

Assignment No.1

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Download all python codes from

<https://github.com/suyogtangade/AI.git>

and latex-tikz codes from

<https://github.com/suyogtangade/AI.git>

$$2\mathbf{P}(\mathbf{A}^\top - \mathbf{B}^\top) = \|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 \quad (1.0.12)$$

P lies on the y-axis

$$\mathbf{P} = y \begin{pmatrix} 0 \\ 1 \end{pmatrix} = y\mathbf{e}_2 \quad (1.0.13)$$

Now substitute this in (1.0.12)

$$2y\mathbf{e}_2(\mathbf{A}^\top - \mathbf{B}^\top) = \|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 \quad (1.0.14)$$

$$y = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2\mathbf{e}_2(\mathbf{A}^\top - \mathbf{B}^\top)} \quad (1.0.15)$$

$$2\mathbf{e}_2(\mathbf{A}^\top - \mathbf{B}^\top) = 2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} (0 \ 8) = 16 \quad (1.0.16)$$

$$\Rightarrow y = \left(\frac{-16}{16} \right) \quad (1.0.17)$$

$$\therefore y = -1 \quad (1.0.18)$$

From (1.0.2) we can write

$$\|\mathbf{P} - \mathbf{B}\|^2 = \|\mathbf{P} - \mathbf{C}\|^2 \quad (1.0.19)$$

$$(\mathbf{P} - \mathbf{A})^\top (\mathbf{P} - \mathbf{A}) = (\mathbf{P} - \mathbf{B})^\top (\mathbf{P} - \mathbf{B}) \quad (1.0.4)$$

$$2\mathbf{P}(\mathbf{B}^\top - \mathbf{C}^\top) = \|\mathbf{B}\|^2 - \|\mathbf{C}\|^2 \quad (1.0.20)$$

$$(\mathbf{P} - \mathbf{A})^\top (\mathbf{P} - \mathbf{A}) = \mathbf{P}^\top \mathbf{P} - \mathbf{P}^\top \mathbf{A} - \mathbf{A}^\top \mathbf{P} + \mathbf{A}^\top \mathbf{A} \quad (1.0.5)$$

P lies on the x-axis

$$(\mathbf{P} - \mathbf{B})^\top (\mathbf{P} - \mathbf{B}) = \mathbf{P}^\top \mathbf{P} - \mathbf{P}^\top \mathbf{B} - \mathbf{B}^\top \mathbf{P} + \mathbf{B}^\top \mathbf{B} \quad (1.0.6)$$

$$\mathbf{P} = x \begin{pmatrix} 1 \\ 0 \end{pmatrix} = x\mathbf{e}_1 \quad (1.0.21)$$

Now substitute this in (1.0.20)

$$2x\mathbf{e}_1(\mathbf{B}^\top - \mathbf{C}^\top) = \|\mathbf{B}\|^2 - \|\mathbf{C}\|^2 \quad (1.0.22)$$

$$x = \frac{\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2}{2\mathbf{e}_1(\mathbf{B}^\top - \mathbf{C}^\top)} \quad (1.0.23)$$

$$2\mathbf{e}_1(\mathbf{B}^\top - \mathbf{C}^\top) = 2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} (4 \ 0) = 8 \quad (1.0.24)$$

$$\Rightarrow x = \left(\frac{24}{8} \right) \quad (1.0.25)$$

$$\therefore x = 3 \quad (1.0.26)$$

The coordinate point equidistance from three points

1 QUESTION No.16(B) (CBSE/2006/SET-2)

Find the co-ordinates of the point equidistant from three given points $\mathbf{A} \begin{pmatrix} 5 \\ 3 \end{pmatrix}$, $\mathbf{B} \begin{pmatrix} 5 \\ -5 \end{pmatrix}$ and $\mathbf{C} \begin{pmatrix} 1 \\ -5 \end{pmatrix}$

Solution:

