Assignment No.1

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Download all python codes from

and latex-tikz codes from

https://github.com/suyogtangade/AI.git

1 Question No.16(b) (cbse/2006/set-2)

Find the co-ordinates of the point equidistant from three given points $\mathbf{A}\begin{pmatrix}5\\3\end{pmatrix}$, $\mathbf{B}\begin{pmatrix}5\\-5\end{pmatrix}$ and $\mathbf{C}\begin{pmatrix}1\\-5\end{pmatrix}$

Solution:

Let the point equidistant from $\mathbf{A}$ & $\mathbf{B}$ & $\mathbf{C}$ be

$$\mathbf{P} = \begin{pmatrix}x\\y\end{pmatrix} \tag{1.0.1}$$

From the given information

$$||\mathbf{P}-\mathbf{A}||^2 = ||\mathbf{P}-\mathbf{B}||^2 = ||\mathbf{P}-\mathbf{C}||^2 \tag{1.0.2}$$

$$\therefore ||\mathbf{P}-\mathbf{A}||^2 = ||\mathbf{P}-\mathbf{B}||^2 \tag{1.0.3}$$

$$\boxed{(\mathbf{P}-\mathbf{A})^\top(\mathbf{P}-\mathbf{A}) = (\mathbf{P}-\mathbf{B})^\top(\mathbf{P}-\mathbf{B})} \tag{1.0.4}$$

$$(\mathbf{P}-\mathbf{A})^\top(\mathbf{P}-\mathbf{A}) = \mathbf{P}^\top\mathbf{P} - \mathbf{P}^\top\mathbf{A} - \mathbf{A}^\top\mathbf{P} + \mathbf{A}^\top\mathbf{A} \tag{1.0.5}$$

$$(\mathbf{P}-\mathbf{B})^\top(\mathbf{P}-\mathbf{B}) = \mathbf{P}^\top\mathbf{P} - \mathbf{P}^\top\mathbf{B} - \mathbf{B}^\top\mathbf{P} + \mathbf{B}^\top\mathbf{B} \tag{1.0.6}$$

Consider the expressions

$$\mathbf{P}^\top\mathbf{P} = ||\mathbf{P}||^2 \tag{1.0.7}$$

$$\mathbf{P}^\top\mathbf{A} = \mathbf{A}^\top\mathbf{P} \tag{1.0.8}$$

Final expression of (1.0.4) can be written as

$$\boxed{||\mathbf{P}||^2 - 2\mathbf{A}^\top\mathbf{P} + \mathbf{A}^\top\mathbf{A} = ||\mathbf{P}||^2 - 2\mathbf{B}^\top\mathbf{P} + \mathbf{B}^\top\mathbf{B}} \tag{1.0.9}$$

$$\implies -2\mathbf{A}^\top\mathbf{P} + 2\mathbf{B}^\top\mathbf{P} = \mathbf{B}^\top\mathbf{B} - \mathbf{A}^\top\mathbf{A} \tag{1.0.10}$$

$$\implies 2\mathbf{P}(\mathbf{A}^\top - \mathbf{B}^\top) = \mathbf{A}^\top\mathbf{A} - \mathbf{B}^\top\mathbf{B} \tag{1.0.11}$$

$$2\mathbf{P}(\mathbf{A}^\top - \mathbf{B}^\top) = ||\mathbf{A}||^2 - ||\mathbf{B}||^2 \tag{1.0.12}$$

P lies on the y-axis

$$\boxed{\mathbf{P} = y\begin{pmatrix}0\\1\end{pmatrix} = y\mathbf{e_2}} \tag{1.0.13}$$

Now substitute this in (1.0.12)

$$2y\mathbf{e_2}(\mathbf{A}^\top - \mathbf{B}^\top) = ||\mathbf{A}||^2 - ||\mathbf{B}||^2 \tag{1.0.14}$$

$$\boxed{\mathbf{y} = \frac{||\mathbf{A}||^2 - ||\mathbf{B}||^2}{2\mathbf{e_2}(\mathbf{A}^\top - \mathbf{B}^\top)}} \tag{1.0.15}$$

$$\mathbf{2e_2}(\mathbf{A}^\top - \mathbf{B}^\top) = 2\begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}0 & 8\end{pmatrix} = 16 \tag{1.0.16}$$

$$\implies \mathbf{y} = \left(\frac{-16}{16}\right) \tag{1.0.17}$$

$$\boxed{\therefore \mathbf{y} = -1} \tag{1.0.18}$$

From (1.0.2) we can write

$$||\mathbf{P}-\mathbf{B}||^2 = ||\mathbf{P}-\mathbf{C}||^2 \tag{1.0.19}$$

$$2\mathbf{P}(\mathbf{B}^\top - \mathbf{C}^\top) = ||\mathbf{B}||^2 - ||\mathbf{C}||^2 \tag{1.0.20}$$

P lies on the x-axis

$$\boxed{\mathbf{P} = x\begin{pmatrix}1\\0\end{pmatrix} = x\mathbf{e_2}} \tag{1.0.21}$$

Now substitute this in (1.0.20)

$$2\mathbf{x}\mathbf{e_2}(\mathbf{B}^\top - \mathbf{C}^\top) = ||\mathbf{B}||^2 - ||\mathbf{C}||^2 \tag{1.0.22}$$

$$\boxed{\mathbf{x} = \frac{||\mathbf{B}||^2 - ||\mathbf{C}||^2}{2\mathbf{e_2}(\mathbf{B}^\top - \mathbf{C}^\top)}} \tag{1.0.23}$$

$$\mathbf{2e_2}(\mathbf{B}^\top - \mathbf{C}^\top) = 2\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}4 & 0\end{pmatrix} = 8 \tag{1.0.24}$$

$$\implies \mathbf{x} = \left(\frac{24}{8}\right) \tag{1.0.25}$$

$$\boxed{\therefore \mathbf{x} = 3} \tag{1.0.26}$$

The coordinate point equidistance from three points is

$$\mathbf{P} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}. \tag{1.0.27}$$

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Fig. 1.1: Fig. 2.25