Supplementary File

Proof of Theorem 1:

Sufficiency:

Suppose that $\forall t''\in\{t'|\ t'\in p_{0i}^{\bullet},\ p_{0i}\in P_0\}$ is not enabled at M. Let $P_M'=\{p'|\ p'\in P_A,\ M(p')=0\}$ and $T=T_A\cup T_0$, where $T_A=\{t_u|\ t_u\in p'^{\bullet},\ p'\in P_A\}$ and $T_0=\{t_v|\ t_v\in p_{0i}^{\bullet},\ p_{0i}\in P_0\}$. By $P_M=\{p'|\ p'\in P_A,\ M(p')\neq 0\}$ and $P_M'=\{p'|\ p'\in P_A,\ M(p')=0\}$, we have that $P_A=P_M\cup P_M'$. According to Definition 5, $\forall t\in T$: $|{}^{\bullet}t\cap(P_A\cup P_0)|=1$. We assume that ${}^{\bullet}t\cap(P_A\cup P_0)=\{p\}$. Hence, if $t\in T_A$, then $p\in P_A$, i.e., $p\in P_M\cup P_M'$; otherwise, if $t\in T_0$, then $p\in P_0$. Thus, there are three cases:

1) When $t\in T_A$ and $p\in P_M'$, by $P_M'=\{p'|\ p'\in P_A,\ M(p')=0\}$, we have $M(p)=0$. Hence, t is not enabled at M.

2) When $t\in T_A$ and $p\in P_M$, since M is a partial deadlock, according to Definition 6, t is not enabled at M.

3) When $t\in T_0$, since $\forall t''\in\{t'|\ t'\in p_{0i}^{\bullet},\ p_{0i}\in P_0\}$ is not enabled at M, t is not enabled at M.

Hence, $\forall t\in T$ is not enabled at M. According to Definition 3, M is a deadlock.

Necessity:

Suppose that M is a deadlock. According to Definition 3, $\forall t\in T$ is not enabled at $M'\in R(M)$. Hence, $\forall t''\in\{t'|\ t'\in p_{0i}^{\bullet},\ p_{0i}\in P_0\}$ is not enabled at M. ■

Proof of Theorem 2:

For $M\in R(M_0)$, we have that

$$AX=M\text{-}M_0.$$

Then, we obtain

$$(AX)^{\mathrm{T}}\pi_r=(M\text{-}M_0)^{\mathrm{T}}\pi_r,$$

i.e.,

$$X^{\mathrm{T}}(A^{\mathrm{T}}\pi_r)=(M\text{-}M_0)^{\mathrm{T}}\pi_r.$$

By $A^{\mathrm{T}}\pi_r=\mathbf{0}$,

$$(M\text{-}M_0)^{\mathrm{T}}\pi_r=0,$$

i.e.,

$$M^{\mathrm{T}}\pi_r=M_0^{\mathrm{T}}\pi_r.$$

Hence,

$$\Sigma_{p\in\|\pi_r\|}\pi_r(p)M(p)=\Sigma_{p\in\|\pi_r\|}\pi_r(p)M_0(p),$$

i.e.,

$$\pi_r(r)M(r)+\Sigma_{p\in\Pi_r}\pi_r(p)M(p)=\pi_r(r)M_0(r)+\Sigma_{p\in\Pi_r}\pi_r(p)M_0(p).$$

By $\pi_r(r)=1$,

$$M(r)+\Sigma_{p\in\Pi_r}\pi_r(p)M(p)=M_0(r)+\Sigma_{p\in\Pi_r}\pi_r(p)M_0(p).$$

By $\Pi_r\subseteq P_A$ and $\forall p\in P_A$: $M_0(p)=0$, we have that:

$$\Sigma_{p\in\Pi_r}\pi_r(p)M_0(p)=0,$$

i.e.,

$$M(r)+\Sigma_{p\in\Pi_r}\pi_r(p)M(p)=M_0(r). \tag{A1}$$

Let $P_r=\{p|\ p\in\Pi_r,\ p\notin\Gamma\}$. (A1) can be written as

$$M(r)+\Sigma_{p\in P_r}\pi_r(p)M(p)+\Sigma_{p\in\Pi_r\cap\Gamma}\pi_r(p)M(p)=M_0(r).$$

According to Definition 8,

$$\Sigma_{p\in\Pi_r\cap\Gamma}\pi_r(p)M(p)=M_0(r). \tag{A2}$$

Hence,

$$M(r)+\Sigma_{p\in P_r}\pi_r(p)M(p)=0.$$

Since $M(r)\geq 0$ and $\Sigma_{p\in P_r}\pi_r(p)M(p)\geq 0$, $M(r)=0$. ■

Theorem A1: If $\tilde{P}_R$ is a CRS at M, then $\forall p\in P_M$ and $\forall t\in p^{\bullet}$, t is not enabled at $\forall M'\in R(M)$.

Proof: Suppose that $\exists t_k\in T$: $M[t_k\rangle M_z$. If $\exists r_u\in\tilde{P}_R$: $r_u\in t_k^{\bullet}$, then $\exists p_v\in\|\pi_u\|$: $p_v\in P_M$ and $p_v\in{}^{\bullet}t_k$. According to Condition 2 of Definition 8, $\exists r_j\in\tilde{P}_R$: $r_j\in{}^{\bullet}t_k$. According to Theorem 2, $M(r_j)=0$. Hence, t_k is not enabled at M. It contradicts that $M[t_k\rangle M_z$. Thus,

$$\forall r\in\tilde{P}_R:\ r\notin t_k^{\bullet}. \tag{A3}$$

From Lemma A1, $\forall r\in\tilde{P}_R$: $M(r)=0$. Hence,

$$\forall r\in\tilde{P}_R:\ r\notin{}^{\bullet}t_k. \tag{A4}$$

By (A3) and (A4), $M_z(r)=M(r)=0$. Through the above analysis,

$$\forall M'\in R(M)\text{ and }\forall r\in\tilde{P}_R:\ M'(r)=M(r)=0. \tag{A5}$$

According to Condition 2 of Definition 8, $\forall p\in P_M$ and $\forall t\in p^{\bullet}$, $\exists r'\in\tilde{P}_R$: $r'\in{}^{\bullet}t$. By (A5), we obtain that $M'(r')=0$. Hence, t is not enabled at $\forall M'\in R(M)$. ■

Proof of Theorem 3:

Sufficiency:

Suppose that $\tilde{P}_R$ is a CRS at M. According to Theorem A1, $\forall p\in P_M$ and $\forall t\in p^{\bullet}$, t is not enabled at $\forall M'\in R(M)$. According to Definition 6, M is a partial deadlock.

Necessity:

Suppose that M is a partial deadlock and there is no CRS at M. Since M is a partial deadlock, $\forall p\in P_M$ and $\forall t\in p^{\bullet}$ such that t is not enabled at $M'\in R(M)$. By $p\in P_M$, $M(p)\neq 0$. Since $\forall t\in p^{\bullet}$ is not enabled at $M'\in R(M)$, $\exists r\in{}^{\bullet}t$: $M(r)=0$. Let

$$\tilde{P}_R=\{r|\ r\in{}^{\bullet}(p^{\bullet}),\ p\in P_M,\ M(r)=0\}. \tag{A6}$$

Hence,

$$\forall r\in\tilde{P}_R,\ \exists p_k\in P_M:\ p_k\in{}^{\bullet}(r^{\bullet}).$$

By $P_M=\{p|\ p\in P_A,\ M(p)\neq 0\}$, we have that:

$$\forall r\in\tilde{P}_R,\ \exists p\in P_A:\ p\in{}^{\bullet}(r^{\bullet}).$$

If $\Gamma\neq P_M$, $\exists p'\in\Gamma$: $p'\notin P_M$. By $\Gamma=\cup_{r'\in\tilde{P}_R}(P_A\cap{}^{\bullet}(r'^{\bullet}))$, we have that:

$$\exists p'\in P_A\cap{}^{\bullet}(r'^{\bullet}):\ p'\notin P_M,$$

where $r'\in\tilde{P}_R$. Let $t'\in r'^{\bullet}$. Then, we obtain that:

$$\exists p'\in P_A\cap{}^{\bullet}t':\ p'\notin P_M.$$

Since $({}^{\bullet}t')\cap P_A=\{p'\}$, i.e.,

$$P_A\cap{}^{\bullet}(r'^{\bullet})=\{p'\}.$$

By $P_M\subseteq P_A$ and $p'\notin P_M$,

$$P_M\cap{}^{\bullet}(r'^{\bullet})=\varnothing.$$

Hence, $\forall p\in P_M$, $p\notin{}^{\bullet}(r'^{\bullet})$, i.e., $r'\notin{}^{\bullet}(p^{\bullet})$. By (A6), we have that $r'\notin\tilde{P}_R$. This contradicts that $r'\in\tilde{P}_R$. Hence, $\Gamma=P_M$. Since $\forall r\in\tilde{P}_R$: $M(r)=0$, by (A1), we obtain that:

$$\Sigma_{p_z\in\Pi_r\setminus\Gamma}\pi_r(p_z)M(p_z)+\Sigma_{p\in\Pi_r\cap\Gamma}\pi_r(p)M(p)=M_0(r). \tag{A7}$$

By $\Gamma=P_M$,

$$\Sigma_{p_j\notin\Gamma}\pi_r(p_j)M(p_j)=0.$$

Thus,

$$\Sigma_{p_z\in\Pi_r\setminus\Gamma}\pi_r(p_z)M(p_z)=0.$$

By (A7),

$$\Sigma_{p\in\Pi_r\cap\Gamma}\pi_r(p)M(p)=M_0(r).$$

Hence, Condition 1 of Definition 8 holds.

Since $\forall p\in P_M$ and $\forall t\in p^{\bullet}$, $\exists r\in{}^{\bullet}t$: $M(r)=0$, by (A6), we have that:

$$\forall p \in P_M \text{ and } \forall t \in p^{\bullet}, \exists r \in \widetilde{P}_R: r \in {}^{\bullet}t.$$

Thus, Condition 2 of Definition 8 holds. We then have that $\widetilde{P}_R$ is a CRS at M. ■

Proof of Lemma 1:

Suppose that $r \in \widetilde{P}_R$ and $p \in \Pi_r$. According to Theorem 2, $M(r)=0$. According to (A1),

$$\Sigma_{p\in\Pi_r}\pi_r(p)M(p)=M_0(r). \quad (A8)$$

Then, (A8) can be written as

$$\Sigma_{p\in\{p|\, p\in\Pi_r,\, p\notin\Gamma\}}\pi_r(p)M(p)+\Sigma_{p\in\Pi_r\cap\Gamma}\pi_r(p)M(p)=M_0(r).$$

According to (A2),

$$\Sigma_{p\in\{p|\, p\in\Pi_r,\, p\notin\Gamma\}}\pi_r(p)M(p)=0.$$

Thus, $\forall p\in\Pi_r$ and $p\notin\Gamma$, $M(p)=0$. ■

Proof of Theorem 4:

Suppose $M\in M_D$, $M\langle t]M'$, and $M'[t'\rangle M_i$, where $t'\in p^{\bullet}$ and $p\in P_{M'}$. Since $M_j\in M_D$, M_j is a partial deadlock. Hence, $\forall p'\in P_{M_j}$ such that $\forall t\in p'^{\bullet}$ is not enabled at $M_j'\in R(M_j)$. Since $\forall p\in P_{M_j}$: $M_j(p)=M_i(p)$, t is not enabled at $M_i'\in R(M_i)$. We then have that M' is a bad marking. ■

Proof of Theorem 5:

Suppose that $\widetilde{P}_R$ is a CRS at M_1, M_2, ..., and M_x. $\forall p\in \Pi_{r'}$, since $P_M=P_{M1}\cup P_{M2}\cup\ldots\cup P_{Mx}$, if $p\notin P_M$, $M_i(p)=M_j(p)=0$. Since $\forall r\in P_R$ and $\forall p\in\|\pi_r\|$: $\pi_r(p)=1$,

$$\Sigma_{p\in\Pi_{r'}\cap P_M}M_i(p)=\Sigma_{p\in\Pi_{r'}\cap P_M}M_j(p)=M_0(r'). \quad (A9)$$

Since $M_i\neq M_j$, $\exists p_u\in\Pi_{r'}\cap P_M$:

$$M_i(p_u)\neq M_j(p_u).$$

Let $P'=\{p_z|\ p_z\in\Pi_{r'}\cap P_M,\ M_i(p_z)=M_j(p_z)\}$ and $P_{V_{r'}}=\{p_y|\ p_y\in\Pi_{r'}\cap P_M,\ M_i(p_y)\neq M_j(p_y)\}$. By (A9), we obtain that:

$$\Sigma_{p\in\Pi_{r'}\cap P_M}M_i(p)=\Sigma_{p_y\in P_{V_{r'}}}M_i(p_y)+\Sigma_{p_z\in P'}M_i(p_z)=M_0(r') \text{ and}$$
$$\Sigma_{p\in\Pi_{r'}\cap P_M}M_j(p)=\Sigma_{p_y\in P_{V_{r'}}}M_j(p_y)+\Sigma_{p_z\in P'}M_j(p_z)=M_0(r'),$$

i.e.,

$$\Sigma_{p_y\in P_{V_{r'}}}M_i(p_y)=M_0(r')\text{-}\Sigma_{p_z\in P'}M_i(p_z) \text{ and}$$
$$\Sigma_{p_y\in P_{V_{r'}}}M_j(p_y)=M_0(r')\text{-}\Sigma_{p_z\in P'}M_j(p_z).$$

Since $\forall p_z\in P'$: $M_i(p_z)=M_j(p_z)$,

$$\Sigma_{p_z\in P'}M_i(p_z)=\Sigma_{p_z\in P'}M_j(p_z).$$

Thus,

$$\Sigma_{p_y\in P_{V_{r'}}}M_i(p_y)=\Sigma_{p_y\in P_{V_{r'}}}M_j(p_y)=M_0(r)\text{-}\Sigma_{p_z\in P'}M_i(p_z).$$

By (2), we obtain that:

$$\Sigma_{p_y\in P_{V_{r'}}}M(p_y)=\Sigma_{p_y\in P_{V_{r'}}}M_i(p_y)=M_0(r)\text{-}\Sigma_{p_z\in P'}M_i(p_z).$$

Since $M_0(r)$ and $\Sigma_{p_z\in P'}M_i(p_z)$ are constants, $\Sigma_{p_y\in P_{V_{r'}}}M(p_y)$ is a constant, i.e., C is a constant. By (1), we have that $\forall p_k\in P_{V_{r'}}$: $M(p_k)=a_k'$. Since a_k' is a variable, so is $M(p_k)$. By $\Pi_r\cap P_M\subseteq P_A$, $P_{V_{r'}}\subseteq P_A$. According to Definition 9, M is an ATI-marking on $P_{V_{r'}}$. For $\forall M_i\in\{M_1, M_2, ..., M_x\}$ and $\forall p_x\in P_M$,

a) if $p_x\in P_{V_{r'}}$, (2) holds; otherwise

b) $M(p_x)=M_i(p_x)$.

By $P_M=P_{M1}\cup P_{M2}\cup\ldots\cup P_{Mx}$, $P_{M_i}\subseteq P_M$. Thus, M_i can be represented by M. ■

Proof of Corollary 1:

Since M is an ATI-partial deadlock of M_1, M_2, ..., and M_x, M is an ATI-marking on P_V. According to Theorem 5, M_i can be represented by M, where $i\in\mathbb{N}_x^+$. Hence,

1) $\forall p_x\in P_M\backslash P_V$, $M(p_x)=M_i(p_x)$; and
2) $\Sigma_{p\in P_V}M(p)=\Sigma_{p\in P_V}M'(p)$.

According to Definition 11, $M\cong M_i$. ■

Proof of Theorem 6:

Suppose $M\in M_A$, $M\langle t]M'$, and $\forall M_i\in R(M')$, $\exists M_j\in M_A$: $M_j\cong M_i$. Since $M_j\in M_A$, M_j is an ATI-partial deadlock on P_V, where $\widetilde{P}_R$ is a CRS at M_j. Since $M_j\cong M_i$, according to Definition 11,

1) $\forall p_x\in P_{M_j}\backslash P_V$, $M_i(p_x)=M_j(p_x)$; and
2) $\Sigma_{p_y\in P_V}M_i(p_y)=\Sigma_{p_y\in P_V}M_j(p_y)$.

