Assignment No.1

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MD/2020/710

Download all python codes from

https://github.com/suyogtangade/AI.git

and latex-tikz codes from

https://github.com/suyogtangade/AI.git

1 Question No.16(b) (cbse/2006/set-2)

Find the co-ordinates of the point equidistant from three given points $\mathbf{A}\begin{pmatrix}5\\3\end{pmatrix}$, $\mathbf{B}\begin{pmatrix}5\\-5\end{pmatrix}$ and $\mathbf{C}\begin{pmatrix}1\\-5\end{pmatrix}$

Solution:

Let the point equidistant from **A** & **B** & **C** be

$$\mathbf{P} = \begin{pmatrix}x\\y\end{pmatrix} \tag{1.0.1}$$

From the given information

$$\|\mathbf{P}-\mathbf{A}\|^2 = \|\mathbf{P}-\mathbf{B}\|^2 = \|\mathbf{P}-\mathbf{C}\|^2 \tag{1.0.2}$$

$$\therefore \|\mathbf{P}-\mathbf{A}\|^2 = \|\mathbf{P}-\mathbf{B}\|^2 \tag{1.0.3}$$

$$\left\|\mathbf{P}-\begin{pmatrix}5\\3\end{pmatrix}\right\|^2 = \left\|\mathbf{P}-\begin{pmatrix}5\\-5\end{pmatrix}\right\|^2 \tag{1.0.4}$$

$$\|\mathbf{P}\|^2 + \|\mathbf{A}\|^2 - 2\mathbf{A}^T\mathbf{P} = \|\mathbf{P}\|^2 + \|\mathbf{B}\|^2 - 2\mathbf{B}^T\mathbf{P} \tag{1.0.5}$$

$$\implies \|\mathbf{P}\|^2 + \left\|\begin{pmatrix}5\\3\end{pmatrix}\right\|^2 - 2\mathbf{A}^T\mathbf{P} \tag{1.0.6}$$

$$= \|\mathbf{P}\|^2 + \left\|\begin{pmatrix}5\\-5\end{pmatrix}\right\|^2 - 2\mathbf{B}^T\mathbf{P} \tag{1.0.7}$$

$$\left\|\begin{pmatrix}5\\3\end{pmatrix}\right\|^2 - \left\|\begin{pmatrix}5\\-5\end{pmatrix}\right\|^2 \tag{1.0.8}$$

$$= 2\begin{pmatrix}5 & 3\end{pmatrix}\mathbf{P} - 2\begin{pmatrix}5 & -5\end{pmatrix}\mathbf{P} \tag{1.0.9}$$

$$\left(\sqrt{5^2} + \sqrt{3^2}\right) - \left(\sqrt{5^2} + \sqrt{-5^2}\right) \tag{1.0.10}$$

$$= \left[\begin{pmatrix}10 & 6\end{pmatrix} - \begin{pmatrix}10 & -10\end{pmatrix}\right]\mathbf{P} \tag{1.0.11}$$

$$\left(\sqrt{34}\right) - \left(\sqrt{50}\right) = \left[\begin{pmatrix}0 & 16\end{pmatrix}\right]\mathbf{P} \tag{1.0.12}$$

$$\mathbf{-16} = \begin{pmatrix}0 & 16\end{pmatrix}\mathbf{P} \tag{1.0.13}$$

Which can be simplified to obtain

$$\begin{pmatrix}16 & 0\end{pmatrix}\mathbf{P} = -16 \implies y = -1 \tag{1.0.14}$$

$$\|\mathbf{P}\|^2 + \|\mathbf{B}\|^2 - 2\mathbf{B}^T\mathbf{P} \tag{1.0.15}$$

$$= \|\mathbf{P}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{C}^T\mathbf{P} \tag{1.0.16}$$

$$\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2 = 2\mathbf{B}^T\mathbf{P} - 2\mathbf{C}^T\mathbf{P} \tag{1.0.17}$$

