

Assignment-07

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Download latex-tikz codes from

<https://github.com/suyogtangade/Assignment7.git>

Download python codes from

<https://github.com/suyogtangade/Assignment7.git>

Question taken from

https://github.com/gadepall/ncert/blob/main/linalg/optimization/gvv_ncert_opt.pdf question 2.14

and,

$$6x + 10y \leq 14 \quad (2.0.5)$$

$$\Rightarrow 3x + 5y \leq 7 \quad (2.0.6)$$

\therefore Our problem is

$$\max_{\mathbf{x}} Z = (6 \ 5) \mathbf{x} \quad (2.0.7)$$

$$s.t. \quad \begin{pmatrix} 10 & 5 \\ 3 & 5 \end{pmatrix} \mathbf{x} \leq \begin{pmatrix} 14 \\ 7 \end{pmatrix} \quad (2.0.8)$$

Lagrangian function is given by

$$\begin{aligned} L(\mathbf{x}, \lambda) &= (6 \ 5) \mathbf{x} + \left\{ \left[(10 \ 5) \mathbf{x} + 14 \right] \right. \\ &\quad \left. + \left[(3 \ 5) \mathbf{x} + 7 \right] \right. \\ &\quad \left. + \left[(-1 \ 0) \mathbf{x} \right] + \left[(0 \ -1) \mathbf{x} \right] \right\} \lambda \end{aligned} \quad (2.0.9)$$

where,

$$\lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \end{pmatrix} \quad (2.0.10)$$

Now,

$$\nabla L(\mathbf{x}, \lambda) = \begin{pmatrix} 6 + (10 \ 5 \ -1 \ 0) \lambda \\ 5 + (3 \ 5 \ 0 \ -1) \lambda \\ (10 \ 5) \mathbf{x} + 14 \\ (3 \ 5) \mathbf{x} + 7 \\ (-1 \ 0) \mathbf{x} \\ (0 \ -1) \mathbf{x} \end{pmatrix} \quad (2.0.11)$$

1 QUESTION No 2.14

There are two types of fertilisers F_1 and F_2 . F_1 consists of 10% nitrogen and 6% phosphoric acid and F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs atleast 14 kg of nitrogen and 14 kg of phosphoric acid for her crop. If F_1 costs Rs 6/kg and F_2 costs Rs 5/kg, determine how much of each type of fertiliser should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost?

2 SOLUTION

Fertiliser	Nitrogen	Phosphoric acid	Quantity	Cost
F1	10 Percentage	6 Percentage	X Kg	Rs 6
F2	5 Percentage	10 Percentage	Y Kg	Rs 5
Total	14 Kg	14Kg		

Lets consider fertiliser F1 requirement is x kg
fertiliser F2 requirement is y kg

$$x \geq 0 \quad (2.0.1)$$

$$y \geq 0 \quad (2.0.2)$$

According to the question,

$$10x + 5y \leq 14 \quad (2.0.3)$$

$$(2.0.4)$$

∴ Lagrangian matrix is given by

$$\begin{pmatrix} 0 & 0 & 10 & 5 & -1 & 0 \\ 0 & 0 & 3 & 5 & 0 & -1 \\ 10 & 5 & 0 & 0 & 0 & 0 \\ 3 & 5 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} -6 \\ -5 \\ 14 \\ 7 \\ 0 \\ 0 \end{pmatrix} \quad (2.0.12)$$

Considering λ_1, λ_2 as only active multiplier,

$$\begin{pmatrix} 0 & 0 & 10 & 5 \\ 0 & 0 & 3 & 5 \\ 10 & 5 & 0 & 0 \\ 3 & 5 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} -6 \\ -5 \\ 14 \\ 7 \end{pmatrix} \quad (2.0.13)$$

resulting in,

$$\begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 & 0 & 10 & 5 \\ 0 & 0 & 3 & 5 \\ 10 & 5 & 0 & 0 \\ 3 & 5 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} -6 \\ -5 \\ 14 \\ 7 \end{pmatrix} \quad (2.0.14)$$

$$\Rightarrow \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{1}{7} & \frac{-1}{7} \\ 0 & 0 & \frac{-3}{35} & \frac{2}{7} \\ \frac{1}{7} & \frac{-1}{7} & 0 & 0 \\ \frac{-3}{35} & \frac{2}{7} & 0 & 0 \end{pmatrix} \begin{pmatrix} -6 \\ -5 \\ 14 \\ 7 \end{pmatrix} \quad (2.0.15)$$

$$\Rightarrow \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{4}{5} \\ \frac{-1}{7} \\ \frac{-32}{35} \end{pmatrix} \quad (2.0.16)$$

$$\therefore \lambda = \left(\frac{-1}{7}, \frac{-32}{35} \right) > \mathbf{0}$$

∴ Optimal solution is given by

$$\mathbf{x} = \begin{pmatrix} 1 \\ \frac{4}{5} \end{pmatrix} \quad (2.0.17)$$

$$Z = (6 \ 5) \mathbf{x} \quad (2.0.18)$$

$$= (6 \ 5) \begin{pmatrix} 1 \\ \frac{4}{5} \end{pmatrix} \quad (2.0.19)$$

$$= 10 \quad (2.0.20)$$

By using cvxpy in python ,

$$\mathbf{x} = \begin{pmatrix} 1 \\ 0.8 \end{pmatrix} \quad (2.0.21)$$

$$Z = 10 \quad (2.0.22)$$

Hence the cost will be minimum If fertilizer F1 used

$$x = 1\text{kg} \text{ and fertilizer F2 used } y = \frac{4}{5}$$

Minimum cost $Z = 10$. This is verified in Fig. 0.

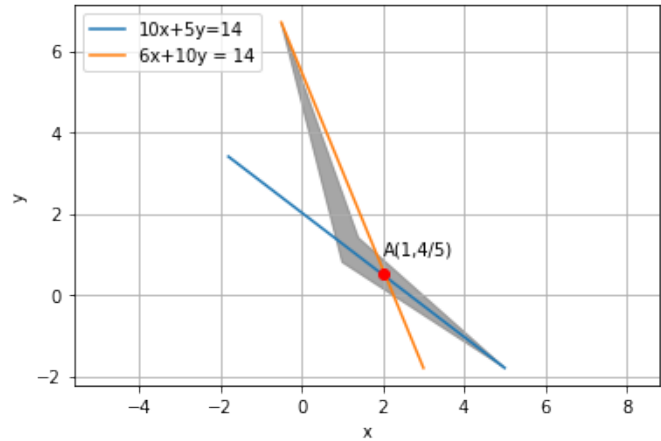


Fig. 0: graphical solution