1	2	3	4	5	6	7	8	9	10	11	12	sum	grade

Discrete Systems Final Exam 2016: Student name ______________ Student ID ______________

1. Let x and y be real numbers such that $0 < x < 10$ and $-2 < y < 2$.
   Is the proposition $\exists x \forall y \; x > y^3$ true or false? *Justify your answer.*

2. Using the Pigeonhole Principle show that if any eight positive integers are chosen, than at least two of them will have the same remainder when divided by 7.

3. Let us consider the integer $I = 1995505$ and other integers generated by rearranging its symbols (for example, 9195505). In addition, the leading 0's have to be omitted, for example, the string "00130" means the number 130, that is "00130"= 130. Find how many different integers I larger than $1000000 \equiv 10^6$ but less than $6000000 \equiv 6 \times 10^6$ can be generated in this way:

$$1000000 < I < 6000000 \,.$$

4. Let M_n denote the number of all possible words of length n on the alphabet $\{a, b, c\}$ which do not contain the substring "aa".

   - Construct a recurrence relation for the number M_n of such words of length n.
   - Construct a formula calculating M_n.

5. Let $R_x = \{x : -1 \leq x \leq 1\}$ and $R_y = \{y : -\infty < y < \infty\}$ be sets of real numbers x, y and f be a relation in $R_x \times R_y$. The relation f is determined by the condition that $(x, y) \in f$ if

$$y = f(x), \qquad \text{where} \quad f(x) = x + \frac{1}{x}.$$

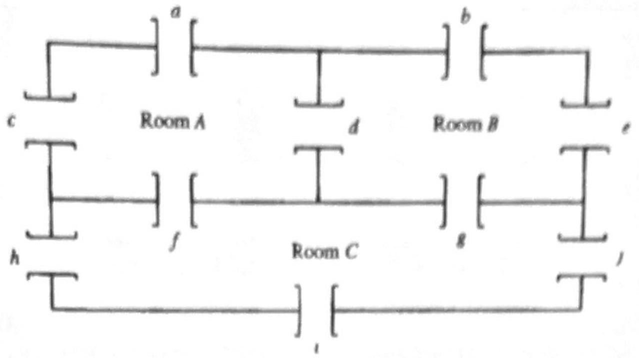
Construct the relation f^{-1} and determine whether it is a function from R_y to R_x. *Justify your answer.*

6. Let $A = \{1, 2, 3, 4\}$ bet a set of 4 elements and

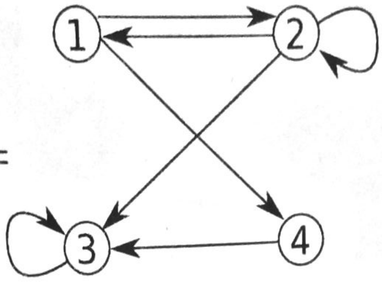
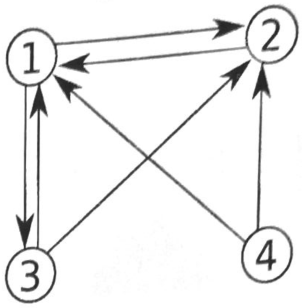
$$R = \{(1,1), (1,2), (2,1), (2,2), (2,4), (4,2), (3,3), (3,4), (4,3)\}$$

be a relation on A. Construct (i) the direct graph of R, (ii) the matrix M_R of R, and determine whether the relation R is (a) symmetric, (b) reflexive, (c) transitive. *Justify your answer.*

7. An inspector of doors has to check the quality of doors in an apartment whose plan is shown in figure below. Is it possible for him to pass (for inspection) through all the doors in such a way that he pass through each door only one time? *Justify your answer.*



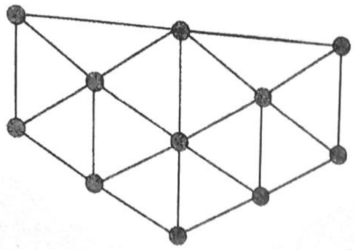
8. Let $A = \{1, 2, 3, 4\}$ be a set of 4 elements and f, g be relations on it that are determined by the following directed graphs.

$f =$  $g =$ 

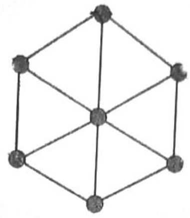
Construct the directed graph of the relation $f \circ g$ on A and its adjacency matrix. *Justify your answer.* You can choose any one of the approaches:

- (a) solution using the definition of the relation composition,
- (b) solution based on the use of the adjacency matrices.

9. Find the chromatic number of the following graphs. *Justify your answer.*

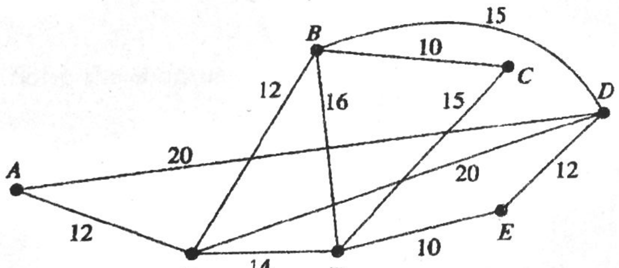


A



B

10. Using Kruskal's algorithm find a minimum spanning tree and its weight for the following weighted graph



11. Find gcd(4550,8085) and lcm(4550,8085). *Justify your answer.*

12. Solve the congruence equation $6x \equiv 1 \pmod 5$. *Justify your answer.*