Hence, $\forall p'\in P_{M_j}$: $M_i(p')\leq M_j(p')$ and $\forall r'\in P_R$: $M_i(r')\leq M_j(r')$. It means that $\forall p'\in P_{M_j}$ occupies fewer resources at M_j than M_i. Since M_j is an ATI-partial deadlock, $\forall p'\in P_{M_j}$ such that $\forall t'\in p'^{\bullet}$ is not enabled at $\forall M_j'\in R(M_j)$. Thus, t' is not enabled at $\forall M_i'\in R(M_i)$. Hence, M' is a bad marking. ■

Proof of Theorem 7:

Suppose that t is dead at M. We then have that there is no marking $M'\in R(M)$ such that t is enabled at M'. If M is neither a partial deadlock nor a bad marking, according to Definition 6, $\exists p\in P_M$ and $\exists t'\in p^{\bullet}$ such that t' is enabled at $M'\in R(M)$. If $\forall p'\in P_M$ and $\forall t''\in p'^{\bullet}$ such that t'' is not enabled at M, then only the output transitions of $p_0\in P_0$ can be fired, which will further reduce the amount of idle resources in the PN, i.e., $\forall M''\in R(M)$ and $\forall r\in P_R$: $M''(r)\leq M(r)$. Notice that the idle resource at M are already insufficient to enable t''. Hence, at any subsequent marking $M''\in R(M)$, the number of tokens in resource places remains insufficient to enable t''. This implies that t'' can never be enabled at $\forall M''\in R(M)$. This contradicts that $\exists p\in P_M$ and $\exists t'\in p^{\bullet}$ such that t' is enabled at $M'\in R(M)$. Hence, t' is enabled at M, and the resulting marking by firing t' is neither a partial deadlock nor a bad marking. By repeating this analysis on each newly generated marking, we can ultimately conclude that all tokens in activity places at M can be returned to idle places, i.e., $M_0\in R(M)$. Since any transition is not dead at initial marking, $\exists M_i\in R(M_0)$: $M_i[t\rangle$. Since $M_0\in R(M)$, we have that $\exists M_i\in R(M)$: $M_i[t\rangle$. This contradicts that t is dead at M. Hence, if t is dead at M, M must be a partial deadlock or a bad marking. ■

Proof of Theorem 8:

Suppose that p_c is a control place for M. According to Definition 14,

$$\|\pi_{p_c}\|\backslash\{p_c\}=\{p|\ p\in\Pi_r\cap P_M,\ r\in\widetilde{P}_R\}, \quad (A10)$$
$$M_0'(p_c)=\Sigma_{r\in\widetilde{P}_R}M_0(r)\text{-}1, \quad (A11)$$

and $\forall p_i\in\|\pi_{p_c}\|\cap P_A$:

$$\pi_{p_c}(p_i)=\Sigma_{r\in P_{R\_p_i}}\pi_r(p_i), \quad (A12)$$

where $P_{R\_p_i}=\{r|\ r\in\widetilde{P}_R,\ p_i\in\|\pi_r\|\}$. Since $\forall M'\in R(M_0')$:

$$M_0'(p_c)=M'(p_c)+\Sigma_{p_i\in\|\pi_{p_c}\|\setminus\{p_c\}}\pi_{p_c}(p_i)M'(p_i),$$

by (A10),

$$M_0'(p_c)=M'(p_c)+\Sigma_{p_i\in\{p_i|\ p_i\in\Pi_r\cap P_M,\ r\in\widetilde{P}_R\}}\pi_{p_c}(p_i)M'(p_i).$$

By (A11) and (A12),

$$M'(p_c)+\Sigma_{p_i\in\{p_i|\ p_i\in\Pi_r\cap P_M,\ r\in\widetilde{P}_R\}}(M'(p_i)\Sigma_{r\in P_{R\_pi}}\pi_r(p_i))<\Sigma_{r\in\widetilde{P}_R}M_0(r).$$

Since $M'(p_c)\geq 0$,

$$\Sigma_{p_i\in\{p_i|\ p_i\in\Pi_r\cap P_M,\ r\in\widetilde{P}_R\}}(M'(p_i)\Sigma_{r\in P_{R\_pi}}\pi_r(p_i))<\Sigma_{r\in\widetilde{P}_R}M_0(r).\quad\text{(A13)}$$

$\forall r\in\widetilde{P}_R$,

$$M_0(r)=M(r)+\Sigma_{p\in\Pi_r}\pi_r(p)M(p)$$
$$=M(r)+\Sigma_{p'\in\Pi_r\cap P_M}\pi_r(p')M(p')+\Sigma_{p''\in\Pi_r\setminus P_M}\pi_r(p'')M(p'').$$

Since $\forall p''\in\Pi_r\setminus P_M$: $M(p'')=0$,

$$M_0(r)=M(r)+\Sigma_{p'\in\Pi_r\cap P_M}\pi_r(p')M(p').$$

Since M is a (ATI-) partial deadlock, according to Theorems 2 and 5, $M(r)=0$. Hence,

$$M_0(r)=\Sigma_{p'\in\Pi_r\cap P_M}\pi_r(p')M(p').$$

Hence,

$$\Sigma_{r\in\widetilde{P}_R}M_0(r)=\Sigma_{r\in\widetilde{P}_R}\Sigma_{p'\in\Pi_r\cap P_M}\pi_r(p')M(p')$$
$$=\Sigma_{p'\in P_M}(M(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p')).\quad\text{(A14)}$$

By (A13),

$$\Sigma_{p_i\in\{p_i|\ p_i\in\Pi_r\cap P_M,\ r\in\widetilde{P}_R\}}(M'(p_i)\Sigma_{r\in P_{R\_pi}}\pi_r(p_i))$$
$$<\Sigma_{p'\in P_M}(M(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p')).\quad\text{(A15)}$$

$\forall p\in P_M$, if $\forall r\in\widetilde{P}_R$: $p\notin\Pi_r$, according to (A10), $p\notin\|\pi_{p_c}\|\setminus\{p_c\}$, i.e., $\pi_{p_c}(p)=0$. Hence, $\forall p_k\in\{p_k|\ p_k\in P_M\setminus\Pi_r,\ r\in\widetilde{P}_R\}$, we have that $\pi_{p_c}(p_k)=0$, i.e.,

$$\Sigma_{p_j\in\{p_j|\ p_j\in P_M\setminus\Pi_r,\ r\in\widetilde{P}_R\}}\pi_{p_c}(p_j)M'(p_j)=0.\quad\text{(A16)}$$

Since

$$\Sigma_{p'\in P_M}\pi_{p_c}(p')M'(p')=\Sigma_{p_j\in\{p_j|\ p_j\in P_M\setminus\Pi_r,\ r\in\widetilde{P}_R\}}\pi_{p_c}(p_j)M'(p_j)+$$
$$\Sigma_{p_i\in\{p_i|\ p_i\in\Pi_r\cap P_M,\ r\in\widetilde{P}_R\}}\pi_{p_c}(p_i)M'(p_i),$$

by (A16),

$$\Sigma_{p'\in P_M}\pi_{p_c}(p')M'(p')=\Sigma_{p_i\in\{p_i|\ p_i\in\Pi_r\cap P_M,\ r\in\widetilde{P}_R\}}\pi_{p_c}(p_i)M'(p_i).$$

By (A12),

$$\Sigma_{p'\in P_M}(M'(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p'))$$
$$=\Sigma_{p_i\in\{p_i|\ p_i\in\Pi_r\cap P_M,\ r\in\widetilde{P}_R\}}(M'(p_i)\Sigma_{r\in P_{R\_pi}}\pi_r(p_i)).\quad\text{(A17)}$$

By (A15) and (A17),

$$\Sigma_{p'\in P_M}(M'(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p'))<\Sigma_{p'\in P_M}(M(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p')),$$

i.e.,

$$\Sigma_{p'\in P_M}((M'(p')-M(p'))\Sigma_{r\in P_{R\_p'}}\pi_r(p'))<0.\quad\text{(A18)}$$

Suppose that $\forall p'\in P_M$: $M'(p)\geq M(p)$. We then have that:

$$M'(p')-M(p')\geq 0.$$

Since $\Sigma_{r\in P_{R\_p'}}\pi_r(p')\geq 0$,

$$\Sigma_{p'\in P_M}((M'(p')-M(p'))\Sigma_{r\in P_{R\_p'}}\pi_r(p'))\geq 0.$$

This contradicts (A18). Hence, $\exists p\in P_M$:

$$M'(p)<M(p).$$ ■

Proof of Theorem 9:

Suppose that p_c is a control place for M. According to Definition 15,

$$\|\pi_{p_c}\|\setminus\{p_c\}=P_M=\{p|\ p\in P_A,\ M(p)\neq 0\},\quad\text{(A19)}$$
$$M_0'(p_c)=\Sigma_{r\in P_R}(M_0(r)-M(r))-1,\quad\text{(A20)}$$

and $\forall p'\in\|\pi_{p_c}\|\cap P_A$:

$$\pi_{p_c}(p')=\Sigma_{r\in P_{R\_p'}}\pi_r(p'),\quad\text{(A21)}$$

where $P_{R\_p}=\{r|\ r\in P_R,\ p'\in\|\pi_r\|\}$. $\forall M'\in R(M_0')$, we have that:

$$M_0'(p_c)=M'(p_c)+\Sigma_{p'\in\|\pi_{p_c}\|\setminus\{p_c\}}\pi_{p_c}(p')M'(p').$$

By (A19), we obtain that:

$$M_0'(p_c)=M'(p_c)+\Sigma_{p'\in P_M}\pi_{p_c}(p')M'(p').$$

By (A21),

$$M_0'(p_c)=M'(p_c)+\Sigma_{p'\in P_M}(M'(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p')).\quad\text{(A22)}$$

By (A20) and (A22),

$$M'(p_c)+\Sigma_{p'\in P_M}(M'(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p'))<\Sigma_{r\in P_R}(M_0(r)-M(r)).$$

Since $M'(p_c)\geq 0$,

$$\Sigma_{p'\in P_M}(M'(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p'))<\Sigma_{r\in P_R}(M_0(r)-M(r)).\quad\text{(A23)}$$

$\forall r\in P_R$,

$$M_0(r)=M(r)+\Sigma_{p'\in\Pi_r}\pi_r(p')M(p').$$

Since $\forall p_i\in\Pi_r\setminus P_M$, $M(p_i)=0$, we have that:

$$M_0(r)=M(r)+\Sigma_{p'\in\Pi_r\cap P_M}\pi_r(p')M(p'),$$

i.e.,

$$M_0(r)-M(r)=\Sigma_{p'\in\Pi_r\cap P_M}\pi_r(p')M(p').$$

Hence,

$$\Sigma_{r\in P_R}(M_0(r)-M(r))=\Sigma_{r\in P_R}\Sigma_{p'\in\Pi_r\cap P_M}\pi_r(p')M(p')$$
$$=\Sigma_{p'\in P_M}(M(p')\Sigma_{r\in P_{R\_pi}}\pi_r(p_i)).\quad\text{(A24)}$$

By (A23),

$$\Sigma_{p'\in P_M}(M'(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p'))<\Sigma_{p'\in P_M}(M(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p')),$$

i.e.,

$$\Sigma_{p'\in P_M}((M'(p')-M(p'))\Sigma_{r\in P_{R\_p'}}\pi_r(p'))<0.\quad\text{(A25)}$$

Suppose that $\forall p'\in P_M$: $M'(p)\geq M(p)$. We then have that:

$$M'(p')-M(p')\geq 0.$$

Since $\Sigma_{r\in P_{R\_p'}}\pi_r(p'))\geq 0$,

$$\Sigma_{p'\in P_M}((M'(p')-M(p'))\Sigma_{r\in P_{R\_p'}}\pi_r(p'))\geq 0.$$

This contradicts (A25). Hence, $\exists p\in P_M$:

$$M'(p)<M(p).$$ ■

Proof of Theorem 10:

Since p_c is a control place for M_i, according to Theorems 8 and 9, $\forall M'\in R(M_0')$, $\exists p'\in P_{Mi}$: $M'(p')<M_i(p')$. Since $\forall p\in P_{Mi}$: $M_i(p)\leq M_j(p)$, we have that $\forall M'\in R(M_0')$, $\exists p'\in P_{Mi}$: $M'(p')<M_j(p')$. Thus, $M_j\notin R(M_0')$. ■

Proof of Theorem 11:

Suppose that $M_i\cong M_j$. Let $P_{Mi}=P_{Mi}'\cup P_{Mi}''$, where $P_{Mi}'=\{p'|\ p'\in P_{Mi},\ M_i(p')$ is a constant$\}$. According to Definition 11, M_i is an ATI-marking and $\forall p_x\in P_{Mi}'$:

$$M_i(p_x)=M_j(p_x).\quad\text{(A26)}$$

Since p_c is a control place for M_i, according to Theorem 8, $\forall M'\in R(M_0')$, $\exists p\in P_{Mi}$:

$$M'(p)<M_i(p).\quad\text{(A27)}$$

Case 1):

Suppose that $p\in P_{Mi}'$. By (A26) and (A27), $\exists p\in P_{Mj}$: $M'(p)<M_j(p)$. Hence, $M_j\notin R(M_0')$.

Case 2):

Suppose that $p\notin P_{Mi}'$, i.e., $\forall p_k\in P_{Mi}'$:

$$M'(p_k)\geq M_i(p_k).\quad\text{(A28)}$$

$\forall M'\in R(M_0')$ and $\forall r\in\widetilde{P}_R$, we have that:

$$\Sigma_{p\in\Pi_r}M'(p)\leq M_0(r).$$

Let $\Pi_r=\Pi_{rc}\cup\Pi_{rv}$, where $\Pi_{rc}=\{p|\ p\in\Pi_r,\ M_i(p)$ is a constant$\}$

and $\Pi_{rv}=\{p|\ p\in\Pi_r,\ M_i(p)$ is a variable$\}$. Since $\forall r\in P_R$ and $\forall p\in\|\pi_r\|$: $\pi_r(p)=1$,

$$\Sigma_{p_k\in\Pi_{rc}}M_i(p_k)+\Sigma_{p_z\in\Pi_{rv}}M_i(p_z)=M_0(r) \tag{A29}$$

and

$$\Sigma_{p_k\in\Pi_{rc}}M'(p_k)+\Sigma_{p_z\in\Pi_{rv}}M'(p_z)\le M_0(r). \tag{A30}$$

Since $P_{Mi}'=\{p'|\ p'\in P_{Mi},\ M_i(p')$ is a constant$\}$, we obtain that $\Pi_{rc}\subseteq P_{Mi}$. By (A28), $\forall p_k\in\Pi_{rc}$: $M'(p_k)\ge M_i(p_k)$. Hence,

$$\Sigma_{p_k\in\Pi_{rc}}M'(p_k)\ge\Sigma_{p_k\in\Pi_{rc}}M_i(p_k). \tag{A31}$$

By (A30),

$$\Sigma_{p_k\in\Pi_{rc}}M_i(p_k)+\Sigma_{p_j\in\Pi_{rv}}M'(p_j)\le M_0(r). \tag{A32}$$

By (A29),

$$\Sigma_{p_k\in\Pi_{rc}}M_i(p_k)=M_0(r)-\Sigma_{p_z\in\Pi_{rv}}M_i(p_z).$$

By (A32),

$$M_0(r)-\Sigma_{p_z\in\Pi_{rv}}M_i(p_z)+\Sigma_{p_j\in\Pi_{rv}}M'(p_j)\le M_0(r).$$

i.e.,

$$\Sigma_{p_z\in\Pi_{rv}}M'(p_z)\le\Sigma_{p_z\in\Pi_{rv}}M_i(p_z). \tag{A33}$$

If $\forall r\in\widetilde{P}_R$: $\Sigma_{p\in P_{Mi}\cap\|\pi_r\|}M_i(p)=\Sigma_{p\in P_{Mi}\cap\|\pi_r\|}M'(p)$, we have that:

$$\Sigma_{r\in\widetilde{P}_R}M_0(r)=\Sigma_{r\in\widetilde{P}_R}\Sigma_{p\in P_{Mi}\cap\|\pi_r\|}M_i(p)=\Sigma_{r\in\widetilde{P}_R}\Sigma_{p\in P_{Mi}\cap\|\pi_r\|}M'(p).$$

Hence,

$$\Sigma_{r\in\widetilde{P}_R}M_0(r)=\Sigma_{r\in\widetilde{P}_R}\Sigma_{p\in\{p|\ p\in\Pi_r\cap P_M,\ r\in\widetilde{P}_R\}}M'(p),$$

i.e,

$$\Sigma_{r\in\widetilde{P}_R}M_0(r)=\Sigma_{p\in\{p|\ p\in\Pi_r\cap P_M,\ r\in\widetilde{P}_R\}}(M'(p)\Sigma_{r\in P_{R\_p}}\pi_r(p)), \tag{A34}$$

where $P_{R\_p}=\{r|\ r\in\widetilde{P}_R,\ p\in\|\pi_r\|\}$. By (A13),

$$\Sigma_{p\in\{p|\ p\in\Pi_r\cap P_M,\ r\in\widetilde{P}_R\}}(M'(p)\Sigma_{r\in P_{R\_p}}\pi_r(p))<\Sigma_{r\in\widetilde{P}_R}M_0(r).$$

This contradicts (A34). Hence, $\exists r'\in\widetilde{P}_R$:

$$\Sigma_{p\in P_{Mi}\cap\|\pi_{r'}\|}M_i(p)\ne\Sigma_{p\in P_{Mi}\cap\|\pi_{r'}\|}M'(p),$$

i.e.,

$$\Sigma_{p_k\in\Pi_{r'c}}M_i(p_k)+\Sigma_{p_z\in\Pi_{r'v}}M_i(p_z)\ne\Sigma_{p_k\in\Pi_{r'c}}M'(p_k)+\Sigma_{p_z\in\Pi_{r'v}}M'(p_z).$$