$$\left\|\mathbf{P}-\begin{pmatrix}5\\-5\end{pmatrix}\right\|^2 = \left\|\mathbf{P}-\begin{pmatrix}1\\-5\end{pmatrix}\right\|^2 \tag{1.0.18}$$

$$\implies \|\mathbf{P}\|^2 + \left\|\begin{pmatrix}5\\-5\end{pmatrix}\right\|^2 - 2\mathbf{B}^T\mathbf{P} \tag{1.0.19}$$

$$= \|\mathbf{P}\|^2 + \left\|\begin{pmatrix}1\\-5\end{pmatrix}\right\|^2 - 2\mathbf{C}^T\mathbf{P} \tag{1.0.20}$$

$$\left\|\begin{pmatrix}5\\-5\end{pmatrix}\right\|^2 - 2\mathbf{B}^T\mathbf{P} = \left\|\begin{pmatrix}1\\-5\end{pmatrix}\right\|^2 - 2\mathbf{C}^T\mathbf{P} \tag{1.0.21}$$

$$\left(\sqrt{5^2} + \sqrt{-5^2}\right) - 2\begin{pmatrix}5 & -5\end{pmatrix}\mathbf{P} \tag{1.0.22}$$

$$= \left(\sqrt{1^2} + \sqrt{-5^2}\right) - 2\begin{pmatrix}1 & -5\end{pmatrix}\mathbf{P} \tag{1.0.23}$$

$$\left(\sqrt{25} + \sqrt{25}\right) - \begin{pmatrix}-10 & 10\end{pmatrix}\mathbf{P} \tag{1.0.24}$$

$$= \left(\sqrt{1} + \sqrt{25}\right) - \begin{pmatrix}-2 & 10\end{pmatrix}\mathbf{P} \tag{1.0.25}$$

$$\left(\sqrt{50}\right) - \begin{pmatrix}-10 & 10\end{pmatrix}\mathbf{P} \tag{1.0.26}$$

$$= \left(\sqrt{26}\right) - \begin{pmatrix}-2 & 10\end{pmatrix}\mathbf{P} \tag{1.0.27}$$

$$24 = \begin{pmatrix}8 & 0\end{pmatrix}\mathbf{P} \tag{1.0.28}$$

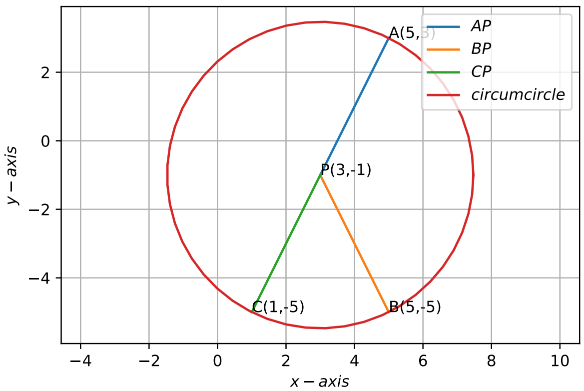

Fig. 1.1: Graphical Solution

Which can be simplified to obtain

$$\begin{pmatrix} 8 & 0 \end{pmatrix} \mathbf{P} = 24 \implies x = 3 \tag{1.0.29}$$

The required point

$$\mathbf{P} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}. \tag{1.0.30}$$

$$\|\mathbf{P} - \mathbf{A}\|^2 = \|\mathbf{P} - \mathbf{B}\|^2 \tag{1.0.31}$$

$$\mathbf{P} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{1.0.32}$$

$$\|\mathbf{P}\|^2 + \|\mathbf{A}\|^2 - 2\mathbf{A}^T\mathbf{P} = \|\mathbf{P}\|^2 + \|\mathbf{B}\|^2 - 2\mathbf{B}^T\mathbf{P} \tag{1.0.33}$$

$$\left(A - B\right)^T \mathbf{P} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2} \tag{1.0.34}$$

$$\left(B - C\right)^T \mathbf{P} = \frac{\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2}{2} \tag{1.0.35}$$

