Assignment-04

Suyog Tangade
MD/2020/710

Download all python codes from

https://github.com/suyogtangade/Assignment4.git

and latex-tikz codes from

https://github.com/suyogtangade/Assignment4.git

Question taken from

https://github.com/gadepall/ncert/blob/main/linalg/linear_forms/gvv_ncert_linear_forms.pdf

1 Linear Forms Exercise 2.5(c)

Find out whether the following pair of linear equations are consistent, or inconsistent.

$$\begin{pmatrix} \frac{3}{2} & \frac{5}{3} \end{pmatrix} \mathbf{x} = 7 \tag{1.0.1}$$

$$\begin{pmatrix} 9 & -10 \end{pmatrix} \mathbf{x} = 14 \tag{1.0.2}$$

2 Solution

$$\begin{pmatrix} \frac{3}{2} & \frac{5}{3} \end{pmatrix} \mathbf{x} = 7 \tag{2.0.1}$$

$$\begin{pmatrix} 9 & -10 \end{pmatrix} \mathbf{x} = 14 \tag{2.0.2}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} \frac{3}{2} & \frac{5}{3} \\ 9 & -10 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 7 \\ 14 \end{pmatrix} \tag{2.0.3}$$

The augmented matrix for the above equation is row reduced as follows:

$$\begin{pmatrix} \frac{3}{2} & \frac{5}{3} & 7 \\ 9 & -10 & 14 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow \frac{2}{3} R_1} \begin{pmatrix} 1 & \frac{10}{9} & \frac{14}{3} \\ 9 & -10 & 14 \end{pmatrix} \tag{2.0.4}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 9R_1} \begin{pmatrix} 1 & \frac{10}{9} & \frac{14}{3} \\ 0 & -20 & -28 \end{pmatrix} \tag{2.0.5}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{-1}{20} R_2} \begin{pmatrix} 1 & \frac{10}{9} & \frac{14}{3} \\ 0 & 1 & \frac{7}{5} \end{pmatrix} \tag{2.0.6}$$

$$\xleftrightarrow{R_1 \leftarrow \frac{-10}{9} R_2 + R_1} \begin{pmatrix} 1 & 0 & \frac{28}{9} \\ 0 & 1 & \frac{7}{5} \end{pmatrix} \tag{2.0.7}$$

So by reduction of the (2×3) matrix

$$\begin{pmatrix} \frac{3}{2} & \frac{5}{3} & 7 \\ 9 & -10 & 14 \end{pmatrix} \tag{2.0.8}$$

gives matrix with 2 non zero row,so i'ts rank is 2.

$$\begin{pmatrix} \frac{3}{2} & \frac{5}{3} \\ 9 & -10 \end{pmatrix} \tag{2.0.9}$$

The rank of the above matrix is also 2.
$\therefore$ lines are Consistent and gives unique solution.

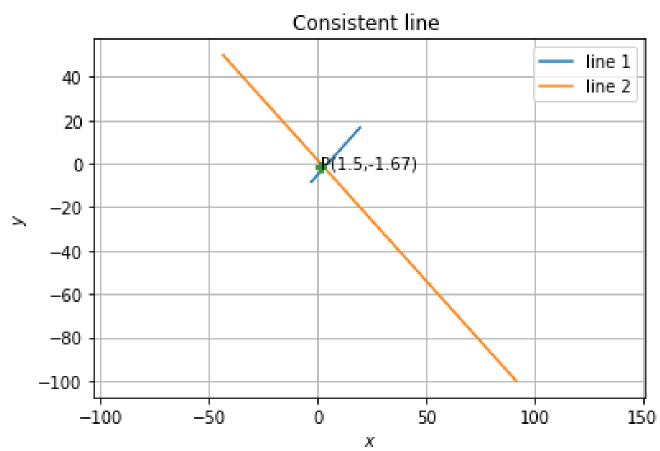

Fig. 2.1: Graphical solution

$\therefore$ This figure verifies that two lines are intersecting at point P$\begin{pmatrix} 1.5 \\ -1.67 \end{pmatrix}$.