Appendix

1. *Algorithm for Finding the Maximal Linearly Independent Set*

For an incidence matrix of a PN, Algorithm A1 shows the steps to find its maximal linearly independent set.

1. *Other Properties Analysis of Modified PNs*

The properties of PNs include behavioral properties and structural properties, in which the former is dependent on the initial marking, and the latter is independent of the initial marking. Reachability, liveness, and persistence are important behavioral properties of PNs, and repetitiveness, siphon, and trap are the structural properties of PNs. Next, we analyze the persistence, repetitiveness, siphon, and trap of modified PNs.

*Definition A*1 [1]: Let *N* be a PN. *N* is persistent if for ∀*M*∈*R*(*M*0) and *ti*, *tj*∈*T*: *M*[*ti*〉, *M*[*tj*〉*Mk*, and *Mk*[*ti*〉.

***Theorem A*1:** Let *N* be a PN and *N'*=(*P'*, *T*, *F'*, *W'*, *M*0*'*) be a modified PN of *N* via Algorithm 1. If *N* is persistent, *N'* is persistent.

*Proof:* If *N* is persistent, for ∀*M*∈*R*(*M*0) and *ti*, *tj*∈*T*, we have that *M*[*ti*〉, *M*[*tj*〉*Mk*, and *Mk*[*ti*〉. According to Theorem 6, for ∀*M*∈*R*(*M*0), ∃*M'*∈*R*(*M*0*'*):



where *c*∈N. For ∀*pr*∈*P'*-*P*, since *pr*•=∅, the number of tokens in *pr* does not affect the fire of transitions. Since *M*[*ti*〉, *M*[*tj*〉*Mk*, and for ∀*p*∈*P*: *M'*(*p*)=*M*(*p*), *M'*[*ti*〉 and *M'*[*tj*〉*Mk'*, where for ∀*p*∈*P*, *Mk'*(*p*)=*Mk*(*p*). By *Mk*[*ti*〉, we have *Mk'*[*ti*〉. Hence, *N'* is persistent. ■

*Example A*1: A persistent PN *N* is shown in Fig. A1(a) with *M*0=[1 0 0 0 0]T. The incidence matrix of *N* is



Since *ℝ*(*A*)=3, the PN is not a UPN. According to Algorithm 1, we can obtain a modified PN *N'* as shown in Fig. A1(b), where *M*0*'*=[1 0 0 0 0 0]T. Since *p*6•=∅, for ∀*M*∈*R*(*M*0) and *ti*, *tj*∈*T*, if *M*[*ti*〉, *M*[*tj*〉*Mk*, and *Mk*[*ti*〉, for ∀*M'*∈*R*(*M*0*'*), we have that *M'*[*ti*〉, *M'*[*tj*〉*Mk'*, and *Mk'*[*ti*〉. Hence, the modified PN is persistent.



**Fig. A1.** Persistent PN (a) and its modified PN (b).

|  |  |
| --- | --- |
| **Algorithm A1**: Finding a maximal linearly independent set | |
|  | Input: An incidence matrix *A* |
|  | Output: A maximal linearly independent set of *A* |
| **1** | Let *i*=1 and *j*=1; |
| **2** | **While** *i≤m*, **do** |
| **3** | **While** *j*≤*n*, **do** |
| **4** | *k=m*; |
| **5** | **While** *A*(*i*, *j*)==0 and *k*≠*i*, **then** |
| **6** | Swap row *i* and row *k* of *A*; |
| **7** | *k*=*k*-1; |
| **8** | **End** |
| **9** | **If** *A*(*i*, *j*)==0, **then** |
| **10** | *j*=*j*+1; |
| **11** | Jump to Line 4; |
| **12** | **If** *j*>*n*, **then** |
| **13** | **Break;** |
| **14** | **End** |
| **15** | **End** |
| **16** | **If** *A*(*i*, *j*)≠1, **then** |
| **17** | **For** *k*=*i*: 1: *m*, **do** |
| **18** | **If** *A*(*k*, *j*)=1, **then** |
| **19** | Swap row *i* and row *k* of *A*; |
| **20** | **Break;** |
| **21** | **End** |
| **22** | **End** |
| **23** | **If** *k*=*m*+1, **then** |
| **24** | *Ai*\*=(1/*A*(*i*, *j*))\**Ai*\*; |
| **22** | **End** |
| **25** | **End** |
| **26** | **For** *k*=1: 1: *m*, **do** |
| **27** | **If** *A*(*k*, *j*)≠0, **then** |
| **28** | *Ak\**=*Ak\**-*A*(*k*, *j*)\**Ai*\*; |
| **29** | **End** |
| **30** | **End** |
| **31** | *j*=*j*+1; |
| **32** | Break; |
| **33** | **End** |
| **34** | **If** *j*≤*n***, then** |
| **35** | *i*=*i*+1; |
| **36** | **End** |
| **37** | **End** |
| **38** | Let *i*=1, *j*=1, and *V'*=∅ |
| **39** | **While** *i*<*m*, **do** |
| **40** | **While** *j*<*n*, **do** |
| **41** | **If** *A*(*i*, *j*)≠0, **then** |
| **42** | *V'*=*V'*∪{*Aj*\*}; |
| **43** | *i*=*i*+1; |
| **44** | **Else if** *A*(*i*, *j*)==0, **then** |
| **45** | *j*=*j*+1; |
| **46** | **End** |
| **47** | **End** |
| **48** | **End** |
| **49** | **Return** *V'*. |

