APPENDIX

A. Algorithm for Finding the Maximal Linearly Independent Set

For an incidence matrix of a PN, Algorithm A1 shows the steps to find its maximal linearly independent set.

B. Other Properties Analysis of Modified PNs

The properties of PNs include behavioral properties and structural properties, in which the former is dependent on the initial marking, and the latter is independent of the initial marking. Reachability, liveness, and persistence are important behavioral properties of PNs, and repetitiveness, siphon, and trap are the structural properties of PNs. Next, we analyze the persistence, repetitiveness, siphon, and trap of modified PNs.

*Definition A*1 [1]: Let N be a PN. N is persistent if for $\forall M\in R(M_0)$ and t_i, $t_j\in T$: $M[t_i\rangle$, $M[t_j\rangle M_k$, and $M_k[t_i\rangle$.

***Theorem A*1:** Let N be a PN and $N'=(P', T, F', W', M_0')$ be a modified PN of N via Algorithm 1. If N is persistent, N' is persistent.

Proof: If N is persistent, for $\forall M\in R(M_0)$ and t_i, $t_j\in T$, we have that $M[t_i\rangle$, $M[t_j\rangle M_k$, and $M_k[t_i\rangle$. According to Theorem 6, for $\forall M\in R(M_0)$, $\exists M'\in R(M_0')$:

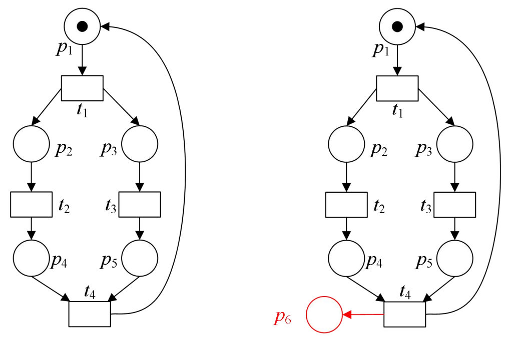
$$M'(p)=\begin{cases} M(p), & \text{if } p\in P \\ c, & \text{otherwise} \end{cases}$$

where $c\in\mathbf{N}$. For $\forall p_r\in P'\text{-}P$, since $p_r^{\bullet}=\varnothing$, the number of tokens in p_r does not affect the fire of transitions. Since $M[t_i\rangle$, $M[t_j\rangle M_k$, and for $\forall p\in P$: $M'(p)=M(p)$, $M'[t_i\rangle$ and $M'[t_j\rangle M_k'$, where for $\forall p\in P$, $M_k'(p)=M_k(p)$. By $M_k[t_i\rangle$, we have $M_k'[t_i\rangle$. Hence, N' is persistent. ■

*Example A*1: A persistent PN N is shown in Fig. A1(a) with $M_0=[1\ 0\ 0\ 0\ 0]^{\mathrm{T}}$. The incidence matrix of N is

$$A=\begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Since $\mathbb{R}(A)=3$, the PN is not a UPN. According to Algorithm 1, we can obtain a modified PN N' as shown in Fig. A1(b), where $M_0'=[1\ 0\ 0\ 0\ 0\ 0]^{\mathrm{T}}$. Since $p_6^{\bullet}=\varnothing$, for $\forall M\in R(M_0)$ and t_i, $t_j\in T$, if $M[t_i\rangle$, $M[t_j\rangle M_k$, and $M_k[t_i\rangle$, for $\forall M'\in R(M_0')$, we have that $M'[t_i\rangle$, $M'[t_j\rangle M_k'$, and $M_k'[t_i\rangle$. Hence, the modified PN is persistent.



(a) A persistent PN N (b) A modified PN of N

Fig. A1. Persistent PN (a) and its modified PN (b).

Algorithm A1: Finding a maximal linearly independent set

Input: An incidence matrix A
Output: A maximal linearly independent set of A
1 Let i=1 and j=1;
2 **While** $i\le m$, **do**
3 **While** $j\le n$, **do**
4 $k=m$;
5 **While** $A_{(i,j)}$==0 and $k\ne i$, **then**
6 Swap row i and row k of A;
7 $k=k$-1;
8 **End**
9 **If** $A_{(i,j)}$==0, **then**
10 $j=j$+1;
11 Jump to Line 4;
12 **If** $j>n$, **then**
13 **Break;**
14 **End**
15 **End**
16 **If** $A_{(i,j)}\ne 1$, **then**
17 **For** $k=i$: 1: m, **do**
18 **If** $A_{(k,j)}$=1, **then**
19 Swap row i and row k of A;
20 **Break;**
21 **End**
22 **End**
23 **If** $k=m$+1, **then**
24 $A_{i*}=(1/A_{(i,j)})*A_{i*}$;
22 **End**
25 **End**
26 **For** k=1: 1: m, **do**
27 **If** $A_{(k,j)}\ne 0$, **then**
28 $A_{k*}=A_{k*}\text{-}A_{(k,j)}*A_{i*}$;
29 **End**
30 **End**
31 $j=j$+1;
32 Break;
33 **End**
34 **If** $j\le n$, **then**
35 $i=i$+1;
36 **End**
37 **End**
38 Let i=1, j=1, and $V'=\varnothing$
39 **While** $i<m$, **do**
40 **While** $j<n$, **do**
41 **If** $A_{(i,j)}\ne 0$, **then**
42 $V'=V'\cup\{A_{j*}\}$;
43 $i=i$+1;
44 **Else if** $A_{(i,j)}$==0, **then**
45 $j=j$+1;
46 **End**
47 **End**
48 **End**
49 **Return** V'.

