Assignment-07

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Download latex-tikz codes from

https://github.com/suyogtangade/Assignment7.git

Download python codes from

https://github.com/suyogtangade/Assignment7.git

Question taken from

https://github.com/gadepall/ncert/blob/main/linalg/optimization/gvv_ncert_opt.pdf question 2.14

1 Question No 2.14

There are two types of fertilisers F_1 and F_2.F_1 consists of 10% nitrogen and 6% phosphoric acid and F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs atleast 14 kg of nitrogen and 14 kg of phosphoric acid for her crop. If F_1 costs Rs 6/kg and F_2 costs Rs 5/kg, determine how much of each type of fertiliser should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost?

2 Solution

Fertiliser	Nitrogen	Phosphoric acid	Quantity	Cost
F1	10 Percentage	6 Percentage	X Kg	Rs 6
F2	5 Percentage	10 Percentage	Y Kg	Rs 5
Total	14 Kg	14Kg		

Lets consider fertiliser F1 requirement is x kg
fertiliser F2 requirement is y kg

$$x \geq 0 \tag{2.0.1}$$

$$y \geq 0 \tag{2.0.2}$$

According to the question,

$$10x + 5y \leq 14 \tag{2.0.3}$$

$$\tag{2.0.4}$$

and,

$$6x + 10y \leq 14 \tag{2.0.5}$$

$$\implies 3x + 5y \leq 7 \tag{2.0.6}$$

$\therefore$ Our problem is

$$\max_{\mathbf{x}} Z = \begin{pmatrix} 6 & 5 \end{pmatrix} \mathbf{x} \tag{2.0.7}$$

$$s.t. \quad \begin{pmatrix} 10 & 5 \\ 3 & 5 \end{pmatrix} \mathbf{x} \leq \begin{pmatrix} 14 \\ 7 \end{pmatrix} \tag{2.0.8}$$

Lagrangian function is given by

$$\begin{aligned} & L(\mathbf{x}, \boldsymbol{\lambda}) \\ & = \begin{pmatrix} 6 & 5 \end{pmatrix} \mathbf{x} + \left\{ \left[\begin{pmatrix} 10 & 5 \end{pmatrix} \mathbf{x} + 14 \right] \right. \\ & + \left[\begin{pmatrix} 3 & 5 \end{pmatrix} \mathbf{x} + 7 \right] \\ & + \left. \left[\begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \right] + \left[\begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \right] \right\} \boldsymbol{\lambda} \end{aligned} \tag{2.0.9}$$

where,

$$\boldsymbol{\lambda} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \end{pmatrix} \tag{2.0.10}$$

Now,

$$\nabla L(\mathbf{x}, \boldsymbol{\lambda}) = \begin{pmatrix} 6 + \begin{pmatrix} 10 & 5 & -1 & 0 \end{pmatrix} \boldsymbol{\lambda} \\ 6 + \begin{pmatrix} 3 & 5 & 0 & -1 \end{pmatrix} \boldsymbol{\lambda} \\ \begin{pmatrix} 10 & 5 \end{pmatrix} \mathbf{x} + 14 \\ \begin{pmatrix} 3 & 5 \end{pmatrix} \mathbf{x} + 7 \\ \begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \\ \begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \end{pmatrix} \tag{2.0.11}$$

$\therefore$ Lagrangian matrix is given by

$$\begin{pmatrix} 0 & 0 & 10 & 5 & -1 & 0 \\ 0 & 0 & 3 & 5 & 0 & -1 \\ 10 & 5 & 0 & 0 & 0 & 0 \\ 3 & 5 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} -6 \\ -5 \\ 14 \\ 7 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.12}$$

Considering λ_1, λ_2 as only active multiplier,

$$\begin{pmatrix} 0 & 0 & 10 & 5 \\ 0 & 0 & 3 & 5 \\ 10 & 5 & 0 & 0 \\ 3 & 5 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} -6 \\ -5 \\ 14 \\ 7 \end{pmatrix} \tag{2.0.13}$$

resulting in,

$$\begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 10 & 5 \\ 0 & 0 & 3 & 5 \\ 10 & 5 & 0 & 0 \\ 3 & 5 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} -6 \\ -5 \\ 14 \\ 7 \end{pmatrix} \tag{2.0.14}$$

$$\implies \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{1}{7} & \frac{-1}{7} \\ 0 & 0 & \frac{-3}{35} & \frac{2}{7} \\ \frac{1}{7} & \frac{-1}{7} & 0 & 0 \\ \frac{-3}{35} & \frac{2}{7} & 0 & 0 \end{pmatrix} \begin{pmatrix} -6 \\ -5 \\ 14 \\ 7 \end{pmatrix} \tag{2.0.15}$$

$$\implies \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{4}{5} \\ \frac{-1}{7} \\ \frac{-32}{35} \end{pmatrix} \tag{2.0.16}$$

$\because \boldsymbol{\lambda} = \begin{pmatrix} \frac{-1}{7} \\ \frac{-23}{35} \end{pmatrix} \succ \mathbf{0}$

$\therefore$ Optimal solution is given by

$$\mathbf{x} = \begin{pmatrix} 1 \\ \frac{4}{5} \end{pmatrix} \tag{2.0.17}$$

$$Z = \begin{pmatrix} 6 & 5 \end{pmatrix} \mathbf{x} \tag{2.0.18}$$

$$= \begin{pmatrix} 6 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ \frac{4}{5} \end{pmatrix} \tag{2.0.19}$$

$$= 10 \tag{2.0.20}$$

By using cvxpy in python ,

$$\mathbf{x} = \begin{pmatrix} 1 \\ 0.8 \end{pmatrix} \tag{2.0.21}$$

$$Z = 10 \tag{2.0.22}$$

Hence the cost will be minimum If fertilizer F1 used $\boxed{x = 1kg}$ and fertilizer F2 used $\boxed{y = \frac{4}{5}}$

Minimum cost $\boxed{Z = 10}$. This is verified in Fig. 0.

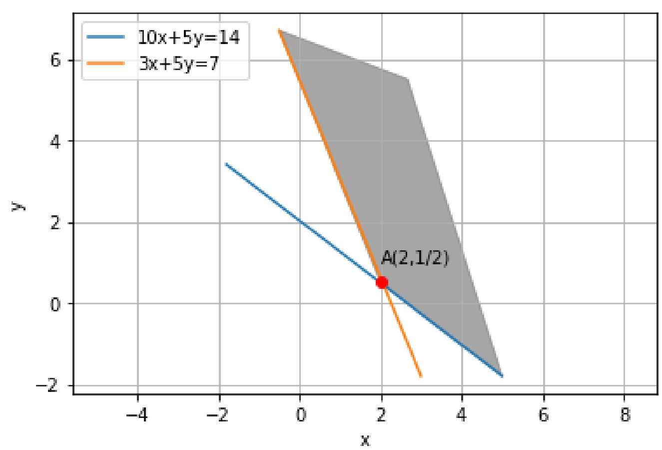

Fig. 0: graphical solution