Assignment No.1

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Download all python codes from

https://github.com/suyogtangade/ASSIGNMENT−01.git

and latex-tikz codes from

https://github.com/suyogtangade/ASSIGNMENT−01.git

1 Question No.16(b) (cbse/2006/set-2)

Find the co-ordinates of the point equidistant from three given points $\mathbf{A}\begin{pmatrix}5\\3\end{pmatrix}$, $\mathbf{B}\begin{pmatrix}5\\-5\end{pmatrix}$ and $\mathbf{C}\begin{pmatrix}1\\-5\end{pmatrix}$

Solution:

Let the point equidistant from **A** & **B** & **C** be

$$\mathbf{P} = \begin{pmatrix}x\\y\end{pmatrix} \tag{1.0.1}$$

$$\|\mathbf{P}-\mathbf{A}\|^2 = \|\mathbf{P}-\mathbf{B}\|^2 \tag{1.0.2}$$

$$\mathbf{P} = \begin{pmatrix}x\\y\end{pmatrix} \tag{1.0.3}$$

$$\|\mathbf{P}\|^2 + \|\mathbf{A}\|^2 - 2\mathbf{A}^T\mathbf{P} \tag{1.0.4}$$

$$= \|\mathbf{P}\|^2 + \|\mathbf{B}\|^2 - 2\mathbf{B}^T\mathbf{P} \tag{1.0.5}$$

$$(\mathbf{A}-\mathbf{B})^T\mathbf{P} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2} \tag{1.0.6}$$

$$(\mathbf{B}-\mathbf{C})^T\mathbf{P} = \frac{\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2}{2} \tag{1.0.7}$$

$$\begin{pmatrix}(\mathbf{A}-\mathbf{B})^{\mathbf{T}}\\(\mathbf{B}-\mathbf{C})^{\mathbf{T}}\end{pmatrix}\mathbf{P} = \frac{1}{2}\begin{pmatrix}\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2\\\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2\end{pmatrix} \tag{1.0.8}$$

$$\mathbf{A}\begin{pmatrix}5\\3\end{pmatrix}, \mathbf{B}\begin{pmatrix}5\\-5\end{pmatrix}, \mathbf{C}\begin{pmatrix}1\\-5\end{pmatrix} \tag{1.0.9}$$

$$\begin{pmatrix}\left(\begin{pmatrix}5\\3\end{pmatrix}-\begin{pmatrix}5\\-5\end{pmatrix}\right)^{\mathbf{T}}\\\left(\begin{pmatrix}5\\-5\end{pmatrix}-\begin{pmatrix}1\\-5\end{pmatrix}\right)^{\mathbf{T}}\end{pmatrix}\mathbf{P} = \frac{1}{2}\begin{pmatrix}\left\|\begin{pmatrix}5\\3\end{pmatrix}\right\|^2 - \left\|\begin{pmatrix}5\\-5\end{pmatrix}\right\|^2\\\left\|\begin{pmatrix}5\\-5\end{pmatrix}\right\|^2 - \left\|\begin{pmatrix}1\\-5\end{pmatrix}\right\|^2\end{pmatrix} \tag{1.0.10}$$

$$\begin{pmatrix}\begin{pmatrix}5 & 3\end{pmatrix}-\begin{pmatrix}5 & -5\end{pmatrix}\\\begin{pmatrix}5 & -5\end{pmatrix}-\begin{pmatrix}1 & -5\end{pmatrix}\end{pmatrix}\mathbf{P} = \tag{1.0.11}$$

$$\frac{1}{2}\begin{pmatrix}\left(\sqrt{5^2}+\sqrt{3^2}\right)^2 - \left(\sqrt{5^2}+\sqrt{-5^2}\right)^2\\\left(\sqrt{5^2}+\sqrt{-5^2}\right)^2 - \left(\sqrt{1^2}+\sqrt{-5^2}\right)^2\end{pmatrix} \tag{1.0.12}$$

$$\begin{pmatrix}0 & 8\\4 & 0\end{pmatrix}\mathbf{P} = \frac{1}{2}\begin{pmatrix}\left(\sqrt{25}+\sqrt{9}\right)^2 - \left(\sqrt{25}+\sqrt{25}\right)^2\\\left(\sqrt{25}+\sqrt{25}\right)^2 - \left(\sqrt{1}+\sqrt{25}\right)^2\end{pmatrix} \tag{1.0.13}$$

$$\begin{pmatrix}0 & 8\\4 & 0\end{pmatrix}\mathbf{P} = \frac{1}{2}\begin{pmatrix}\left(\sqrt{34}\right)^2 - \left(\sqrt{50}\right)^2\\\left(\sqrt{50}\right)^2 - \left(\sqrt{26}\right)^2\end{pmatrix} \tag{1.0.14}$$

$$\begin{pmatrix}0 & 8\\4 & 0\end{pmatrix}\mathbf{P} = \frac{1}{2}\begin{pmatrix}\left(\sqrt{-16}\right)^2\\\left(\sqrt{24}\right)^2\end{pmatrix} \tag{1.0.15}$$

$$\begin{pmatrix}0 & 8\\4 & 0\end{pmatrix}\mathbf{P} = \begin{pmatrix}-8\\12\end{pmatrix} \tag{1.0.16}$$

$$\begin{pmatrix}0 & 8\\4 & 0\end{pmatrix}\begin{pmatrix}\mathbf{x}\\\mathbf{y}\end{pmatrix} = \begin{pmatrix}-8\\12\end{pmatrix} \tag{1.0.17}$$

$$8\mathbf{y} = -8 \implies \mathbf{y} = -1 \tag{1.0.18}$$

$$4\mathbf{x} = 12 \implies \mathbf{x} = 3 \tag{1.0.19}$$

co-ordinates for the point equidistant from three

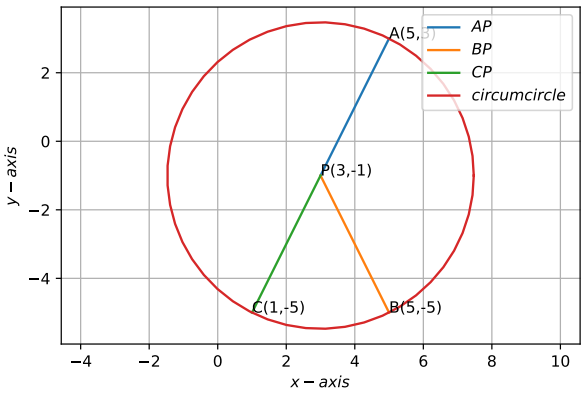

Fig. 1.1: Graphical Solution

given points $\mathbf{A}\begin{pmatrix}5\\3\end{pmatrix}$, $\mathbf{B}\begin{pmatrix}5\\-5\end{pmatrix}$ and $\mathbf{C}\begin{pmatrix}1\\-5\end{pmatrix}$ are

$$\mathbf{P} = \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}3\\-1\end{pmatrix} \tag{1.0.20}$$