*Definition A*2 [1]: Let *N* be a PN. *N* is repetitive if ∃σ: *M*0[σ〉 and ∀*t*∈*T* fires infinitely often in σ.

***Theorem A*2:** Let *N* be a PN and *N'*=(*P'*, *T*, *F'*, *W'*, *M*0*'*) be its modified PN via Algorithm 1. If *N* is repetitive, *N'* is repetitive.

*Proof:* If *N* is repetitive, ∃σ: *M*0[σ〉, and ∀*t*∈*T* fires infinitely often in σ. According to Theorem 6, if *M*0[σ〉, *M*0*'*[σ〉. Since ∀*t*∈*T* fires infinitely often in σ, *N'* is repetitive. ■

*Example A*2: From Fig. A1(a), we have that there is an LFS σ=*t*1*t*2*t*3*t*4*t*1*t*2*t*3*t*4... in *N* such that *M*0[σ〉 and ∀*t*∈*T* fires infinitely often in σ. Hence, *N* is repetitive. From Fig. A1(b), we have that *M*0*'*[σ〉 and ∀*t*∈*T* fires infinitely often

in σ. Thus, the modified PN *N'* is repetitive.

*Definition A*3 [1]: Let *N* be a PN and *P'*⊆*P*. *P'* is called a siphon/trap if •*P'*⊆*P'*•/*P'*•⊆•*P'*.

***Theorem A*3:** Let *N* be a PN and *N'*=(*P'*, *T*, *F'*, *W'*, *M*0*'*) be a modified PN of *N*. If *Ṕ* is a siphon/trap in *N*, so is *Ṕ* in *N'*.

*Proof:* Since *Ṕ* is a siphon/trap in *N*, •*Ṕ*⊆*Ṕ*•/*Ṕ*•⊆•*Ṕ*. According to the definition of the modified PN, we have that for ∀*pk*∈*Ṕ*, if *tj*∈•*pk* and *tr*∈*pk*• in *N*, *tj*∈•*pk* and *tr*∈*pk*• in *N'*.

Hence, if *Tj* and *Tr* are the pre-set and post-set of *pk* in *N*, *Tj* and *Tr* are the pre-set and post-set of *pk* in *N'*. Hence, if •*Ṕ*⊆*Ṕ*•/*Ṕ*•⊆•*Ṕ* in *N*, •*Ṕ*⊆*Ṕ*•/*Ṕ*•⊆•*Ṕ* in *N'*, i.e., if *Ṕ* is a siphon/trap in *N*, *Ṕ* is a siphon/trap in *N'*. ■

*Example A*3: A PN *N* is shown in Fig. A2(a). The incidence matrix of *N* is



Since *ℝ*(*A*)=4, the PN is not a UPN. According to Algorithm 1, we can obtain a modified PN *N'* as shown in Fig. A2(b). Suppose that *Ṕ*={*p*1, *p*2, *p*3, *p*4}. In PN *N*, since •*Ṕ*={*t*1, *t*2, *t*3, *t*4, *t*5} and *Ṕ*•={*t*1, *t*2, *t*3, *t*4, *t*5}, •*Ṕ*⊆•*Ṕ* and *Ṕ*•⊆•*Ṕ*, i.e., *Ṕ* is both a siphon and a trap in *N*. From the figure, we have that •*Ṕ*⊆•*Ṕ* and *Ṕ*•⊆•*Ṕ* in *N'*, *Ṕ* is both a siphon and a trap in *N'*.

**Fig. A2.** PN with siphons and traps.