ALGORITHMS ANS DATA STRUCTURES II

Lecture 7

Greedy Algorithms,

File compression,

Huffman codes.

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GREEDY ALGORITHMS

- Algorithms for optimization problems typically go through a sequence of steps, with a set of choices at each step.

- A **greedy algorithm** always makes the choice that looks best at the moment. That is, it makes a **locally optimal** choice in the hope that this choice will lead to a globally optimal solution.

GREEDY ALGORITHMS

- Examples of greedy algorithms:
  - **Prim's** algorithm (for the minimum spanning tree problem).
  - **Kruskal's** algorithm (for the minimum spanning tree problem).
  - **Dijkstra's** algorithm (for the single source shortest path problem).

GREEDY ALGORITHMS

- A greedy algorithm obtains an optimal solution to a problem by making a **sequence** of choices.
- At each decision point, the algorithm makes a choice that **seems best** at the moment.
- This strategy **does not always** produce an optimal solution.

FILE COMPRESSION

- **Objective:** conserving of space (the memory space needed for storing the file)

- **Method:** use the redundancy (save space by exploiting the fact that most files have a relatively low "information content")

RUN-LENGTH ENCODING

- The simplest type of redundancy in a file is long runs (sequences) of repeated characters (no digits are allowed) e.g.: **AAAABBBAABBBBBBCCCCCCCCDABCBAAABBBBCCCD**

- Above string can be encoded as : **4A3BAA5B8CDABCB3A4B3CD** (every digit denotes the number of repetitions for the character that follows it)

RUN-LENGTH ENCODING

- If we want to use the same character set for encoding the lengths we need an "escape character" **(Q)** which signals the beginning of a (count, character) pair

- **AAAABBBAABBBBBBCCCCCCCCCDABCBAAA BBBBCCCD** can be encoded as: **QDABBBAAQEBQHCDABCBAAAQDBCCCD**

- Repetitions: **D**=4,**E**=5,**F**=6,**G**=7,**H**=8… **QQ** is used for encoding a single **Q**

RUN-LENGTH ENCODING

- Bitmap encoding example

00000000000000000000000000011111111111111000000000	28 14 9
00000000000000000000000000111111111111111110000000	26 18 7
00000000000000000000000001111111111111111111110000	23 24 4
00000000000000000000000011111111111111111111111000	22 26 3
00000000000000000000000111111111111111111111111110	20 30 1
00000000000000000000011111110000000000000000001111111	19 7 18 7
00000000000000000000111110000000000000000000000011111	19 5 22 5
00000000000000000000111000000000000000000000000000111	19 3 26 3
00000000000000000000111000000000000000000000000000111	19 3 26 3
00000000000000000000111000000000000000000000000000111	19 3 26 3
00000000000000000000111000000000000000000000000000111	19 3 26 3
00000000000000000000111100000000000000000000000001110	20 4 23 3 1
00000000000000000000011100000000000000000000000111000	22 3 20 3 3
01111111111111111111111111111111111111111111111111111	1 50
01111111111111111111111111111111111111111111111111111	1 50
01111111111111111111111111111111111111111111111111111	1 50
01111111111111111111111111111111111111111111111111111	1 50
01111111111111111111111111111111111111111111111111111	1 50
01100000000000000000000000000000000000000000000000011	1 2 46 2

VARIABLE-LENGTH ENCODING

- Assume that the 26 letters **A, B, ... Z** are assigned the 5-bits binary codes as follows:

  1. **A : 00001**, 2. **B : 00010**,
  3. **C : 00011**, ... , 26.**Z : 11010**.

- Given **ABRACADABRA**, using the 5-bits binary representation, it is encoded as the following:

VARIABLE-LENGTH ENCODING

ABRACADABRA:

00001000101001000001000011000010010000010000101001000001

- To decode this message, read off 5 bits at a time and convert according to the above assignment.
- In the sequence, **D** appears once and **A** appears 5 times, while they are both assigned 5 bits in the binary codes.

VARIABLE-LENGTH ENCODING

- We may give an assignment like
  A : 0, B : 1, R : 01, C : 10, D : 11,
  and **ABRACADABRA** is encoded as
  0␣1␣01␣0␣10␣0␣11␣0␣1␣01␣0
- This uses **15** bits compared with the **55** bits before. But, this code depends on the blanks to delimit the characters. Without the blanks, the string **01010100110101 0** could be decoded as **RRRARBRRA** or as several other strings.