Let the point equidistant from A & B & C be

$$\mathbf{P} = \begin{pmatrix} x \\ y \end{pmatrix} \quad (1.0.1)$$

From the given information

$$\|\mathbf{P} - \mathbf{A}\|^2 = \|\mathbf{P} - \mathbf{B}\|^2 = \|\mathbf{P} - \mathbf{C}\|^2 \quad (1.0.2)$$

$$\therefore \|\mathbf{P} - \mathbf{A}\|^2 = \|\mathbf{P} - \mathbf{B}\|^2 \quad (1.0.3)$$

Consider the expressions

$$\mathbf{P}^\top \mathbf{P} = \|\mathbf{P}\|^2 \quad (1.0.7)$$

$$\mathbf{P}^\top \mathbf{A} = \mathbf{A}^\top \mathbf{P} \quad (1.0.8)$$

Final expression of (1.0.4) can be written as

$$\|\mathbf{P}\|^2 - 2\mathbf{A}^\top \mathbf{P} + \mathbf{A}^\top \mathbf{A} = \|\mathbf{P}\|^2 - 2\mathbf{B}^\top \mathbf{P} + \mathbf{B}^\top \mathbf{B} \quad (1.0.9)$$

$$\Rightarrow -2\mathbf{A}^\top \mathbf{P} + 2\mathbf{B}^\top \mathbf{P} = \mathbf{B}^\top \mathbf{B} - \mathbf{A}^\top \mathbf{A} \quad (1.0.10)$$

$$\Rightarrow 2\mathbf{P}(\mathbf{A}^\top - \mathbf{B}^\top) = \mathbf{A}^\top \mathbf{A} - \mathbf{B}^\top \mathbf{B} \quad (1.0.11)$$

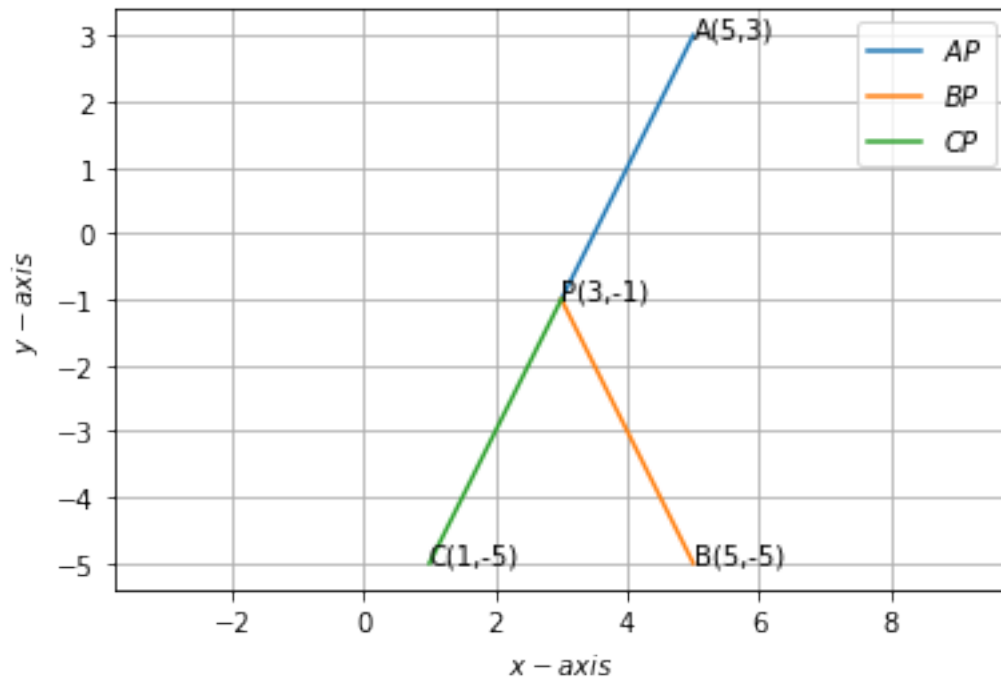


Fig. 1.1: Graphical Solution

is

$$\mathbf{P} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}. \quad (1.0.27)$$