By (A29) and (A30),

$$\Sigma_{p_k\in\Pi_{r'c}}M'(p_k)+\Sigma_{p_z\in\Pi_{r'v}}M'(p_z)<\Sigma_{p_k\in\Pi_{r'c}}M_i(p_k)+\Sigma_{p_z\in\Pi_{r'v}}M_i(p_z),$$

i.e.,

$$\Sigma_{p_k\in\Pi_{r'c}}M'(p_k)-\Sigma_{p_k\in\Pi_{r'c}}M_i(p_k)<\Sigma_{p_z\in\Pi_{r'v}}M_i(p_z)-\Sigma_{p_z\in\Pi_{r'v}}M'(p_z).$$

By (A31), $0<\Sigma_{p_z\in\Pi_{r'v}}M_i(p_z)-\Sigma_{p_z\in\Pi_{r'v}}M'(p_z)$, i.e.,

$$\Sigma_{p_z\in\Pi_{r'v}}M'(p_z)<\Sigma_{p_z\in\Pi_{r'v}}M_i(p_z).$$

Since $M_i\cong M_j$, according to Definition 11,

$$\Sigma_{p_z\in\Pi_{r'v}}M_i(p_z)=\Sigma_{p_z\in\Pi_{r'v}}M_j(p_z).$$

Hence,

$$\Sigma_{p_z\in\Pi_{r'v}}M'(p_z)<\Sigma_{p_z\in\Pi_{r'v}}M_j(p_z).$$

Thus, $M_j\notin R(M_0')$. ■

Proof of Corollary 2:

Since M is an ATI-partial deadlock of M_1, M_2, ..., and M_x, according to Corollary 1, $M\cong M_i$, where $i\in\mathbb{N}_x^+$. Since p_c is a control place for M, $M_i\notin R(M_0')$ according to Theorem 11. ■

Proof of Theorem 12:

Since $M_i[t\rangle M_k$, $\forall p_u\in{}^\bullet t$: $M(p_u)\ne0$. If t is not enabled at M_j, since $\forall p\in P$:

$$M_i(p)=M_j(p),$$

$\exists p_c\in P_C$:

$$p_c\in{}^\bullet t \text{ and } M_j(p_c)<W(p_c,\ t).$$

Suppose that p_c is a control place of M. Since $M_j\in R(M_0')$, we then have that:

$$M_0'(p_c)-M_j(p_c)=\Sigma_{p'\in\|\pi_{p_c}\|\setminus\{p_c\}}\pi_{p_c}(p')M_j(p'),$$

If M is a (ATI-) partial deadlock, by (A10) and (A12),

$$M_0'(p_c)-M_j(p_c)=\Sigma_{p'\in\{p'|\ p'\in\Pi_r\cap P_M,\ r\in\widetilde{P}_R\}}(M_j(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p')).$$

By (A17),

$$M_0'(p_c)-M_j(p_c)=\Sigma_{p'\in P_M}(M'(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p')).$$

If M is a bad marking, by (A22), we can also obtain that:

$$M_0'(p_c)-M_j(p_c)=\Sigma_{p'\in P_M}(M_j(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p')).$$

Notice that if M is a (ATI-) partial deadlock, $P_{R\_p'}=\{r|\ r\in\widetilde{P}_R,\ p'\in\|\pi_r\|\}$; otherwise, if M is a bad marking, $P_{R\_p'}=\{r|\ r\in P_R,\ p'\in|\pi_r|\}$. Since $M_j(p_c)<W(p_c,\ t)$, we have that:

$$\Sigma_{p'\in P_M}(M_j(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p'))>M_0'(p_c)-W(p_c,\ t).$$

Since $\forall p\in P$: $M_i(p)=M_j(p)$,

$$\Sigma_{p'\in P_M}(M_j(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p'))=\Sigma_{p'\in P_M}(M_i(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p')),$$

i.e.,

$$\Sigma_{p'\in P_M}(M_i(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p'))>M_0'(p_c)-W(p_c,\ t). \tag{A35}$$

Let $\{p_v\}={}^\bullet t\cap P_M$ and $\{p_u\}=t^\bullet\cap P_M$. Since $M_i[t\rangle M_k$,

$$M_k(p_v)=M_i(p_v)-1,\ M_k(p_u)=M_i(p_u)+1, \text{ and}$$
$$\forall p\in P_M\setminus\{p_u,\ p_v\}:\ M_k(p_v)=M_i(p_v).$$

Since

$$\Sigma_{p'\in P_M}(M_k(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p'))=\Sigma_{p\in P_M\setminus\{p_u,\ p_v\}}(M_k(p)\Sigma_{r\in P_{R\_p}}\pi_r(p))+$$
$$M_k(p_u)\Sigma_{r\in P_{R\_pu}}\pi_r(p)+M_k(p_v)\Sigma_{r\in P_{R\_pv}}\pi_r(p),$$

we have that:

$$\Sigma_{p'\in P_M}(M_k(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p'))=\Sigma_{p\in P_M\setminus\{p_u,\ p_v\}}(M_i(p)\Sigma_{r\in P_{R\_p}}\pi_r(p))$$
$$+(M_i(p_v)-1)\Sigma_{r\in P_{R\_pv}}\pi_r(p_v)+(M_i(p_u)+1)\Sigma_{r\in P_{R\_pu}}\pi_r(p_u),$$

i.e.,

$$\Sigma_{p'\in P_M}(M_k(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p'))=\Sigma_{p'\in P_M}(M_i(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p'))$$
$$+\Sigma_{r\in P_{R\_pu}}\pi_r(p_u)-\Sigma_{r\in P_{R\_pv}}\pi_r(p_v).$$

According to Definitions 14 and 15,

$$\Sigma_{r\in P_{R\_pu}}\pi_r(p_u)-\Sigma_{r\in P_{R\_pv}}\pi_r(p_v)=W(p_c,\ t).$$

Hence,

$$\Sigma_{p'\in P_M}(M_k(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p'))=$$
$$\Sigma_{p'\in P_M}(M_i(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p'))+W(p_c,\ t). \tag{A36}$$

By (A35) and A(36),

$$\Sigma_{p'\in P_M}(M_k(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p'))>M_0'(p_c). \tag{A37}$$

If M is a partial deadlock, according to Definition 14,

$$M_0'(p_c)=\Sigma_{r\in\widetilde{P}_R}M_0(r)-1.$$

If M is a bad marking, according to Definition 15,

$$M_0'(p_c)=\Sigma_{r\in P_R}(M_0(r)-M(r))-1.$$

By (A14) and (A24),

$$M_0'(p_c)=\Sigma_{p'\in P_M}(M(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p'))-1.$$

By (A37),

$$\Sigma_{p'\in P_M}(M_k(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p'))>\Sigma_{p'\in P_M}(M(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p'))-1,$$

i.e.,

$$-1<\Sigma_{p'\in P_M}((M_k(p')-M(p'))\Sigma_{r\in P_{R\_p'}}\pi_r(p'). \tag{A38}$$

By (A18) and (A25), $\forall M'\in R(M_0')$:

$$\Sigma_{p'\in P_M}(M'(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p'))<\Sigma_{p'\in P_M}(M(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p')).$$

Since $M_k\in R(M_0')$,

$$\Sigma_{p'\in P_M}(M_k(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p'))<\Sigma_{p'\in P_M}(M(p')\Sigma_{r\in P_{R\_p'}}\pi_r(p')).$$

Hence,

$$\Sigma_{p'\in P_M}((M_k(p')-M(p'))\Sigma_{r\in P_{R\_p'}}\pi_r(p')<0.$$

By (A38),

$$-1<\Sigma_{p'\in P_M}((M_k(p')-M(p'))\Sigma_{r\in P_{R\_p'}}\pi_r(p')<0. \tag{A39}$$

Since $M_k(p')-M(p')\in\mathbb{N}$ and $\Sigma_{r\in P_{R\_p}}\pi_r(p')\in\mathbb{N}$,

$$\Sigma_{p'\in P_M}((M_k(p')-M(p'))\Sigma_{r\in P_{R\_p}}\pi_r(p')\in\mathbb{N}.$$

This contradicts (A39). Hence, t is enabled at M_j. ■

Proof of Theorem 13:

At M_0', $\forall p\in P$: $M_0(p)=M_0'(p)$ and $\forall p_c\in P_C$: $M_0'(p_c)\neq 0$. Hence, $\forall t\in T$, if $M_0[t\rangle M_i$, $M_0'[t\rangle M_i'$ and $\forall p\in P$: $M_i(p)=M_i'(p)$. At M_i, if $M_i[t'\rangle M_j$ and M_j is not a partial deadlock or bad marking, $\forall p'\in {}^\bullet t'$: $M_i(p')\neq 0$. According to Theorem 12, $M_i'[t'\rangle M_j'$, where $\forall p\in P$: $M_j(p)=M_j'(p)$. By analogy, if $M_0[\sigma\rangle M$ and M is not a partial deadlock or bad marking, $M_0'[\sigma\rangle M'$, where $\forall p\in P$: $M(p)=M'(p)$. ■

Proof of Theorem 14:

At M_0', since $\forall p\in P$: $M_0(p)=M_0'(p)$, if $M_0'[t\rangle M_i'$, $M_0[t\rangle M_i$, where $\forall p\in P$: $M_i(p)=M_i'(p)$. Similarly, at M_i', since $\forall p\in P$: $M_i(p)=M_i'(p)$, if $M_i'[t'\rangle M_j'$, $M_i[t'\rangle M_j$ and $\forall p\in P$: $M_j(p)=M_j'(p)$. By analogy, if $M_0'[\sigma\rangle M'$, $M_0[\sigma\rangle M$, where $\forall p\in P$: $M(p)=M'(p)$. ■

Proof of Theorem 15:

Suppose that there is no partial deadlock in N_c. If there is a deadlock M in N_c, according to Definition 3, $\forall t\in T$ is not enabled at M. If $P_M=\varnothing$, $\forall p'\in P_A$: $M(p')=0$. Hence, M is an initial marking. This contradicts that the initial marking is not an deadlock. Thus, $\forall p\in P_M$ satisfies that $P_M\neq\varnothing$ and $\forall t\in p^\bullet$ is not enabled at $M'\in R(M)$. According to Definition 6, M is a partial deadlock. This contradicts that there is no partial deadlock in N_c. Hence, there is no deadlock in N_c. ■

Proof of Theorem 16:

Since N is an S^{3}PR, $\forall(y, x)\in F$: $W(x, y)=1$ and $\forall p\in P_A$: $\Sigma_{r\in P_R}\pi_r(p)=1$. Let M_I be a set of illegal markings of N. $\forall M\in M_I$, if p_c is a control place for M, according to Definitions 14 and 15, a) $\forall t\in T_I\backslash T_O$: $W(p_c, t)=1$; b) $\forall t\in T_O\backslash T_I$: $W(t, p_c)=1$; and c) $\forall t\in T_I\cap T_O$: $W(p_c, t)=W(t, p_c)=0$. Hence, $\forall(y', x')\in F'$: $W(x', y')=1$, i.e., N_c is an ordinary PN. ■

The conditions satisfied by the initial state:

In this work, we assume that initial marking M_0 satisfies: 1) $\forall p\in P_0$: $M_0(p)\geq\sum_{r\in P_R}M_0(r)$; and 2) $\forall p'\in P_A$: $M_0(p')=0$.

In an ordinary S^{4}PR, the idle places can be viewed as buffers for input and output of parts and the number of tokens in idle places can be regarded as the number of parts waiting to be processed. Hence, the first assumption (i.e., $\forall p\in P_0$: $M_0(p)\geq\sum_{r\in P_R}M_0(r)$) means that the number of parts to be processed is greater than the number of resources, which is the most common situation. In addition, the number of partial deadlocks in a system is not only related to the number of resources but also to the number of parts, i.e., the number of tokens in activity places. For example, in the PN in Fig. A1(a), activity places are p_1 and p_5. If the initial marking is $M_0=2p_1+2p_5+r_1+r_2$, we have that there are two tokens in p_1 and p_5, respectively. In this case, there are two deadlocks, i.e., $M_1=p_1+p_2+p_5+p_6$ and $M_2=p_2+p_3+2p_5$. However, if the initial marking is $M_0'=p_1+p_5+r_1+r_2$, there is one token in p_1 and p_5, respectively. In this case, only one deadlock $M_3=p_2+p_6$ exists in this PN. Since M_0 and M_0' have the same number of resources, the number of tokens in activity places affects the number of deadlocks in the system. It means that if only deadlocks reachable from M_0' are controlled, the system will suffer new deadlocks when the number of parts increases. However, when the number of tokens in activity places is not less than the total number of resources, the system no longer generates new deadlocks due to the increase in the number of tokens in activity places. Hence, to avoid this situation, we present the first assumption. Then, we have that the number of tokens in idle places does not affect the firing of transitions. Hence, we do not need to consider the idle places. In this work, we detect partial deadlocks through analyzing the occupation of resources by different activity places. The existence of tokens in activity places at an initial marking means that the activity places occupy resources of the system. Hence, in order to analyze the partial deadlocks, we need to first calculate the resources occupied by activity places. For example, for the PN in Fig. A1(a), suppose that the initial marking $M_0=2p_1+p_5+p_8+r_2$. Although no token exists in r_1 at the initial marking, this does not mean that such resources do not exist in the system. In fact, the resource in r_1 is occupied by p_8. Hence, when analyzing deadlocks of the PN, if we want to know the actual number of resources in the system, it is necessary to first determine the number of resources occupied by activity places. To simplify this process, we assume that the activity places do not occupy any resource at an initial marking. Hence, the second assumption (i.e., $\forall p'\in P_A$: $M_0(p')=0$) is introduced. In addition, the deadlock control process is realized during the system design phase. In this process, the number of resources in the system is known, and there are no parts being processed. Hence, the assumption is appropriate.

Partial Deadlock Detection Algorithm:

Given an ordinary S^{4}PR, for each $P_R'\subseteq P_R$, if it is a CRS at marking M, then P_R' satisfies Conditions 1 and 2 of Definition 7 at M. From Condition 1, we have that $\forall r\in P_R'$: $\sum_{p\in\Pi_r\cap\Gamma}\pi_r(p)M(p)=M_0(r)$, i.e., $\forall r\in P_R'$: $\exists p\in\Pi_r\cap\Gamma$, $M(p)\neq 0$. Hence,

a) $\forall r\in P_R'$: $\Pi_r\cap\Gamma\neq\varnothing$; and

b) $\exists P'\subseteq\cup_{r'\in P_R}(\Gamma\cap\Pi_{r'})$ such that $P'\subseteq P_M$, where $\forall r\in P_R'$: $\Pi_r\cap P'\neq\varnothing$ and $|\Pi_r\cap P'|\leq M_0(r)$.

From a), if $\exists r\in P_R'$: $\Gamma\cap\Pi_r=\varnothing$, P_R' is not a CRS (Lines 1-3). Otherwise, according to b), we need to determine whether

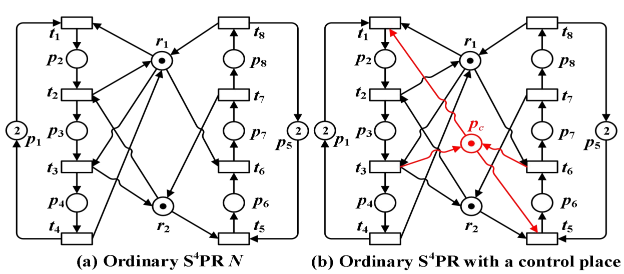

Fig. A1. (a) Ordinary S^{4}PR and (b) ordinary S^{4}PR with a control place.

there exists $P'\subseteq\cup_{r'\in P_R'}(\Gamma\cap\Pi_{r'})$ such that $P'\subseteq P_M$. For each $P'\subseteq\cup_{r\in P_R'}(\Gamma\cap\Pi_r)$, if $P'\subseteq P_M$, according to b),

$$\Pi_r\cap P'\neq\varnothing \text{ and } |\Pi_r\cap P'|\leq M_0(r). \tag{A40}$$

From Condition 2 of Definition 7, P' satisfies that

$$\forall p'\in P' \text{ and } \forall t\in p'^{\bullet}: \exists r\in P_R'\cap{}^{\bullet}t. \tag{A41}$$

Hence, for each $P'\subseteq\cup_{r\in P_R'}(\Gamma\cap\Pi_r)$ that satisfies (A40) and (A41), we obtain $P'\subseteq P_M$. There are the following two cases.