$$\left[\begin{pmatrix} (A - B)^T \\ (B - C)^T \end{pmatrix}\right] \mathbf{P} = \frac{1}{2} \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2} \tag{1.0.36}$$

$$\mathbf{A}\begin{pmatrix} 5 \\ 3 \end{pmatrix}, \mathbf{B}\begin{pmatrix} 5 \\ -5 \end{pmatrix}, \mathbf{C}\begin{pmatrix} 1 \\ -5 \end{pmatrix} \tag{1.0.37}$$

$$\left(A - B\right)^T \mathbf{P} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2} \tag{1.0.38}$$

$$\left[\begin{pmatrix} 5 & 3 \end{pmatrix} - \begin{pmatrix} 5 & -5 \end{pmatrix}\right] \mathbf{P} = \frac{\left\|\begin{pmatrix} 5 \\ 3 \end{pmatrix}\right\|^2 - \left\|\begin{pmatrix} 5 \\ -5 \end{pmatrix}\right\|^2}{2} \tag{1.0.39}$$

$$\begin{pmatrix} 0 & 8 \end{pmatrix} \mathbf{P} = \left[\frac{\left(\sqrt{5^2} + \sqrt{3^2}\right) - \left(\sqrt{5^2} + \sqrt{-5^2}\right)}{2}\right] \tag{1.0.40}$$

$$\begin{pmatrix} 0 & 8 \end{pmatrix} \mathbf{P} = \left[\frac{\left(\sqrt{25} + \sqrt{9}\right) - \left(\sqrt{25} + \sqrt{25}\right)}{2}\right] \tag{1.0.41}$$

$$\begin{pmatrix} 0 & 8 \end{pmatrix} \mathbf{P} = \left[\frac{\left(\sqrt{34}\right) - \left(\sqrt{50}\right)}{2}\right] \tag{1.0.42}$$

$$\begin{pmatrix} 0 & 8 \end{pmatrix} \mathbf{P} = \frac{-16}{2} \tag{1.0.43}$$

$$\begin{pmatrix} 0 & 8 \end{pmatrix} \mathbf{P} = \mathbf{-8} \tag{1.0.44}$$

$$\implies \mathbf{P} = \mathbf{y} = -1 \tag{1.0.45}$$

$$\left(B - C\right)^T \mathbf{P} = \frac{\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2}{2} \tag{1.0.46}$$

$$\left[\begin{pmatrix} 5 & -5 \end{pmatrix} - \begin{pmatrix} 1 & -5 \end{pmatrix}\right] \mathbf{P} = \frac{\left\|\begin{pmatrix} 5 \\ -5 \end{pmatrix}\right\|^2 - \left\|\begin{pmatrix} 1 \\ -5 \end{pmatrix}\right\|^2}{2} \tag{1.0.47}$$

$$\begin{pmatrix} 4 & 0 \end{pmatrix} \mathbf{P} = \left[\frac{\left(\sqrt{5^2} + \sqrt{-5^2}\right) - \left(\sqrt{1^2} + \sqrt{-5^2}\right)}{2}\right] \tag{1.0.48}$$

$$\begin{pmatrix} 4 & 0 \end{pmatrix} \mathbf{P} = \left[\frac{\left(\sqrt{25} + \sqrt{25}\right) - \left(\sqrt{1} + \sqrt{25}\right)}{2}\right] \tag{1.0.49}$$

$$\begin{pmatrix} 4 & 0 \end{pmatrix} \mathbf{P} = \left[\frac{\left(\sqrt{50}\right) - \left(\sqrt{26}\right)}{2}\right] \tag{1.0.50}$$

$$\begin{pmatrix} 4 & 0 \end{pmatrix} \mathbf{P} = \frac{24}{2} \tag{1.0.51}$$

$$\begin{pmatrix} 4 & 0 \end{pmatrix} \mathbf{P} = \mathbf{12} \tag{1.0.52}$$

$$\implies \mathbf{P} = \mathbf{x} = 3 \tag{1.0.53}$$