqrt{25})^2}{2} \right] \quad (1.0.21)$$

$$(4 \ 0) \mathbf{P} = \left[\frac{(\sqrt{50})^2 - (\sqrt{26})^2}{2} \right] \quad (1.0.22)$$

$$(4 \ 0) \mathbf{P} = \frac{24}{2} \quad (1.0.23)$$

$$(4 \ 0) \mathbf{P} = \mathbf{12} \quad (1.0.24)$$

$$\implies \mathbf{P} = \mathbf{x} = 3 \quad (1.0.25)$$

$$\mathbf{P} = \begin{pmatrix} x \\ y \end{pmatrix} \quad (1.0.26)$$

$$\mathbf{P} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}. \quad (1.0.27)$$