Case 1:

When $P'=P_M$, since P_R' is a CRS, we have that $P_M=P'$ and $\widetilde{P}_R=P_R'$ (Lines 4-6). Since M is a partial deadlock, it satisfies the following conditions:

i) $\forall p_v\in P_A\backslash P_M$, by $P_M=\{p|\ p\in P_A, M(p)\neq 0\}$, we obtain that:

$$M(p_v)=0.$$

ii) $\forall p_v\in P_M$, if $p_v\in\cup_{r\in\widetilde{P}_R}(\Pi_r\cap\Gamma)$,

$$M(p_v)=a_v,$$

where $\Sigma_{v\in\{v|\ p_v\in\Pi_r\cap\Gamma\}}\pi_r(p_v)a_v=M_0(r)$ and $a_v\neq 0$.

iii) $\forall p_v\in P_M$, if $p_v\notin\cup_{r\in\widetilde{P}_R}(\Pi_r\cap\Gamma)$,

$$M(p_v)=a_v',$$

where $P_{Rv}=\{r'|\ p_v\in\Pi_{r'}\}$ and $0<\pi_{r'}(p_v)a_v'\leq\min\{M_0(r')|\ r'\in P_{Rv}\}$.

iv) $\forall p_v\in\widetilde{P}_R$, according to Lemma 1,

$$M(p_v)=0.$$

v) $\forall p_v\in P_R\backslash\widetilde{P}_R$,

$$M(p_v)=M_0(p_v)-\Sigma_{p_z\in\Pi_{p_v}}Y_{p_v}(p_z)M(p_z).$$

vi) $\forall p_v\in P_{0i}$,

$$M(p_v)=M_0(p_v)-\Sigma_{p_z\in P_A\cap P_i}M(p_z).$$

In summary, we have that:

$$M(p_v)=\begin{cases} 0, & if\ p_v\in P_A\backslash P_M \\ a_v, & if\ p_v\in\bigcup_{r\in\widetilde{P}_R}(\Pi_r\cap\Gamma) \\ a_v', & if\ p_v\in P_M\backslash\bigcup_{r\in\widetilde{P}_R}(\Pi_r\cap\Gamma) \\ 0, & if\ p_v\in\widetilde{P}_R \\ M_0(p_v)-\Sigma_{p_z\in\Pi_{p_v}}\pi_{pv}(p_z)M(p_z), & if\ p_v\in P_R\backslash\widetilde{P}_R \\ M_0(p_v)-\Sigma_{p_z\in P_A\cap P_i}M(p_z), & if\ p_v\in P_{0i} \end{cases} \tag{A42}$$

where $P_{Rv}=\{r'|\ p_v\in\Pi_{r'}\}$, $\Sigma_{v\in\{v|\ p_v\in\Pi_r\cap\Gamma\}}\pi_r(p_v)a_v=M_0(r)$, $a_v\neq 0$, and $0<\pi_{r'}(p_v)a_v'\leq\min\{M_0(r')|\ r'\in P_{Rv}\}$. Hence, we can obtain partial deadlocks by solving (A42) (Line 7).

Case 2:

Suppose that $P'=P_M$. By $P'\subseteq P_M$, $\exists p''\notin P'$: $p''\in P_M$, i.e., $\exists P''\subseteq P_A$: $P'\cap P''=\varnothing$ and $P''\subseteq P_M$. Since P_R' is a CRS, according to Condition 2 of Definition 7, we have that P'' satisfies (A41). Hence, we have that $P_M=P'\cup P''$ and $\widetilde{P}_R=P_R'$ (Lines 9 and 10). Then, we can find partial deadlocks by solving (A42) (Line 11).

Our previous work has proposed a polynomial-time algorithm (i.e., Backward Algorithm, BA) to determine the reachability of a marking when there are a finite number of solutions to a state equation [45]. According to Property 2, for each obtained partial deadlock, there are a finite number of NISs X that satisfy its state equation, where $\forall p\in P_0$ and $t_i\in{}^{\bullet}p$, $X(i)=0$. Hence, we can use BA to determine the reachability of each obtained partial deadlock in polynomial time. Finally, all reachable partial deadlocks are detected (Lines 16-21).

Algorithm A1: Partial deadlock detection for ordinary S^4PR

Input: $N=(P_0\cup P_A\cup P_R, T, F, M)$
Output: Set of partial deadlocks M_D and set of CRSs CS
Let $D=M_D=CS=\varnothing$;
1 **For** each $P_R'\subseteq P_R$, **do**
2 **If** $\exists r\in P_R'$: $\Gamma\cap\Pi_r=\varnothing$, **then**
3 Jump to line 1;
4 **Else if** $\forall r\in P_R'$: $\Gamma\cap\Pi_r\neq\varnothing$, **then**
5 **For** each P', where P' is a subset of $\cup_{r\in P_R'}(\Gamma\cap\Pi_r)$ that satisfies (A40) and (A41), **do**
6 Let $P_M=P'$ and $P_R'=\widetilde{P}_R$;
7 Find all partial deadlocks by solving (A42) and put them into D;
8 $CS=CS\cup\{\widetilde{P}_R\}$;
9 **For** each P'', where P'' is a subset of $P_A\backslash P'$ that satisfies (A41), **do**
10 Let $P_M=P_M\cup P''$;
11 Find all partial deadlocks by solving (A42) and put them into D;
12 **End**
13 **End**
14 **End**
15 **End**
16 **For** each $M\in D$, **do**
17 Determine the reachability of M via Backward Algorithm [45];
18 **If** M is reachable, **then**
19 $M_D=M_D\cup\{M\}$;
20 **End**
21 **End**
22 **Return** M_D and CS;
23 **End**

Since there are $|P_R|$ resource places in P_R, there are at most $2^{|P_R|}$-1 subsets of P_R. Hence, lines 2-14 need to be repeated $2^{|P_R|}$-1 times. Suppose that there are x subsets of $\cup_{r\in P_R'}(\Gamma\cap\Pi_r)$ that satisfy (A40) and (A41) and y subsets of $P_A\backslash P'$ that satisfy (A41). We then have that lines 5-13 and lines 9-12 need to be repeated x and y times, respectively. The time complexity of lines 1-15 is $O(2^{|P_R|}xy)$. Notice that x and y depend on the ordinary S^4PR's structure. Since $\cup_{r\in P_R'}(\Gamma\cap\Pi_r)\subseteq P_A$ and $P_A\backslash P'\subseteq P_A$, in theory, x and y are at most $2^{|P_A|}$. In this case, $\cup_{r\in P_R'}(\Gamma\cap\Pi_r)=P_A$ and $P_A\backslash P'=P_A$. When $\cup_{r\in P_R'}(\Gamma\cap\Pi_r)=P_A$, since P_R' is any subset of P_R, $\forall\{r\}\subseteq P_R$ satisfies $\Gamma\cap\Pi_r=P_A$. Hence, $\forall p\in P_A$: $p\in\Gamma\cap\Pi_r$, i.e., $\forall p\in P_A$: $p\in{}^{\bullet}(r^{\bullet})$. Suppose that $p'\in P_A$ and $(p'^{\bullet})^{\bullet}\in P_0$. We then have that $p'\in{}^{\bullet}(r^{\bullet})$. This contradicts the definition of ordinary S^4PR. When $P_A\backslash P'=P_A$, we obtain that $P'=\varnothing$. According to Line 5, lines 5-13 do not need to be performed. Hence, in fact, x and y are much less than $2^{|P_A|}$. Suppose that there are d markings in D. From Lines 7 and 11, d is related to the number of solutions that satisfy the integer linear program (A42). According to (A42), it depends on the number of tokens in resource places and that of elements in each resource place's support. Since the time complexity of BA is T_1, the time complexity of lines 16-21 is $O(dT_1)$, where $T_1=O(3|P||T|(c+1)+((2c+1)|P||T|+(2n_1+1)|P|+2|T|^3)n_1/2)$ and c and n_1 are the number of directed circuits and the sum of the non-zero elements in a solution to a state equation, respectively. The time complexity of Algorithm A1 is $O(2^{|P_R|}xy+dT_1)$.

In the existing work, only reachability tree/graph-based methods can obtain reachable partial deadlocks. Next, we compare the proposed method with those based on reachability tree/graph. When we detect partial deadlocks by traversing a reachability tree/graph, in the worst case, $|T|$-1 transitions can be fired at each marking and generate $|T|$-1 different markings, then the state space grows at a rate of $(|T|\text{-}1)^{(|T|\text{-}1)}$. Hence, its time complexity is $O(|T|^{|T|})$. Thus, when the number of transitions is more than that of resource places and the number of tokens in these resource places, which is usually the case in practical AMS, our approach is more efficient to obtain reachable partial deadlocks and determine whether an ordinary S^{4}PR is deadlock-free.

Bad marking Detection Algorithm (1):

Algorithm A2: Bad Marking Detection (1)

```
Input: An S^4PR and a set of partial deadlocks M_D
Output: A set of bad markings M_B
Let M_I=M_D and M_B=∅;
For each M∈M_I, do
    For each backward-enabled transition t at M, do
        t backward fires at M, where M⟨t]M′;
        For each enabled transition t′∈T_{M′} at M′, where T_{M′}={t′|
          t′∈p•, p∈P_{M′}}, do
            Let I=0 and t fires at M′, where M′[t′⟩M_i;
            For each M_j∈M_I, do
                If ∀p∈P_{M_j}: M_j(p)=M_i(p), then
                    Let I=1;
                    break;
                End
            End
            If I=0, then
                Break;
            End
        End
        If I=1, then
            Let M_I=M_I∪{M′} and M_B=M_B∪{M′};
        End
    End
End
Return M_B;
End
```

Analysis of the time complexity of transition firing:

For an individual marking M, if t fires at M to generate a new marking M', we have that a) $\forall r\in{}^\bullet t\cap P_R$: $M'(r)=M(r)$-1, b) $\forall r'\in t^\bullet\cap P_R$: $M'(r')=M(r')$+1, c) $\forall p\in{}^\bullet t\cap P_A$: $M'(p)=M(p)$-1, and d) $\forall p'\in t^\bullet\cap P_A$: $M'(p')=M(p')$+1. Since $|{}^\bullet t\cap P_R|\leq|P_R|$ and $|t^\bullet\cap P_R|\leq|P_R|$, the total time complexity of a) and b) is $O(2|P_R|)$. Since $|{}^\bullet t\cap P_A|=|t^\bullet\cap P_A|=1$, we have that the total time complexity of c) and d) is $O(2)$. Hence, the time complexity of firing a transition at an individual marking is

$$O(2|P_R|+2)=O(|P_R|).$$

For an ATI-marking M_A, if t fires at M_A generating a new marking M_A', according to Definition 12, we have that

a) $\forall r\in{}^\bullet t\cap P_R$, if $M(r)=a'$ is a variable, where $a'\in\mathbb{N}_z$, then $M'(r)=a''$, where $a''\in\mathbb{N}_{z\text{-}1}$; otherwise, $M'(r)=M(r)$-1;

b) $\forall r'\in t^\bullet\cap P_R$, if $M(r)=a'$ is a variable, where $a'\in\mathbb{N}_z$, then $M'(p)=a''$, where $a''\in\mathbb{N}_{z+1}$; otherwise, $M'(r')=M(r')$ +1.

Since $|{}^\bullet t\cap P_R|\leq|P_R|$ and $|t^\bullet\cap P_R|\leq|P_R|$, the total time complexity of a) and b) is $O(2|P_R|)$.

c) $\forall p\in{}^\bullet t\cap P_A$, if $M(p)=a'$ is a variable, where $a'\in\mathbb{N}_z$, then $M'(p)=a''$, where $a''\in\mathbb{N}_{z\text{-}1}$; if $M(p)$ is a variable, where $p\in P_V$ and $\Sigma_{p_i\in P_V}M(p_i)=1$, then $\forall p_i\in P_V$: $M'(p_i)=0$; if $M(p')$ is a variable, where $p'\in P_V$ and $\Sigma_{p_i\in P_V}M(p_i)=C>1$, then $\Sigma_{p_i\in P_V}M'(p_i)=C\text{-}1$, otherwise $M'(p)=M(p)$-1.

d) $\forall p'\in t^\bullet\cap P_A$, $M(p')=a'$ is a variable, where $a'\in\mathbb{N}_z$, then $M'(p')=a''$, where $a''\in\mathbb{N}_{z+1}$; if $M(p')$ is a variable, where $p'\in P_V$ and $\Sigma_{p_i\in P_V}M(p_i)=C$, then $\Sigma_{p_i\in P_V}M'(p_i)=C+1$; otherwise, $M'(p')=M(p')+1$.

Since $|{}^\bullet t\cap P_A|=1$ and there are at most $|P_R|$ P_V such that $p\in P_V$, the total time complexity of c) and d) is $O(2|P_R|)$. Hence, the time complexity of firing a transition at an individual marking is

$$O(4|P_R|)=O(|P_R|).$$

Thus, the time complexity of firing a transition at an ATI-marking is comparable to that of firing a transition at an individual marking. Similarly, we can obtain that the time complexity of backward firing a transition from an ATI-marking is about the same as that of backward firing a transition at an individual marking.

Case study of ES^{3}PR:

From Fig. A2(a), for any transition, the weight of its input arc is 1. However, for transition t_4, the weight of the arc from t_4 to r_1 is 3. Hence, it is an ES^{3}PR. For this net, given initial marking $M_0=4p_1+4p_5+3r_1+r_2$, $M_1=p_1+3p_2+4p_5+r_2$ and $M_2=2p_1+p_2+p_3+4p_5+r_2$ are partial deadlocks. Let $\widetilde{P}_R=\{r_1\}$. Since $\Pi_{r1}=\{p_2, p_3, p_4\}$ and $\Gamma=\{p_2, p_3\}$, we have that $\Pi_{r1}\cap\Gamma=\{p_2, p_3\}$. By $\pi_1(p_2)=1$ and $\pi_1(p_3)=2$,

$$\textstyle\sum_{p\in\Pi_{r1}\cap\Gamma}\pi_1(p)M_1(p)=M_1(p_2)=M_0(r_1) \text{ and}$$

$$\textstyle\sum_{p\in\Pi_{r1}\cap\Gamma}\pi_1(p)M_2(p)=M_2(p_2)+2M_2(p_3)=M_0(r_1).$$

Since $p_2^\bullet=\{t_2\}$, $p_3^\bullet=\{t_3\}$, $r_1\in{}^\bullet t_2$, $r_1\in{}^\bullet t_3$, and $r_1\in\widetilde{P}_R$,

$$\forall p\in P_{M1} \text{ and } \forall t\in p^\bullet, \exists r\in\widetilde{P}_R: r\in{}^\bullet t, \text{ and}$$

$$\forall p\in P_{M2} \text{ and } \forall t\in p^\bullet, \exists r\in\widetilde{P}_R: r\in{}^\bullet t.$$

According to Definition 8, $\widetilde{P}_R$ is a CRS at M_1 and M_2. Then, bad marking $M_3=2p_1+2p_2+4p_5+r_1+r_2$ can be obtained via Algorithm A2.

For partial deadlock M_2, $P_{M2}=\{p_2, p_3\}$, ${}^\bullet p_2=\{t_1\}$, ${}^\bullet p_3=\{t_2\}$, $p_2^\bullet=\{t_2\}$, and $p_3^\bullet=\{t_3\}$. Hence, $T_I=\{t|\ t\in{}^\bullet p, p\in P_{M2}\}=\{t_1, t_2\}$ and $T_O=\{t|\ t\in p^\bullet, p\in P_{M2}\}=\{t_2, t_3\}$. We design a control place p_c for M_2. For $t_1\in T_I\backslash T_O$, since $t_1^\bullet\cap P_{M2}=\{p_2\}$, according to Condition 1) of Definition 14,

$$t_1\in p_c^\bullet \text{ and } W(p_c, t_1)=\Sigma_{r\in\widetilde{P}_R}\pi_r(p_2)=1.$$

For $t_3\in T_O\backslash T_I$, since ${}^\bullet t_3\cap P_{M2}=\{p_3\}$, according to Condition 2) of Definition 14,

$$t_3\in{}^\bullet p_c \text{ and } W(t_3, p_c)=\Sigma_{r\in\widetilde{P}_R}\pi_r(p_3)=2.$$

For $t_2\in T_I\cap T_O$, since $\{p_2\}={}^\bullet t_2\cap P_{M2}$, $\{p_3\}=t_2^\bullet\cap P_{M2}$, and $\Sigma_{r\in\widetilde{P}_R}\pi_r(p_2)\text{-}\Sigma_{r\in\widetilde{P}_R}\pi_r(p_3)<0$, according to Condition 3) of Defi-

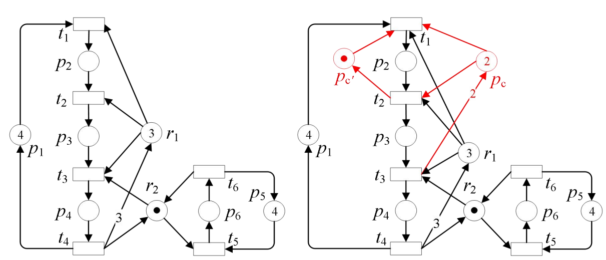

Fig. A2. (a) Example of E^{3}PR and (b) its controlled net.

nition 14,

$$t_2 \in p_c^{\bullet} \text{ and } W(p_c, t_2)=\Sigma_{r\in \tilde{P}_k}\pi_r(p_3)-\Sigma_{r\in \tilde{P}_k}\pi_r(p_2)=1.$$

By $\Sigma_{r\in \tilde{P}_k}M_0(r)$-1=2, $M_0'(p_c)$=2 according to 5) of Definition 14.