VARIABLE-LENGTH ENCODING

- How to encode the string compactly without using the delimiters?
- If the code of a character is **not** the prefix of any code of other characters, then delimiters are not needed.
  - **0**, **00**, **001**, and **0011** are prefix of **00110**.
  - In **A** : **0**, **B** : **1**, **R** : **01**, **C** : **10**, **D** : **11**, the code of **A** is a prefix of **R**, the code of **B** is a prefix of **C** and **D**.

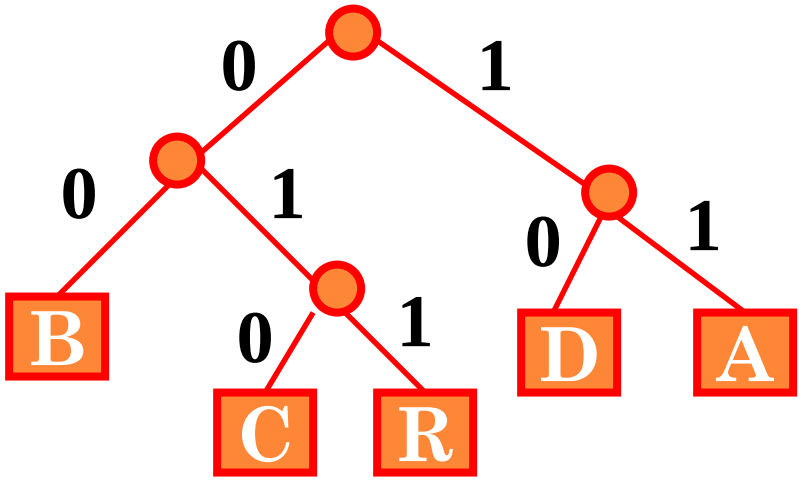
VARIABLE-LENGTH ENCODING

- A binary tree with *M* leaves can be used to encode any message with *M* different characters (one leaf holds one character).

- The code for each character is determined by the path from the root to the leaf that holds that character, with 0 for go 'left' and 1 for go 'right'.

VARIABLE-LENGTH ENCODING

- **Example:** The following binary tree encodes A as 11, B as 00, C as 010, D as 10, and R as 011.



ABRACADABRA is encoded as

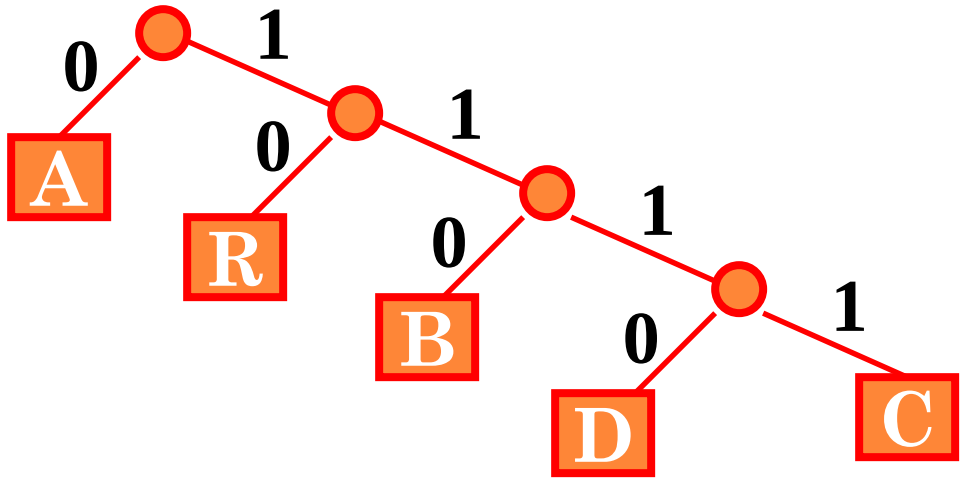
11000111101011011000111

VARIABLE-LENGTH ENCODING

- The binary tree representation **guarantees** that no character code is a prefix of another, so the string is uniquely decodable from the tree.
- **Decoding:** Starting at the root, proceed down the tree according to the bits of the message: each time a leaf is encountered, output the character at that leaf and restart at the root.

VARIABLE-LENGTH ENCODING

- The following is another binary tree for encoding **ABRACADABRA**.



VARIABLE-LENGTH ENCODING

- **Question:** which binary tree is the best one to use (gives the shortest binary sequence for a given string of characters)?

- D. Huffman gave an elegant method (called **Huffman encoding**) to compute a binary tree which leads to a bit string of minimal length for any given string of characters.