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*Definition A*2 [1]: Let N be a PN. N is repetitive if $\exists\sigma$: $M_0[\sigma\rangle$ and $\forall t\in T$ fires infinitely often in σ.

Theorem A2: Let N be a PN and $N'=(P', T, F', W', M_0')$ be its modified PN via Algorithm 1. If N is repetitive, N' is repetitive.

Proof: If N is repetitive, $\exists\sigma$: $M_0[\sigma\rangle$, and $\forall t\in T$ fires infinitely often in σ. According to Theorem 6, if $M_0[\sigma\rangle$, $M_0'[\sigma\rangle$. Since $\forall t\in T$ fires infinitely often in σ, N' is repetitive. ■

Example A2: From Fig. A1(a), we have that there is an LFS $\sigma=t_1t_2t_3t_4t_1t_2t_3t_4...$ in N such that $M_0[\sigma\rangle$ and $\forall t\in T$ fires infinitely often in σ. Hence, N is repetitive. From Fig. A1(b), we have that $M_0'[\sigma\rangle$ and $\forall t\in T$ fires infinitely often in σ. Thus, the modified PN N' is repetitive.

Definition A3 [1]: Let N be a PN and $P'\subseteq P$. P' is called a siphon/trap if ${}^\bullet P'\subseteq P'^\bullet/P'^\bullet\subseteq{}^\bullet P'$.

Theorem A3: Let N be a PN and $N'=(P', T, F', W', M_0')$ be a modified PN of N. If $\acute{P}$ is a siphon/trap in N, so is $\acute{P}$ in N'.

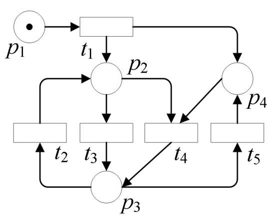
Proof: Since $\acute{P}$ is a siphon/trap in N, ${}^\bullet\acute{P}\subseteq\acute{P}^\bullet/\acute{P}^\bullet\subseteq{}^\bullet\acute{P}$. According to the definition of the modified PN, we have that for $\forall p_k\in\acute{P}$, if $t_j\in{}^\bullet p_k$ and $t_r\in p_k^\bullet$ in N, $t_j\in{}^\bullet p_k$ and $t_r\in p_k^\bullet$ in N'.

Hence, if T_j and T_r are the pre-set and post-set of p_k in N, T_j and T_r are the pre-set and post-set of p_k in N'. Hence, if ${}^\bullet\acute{P}\subseteq\acute{P}^\bullet/\acute{P}^\bullet\subseteq{}^\bullet\acute{P}$ in N, ${}^\bullet\acute{P}\subseteq\acute{P}^\bullet/\acute{P}^\bullet\subseteq{}^\bullet\acute{P}$ in N', i.e., if $\acute{P}$ is a siphon/trap in N, $\acute{P}$ is a siphon/trap in N'. ■

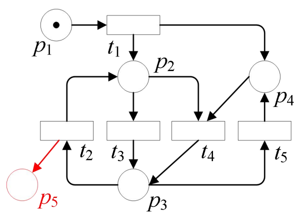
Example A3: A PN N is shown in Fig. A2(a). The incidence matrix of N is

$$A=\begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 \\ 0 & -1 & 1 & 1 & -1 \\ 1 & 0 & 0 & -1 & 1 \end{bmatrix}$$

Since $\mathbb{R}(A)=4$, the PN is not a UPN. According to Algorithm 1, we can obtain a modified PN N' as shown in Fig. A2(b). Suppose that $\acute{P}=\{p_1, p_2, p_3, p_4\}$. In PN N, since ${}^\bullet\acute{P}=\{t_1, t_2, t_3, t_4, t_5\}$ and $\acute{P}^\bullet=\{t_1, t_2, t_3, t_4, t_5\}$, ${}^\bullet\acute{P}\subseteq{}^\bullet\acute{P}$ and $\acute{P}^\bullet\subseteq{}^\bullet\acute{P}$, i.e., $\acute{P}$ is both a siphon and a trap in N. From the figure, we have that ${}^\bullet\acute{P}\subseteq{}^\bullet\acute{P}$ and $\acute{P}^\bullet\subseteq{}^\bullet\acute{P}$ in N', $\acute{P}$ is both a siphon and a trap in N'.



(a) A PN N with siphons and traps



(b) A modified PN of N

Fig. A2. PN with siphons and traps.