For bad marking M_3, $P_{M3}=\{p_2\}$, ${}^{\bullet}p_2=\{t_1\}$, and $p_2^{\bullet}=\{t_2\}$. We design a control place p_c' for M_3 according to Definition 15. For $t_1 \in T_I \backslash T_O$, since $t_1^{\bullet}\cap P_{M3}=\{p_2\}$, according to Condition 1) of Definition 15,

$$t_1 \in p_c'^{\bullet} \text{ and } W(p_c', t_1)=\Sigma_{r\in \tilde{P}_k}\pi_r(p_2)=1.$$

For $t_2 \in T_O \backslash T_I$, since ${}^{\bullet}t_2\cap P_{M3}=\{p_2\}$, according to Condition 2) of Definition 15,

$$t_2 \in {}^{\bullet}p_c' \text{ and } W(t_2, p_c')=\Sigma_{r\in \tilde{P}_k}\pi_r(p_2)=1.$$

Since $\Sigma_{r\in P_R}(M_0(r)-M_3(r))$-1=1, $M_0'(p_c')$=1 according to Condition 5) of Definition 15.

For partial deadlock M_1 and bad marking M_3, since $P_{M1}=P_{M3}=\{p_2\}$ and $M_3(p_2)<M_1(p_2)$, according to Theorem 10, after adding the control place p_c' for M_3, M_1 is also prevented. Finally, a controlled ES^{3}PR is shown in Fig. A2(b).

Flowchart of deadlock control method:

The flowchart of the deadlock control method proposed in this paper is shown in Fig. A3.

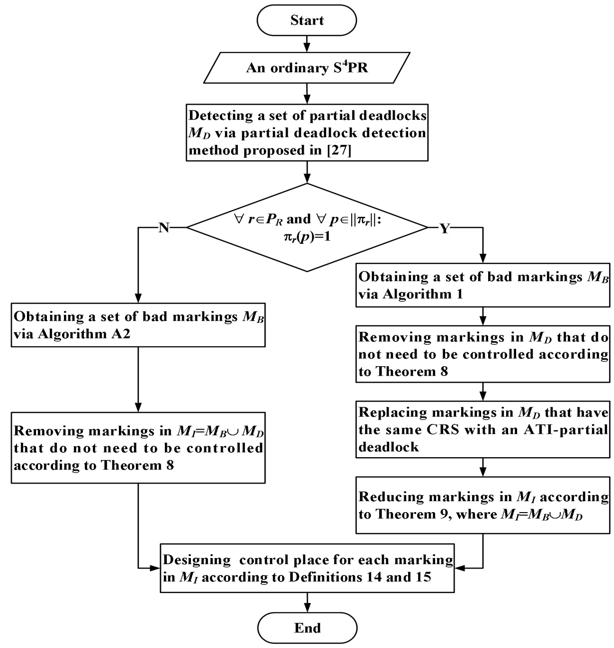

Fig. A3. Flow chart of a deadlock control strategy.

Table A1
Partial Deadlocks in Fig. 5

Partial deadlocks	CRSs
$M_1=5p_1+p_3+5p_8+p_{11}+r_1+r_3+r_4+r_6$ $M_2=6p_1+4p_8+p_{11}+p_{12}+r_1+r_3+r_4+r_6$	$\tilde{P}_{R1}=\{r_2, r_5\}$
$M_3=3p_1+p_2+p_3+p_4+6p_8+r_3+r_4+r_6$ $M_4=4p_1+p_2+p_4+5p_8+p_{12}+r_3+r_4+r_6$ $M_5=4p_1+p_3+p_4+5p_8+p_{11}+r_3+r_4+r_6$ $M_6=5p_1+p_4+4p_8+p_{11}+p_{12}+r_3+r_4+r_6$	$\tilde{P}_{R2}=\{r_1, r_2, r_5\}$
$M_7=5p_1+p_3+4p_8+p_{10}+p_{11}+r_1+r_3+r_6$ $M_8=6p_1+3p_8+p_{10}+p_{11}+p_{12}+r_1+r_3+r_6$	$\tilde{P}_{R3}=\{r_2, r_4, r_5\}$
$M_9=3p_1+p_2+p_3+p_4+5p_8+p_{10}+r_3+r_6$ $M_{10}=4p_1+p_2+p_4+4p_8+p_{10}+p_{12}+r_3+r_6$ $M_{11}=4p_1+p_3+p_4+4p_8+p_{10}+p_{11}+r_3+r_6$ $M_{12}=5p_1+p_4+3p_8+p_{10}+p_{11}+p_{12}+r_3+r_6$	$\tilde{P}_{R4}=\{r_1, r_2, r_4, r_5\}$
$M_{13}=5p_1+p_3+3p_8+p_9+p_{10}+p_{11}+r_1+r_3$ $M_{14}=6p_1+2p_8+p_9+p_{10}+p_{11}+p_{12}+r_1+r_3$	$\tilde{P}_{R5}=\{r_2, r_4, r_5, r_6\}$
$M_{15}=4p_1+p_5+p_6+4p_8+p_9+p_{10}+r_1+r_2$	$\tilde{P}_{R6}=\{r_3, r_4, r_5, r_6\}$
$M_{16}=3p_1+p_4+p_5+p_6+4p_8+p_9+p_{10}+r_2$	$\tilde{P}_{R7}=\{r_1, r_3, r_4, r_5, r_6\}$
$M_{17}=3p_1+p_2+p_3+p_4+4p_8+p_9+p_{10}+r_3$ $M_{18}=4p_1+p_2+p_4+3p_8+p_9+p_{10}+p_{12}+r_3$ $M_{19}=4p_1+p_3+p_4+3p_8+p_9+p_{10}+p_{11}+r_3$ $M_{20}=5p_1+p_4+2p_8+p_9+p_{10}+p_{11}+p_{12}+r_3$	$\tilde{P}_{R8}=\{r_1, r_2, r_4, r_5, r_6\}$
$M_{21}=4p_1+p_3+p_6+3p_8+p_9+p_{10}+p_{11}+r_1$ $M_{22}=5p_1+p_6+2p_8+p_9+p_{10}+p_{11}+p_{12}+r_1$ $M_{23}=3p_1+p_3+p_5+p_6+4p_8+p_9+p_{10}+r_1$ $M_{24}=4p_1+p_5+p_6+3p_8+p_9+p_{10}+p_{12}+r_1$	$\tilde{P}_{R9}=\{r_2, r_3, r_4, r_5, r_6\}$
$M_{25}=2p_1+p_2+p_3+p_4+p_6+4p_8+p_9+p_{10}$ $M_{26}=3p_1+p_2+p_4+p_6+3p_8+p_9+p_{10}+p_{12}$ $M_{27}=3p_1+p_3+p_4+p_6+3p_8+p_9+p_{10}+p_{11}$ $M_{28}=4p_1+p_4+p_6+2p_8+p_9+p_{10}+p_{11}+p_{12}$ $M_{29}=3p_1+p_4+p_5+p_6+3p_8+p_9+p_{10}+p_{12}$ $M_{30}=2p_1+p_3+p_4+p_5+p_6+4p_8+p_9+p_{10}$	$\tilde{P}_{R10}=\{r_1, r_2, r_3, r_4, r_5, r_6\}$

Table A2
Partial Deadlocks in Fig. 6

CRSs	Partial deadlocks
$\widetilde{P}_{R1}=\{r_4, r_6, r_7\}$ $\acute{M}_1=a_1p_1+a_3p_3+a_4p_4+a_5p_5+a_7p_7+a_8p_8+12p_9+p_{11}+a_{13}p_{13}+a_{14}p_{14}+3r_1+r_2+a_3'r_3+a_5'r_5$, where $a_3+a_7\in\{0, 1\}$, $a_4+a_8=3$, $a_5+a_{14}=3$	$M_1=7p_1+3p_5+3p_8+12p_9+p_{11}+13p_{13}+3r_1+r_2+2r_3+r_5$ $M_2=7p_1+3p_4+3p_5+12p_9+p_{11}+13p_{13}+3r_1+r_2+2r_3+r_5$ $M_3=7p_1+2p_4+3p_5+p_8+12p_9+p_{11}+13p_{13}+3r_1+r_2+2r_3+r_5$ $M_4=7p_1+p_4+3p_5+2p_8+12p_9+p_{11}+13p_{13}+3r_1+r_2+2r_3+r_5$ $M_5=8p_1+2p_5+3p_8+12p_9+p_{11}+12p_{13}+p_{14}+3r_1+r_2+2r_3$ $M_6=8p_1+3p_4+2p_5+12p_9+p_{11}+12p_{13}+p_{14}+3r_1+r_2+2r_3$ $M_7=8p_1+2p_4+2p_5+p_8+12p_9+p_{11}+12p_{13}+p_{14}+3r_1+r_2+2r_3$ $M_8=8p_1+p_4+2p_5+2p_8+12p_9+p_{11}+12p_{13}+p_{14}+3r_1+r_2+2r_3$ $M_9=6p_1+p_3+2p_4+3p_5+p_8+12p_9+p_{11}+13p_{13}+3r_1+r_2+r_3+r_5$ $M_{10}=6p_1+p_3+p_4+3p_5+2p_8+12p_9+p_{11}+13p_{13}+3r_1+r_2+r_3+r_5$ $M_{11}=6p_1+p_3+3p_5+3p_8+12p_9+p_{11}+13p_{13}+3r_1+r_2+r_3+r_5$ $M_{12}=6p_1+3p_5+p_7+3p_8+12p_9+p_{11}+13p_{13}+3r_1+r_2+r_3+r_5$ $M_{13}=6p_1+3p_4+3p_5+p_7+12p_9+p_{11}+13p_{13}+3r_1+r_2+r_3+r_5$ $M_{14}=6p_1+2p_4+3p_5+p_7+p_8+12p_9+p_{11}+13p_{13}+3r_1+r_2+r_3+r_5$ $M_{15}=6p_1+p_4+3p_5+p_7+2p_8+12p_9+p_{11}+13p_{13}+3r_1+r_2+r_3+r_5$ $M_{16}=7p_1+2p_5+p_7+3p_8+12p_9+p_{11}+12p_{13}+p_{14}+3r_1+r_2+r_3$ $M_{17}=7p_1+3p_4+2p_5+p_7+12p_9+p_{11}+12p_{13}+p_{14}+3r_1+r_2+r_3$ $M_{18}=7p_1+2p_4+2p_5+p_7+p_8+12p_9+p_{11}+12p_{13}+p_{14}+3r_1+r_2+r_3$ $M_{19}=7p_1+p_3+2p_5+3p_8+12p_9+p_{11}+12p_{13}+p_{14}+3r_1+r_2+r_3$ $M_{20}=7p_1+p_4+2p_5+p_7+2p_8+12p_9+p_{11}+12p_{13}+p_{14}+3r_1+r_2+r_3$ $M_{21}=7p_1+p_3+2p_4+2p_5+p_8+12p_9+p_{11}+12p_{13}+p_{14}+3r_1+r_2+r_3$ $M_{22}=7p_1+p_3+p_4+2p_5+2p_8+12p_9+p_{11}+12p_{13}+p_{14}+3r_1+r_2+r_3$ $M_{23}=7p_1+p_3+3p_4+2p_5+12p_9+p_{11}+12p_{13}+p_{14}+3r_1+r_2+r_3$ $M_{24}=6p_1+p_3+3p_4+3p_5+12p_9+p_{11}+13p_{13}+3r_1+r_2+r_3+r_5$
$\widetilde{P}_{R2}=\{r_4, r_5, r_6\}$ $\acute{M}_2=a_1p_1+a_3p_3+a_4p_4+2p_5+a_7p_7+a_8p_8+13p_9+12p_{13}+p_{14}+3r_1+r_2+a_3'r_3+r_7$, where $a_3+a_7\in\{0, 1\}$, $a_4+a_8=3$	$M_{25}=8p_1+2p_5+3p_8+13p_9+12p_{13}+p_{14}+3r_1+r_2+2r_3+r_7$ $M_{26}=8p_1+3p_4+2p_5+13p_9+12p_{13}+p_{14}+3r_1+r_2+2r_3+r_7$ $M_{27}=8p_1+2p_4+2p_5+p_8+13p_9+12p_{13}+p_{14}+3r_1+r_2+2r_3+r_7$ $M_{28}=7p_1+2p_5+p_7+3p_8+13p_9+12p_{13}+p_{14}+3r_1+r_2+r_3+r_7$ $M_{29}=7p_1+3p_4+2p_5+p_7+13p_9+12p_{13}+p_{14}+3r_1+r_2+r_3+r_7$ $M_{30}=7p_1+2p_4+2p_5+p_7+p_8+13p_9+12p_{13}+p_{14}+3r_1+r_2+r_3+r_7$ $M_{31}=8p_1+p_4+2p_5+2p_8+13p_9+12p_{13}+p_{14}+3r_1+r_2+2r_3+r_7$ $M_{32}=7p_1+p_4+2p_5+p_7+2p_8+13p_9+12p_{13}+p_{14}+3r_1+r_2+r_3+r_7$ $M_{33}=7p_1+p_3+2p_5+3p_8+13p_9+12p_{13}+p_{14}+3r_1+r_2+r_3+r_7$ $M_{34}=7p_1+p_3+3p_4+2p_5+13p_9+12p_{13}+p_{14}+3r_1+r_2+r_3+r_7$ $M_{35}=7p_1+p_3+2p_4+2p_5+p_8+13p_9+12p_{13}+p_{14}+3r_1+r_2+r_3+r_7$ $M_{36}=7p_1+p_3+p_4+2p_5+2p_8+13p_9+12p_{13}+p_{14}+3r_1+r_2+r_3+r_7$
$\widetilde{P}_{R3}=\{r_3, r_6\}$ $\acute{M}_3=11p_1+a_3p_3+a_7p_7+13p_9+10p_{13}+3p_{15}+3r_1+r_2+r_5+r_7$, where $a_3+a_7=2$	$M_{37}=11p_1+2p_7+13p_9+10p_{13}+3p_{15}+3r_1+r_2+r_5+r_7$ $M_{38}=11p_1+2p_3+13p_9+10p_{13}+3p_{15}+3r_1+r_2+r_5+r_7$ $M_{39}=11p_1+p_3+p_7+13p_9+10p_{13}+3p_{15}+3r_1+r_2+r_5+r_7$
$\widetilde{P}_{R4}=\{r_3, r_6, r_7\}$ $\acute{M}_4=11p_1+a_3p_3+a_7p_7+12p_9+p_{11}+10p_{13}+3p_{15}+3r_1+r_2+r_5$, where $a_3+a_7=2$	$M_{40}=11p_1+2p_7+12p_9+p_{11}+10p_{13}+3p_{15}+3r_1+r_2+r_5$ $M_{41}=11p_1+2p_3+12p_9+p_{11}+10p_{13}+3p_{15}+3r_1+r_2+r_5$ $M_{42}=11p_1+p_3+p_7+12p_9+p_{11}+10p_{13}+3p_{15}+3r_1+r_2+r_5$
$\widetilde{P}_{R5}=\{r_3, r_4, r_5, r_6\}$ $\acute{M}_5=a_1p_1+a_3p_3+a_4p_4+a_5p_5+a_7p_7+a_8p_8+13p_9+a_{13}p_{13}+p_{14}+a_{15}p_{15}+3r_1+r_2+r_7$, where $a_3+a_7=2$, $a_4+a_8+a_{15}=3$, $a_5+a_{15}=2$,	$M_{43}=10p_1+2p_7+p_8+13p_9+10p_{13}+p_{14}+2p_{15}+3r_1+r_2+r_7$ $M_{44}=6p_1+3p_4+2p_5+2p_7+13p_9+12p_{13}+p_{14}+3r_1+r_2+r_7$ $M_{45}=8p_1+2p_4+p_5+2p_7+13p_9+11p_{13}+p_{14}+p_{15}+3r_1+r_2+r_7$ $M_{46}=6p_1+2p_4+2p_5+2p_7+p_8+13p_9+12p_{13}+p_{14}+3r_1+r_2+r_7$ $M_{47}=8p_1+p_4+p_5+2p_7+p_8+13p_9+11p_{13}+p_{14}+p_{15}+3r_1+r_2+r_7$ $M_{48}=6p_1+2p_3+2p_5+3p_8+13p_9+12p_{13}+p_{14}+3r_1+r_2+r_7$ $M_{49}=8p_1+2p_3+p_5+2p_8+13p_9+11p_{13}+p_{14}+p_{15}+3r_1+r_2+r_7$ $M_{50}=6p_1+2p_3+3p_4+2p_5+13p_9+12p_{13}+p_{14}+3r_1+r_2+r_7$ $M_{51}=8p_1+2p_3+2p_4+p_5+13p_9+11p_{13}+p_{14}+p_{15}+3r_1+r_2+r_7$ $M_{52}=6p_1+2p_3+2p_4+2p_5+p_8+13p_9+12p_{13}+p_{14}+3r_1+r_2+r_7$ $M_{53}=6p_1+2p_3+p_4+2p_5+2p_8+13p_9+12p_{13}+p_{14}+3r_1+r_2+r_7$ $M_{54}=6p_1+p_4+2p_5+2p_7+2p_8+13p_9+12p_{13}+p_{14}+3r_1+r_2+r_7$ $M_{55}=8p_1+2p_3+p_4+p_5+p_8+13p_9+11p_{13}+p_{14}+p_{15}+3r_1+r_2+r_7$ $M_{56}=6p_1+p_3+2p_5+p_7+3p_8+13p_9+12p_{13}+p_{14}+3r_1+r_2+r_7$ $M_{57}=8p_1+p_3+p_5+p_7+2p_8+13p_9+11p_{13}+p_{14}+p_{15}+3r_1+r_2+r_7$ $M_{58}=6p_1+p_3+3p_4+2p_5+p_7+13p_9+12p_{13}+p_{14}+3r_1+r_2+r_7$ $M_{59}=8p_1+p_3+2p_4+p_5+p_7+13p_9+11p_{13}+p_{14}+p_{15}+3r_1+r_2+r_7$ $M_{60}=6p_1+p_3+2p_4+2p_5+p_7+p_8+13p_9+12p_{13}+p_{14}+3r_1+r_2+r_7$ $M_{61}=6p_1+p_3+p_4+2p_5+p_7+2p_8+13p_9+12p_{13}+p_{14}+3r_1+r_2+r_7$ $M_{62}=8p_1+p_3+p_4+p_5+p_7+p_8+13p_9+11p_{13}+p_{14}+p_{15}+3r_1+r_2+r_7$ $M_{63}=10p_1+p_4+2p_7+13p_9+10p_{13}+p_{14}+2p_{15}+3r_1+r_2+r_7$ $M_{64}=10p_1+2p_3+p_8+13p_9+10p_{13}+p_{14}+2p_{15}+3r_1+r_2+r_7$ $M_{65}=10p_1+2p_3+p_4+13p_9+10p_{13}+p_{14}+2p_{15}+3r_1+r_2+r_7$ $M_{66}=10p_1+p_3+p_7+p_8+13p_9+10p_{13}+p_{14}+2p_{15}+3r_1+r_2+r_7$ $M_{67}=6p_1+2p_5+2p_7+3p_8+13p_9+12p_{13}+p_{14}+3r_1+r_2+r_7$ $M_{68}=8p_1+p_5+2p_7+2p_8+13p_9+11p_{13}+p_{14}+p_{15}+3r_1+r_2+r_7$ $M_{69}=10p_1+p_3+p_4+p_7+13p_9+10p_{13}+p_{14}+2p_{15}+3r_1+r_2+r_7$