HUFFMAN ENCODING

- **Algorithm:** Let S be the set of characters that appear in a given string.

  Step 1: Count the frequency of each character of S in the given string. Consider S as a set of roots of binary trees, each tree has one node.

  Step 2: Find two roots with the minimum frequencies and generate a new node as the root of the two roots. The new root has the frequency of the sum of frequencies of its two children. Delete the two children from S and add the new root to S.

  Step 3: If $|S| \neq 1$, go to **Step 2**.

HUFFMAN ENCODING

- Given the string **ABRACADABRA** we have:

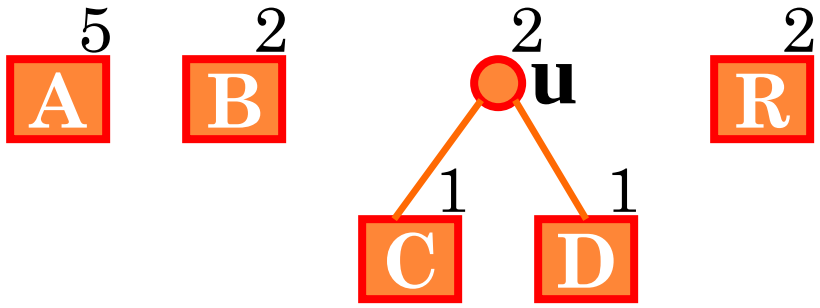
$S = \{\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{R}\},$

$w(\mathbf{A}) = 5,\ \ w(\mathbf{B}) = 2,\ \ w(\mathbf{C}) = 1,$

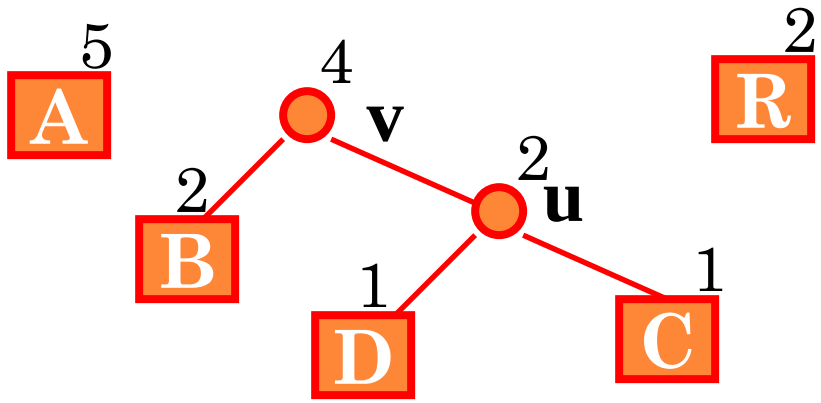
$w(\mathbf{D}) = 1,\ \ w(\mathbf{R}) = 2.$

