Assignment No.1

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Download all python codes from

and latex-tikz codes from

https://github.com/suyogtangade/AI.git

1 Question No.16(b) (cbse/2006/set-2)

Find the co-ordinates of the point equidistant from three given points $\mathbf{A}\begin{pmatrix}5\\3\end{pmatrix}$, $\mathbf{B}\begin{pmatrix}5\\-5\end{pmatrix}$ and $\mathbf{C}\begin{pmatrix}1\\-5\end{pmatrix}$

Solution:

Let the point equidistant from $\mathbf{A}$ & $\mathbf{B}$ & $\mathbf{C}$ be

$$\mathbf{P} = \begin{pmatrix}x\\y\end{pmatrix} \tag{1.0.1}$$

From the given information

$$||\mathbf{P}-\mathbf{A}||^2 = ||\mathbf{P}-\mathbf{B}||^2 = ||\mathbf{P}-\mathbf{C}||^2 \tag{1.0.2}$$

$$\therefore ||\mathbf{P}-\mathbf{A}||^2 = ||\mathbf{P}-\mathbf{B}||^2 \tag{1.0.3}$$

$$\left\|\mathbf{P}-\begin{pmatrix}5\\3\end{pmatrix}\right\|^2 = \left\|\mathbf{P}-\begin{pmatrix}5\\-5\end{pmatrix}\right\|^2 \tag{1.0.4}$$

$$||\mathbf{P}||^2 + ||\mathbf{A}||^2 - 2\mathbf{A}^T\mathbf{P} = ||\mathbf{P}||^2 + ||\mathbf{B}||^2 - 2\mathbf{B}^T\mathbf{P} \tag{1.0.5}$$

$$\implies ||\mathbf{P}||^2 + \left\|\begin{pmatrix}5\\3\end{pmatrix}\right\|^2 - 2\mathbf{A}^T\mathbf{P} \tag{1.0.6}$$

$$= ||\mathbf{P}||^2 + \left\|\begin{pmatrix}5\\-5\end{pmatrix}\right\|^2 - 2\mathbf{B}^T\mathbf{P} \tag{1.0.7}$$

$$\left\|\begin{pmatrix}5\\3\end{pmatrix}\right\|^2 - \left\|\begin{pmatrix}5\\-5\end{pmatrix}\right\|^2 = 2\begin{pmatrix}5 & 3\end{pmatrix}\mathbf{P} - 2\begin{pmatrix}5 & -5\end{pmatrix}\mathbf{P} \tag{1.0.8}$$

$$\left(\sqrt{5^2}+\sqrt{3^2}\right) - \left(\sqrt{5^2}+\sqrt{-5^2}\right) = \left[\begin{pmatrix}10 & 6\end{pmatrix} - \begin{pmatrix}10 & -10\end{pmatrix}\right]\mathbf{P} \tag{1.0.9}$$

$$\left(\sqrt{34}\right) - \left(\sqrt{50}\right) = \left[\begin{pmatrix}16 & 0\end{pmatrix}\right]\mathbf{P} \tag{1.0.10}$$

$$\mathbf{-16} = \begin{pmatrix}16 & 0\end{pmatrix}\mathbf{P} \tag{1.0.11}$$

Which can be simplified to obtain

$$\begin{pmatrix}0 & 16\end{pmatrix}\mathbf{P} = -16 \implies y = -1 \tag{1.0.12}$$

$$||\mathbf{P}||^2 + ||\mathbf{B}||^2 - 2\mathbf{B}^T\mathbf{P} = ||\mathbf{P}||^2 + ||\mathbf{C}||^2 - 2\mathbf{C}^T\mathbf{P} \tag{1.0.13}$$

$$||\mathbf{B}||^2 - ||\mathbf{C}||^2 = 2\mathbf{B}^T\mathbf{P} - 2\mathbf{C}^T\mathbf{P} \tag{1.0.14}$$

$$\left\|\mathbf{P}-\begin{pmatrix}5\\-5\end{pmatrix}\right\|^2 = \left\|\mathbf{P}-\begin{pmatrix}1\\-5\end{pmatrix}\right\|^2 \tag{1.0.15}$$

$$\implies ||\mathbf{P}||^2 + \left\|\begin{pmatrix}5\\-5\end{pmatrix}\right\|^2 - 2\mathbf{B}^T\mathbf{P} \tag{1.0.16}$$

$$= ||\mathbf{P}||^2 + \left\|\begin{pmatrix}1\\-5\end{pmatrix}\right\|^2 - 2\mathbf{C}^T\mathbf{P} \tag{1.0.17}$$

$$\left\|\begin{pmatrix}5\\-5\end{pmatrix}\right\|^2 - 2\mathbf{B}^T\mathbf{P} = \left\|\begin{pmatrix}1\\-5\end{pmatrix}\right\|^2 - 2\mathbf{C}^T\mathbf{P} \tag{1.0.18}$$

$$\left(\sqrt{5^2}+\sqrt{-5^2}\right) - 2\begin{pmatrix}5 & -5\end{pmatrix}\mathbf{P} = \left(\sqrt{1^2}+\sqrt{-5^2}\right) - 2\begin{pmatrix}1 & -5\end{pmatrix}\mathbf{P} \tag{1.0.19}$$

$$\left(\sqrt{25}+\sqrt{25}\right) - \begin{pmatrix}-10 & 10\end{pmatrix}\mathbf{P} = \left(\sqrt{1}+\sqrt{25}\right) - \begin{pmatrix}-2 & 10\end{pmatrix}\mathbf{P} \tag{1.0.20}$$

$$\left(\sqrt{50}\right) - \begin{pmatrix}-10 & 10\end{pmatrix}\mathbf{P} = \left(\sqrt{26}\right) - \begin{pmatrix}-2 & 10\end{pmatrix}\mathbf{P} \tag{1.0.21}$$

$$24 = \begin{pmatrix}0 & 8\end{pmatrix}\mathbf{P} \tag{1.0.22}$$

Which can be simplified to obtain

$$\begin{pmatrix}8 & 0\end{pmatrix}\mathbf{P} = 24 \implies x = 3 \tag{1.0.23}$$

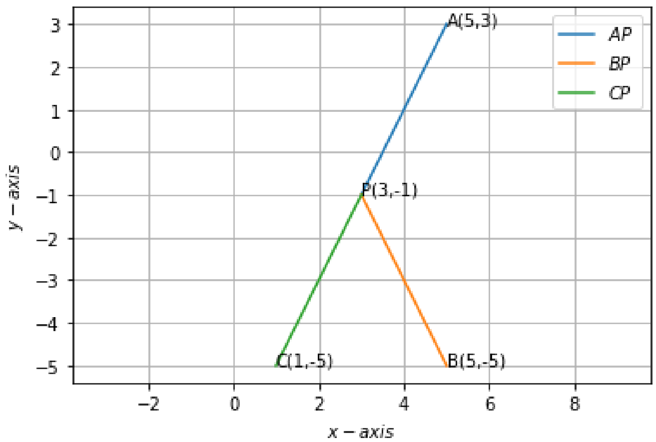

Fig. 1.1: Graphical Solution

The required point

$$\mathbf{P} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}. \tag{1.0.24}$$

The coordinate point equidistance from three points is