$\widetilde{P}_{R6}=\{r_3, r_4, r_6, r_7\}$ $\acute{M}_6=a_1p_1+a_3p_3+a_4p_4+a_5p_5+a_7p_7+a_8p_8+12p_9+p_{11}+a_{13}p_{13}+a_{14}p_{14}+a_{15}p_{15}+3r_1+r_2+a_5'r_5$, where $a_3+a_7=2$, $a_4+a_8+a_{15}=3$, $a_5+a_{14}+a_{15}=3$	$M_{70}=10p_1+p_3+p_7+p_8+12p_9+p_{11}+10p_{13}+p_{14}+2p_{15}+3r_1+r_2$ $M_{71}=10p_1+p_3+p_4+p_7+12p_9+p_{11}+10p_{13}+p_{14}+2p_{15}+3r_1+r_2$ $M_{72}=5p_1+p_3+3p_5+p_7+3p_8+12p_9+p_{11}+13p_{13}+3r_1+r_2+r_5$ $M_{73}=7p_1+p_3+2p_5+p_7+2p_8+12p_9+p_{11}+12p_{13}+p_{15}+3r_1+r_2+r_5$ $M_{74}=9p_1+p_3+p_5+p_7+p_8+12p_9+p_{11}+11p_{13}+2p_{15}+3r_1+r_2+r_5$ $M_{75}=5p_1+p_3+3p_4+3p_5+p_7+12p_9+p_{11}+13p_{13}+3r_1+r_2+r_5$ $M_{76}=7p_1+p_3+2p_4+2p_5+p_7+12p_9+p_{11}+12p_{13}+p_{15}+3r_1+r_2+r_5$ $M_{77}=9p_1+p_3+p_4+p_5+p_7+12p_9+p_{11}+11p_{13}+2p_{15}+3r_1+r_2+r_5$ $M_{78}=5p_1+p_3+2p_4+3p_5+p_7+p_8+12p_9+p_{11}+13p_{13}+3r_1+r_2+r_5$ $M_{79}=5p_1+p_3+p_4+3p_5+p_7+2p_8+12p_9+p_{11}+13p_{13}+3r_1+r_2+r_5$ $M_{80}=7p_1+p_3+p_4+2p_5+p_7+p_8+12p_9+p_{11}+12p_{13}+p_{15}+3r_1+r_2+r_5$ $M_{81}=6p_1+p_3+2p_5+p_7+3p_8+12p_9+p_{11}+12p_{13}+p_{14}+3r_1+r_2$ $M_{82}=8p_1+p_3+p_5+p_7+2p_8+12p_9+p_{11}+11p_{13}+p_{14}+p_{15}+3r_1+r_2$ $M_{83}=6p_1+p_3+3p_4+2p_5+p_7+12p_9+p_{11}+12p_{13}+p_{14}+3r_1+r_2$ $M_{84}=8p_1+p_3+2p_4+p_5+p_7+12p_9+p_{11}+11p_{13}+p_{14}+p_{15}+3r_1+r_2$ $M_{85}=6p_1+p_3+2p_4+2p_5+p_7+p_8+12p_9+p_{11}+12p_{13}+p_{14}+3r_1+r_2$ $M_{86}=6p_1+p_3+p_4+2p_5+p_7+2p_8+12p_9+p_{11}+12p_{13}+p_{14}+3r_1+r_2$ $M_{87}=8p_1+p_3+p_4+p_5+p_7+p_8+12p_9+p_{11}+11p_{13}+p_{14}+p_{15}+3r_1+r_2$ $M_{88}=10p_1+2p_7+p_8+12p_9+p_{11}+10p_{13}+p_{14}+2p_{15}+3r_1+r_2$ $M_{89}=10p_1+p_4+2p_7+12p_9+p_{11}+10p_{13}+p_{14}+2p_{15}+3r_1+r_2$ $M_{90}=5p_1+3p_5+2p_7+3p_8+12p_9+p_{11}+13p_{13}+3r_1+r_2+r_5$ $M_{91}=7p_1+2p_5+2p_7+2p_8+12p_9+p_{11}+12p_{13}+p_{15}+3r_1+r_2+r_5$ $M_{92}=9p_1+p_5+2p_7+p_8+12p_9+p_{11}+11p_{13}+2p_{15}+3r_1+r_2+r_5$ $M_{93}=5p_1+3p_4+3p_5+2p_7+12p_9+p_{11}+13p_{13}+3r_1+r_2+r_5$ $M_{94}=7p_1+2p_4+2p_5+2p_7+12p_9+p_{11}+12p_{13}+p_{15}+3r_1+r_2+r_5$ $M_{95}=9p_1+p_4+p_5+2p_7+12p_9+p_{11}+11p_{13}+2p_{15}+3r_1+r_2+r_5$ $M_{96}=5p_1+2p_4+3p_5+2p_7+p_8+12p_9+p_{11}+13p_{13}+3r_1+r_2+r_5$ $M_{97}=5p_1+p_4+3p_5+2p_7+2p_8+12p_9+p_{11}+13p_{13}+3r_1+r_2+r_5$ $M_{98}=7p_1+p_4+2p_5+2p_7+p_8+12p_9+p_{11}+12p_{13}+p_{15}+3r_1+r_2+r_5$ $M_{99}=6p_1+2p_5+2p_7+3p_8+12p_9+p_{11}+12p_{13}+p_{14}+3r_1+r_2$ $M_{100}=8p_1+p_5+2p_7+2p_8+12p_9+p_{11}+11p_{13}+p_{14}+p_{15}+3r_1+r_2$ $M_{101}=6p_1+3p_4+2p_5+2p_7+12p_9+p_{11}+12p_{13}+p_{14}+3r_1+r_2$ $M_{102}=8p_1+2p_4+p_5+2p_7+12p_9+p_{11}+11p_{13}+p_{14}+p_{15}+3r_1+r_2$ $M_{103}=6p_1+2p_4+2p_5+2p_7+p_8+12p_9+p_{11}+12p_{13}+p_{14}+3r_1+r_2$ $M_{104}=6p_1+p_4+2p_5+2p_7+2p_8+12p_9+p_{11}+12p_{13}+p_{14}+3r_1+r_2$ $M_{105}=8p_1+p_4+p_5+2p_7+p_8+12p_9+p_{11}+11p_{13}+p_{14}+p_{15}+3r_1+r_2$ $M_{106}=10p_1+2p_3+p_8+12p_9+p_{11}+10p_{13}+p_{14}+2p_{15}+3r_1+r_2$ $M_{107}=10p_1+2p_3+p_4+12p_9+p_{11}+10p_{13}+p_{14}+2p_{15}+3r_1+r_2$ $M_{108}=5p_1+2p_3+3p_5+3p_8+12p_9+p_{11}+13p_{13}+3r_1+r_2+r_5$ $M_{109}=7p_1+2p_3+2p_5+2p_8+12p_9+p_{11}+12p_{13}+p_{15}+3r_1+r_2+r_5$ $M_{110}=9p_1+2p_3+p_5+p_8+12p_9+p_{11}+11p_{13}+2p_{15}+3r_1+r_2+r_5$ $M_{111}=5p_1+2p_3+3p_4+3p_5+12p_9+p_{11}+13p_{13}+3r_1+r_2+r_5$ $M_{112}=7p_1+2p_3+2p_4+2p_5+12p_9+p_{11}+12p_{13}+p_{15}+3r_1+r_2+r_5$ $M_{113}=9p_1+2p_3+p_4+p_5+12p_9+p_{11}+11p_{13}+2p_{15}+3r_1+r_2+r_5$ $M_{114}=5p_1+2p_3+2p_4+3p_5+p_8+12p_9+p_{11}+13p_{13}+3r_1+r_2+r_5$ $M_{115}=5p_1+2p_3+p_4+3p_5+2p_8+12p_9+p_{11}+13p_{13}+3r_1+r_2+r_5$ $M_{116}=7p_1+2p_3+p_4+2p_5+p_8+12p_9+p_{11}+12p_{13}+p_{15}+3r_1+r_2+r_5$ $M_{117}=6p_1+2p_3+2p_5+3p_8+12p_9+p_{11}+12p_{13}+p_{14}+3r_1+r_2$ $M_{118}=8p_1+2p_3+p_5+2p_8+12p_9+p_{11}+11p_{13}+p_{14}+p_{15}+3r_1+r_2$ $M_{119}=6p_1+2p_3+3p_4+2p_5+12p_9+p_{11}+12p_{13}+p_{14}+3r_1+r_2$ $M_{120}=8p_1+2p_3+2p_4+p_5+12p_9+p_{11}+11p_{13}+p_{14}+p_{15}+3r_1+r_2$ $M_{121}=6p_1+2p_3+2p_4+2p_5+p_8+12p_9+p_{11}+12p_{13}+p_{14}+3r_1+r_2$ $M_{122}=6p_1+2p_3+p_4+2p_5+2p_8+12p_9+p_{11}+12p_{13}+p_{14}+3r_1+r_2$ $M_{123}=8p_1+2p_3+p_4+p_5+p_8+12p_9+p_{11}+11p_{13}+p_{14}+p_{15}+3r_1+r_2$
$\widetilde{P}_{R7}=\{r_2, r_3\}$ $\acute{M}_7=12p_1+p_2+13p_9+11p_{13}+a_{15}p_{15}+2p_{16}+2r_1+a_4'r_4+r_5+a_6'r_6+r_7$, where $a_{15}\in\{0, 1, 2\}$	$M_{124}=12p_1+p_2+13p_9+11p_{13}+2p_{16}+2r_1+3r_4+r_5+3r_6+r_7$ $M_{125}=12p_1+p_2+13p_9+10p_{13}+p_{15}+2p_{16}+2r_1+2r_4+r_5+2r_6+r_7$ $M_{126}=12p_1+p_2+13p_9+9p_{13}+2p_{15}+2p_{16}+2r_1+r_4+r_5+r_6+r_7$
$\widetilde{P}_{R8}=\{r_2, r_3, r_6\}$ $\acute{M}_8=a_1p_1+p_2+a_3p_3+a_7p_7+13p_9+a_{13}p_{13}+3p_{15}+a_{16}p_{16}+2r_1+r_5+r_7$, where $a_3+a_7+a_{16}=2$	$M_{127}=12p_1+p_2+13p_9+8p_{13}+3p_{15}+2p_{16}+2r_1+r_5+r_7$ $M_{128}=10p_1+p_2+2p_7+13p_9+10p_{13}+3p_{15}+2r_1+r_5+r_7$ $M_{129}=11p_1+p_2+p_7+13p_9+9p_{13}+3p_{15}+p_{16}+2r_1+r_5+r_7$ $M_{130}=10p_1+p_2+2p_3+13p_9+10p_{13}+3p_{15}+2r_1+r_5+r_7$ $M_{131}=11p_1+p_2+p_3+13p_9+9p_{13}+3p_{15}+p_{16}+2r_1+r_5+r_7$ $M_{132}=10p_1+p_2+p_3+p_7+13p_9+10p_{13}+3p_{15}+2r_1+r_5+r_7$
$\widetilde{P}_{R9}=\{r_2, r_3, r_6, r_7\}$ $\acute{M}_9=a_1p_1+p_2+a_3p_3+a_7p_7+12p_9+p_{11}+a_{13}p_{13}+3p_{15}+a_{16}p_{16}+2r_1+r_5$, where $a_3+a_7+a_{16}=2$	$M_{133}=12p_1+p_2+12p_9+p_{11}+8p_{13}+3p_{15}+2p_{16}+2r_1+r_5$ $M_{134}=10p_1+p_2+2p_7+12p_9+p_{11}+10p_{13}+3p_{15}+2r_1+r_5$ $M_{135}=11p_1+p_2+p_7+12p_9+p_{11}+9p_{13}+3p_{15}+p_{16}+2r_1+r_5$ $M_{136}=10p_1+p_2+2p_3+12p_9+p_{11}+10p_{13}+3p_{15}+2r_1+r_5$ $M_{137}=11p_1+p_2+p_3+12p_9+p_{11}+9p_{13}+3p_{15}+p_{16}+2r_1+r_5$ $M_{138}=10p_1+p_2+p_3+p_7+12p_9+p_{11}+10p_{13}+3p_{15}+2r_1+r_5$

