

Assignment No.1

Suyog Tangade
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Download all python codes from

<https://github.com/suyogtangade/AI.git>

and latex-tikz codes from

<https://github.com/suyogtangade/AI.git>

1 QUESTION No.16(B) (CBSE/2006/SET-2)

Find the co-ordinates of the point equidistant from three given points $\mathbf{A}\begin{pmatrix} 5 \\ 3 \end{pmatrix}$, $\mathbf{B}\begin{pmatrix} 5 \\ -5 \end{pmatrix}$ and $\mathbf{C}\begin{pmatrix} 1 \\ -5 \end{pmatrix}$

Solution:

Let the point equidistant from \mathbf{A} & \mathbf{B} & \mathbf{C} be

$$\mathbf{P} = \begin{pmatrix} x \\ y \end{pmatrix} \quad (1.0.1)$$

From the given information

$$\|\mathbf{P} - \mathbf{A}\|^2 = \|\mathbf{P} - \mathbf{B}\|^2 = \|\mathbf{P} - \mathbf{C}\|^2 \quad (1.0.2)$$

$$\therefore \|\mathbf{P} - \mathbf{A}\|^2 = \|\mathbf{P} - \mathbf{B}\|^2 \quad (1.0.3)$$

$$\left\| \mathbf{P} - \begin{pmatrix} 5 \\ 3 \end{pmatrix} \right\|^2 = \left\| \mathbf{P} - \begin{pmatrix} 5 \\ -5 \end{pmatrix} \right\|^2 \quad (1.0.4)$$

$$\|\mathbf{P}\|^2 + \|\mathbf{A}\|^2 - 2\mathbf{A}^T \mathbf{P} = \|\mathbf{P}\|^2 + \|\mathbf{B}\|^2 - 2\mathbf{B}^T \mathbf{P} \quad (1.0.5)$$

$$\Rightarrow \|\mathbf{P}\|^2 + \left\| \begin{pmatrix} 5 \\ 3 \end{pmatrix} \right\|^2 - 2\mathbf{A}^T \mathbf{P} \quad (1.0.6)$$

$$= \|\mathbf{P}\|^2 + \left\| \begin{pmatrix} 5 \\ -5 \end{pmatrix} \right\|^2 - 2\mathbf{B}^T \mathbf{P} \quad (1.0.7)$$

$$\left\| \begin{pmatrix} 5 \\ 3 \end{pmatrix} \right\|^2 - \left\| \begin{pmatrix} 5 \\ -5 \end{pmatrix} \right\|^2 \quad (1.0.8)$$

$$= 2(5 \ 3)\mathbf{P} - 2(5 \ -5)\mathbf{P} \quad (1.0.9)$$

$$(\sqrt{5^2 + 3^2}) - (\sqrt{5^2 + (-5)^2}) \quad (1.0.10)$$

$$= \left[(10 \ 6) - (10 \ -10) \right] \mathbf{P} \quad (1.0.11)$$

$$(\sqrt{34}) - (\sqrt{50}) = \left[(0 \ 16) \right] \mathbf{P} \quad (1.0.12)$$

$$-16 = (0 \ 16) \mathbf{P} \quad (1.0.13)$$

Which can be simplified to obtain

$$(16 \ 0) \mathbf{P} = -16 \Rightarrow y = -1 \quad (1.0.14)$$

$$\|\mathbf{P}\|^2 + \|\mathbf{B}\|^2 - 2\mathbf{B}^T \mathbf{P} \quad (1.0.15)$$

$$= \|\mathbf{P}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{C}^T \mathbf{P} \quad (1.0.16)$$

$$\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2 = 2\mathbf{B}^T \mathbf{P} - 2\mathbf{C}^T \mathbf{P} \quad (1.0.17)$$

$$\left\| \mathbf{P} - \begin{pmatrix} 5 \\ -5 \end{pmatrix} \right\|^2 = \left\| \mathbf{P} - \begin{pmatrix} 1 \\ -5 \end{pmatrix} \right\|^2 \quad (1.0.18)$$

$$\Rightarrow \|\mathbf{P}\|^2 + \left\| \begin{pmatrix} 5 \\ -5 \end{pmatrix} \right\|^2 - 2\mathbf{B}^T \mathbf{P} \quad (1.0.19)$$

$$= \|\mathbf{P}\|^2 + \left\| \begin{pmatrix} 1 \\ -5 \end{pmatrix} \right\|^2 - 2\mathbf{C}^T \mathbf{P} \quad (1.0.20)$$

$$\left\| \begin{pmatrix} 5 \\ -5 \end{pmatrix} \right\|^2 - 2\mathbf{B}^T \mathbf{P} = \left\| \begin{pmatrix} 1 \\ -5 \end{pmatrix} \right\|^2 - 2\mathbf{C}^T \mathbf{P} \quad (1.0.21)$$

$$(\sqrt{5^2 + (-5)^2}) - 2(5 \ -5)\mathbf{P} \quad (1.0.22)$$

$$= (\sqrt{1^2 + (-5)^2}) - 2(1 \ -5)\mathbf{P} \quad (1.0.23)$$

$$(\sqrt{25} + \sqrt{25}) - (-10 \ 10)\mathbf{P} \quad (1.0.24)$$

$$= (\sqrt{1} + \sqrt{25}) - (-2 \ 10)\mathbf{P} \quad (1.0.25)$$

$$(\sqrt{50}) - (-10 \ 10)\mathbf{P} \quad (1.0.26)$$

$$= (\sqrt{26}) - (-2 \ 10)\mathbf{P} \quad (1.0.27)$$

$$24 = (8 \ 0)\mathbf{P} \quad (1.0.28)$$

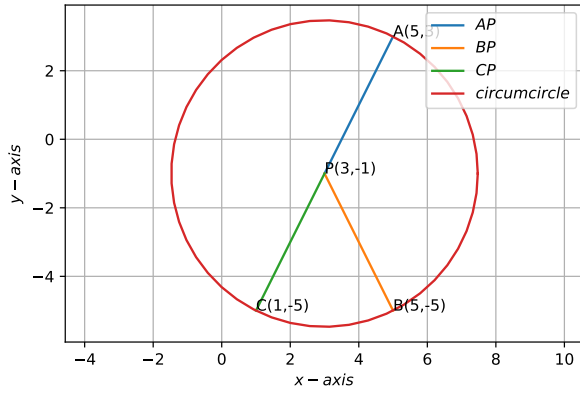


Fig. 1.1: Graphical Solution

Which can be simplified to obtain

$$\begin{pmatrix} 8 & 0 \end{pmatrix} \mathbf{P} = 24 \implies x = 3 \quad (1.0.29)$$

The required point

$$\mathbf{P} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}. \quad (1.0.30)$$