HUFFMAN ENCODING

S = {A, B, u, R},

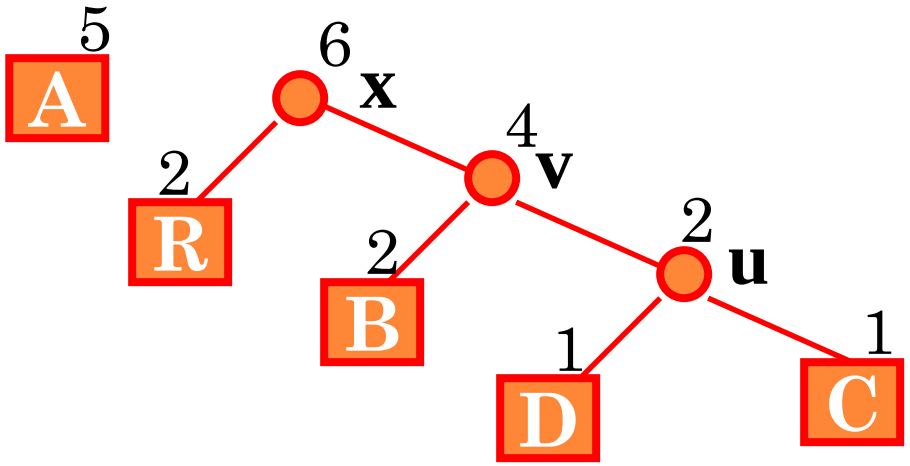


S = {A, v, R},

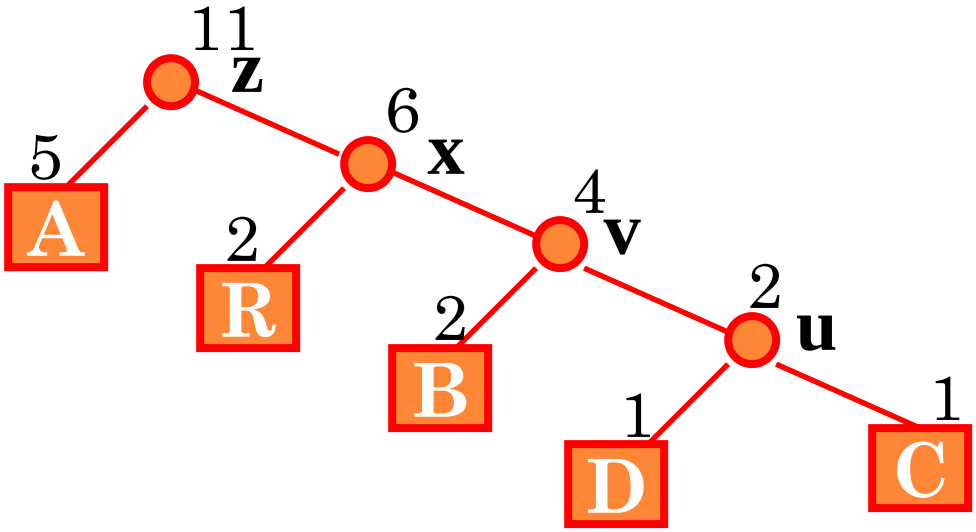


HUFFMAN ENCODING

S = {**A, x**},

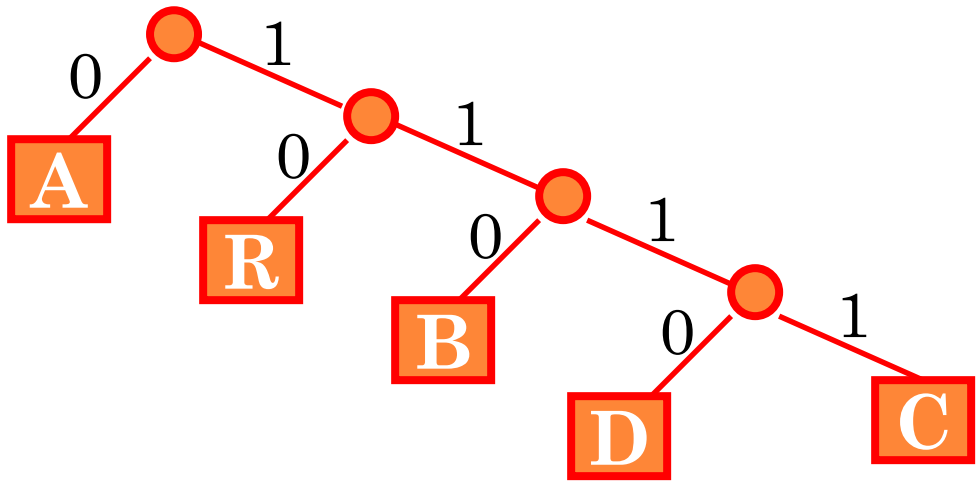


S = {**z**},



HUFFMAN ENCODING

- Encoding tree:



- We assign the binary codes to the characters of S as: **A : 0, B : 110, C : 1111, D : 1110,** and **R : 10.**