$\widetilde{P}_{R10}=\{r_2, r_3, r_4, r_5, r_6\}$ $\acute{M}_{10}=a_1p_1+p_2+a_3p_3+a_4p_4+a_5p_5+a_7p_7+a_8p_8+13p_9+a_{13}p_{13}+p_{14}+a_{15}p_{15}+a_{16}p_{16}+2r_1+r_7$, where $a_3+a_7+a_{16}=2$, $a_5+a_{15}=2$, and $a_4+a_8+a_{15}=3$	$M_{139}=11p_1+p_2+p_8+13p_9+8p_{13}+p_{14}+2p_{15}+2p_{16}+2r_1+r_7$ $M_{140}=11p_1+p_2+p_4+13p_9+8p_{13}+p_{14}+2p_{15}+2p_{16}+2r_1+r_7$ $M_{141}=9p_1+p_2+2p_7+p_8+13p_9+10p_{13}+p_{14}+2p_{15}+2r_1+r_7$ $M_{142}=9p_1+p_2+p_4+2p_7+13p_9+10p_{13}+p_{14}+2p_{15}+2r_1+r_7$ $M_{143}=10p_1+p_2+p_7+p_8+13p_9+9p_{13}+p_{14}+2p_{15}+p_{16}+2r_1+r_7$ $M_{144}=10p_1+p_2+p_4+p_7+13p_9+9p_{13}+p_{14}+2p_{15}+p_{16}+2r_1+r_7$ $M_{145}=9p_1+p_2+2p_3+p_4+13p_9+10p_{13}+p_{14}+2p_{15}+2r_1+r_7$ $M_{146}=9p_1+p_2+2p_3+p_8+13p_9+10p_{13}+p_{14}+2p_{15}+2r_1+r_7$ $M_{147}=10p_1+p_2+p_3+p_8+13p_9+9p_{13}+p_{14}+2p_{15}+p_{16}+2r_1+r_7$ $M_{148}=9p_1+p_2+p_3+p_7+p_8+13p_9+10p_{13}+p_{14}+2p_{15}+2r_1+r_7$ $M_{149}=9p_1+p_2+p_3+p_4+p_7+13p_9+10p_{13}+p_{14}+2p_{15}+2r_1+r_7$ $M_{150}=10p_1+p_2+p_3+p_4+13p_9+9p_{13}+p_{14}+2p_{15}+p_{16}+2r_1+r_7$ $M_{151}=9p_1+p_2+2p_4+p_5+13p_9+9p_{13}+p_{14}+p_{15}+2p_{16}+2r_1+r_7$ $M_{152}=9p_1+p_2+p_5+2p_8+13p_9+9p_{13}+p_{14}+p_{15}+2p_{16}+2r_1+r_7$ $M_{153}=7p_1+p_2+3p_4+2p_5+13p_9+10p_{13}+p_{14}+2p_{16}+2r_1+r_7$ $M_{154}=7p_1+p_2+2p_5+3p_8+13p_9+10p_{13}+p_{14}+2p_{16}+2r_1+r_7$ $M_{155}=7p_1+p_2+2p_4+2p_5+p_8+13p_9+10p_{13}+p_{14}+2p_{16}+2r_1+r_7$ $M_{156}=7p_1+p_2+p_4+2p_5+2p_8+13p_9+10p_{13}+p_{14}+2p_{16}+2r_1+r_7$ $M_{157}=9p_1+p_2+p_4+p_5+p_8+13p_9+9p_{13}+p_{14}+p_{15}+2p_{16}+2r_1+r_7$ $M_{158}=5p_1+p_2+2p_5+2p_7+3p_8+13p_9+12p_{13}+p_{14}+2r_1+r_7$ $M_{159}=7p_1+p_2+p_5+2p_7+2p_8+13p_9+11p_{13}+p_{14}+p_{15}+2r_1+r_7$ $M_{160}=5p_1+p_2+3p_4+2p_5+2p_7+13p_9+12p_{13}+p_{14}+2r_1+r_7$ $M_{161}=7p_1+p_2+2p_4+p_5+2p_7+13p_9+11p_{13}+p_{14}+p_{15}+2r_1+r_7$ $M_{162}=5p_1+p_2+2p_4+2p_5+2p_7+p_8+13p_9+12p_{13}+p_{14}+2r_1+r_7$ $M_{163}=5p_1+p_2+p_4+2p_5+2p_7+2p_8+13p_9+12p_{13}+p_{14}+2r_1+r_7$ $M_{164}=7p_1+p_2+p_4+p_5+2p_7+p_8+13p_9+11p_{13}+p_{14}+p_{15}+2r_1+r_7$ $M_{165}=6p_1+p_2+2p_5+p_7+3p_8+13p_9+11p_{13}+p_{14}+p_{16}+2r_1+r_7$ $M_{166}=8p_1+p_2+p_5+p_7+2p_8+13p_9+10p_{13}+p_{14}+p_{15}+p_{16}+2r_1+r_7$ $M_{167}=6p_1+p_2+3p_4+2p_5+p_7+13p_9+11p_{13}+p_{14}+p_{16}+2r_1+r_7$ $M_{168}=8p_1+p_2+2p_4+p_5+p_7+13p_9+10p_{13}+p_{14}+p_{15}+p_{16}+2r_1+r_7$ $M_{169}=6p_1+p_2+2p_4+2p_5+p_7+p_8+13p_9+11p_{13}+p_{14}+p_{16}+2r_1+r_7$ $M_{170}=6p_1+p_2+p_4+2p_5+p_7+2p_8+13p_9+11p_{13}+p_{14}+p_{16}+2r_1+r_7$ $M_{171}=8p_1+p_2+p_4+p_5+p_7+p_8+13p_9+10p_{13}+p_{14}+p_{15}+p_{16}+2r_1+r_7$ $M_{172}=5p_1+p_2+2p_3+2p_5+3p_8+13p_9+12p_{13}+p_{14}+2r_1+r_7$ $M_{173}=7p_1+p_2+2p_3+p_5+2p_8+13p_9+11p_{13}+p_{14}+p_{15}+2r_1+r_7$ $M_{174}=5p_1+p_2+2p_3+3p_4+2p_5+13p_9+12p_{13}+p_{14}+2r_1+r_7$ $M_{175}=7p_1+p_2+2p_3+2p_4+p_5+13p_9+11p_{13}+p_{14}+p_{15}+2r_1+r_7$ $M_{176}=5p_1+p_2+2p_3+2p_4+2p_5+p_8+13p_9+12p_{13}+p_{14}+2r_1+r_7$ $M_{177}=5p_1+p_2+2p_3+p_4+2p_5+2p_8+13p_9+12p_{13}+p_{14}+2r_1+r_7$ $M_{178}=7p_1+p_2+2p_3+p_4+p_5+p_8+13p_9+11p_{13}+p_{14}+p_{15}+2r_1+r_7$ $M_{179}=6p_1+p_2+p_3+2p_5+3p_8+13p_9+11p_{13}+p_{14}+p_{16}+2r_1+r_7$ $M_{180}=8p_1+p_2+p_3+p_5+2p_8+13p_9+10p_{13}+p_{14}+p_{15}+p_{16}+2r_1+r_7$ $M_{181}=6p_1+p_2+p_3+3p_4+2p_5+13p_9+11p_{13}+p_{14}+p_{16}+2r_1+r_7$ $M_{182}=8p_1+p_2+p_3+2p_4+p_5+13p_9+10p_{13}+p_{14}+p_{15}+p_{16}+2r_1+r_7$ $M_{183}=6p_1+p_2+p_3+2p_4+2p_5+p_8+13p_9+11p_{13}+p_{14}+p_{16}+2r_1+r_7$ $M_{184}=6p_1+p_2+p_3+p_4+2p_5+2p_8+13p_9+11p_{13}+p_{14}+p_{16}+2r_1+r_7$ $M_{185}=8p_1+p_2+p_3+p_4+p_5+p_8+13p_9+10p_{13}+p_{14}+p_{15}+p_{16}+2r_1+r_7$ $M_{186}=5p_1+p_2+p_3+2p_5+p_7+3p_8+13p_9+12p_{13}+p_{14}+2r_1+r_7$ $M_{187}=7p_1+p_2+p_3+p_5+p_7+2p_8+13p_9+11p_{13}+p_{14}+p_{15}+2r_1+r_7$ $M_{188}=5p_1+p_2+p_3+3p_4+2p_5+p_7+13p_9+12p_{13}+p_{14}+2r_1+r_7$ $M_{189}=7p_1+p_2+p_3+2p_4+p_5+p_7+13p_9+11p_{13}+p_{14}+p_{15}+2r_1+r_7$ $M_{190}=5p_1+p_2+p_3+2p_4+2p_5+p_7+p_8+13p_9+12p_{13}+p_{14}+2r_1+r_7$ $M_{191}=5p_1+p_2+p_3+p_4+2p_5+p_7+2p_8+13p_9+12p_{13}+p_{14}+2r_1+r_7$ $M_{192}=7p_1+p_2+p_3+p_4+p_5+p_7+p_8+13p_9+11p_{13}+p_{14}+p_{15}+2r_1+r_7$
$\widetilde{P}_{R11}=\{r_2, r_3, r_4, r_6, r_7\}$ $\acute{M}_{11}=a_1p_1+p_2+a_3p_3+a_4p_4+a_5p_5+a_7p_7+a_8p_8+a_{14}p_{14}+a_{15}p_{15}+a_{16}p_{16}+12p_9+p_{11}+a_{13}p_{13}+2r_1+a_5'r_5$ where $a_3+a_7+a_{16}=2$, $a_5+a_{14}+a_{15}=3$, and $a_4+a_8+a_{15}=3$	$M_{193}=8p_1+p_2+2p_5+2p_8+12p_9+p_{11}+10p_{13}+p_{15}+2p_{16}+2r_1+r_5$ $M_{194}=10p_1+p_2+p_5+p_8+12p_9+p_{11}+9p_{13}+2p_{15}+2p_{16}+2r_1+r_5$ $M_{195}=6p_1+p_2+3p_4+3p_5+12p_9+p_{11}+11p_{13}+2p_{16}+2r_1+r_5$ $M_{196}=8p_1+p_2+2p_4+2p_5+12p_9+p_{11}+10p_{13}+p_{15}+2p_{16}+2r_1+r_5$ $M_{197}=10p_1+p_2+p_4+p_5+12p_9+p_{11}+9p_{13}+2p_{15}+2p_{16}+2r_1+r_5$ $M_{198}=6p_1+p_2+2p_4+3p_5+p_8+12p_9+p_{11}+11p_{13}+2p_{16}+2r_1+r_5$ $M_{199}=6p_1+p_2+p_4+3p_5+2p_8+12p_9+p_{11}+11p_{13}+2p_{16}+2r_1+r_5$ $M_{200}=8p_1+p_2+p_4+2p_5+p_8+12p_9+p_{11}+10p_{13}+p_{15}+2p_{16}+2r_1+r_5$ $M_{201}=7p_1+p_2+2p_5+3p_8+12p_9+p_{11}+10p_{13}+p_{14}+2p_{16}+2r_1$ $M_{202}=9p_1+p_2+p_5+2p_8+12p_9+p_{11}+9p_{13}+p_{14}+p_{15}+2p_{16}+2r_1$ $M_{203}=7p_1+p_2+3p_4+2p_5+12p_9+p_{11}+10p_{13}+p_{14}+2p_{16}+2r_1$ $M_{204}=9p_1+p_2+2p_4+p_5+12p_9+p_{11}+9p_{13}+p_{14}+p_{15}+2p_{16}+2r_1$ $M_{205}=7p_1+p_2+2p_4+2p_5+p_8+12p_9+p_{11}+10p_{13}+p_{14}+2p_{16}+2r_1$ $M_{206}=7p_1+p_2+p_4+2p_5+2p_8+12p_9+p_{11}+10p_{13}+p_{14}+2p_{16}+2r_1$ $M_{207}=9p_1+p_2+p_4+p_5+p_8+12p_9+p_{11}+9p_{13}+p_{14}+p_{15}+2p_{16}+2r_1$ $M_{208}=9p_1+p_2+2p_7+p_8+12p_9+p_{11}+10p_{13}+p_{14}+2p_{15}+2r_1$

$\widetilde{P}_{R11}=\{r_2, r_3, r_4, r_6, r_7\}$ $\acute{M}_{11}=a_1p_1+p_2+a_3p_3+a_4p_4+a_5p_5+a_7p_7+a_8p_8+a_{14}p_{14}+a_{15}p_{15}+a_{16}p_{16}+12p_9+p_{11}+a_{13}p_{13}+2r_1+a_5'r_5$ where $a_3+a_7+a_{16}=2$, $a_5+a_{14}+a_{15}=3$, and $a_4+a_8+a_{15}=3$	$M_{209}=9p_1+p_2+p_4+2p_7+12p_9+p_{11}+10p_{13}+p_{14}+2p_{15}+2r_1$ $M_{210}=4p_1+p_2+3p_5+2p_7+3p_8+12p_9+p_{11}+13p_{13}+2r_1+r_5$ $M_{211}=6p_1+p_2+2p_5+2p_7+2p_8+12p_9+p_{11}+12p_{13}+p_{15}+2r_1+r_5$ $M_{212}=8p_1+p_2+p_5+2p_7+p_8+12p_9+p_{11}+11p_{13}+2p_{15}+2r_1+r_5$ $M_{213}=4p_1+p_2+3p_4+3p_5+2p_7+12p_9+p_{11}+13p_{13}+2r_1+r_5$ $M_{214}=6p_1+p_2+2p_4+2p_5+2p_7+12p_9+p_{11}+12p_{13}+p_{15}+2r_1+r_5$ $M_{215}=8p_1+p_2+p_4+p_5+2p_7+12p_9+p_{11}+11p_{13}+2p_{15}+2r_1+r_5$ $M_{216}=4p_1+p_2+2p_4+3p_5+2p_7+p_8+12p_9+p_{11}+13p_{13}+2r_1+r_5$ $M_{217}=4p_1+p_2+p_4+3p_5+2p_7+2p_8+12p_9+p_{11}+13p_{13}+2r_1+r_5$ $M_{218}=6p_1+p_2+p_4+2p_5+2p_7+p_8+12p_9+p_{11}+12p_{13}+p_{15}+2r_1+r_5$ $M_{219}=5p_1+p_2+2p_5+2p_7+3p_8+12p_9+p_{11}+12p_{13}+p_{14}+2r_1$ $M_{220}=7p_1+p_2+p_5+2p_7+2p_8+12p_9+p_{11}+11p_{13}+p_{14}+p_{15}+2r_1$ $M_{221}=5p_1+p_2+3p_4+2p_5+2p_7+12p_9+p_{11}+12p_{13}+p_{14}+2r_1$ $M_{222}=7p_1+p_2+2p_4+p_5+2p_7+12p_9+p_{11}+11p_{13}+p_{14}+p_{15}+2r_1$ $M_{223}=5p_1+p_2+2p_4+2p_5+2p_7+p_8+12p_9+p_{11}+12p_{13}+p_{14}+2r_1$ $M_{224}=5p_1+p_2+p_4+2p_5+2p_7+2p_8+12p_9+p_{11}+12p_{13}+p_{14}+2r_1$ $M_{225}=7p_1+p_2+p_4+p_5+2p_7+p_8+12p_9+p_{11}+11p_{13}+p_{14}+p_{15}+2r_1$ $M_{226}=10p_1+p_2+p_7+p_8+12p_9+p_{11}+9p_{13}+p_{14}+2p_{15}+p_{16}+2r_1$ $M_{227}=10p_1+p_2+p_4+p_7+12p_9+p_{11}+9p_{13}+p_{14}+2p_{15}+p_{16}+2r_1$ $M_{228}=5p_1+p_2+3p_5+p_7+3p_8+12p_9+p_{11}+12p_{13}+p_{16}+2r_1+r_5$ $M_{229}=7p_1+p_2+2p_5+p_7+2p_8+12p_9+p_{11}+11p_{13}+p_{15}+p_{16}+2r_1+r_5$ $M_{230}=9p_1+p_2+p_5+p_7+p_8+12p_9+p_{11}+10p_{13}+2p_{15}+p_{16}+2r_1+r_5$ $M_{231}=5p_1+p_2+3p_4+3p_5+p_7+12p_9+p_{11}+12p_{13}+p_{16}+2r_1+r_5$ $M_{232}=7p_1+p_2+2p_4+2p_5+p_7+12p_9+p_{11}+11p_{13}+p_{15}+p_{16}+2r_1+r_5$ $M_{233}=9p_1+p_2+p_4+p_5+p_7+12p_9+p_{11}+10p_{13}+2p_{15}+p_{16}+2r_1+r_5$ $M_{234}=5p_1+p_2+2p_4+3p_5+p_7+p_8+12p_9+p_{11}+12p_{13}+p_{16}+2r_1+r_5$ $M_{235}=5p_1+p_2+p_4+3p_5+p_7+2p_8+12p_9+p_{11}+12p_{13}+p_{16}+2r_1+r_5$ $M_{236}=7p_1+p_2+p_4+2p_5+p_7+p_8+12p_9+p_{11}+11p_{13}+p_{15}+p_{16}+2r_1+r_5$ $M_{237}=6p_1+p_2+2p_5+p_7+3p_8+12p_9+p_{11}+11p_{13}+p_{14}+p_{16}+2r_1$ $M_{238}=8p_1+p_2+p_5+p_7+2p_8+12p_9+p_{11}+10p_{13}+p_{14}+p_{15}+p_{16}+2r_1$ $M_{239}=6p_1+p_2+3p_4+2p_5+p_7+12p_9+p_{11}+11p_{13}+p_{14}+p_{16}+2r_1$ $M_{240}=8p_1+p_2+2p_4+p_5+p_7+12p_9+p_{11}+10p_{13}+p_{14}+p_{15}+p_{16}+2r_1$ $M_{241}=6p_1+p_2+2p_4+2p_5+p_7+p_8+12p_9+p_{11}+11p_{13}+p_{14}+p_{16}+2r_1$ $M_{242}=6p_1+p_2+p_4+2p_5+p_7+2p_8+12p_9+p_{11}+11p_{13}+p_{14}+p_{16}+2r_1$ $M_{243}=8p_1+p_2+p_4+p_5+p_7+p_8+12p_9+p_{11}+10p_{13}+p_{14}+p_{15}+p_{16}+2r_1$ $M_{244}=9p_1+p_2+2p_3+p_8+12p_9+p_{11}+10p_{13}+p_{14}+2p_{15}+2r_1$ $M_{245}=9p_1+p_2+2p_3+p_4+12p_9+p_{11}+10p_{13}+p_{14}+2p_{15}+2r_1$ $M_{246}=5p_1+p_2+p_3+3p_4+2p_5+p_7+12p_9+p_{11}+12p_{13}+p_{14}+2r_1$ $M_{247}=7p_1+p_2+p_3+2p_4+p_5+p_7+12p_9+p_{11}+11p_{13}+p_{14}+p_{15}+2r_1$ $M_{248}=5p_1+p_2+p_3+2p_4+2p_5+p_7+p_8+12p_9+p_{11}+12p_{13}+p_{14}+2r_1$ $M_{249}=5p_1+p_2+p_3+p_4+2p_5+p_7+2p_8+12p_9+p_{11}+12p_{13}+p_{14}+2r_1$ $M_{250}=7p_1+p_2+p_3+p_4+p_5+p_7+p_8+12p_9+p_{11}+11p_{13}+p_{14}+p_{15}+2r_1$ $M_{251}=11p_1+p_2+p_8+12p_9+p_{11}+8p_{13}+p_{14}+2p_{15}+2p_{16}+2r_1$ $M_{252}=11p_1+p_2+p_4+12p_9+p_{11}+8p_{13}+p_{14}+2p_{15}+2p_{16}+2r_1$ $M_{253}=6p_1+p_2+3p_5+3p_8+12p_9+p_{11}+11p_{13}+2p_{16}+2r_1+r_5$ $M_{254}=4p_1+p_2+2p_3+3p_5+3p_8+12p_9+p_{11}+13p_{13}+2r_1+r_5$ $M_{255}=6p_1+p_2+2p_3+2p_5+2p_8+12p_9+p_{11}+12p_{13}+p_{15}+2r_1+r_5$ $M_{256}=8p_1+p_2+2p_3+p_5+p_8+12p_9+p_{11}+11p_{13}+2p_{15}+2r_1+r_5$ $M_{257}=4p_1+p_2+2p_3+3p_4+3p_5+12p_9+p_{11}+13p_{13}+2r_1+r_5$ $M_{258}=6p_1+p_2+2p_3+2p_4+2p_5+12p_9+p_{11}+12p_{13}+p_{15}+2r_1+r_5$ $M_{259}=8p_1+p_2+2p_3+p_4+p_5+12p_9+p_{11}+11p_{13}+2p_{15}+2r_1+r_5$ $M_{260}=4p_1+p_2+2p_3+2p_4+3p_5+p_8+12p_9+p_{11}+13p_{13}+2r_1+r_5$ $M_{261}=4p_1+p_2+2p_3+p_4+3p_5+2p_8+12p_9+p_{11}+13p_{13}+2r_1+r_5$ $M_{262}=6p_1+p_2+2p_3+p_4+2p_5+p_8+12p_9+p_{11}+12p_{13}+p_{15}+2r_1+r_5$ $M_{263}=5p_1+p_2+2p_3+2p_5+3p_8+12p_9+p_{11}+12p_{13}+p_{14}+2r_1$ $M_{264}=7p_1+p_2+2p_3+p_5+2p_8+12p_9+p_{11}+11p_{13}+p_{14}+p_{15}+2r_1$ $M_{265}=5p_1+p_2+2p_3+3p_4+2p_5+12p_9+p_{11}+12p_{13}+p_{14}+2r_1$ $M_{266}=7p_1+p_2+2p_3+2p_4+p_5+12p_9+p_{11}+11p_{13}+p_{14}+p_{15}+2r_1$ $M_{267}=5p_1+p_2+2p_3+2p_4+2p_5+p_8+12p_9+p_{11}+12p_{13}+p_{14}+2r_1$ $M_{268}=5p_1+p_2+2p_3+p_4+2p_5+2p_8+12p_9+p_{11}+12p_{13}+p_{14}+2r_1$ $M_{269}=7p_1+p_2+2p_3+p_4+p_5+p_8+12p_9+p_{11}+11p_{13}+p_{14}+p_{15}+2r_1$ $M_{270}=10p_1+p_2+p_3+p_8+12p_9+p_{11}+9p_{13}+p_{14}+2p_{15}+p_{16}+2r_1$ $M_{271}=10p_1+p_2+p_3+p_4+12p_9+p_{11}+9p_{13}+p_{14}+2p_{15}+p_{16}+2r_1$ $M_{272}=5p_1+p_2+p_3+3p_5+3p_8+12p_9+p_{11}+12p_{13}+p_{16}+2r_1+r_5$ $M_{273}=7p_1+p_2+p_3+2p_5+2p_8+12p_9+p_{11}+11p_{13}+p_{15}+p_{16}+2r_1+r_5$ $M_{274}=9p_1+p_2+p_3+p_5+p_8+12p_9+p_{11}+10p_{13}+2p_{15}+p_{16}+2r_1+r_5$ $M_{275}=5p_1+p_2+p_3+3p_4+3p_5+12p_9+p_{11}+12p_{13}+p_{16}+2r_1+r_5$ $M_{276}=7p_1+p_2+p_3+2p_4+2p_5+12p_9+p_{11}+11p_{13}+p_{15}+p_{16}+2r_1+r_5$ $M_{277}=9p_1+p_2+p_3+p_4+p_5+12p_9+p_{11}+10p_{13}+2p_{15}+p_{16}+2r_1+r_5$ $M_{278}=5p_1+p_2+p_3+2p_4+3p_5+p_8+12p_9+p_{11}+12p_{13}+p_{16}+2r_1+r_5$ $M_{279}=5p_1+p_2+p_3+p_4+3p_5+2p_8+12p_9+p_{11}+12p_{13}+p_{16}+2r_1+r_5$ $M_{280}=7p_1+p_2+p_3+p_4+2p_5+p_8+12p_9+p_{11}+11p_{13}+p_{15}+p_{16}+2r_1+r_5$ $M_{281}=6p_1+p_2+p_3+2p_5+3p_8+12p_9+p_{11}+11p_{13}+p_{14}+p_{16}+2r_1$ $M_{282}=8p_1+p_2+p_3+p_5+2p_8+12p_9+p_{11}+10p_{13}+p_{14}+p_{15}+p_{16}+2r_1$