HUFFMAN ENCODING

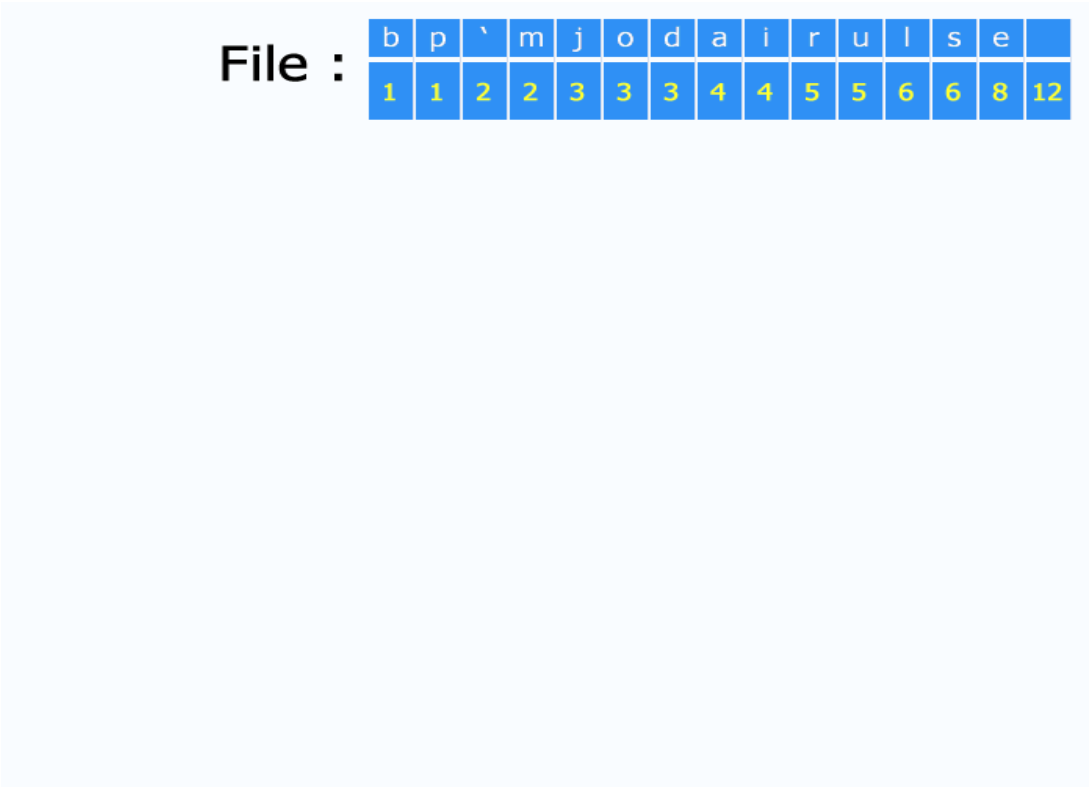
- Algorithm implementation using minimum priority queue:

```
def HUFFMAN (S)
// Input: S – set of n letters with attribute freq giving their
// frequency.  It is also the key for Q.
    n = |S|
    Q = MIN-PRIORITY-QUEUE (S)
    for i = 1 to n -1
        allocate new node z
        z.left = x = EXTRACT-MIN (Q)
        z.right = y = EXTRACT-MIN (Q)
        z.freq = x.freq + y.freq
        INSERT (Q, z)
    return EXTRACT-MIN (Q)
```

HUFFMAN ENCODING

- Animated example:

"j'aime aller sur le bord de l'eau les jeudis ou les jours impairs"

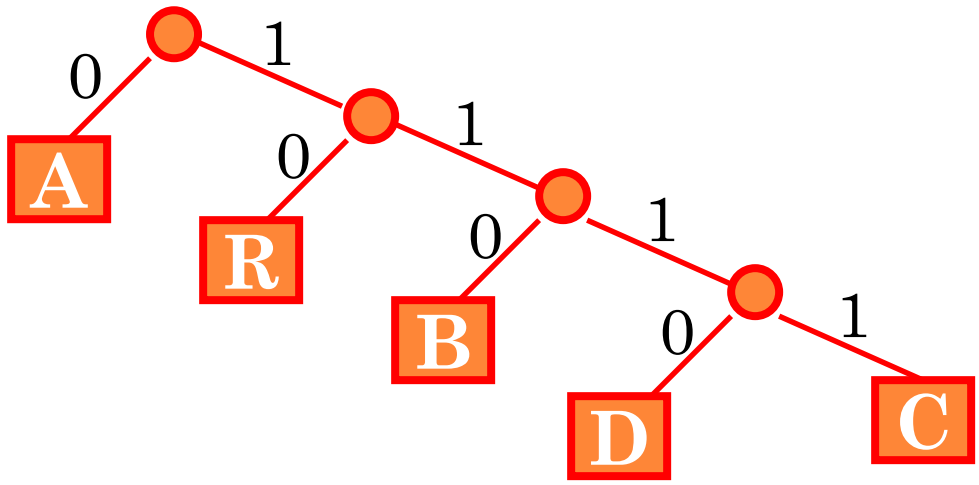


HUFFMAN ENCODING

- **Decoding**:
  - Starting with the first bit in the stream, one then uses successive bits from the stream to determine whether to go left or right in the decoding tree.
  - When we reach a leaf (character) of the tree we place that character to the output (decoded) stream.
  - Note: Compressed file must contain the table of codes.

HUFFMAN ENCODING

- Decoding example:



0110100111101110100110100

A B RA C A D A B RA

That's All for Today!