$\widetilde{P}_{R11}=\{r_2, r_3, r_4, r_6, r_7\}$ $\acute{M}_{11}=a_1p_1+p_2+a_3p_3+a_4p_4+a_5p_5+a_7p_7+a_8p_8+a_{14}p_{14}+a_{15}p_{15}+a_{16}p_{16}+12p_9+p_{11}+a_{13}p_{13}+2r_1+a_5'r_5$ where $a_3+a_7+a_{16}=2$, $a_5+a_{14}+a_{15}=3$, and $a_4+a_8+a_{15}=3$	$M_{283}=6p_1+p_2+p_3+3p_4+2p_5+12p_9+p_{11}+11p_{13}+p_{14}+p_{16}+2r_1$ $M_{284}=8p_1+p_2+p_3+2p_4+p_5+12p_9+p_{11}+10p_{13}+p_{14}+p_{15}+p_{16}+2r_1$ $M_{285}=6p_1+p_2+p_3+2p_4+2p_5+p_8+12p_9+p_{11}+11p_{13}+p_{14}+p_{16}+2r_1$ $M_{286}=6p_1+p_2+p_3+p_4+2p_5+2p_8+12p_9+p_{11}+11p_{13}+p_{14}+p_{16}+2r_1$ $M_{287}=8p_1+p_2+p_3+p_4+p_5+p_8+12p_9+p_{11}+10p_{13}+p_{14}+p_{15}+p_{16}+2r_1$ $M_{288}=9p_1+p_2+p_3+p_7+p_8+12p_9+p_{11}+10p_{13}+p_{14}+2p_{15}+2r_1$ $M_{289}=9p_1+p_2+p_3+p_4+p_7+12p_9+p_{11}+10p_{13}+p_{14}+2p_{15}+2r_1$ $M_{290}=4p_1+p_2+p_3+3p_5+p_7+3p_8+12p_9+p_{11}+13p_{13}+2r_1+r_5$ $M_{291}=6p_1+p_2+p_3+2p_5+p_7+2p_8+12p_9+p_{11}+12p_{13}+p_{15}+2r_1+r_5$ $M_{292}=8p_1+p_2+p_3+p_5+p_7+p_8+12p_9+p_{11}+11p_{13}+2p_{15}+2r_1+r_5$ $M_{293}=4p_1+p_2+p_3+3p_4+3p_5+p_7+12p_9+p_{11}+13p_{13}+2r_1+r_5$ $M_{294}=6p_1+p_2+p_3+2p_4+2p_5+p_7+12p_9+p_{11}+12p_{13}+p_{15}+2r_1+r_5$ $M_{295}=8p_1+p_2+p_3+p_4+p_5+p_7+12p_9+p_{11}+11p_{13}+2p_{15}+2r_1+r_5$ $M_{296}=4p_1+p_2+p_3+2p_4+3p_5+p_7+p_8+12p_9+p_{11}+13p_{13}+2r_1+r_5$ $M_{297}=4p_1+p_2+p_3+p_4+3p_5+p_7+2p_8+12p_9+p_{11}+13p_{13}+2r_1+r_5$ $M_{298}=6p_1+p_2+p_3+p_4+2p_5+p_7+p_8+12p_9+p_{11}+12p_{13}+p_{15}+2r_1+r_5$ $M_{299}=5p_1+p_2+p_3+2p_5+p_7+3p_8+12p_9+p_{11}+12p_{13}+p_{14}+2r_1$ $M_{300}=7p_1+p_2+p_3+p_5+p_7+2p_8+12p_9+p_{11}+11p_{13}+p_{14}+p_{15}+2r_1$

Table A3
Bad markings of PN in Fig. 6

$M_1''=a_1p_1+p_2+a_4p_4+a_5p_5+a_8p_8+12p_9+p_{11}+a_{13}p_{13}+a_{14}p_{14}+2r_1+2r_3+a_5'r_5$, where $a_4+a_8=3$ and $a_5+a_{14}=3$
$M_2''=a_1p_1+p_2+a_4p_4+2p_5+a_8p_8+13p_9+12p_{13}+p_{14}+2r_1+2r_3+r_7$,
where $a_4+a_8=3$
$M_3''=a_1p_1+p_2+a_3p_3+a_4p_4+2p_5+a_7p_7+a_8p_8+13p_9+12p_{13}+p_{14}+2r_1+r_3+r_7$,
where $a_3+a_7=1$ and $a_4+a_8=3$
$M_4''=a_1p_1+p_2+a_3p_3+a_4p_4+2p_5+a_7p_7+a_8p_8+12p_9+p_{11}+12p_{13}+p_{14}+2r_1+r_3$,
where $a_3+a_7=1$ and $a_4+a_8=3$
$M_5''=12p_1+p_2+13p_9+a_{13}p_{13}+p_{14}+a_{15}p_{15}+2p_{16}+2r_1+a_4'r_4+a_6'r_6+r_7$,
where $a_{15}\in\{0, 1\}$
$M_6''=12p_1+p_2+13p_9+8p_{13}+p_{14}+2p_{15}+2p_{16}+2r_1+r_6+r_7$
$M_7''=12p_1+p_2+12p_9+p_{11}+8p_{13}+p_{14}+2p_{15}+2p_{16}+2r_1+r_6$
$M_8''=12p_1+p_2+12p_9+p_{10}+8p_{13}+p_{14}+2p_{15}+2p_{16}+2r_1$
$M_9''=12p_1+p_2+12p_9+p_{11}+9p_{13}+2p_{15}+2p_{16}+2r_1+r_4+r_5+r_6$
$M_{10}''=12p_1+p_2+12p_9+p_{10}+9p_{13}+2p_{15}+2p_{16}+2r_1+r_4+r_5$
$M_{11}''=12p_1+p_2+12p_9+p_{11}+9p_{13}+p_{14}+p_{15}+2p_{16}+2r_1+r_4+2r_6$
$M_{12}''=12p_1+p_2+12p_9+p_{10}+9p_{13}+p_{14}+p_{15}+2p_{16}+2r_1+r_4+r_6$
$M_{13}''=12p_1+p_2+12p_9+p_{11}+10p_{13}+p_{15}+2p_{16}+2r_1+2r_4+r_5+2r_6$
$M_{14}''=12p_1+p_2+12p_9+p_{10}+10p_{13}+p_{15}+2p_{16}+2r_1+2r_4+r_5+r_6$
$M_{15}''=12p_1+p_2+12p_9+p_{11}+10p_{13}+p_{14}+2p_{16}+2r_1+2r_4+3r_6$
$M_{16}''=12p_1+p_2+12p_9+p_{10}+10p_{13}+p_{14}+2p_{16}+2r_1+2r_4+2r_6$
$M_{17}''=12p_1+p_2+12p_9+p_{11}+11p_{13}+2p_{16}+2r_1+3r_4+r_5+3r_6$
$M_{18}''=12p_1+p_2+12p_9+p_{10}+11p_{13}+2p_{16}+2r_1+3r_4+r_5+2r_6$

Table A4
ATI-partial deadlocks of PN in Fig. 6

CRSs	ATI-partial deadlocks
$\widetilde{P}_{R1}=\{r_4, r_6, r_7\}$	$\acute{M}_1=7p_1+a_4p_4+3p_5+a_8p_8+12p_9+p_{11}+13p_{13}+3r_1+r_2+2r_3+r_5$, where $a_4+a_8=3$
$\widetilde{P}_{R2}=\{r_4, r_5, r_6\}$	$\acute{M}_2=8p_1+a_4p_4+2p_5+a_8p_8+13p_9+12p_{13}+p_{14}+3r_1+r_2+2r_3+r_7$, where $a_4+a_8=3$
$\widetilde{P}_{R3}=\{r_3, r_6\}$	$\acute{M}_3=11p_1+a_3p_3+a_7p_7+13p_9+10p_{13}+3p_{15}+3r_1+r_2+r_5+r_7$, where $a_3+a_7=2$
$\widetilde{P}_{R5}=\{r_3, r_4, r_5, r_6\}$	$\acute{M}_4=a_1p_1+a_3p_3+a_4p_4+a_5p_5+a_7p_7+a_8p_8+13p_9+a_{13}p_{13}+p_{14}+a_{15}p_{15}+3r_1+r_2+r_7$, where $a_3+a_7=2$, $a_4+a_8+a_{15}=3$, and $a_5+a_{15}=2$,
$\widetilde{P}_{R6}=\{r_3, r_4, r_6, r_7\}$	$\acute{M}_5=a_1p_1+a_3p_3+a_4p_4+a_5p_5+a_7p_7+a_8p_8+12p_9+p_{11}+a_{13}p_{13}+a_{15}p_{15}+3r_1+r_2+r_5$, where $a_3+a_7=2$, $a_4+a_8+a_{15}=3$, and $a_5+a_{15}=3$
$\widetilde{P}_{R7}=\{r_2, r_3\}$	$\acute{M}_6=12p_1+p_2+13p_9+11p_{13}+2p_{16}+2r_1+3r_4+r_5+3r_6+r_7$
$\widetilde{P}_{R8}=\{r_2, r_3, r_6\}$	$\acute{M}_7=11p_1+p_2+a_3p_3+a_7p_7+13p_9+9p_{13}+3p_{15}+p_{16}+2r_1+r_5+r_7$, where $a_3+a_7=1$
$\widetilde{P}_{R10}=\{r_2, r_3, r_4, r_5, r_6\}$	$\acute{M}_8=a_1p_1+p_2+a_3p_3+a_4p_4+a_5p_5+a_7p_7+a_8p_8+13p_9+a_{13}p_{13}+p_{14}+a_{15}p_{15}+a_{16}p_{16}+2r_1+r_7$, where $a_3+a_7+a_{16}=2$, $a_4+a_8+a_{15}=3$, and $a_5+a_{15}=2$
$\widetilde{P}_{R11}=\{r_2, r_3, r_4, r_6, r_7\}$	$\acute{M}_9=a_1p_1+p_2+a_3p_3+a_4p_4+a_5p_5+a_7p_7+a_8p_8+12p_9+p_{11}+a_{13}p_{13}+a_{15}p_{15}+a_{16}p_{16}+2r_1+r_5$, where $a_3+a_7+a_{16}=2$, $a_4+a_8+a_{15}=3$, and $a_5+a_{15}=3$

Table A5
Information of control places for PN in Fig. 6

Markings	Control places	Pre-set	Post-set	$M_0(p_c)$
$\acute{M}_1$	p_{c1}	t_5, t_{12}	t_3, t_8, t_{11}	6
$\acute{M}_2$	p_{c2}	t_5, $2t_{15}$	t_3, t_8, $2t_{14}$	6
$\acute{M}_3$	p_{c3}	t_3, t_8, t_{16}	t_2, t_7, t_{15}	4
$\acute{M}_4$	p_{c4}	t_5, $2t_{16}$	t_2, t_7, $2t_{14}$	8
$\acute{M}_5$	p_{c5}	t_5, t_{12}, $2t_{16}$	t_2, t_7, t_{11}, $2t_{15}$	8
$\acute{M}_6$	p_{c6}	t_2, t_7, t_{17}	t_1, t_{16}	2
$\acute{M}_7$	p_{c7}	t_3, t_8, t_{17}	t_1, t_{15}	5
$\acute{M}_8$	p_{c8}	t_5, t_{16}, t_{17}	t_1, $2t_{14}$	9
$\acute{M}_9$	p_{c9}	t_5, t_{12}, t_{16}, t_{17}	t_1, t_{11}, $2t_{15}$	9