Exercise 9. Answer Sheet

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Problem 1. (40 points) Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions is <5, 10, 3, 12, 5, 50, 6>. Show your work (costs matrix **m,** number of multiplications).

Table of length 2:

	B (10 * 3)	C (3 * 12)	D (12 * 5)	E (5 * 50)	F (50 * 6)
A (5 * 10)	150				
B (10 * 3)	-	360			
C (3 * 12)	-	-	180		
D (12 * 5)	-	-	-	3000	
E (5 * 50)	-	-	-	-	1500

M_{AB} = 5 * 10 * 3 = 150
M_{BC} = 10 * 3 * 12 = 360
M_{CD} = 3 * 12 * 5 = 180
M_{DE} = 12 * 5 * 50 = 3000
M_{EF} = 5 * 50 * 6 = 1500

Next, by calculating the table of length 3, it gives the matrix below.

	B (10 * 3)	C (3 * 12)	D (12 * 5)	E (5 * 50)	F (50 * 6)
A (5 * 10)	150 [A][B]	330 [AB][C]			
B (10 * 3)	-	360 [B][C]	330 [B][CD]		
C (3 * 12)	-	-	180 [C][D]	930 [CD][E]	
D (12 * 5)	-	-	-	3000 [D][E]	1860 [D][EF]
E (5 * 50)	-	-	-	-	1500 [E][F]

M_{AC} (A*(B*C)) = 0 + 360 + 5 * 10 * 12 = 960
M_{AC} ((A*B)*C) = 150 + 0 + 5 * 3 * 12 = 330
M_{BD} (B*(C*D)) = 0 + 180 + 10 * 3 * 5 = 330
M_{BD} ((B*C)*D) = 360 + 0 + 10 * 12 * 5 = 960
M_{CE} (C*(D*E)) = 0 + 3000 + 3 * 12 * 50 = 4800
M_{CE} ((C*D)*E) = 180 + 0 + 3 * 5 * 50 = 930
M_{DF} (D*(E*F)) = 0 + 1500 + 12 * 5 * 6 = 1860
M_{DF} ((D*E)*F) = 3000 + 0 + 12 * 50 * 6 = 3360

Next, by calculating the table of length 4, it gives the matrix below.

	B (10 * 3)	C (3 * 12)	D (12 * 5)	E (5 * 50)	F (50 * 6)
A (5 * 10)	150 [A][B]	330 [AB][C]	405 [AB][CD]		
B	-	360	330	2430	

(10 * 3)		[B][C]	[B][CD]	[B][CDE]
C (3 * 12)	-	-	180 [C][D]	930 [CD][E]	1770 [CD][EF]
D (12 * 5)	-	-	-	3000 [D][E]	1860 [D][EF]
E (5 * 50)	-	-	-	-	1500 [E][F]

M_{AD} (A*(B*C*D)) = 0 + 330 + 5 * 10 * 5 = 580
M_{AD} ((A*B)*(C*D)) = 150 + 180 + 5 * 3 * 5 = 405
M_{AD} ((A*B*C)*D) = 330 + 0 + 5 * 12 * 5 = 630
M_{BE} (B*(C*D*E)) = 0 + 930 + 10 * 3 * 50 = 2430
M_{BE} ((B*C)*(D*E)) = 360 + 3000 + 10 * 12 * 50 = 3960
M_{BE} ((B*C*D)*E) = 330 + 0 + 10 * 5 * 50 = 2830
M_{CF} (C*(D*E*F)) = 0 + 1860 + 3 * 12 * 6 = 2076
M_{CF} ((C*D)*(E*F)) = 180 + 1500 + 3 * 5 * 6 = 1770
M_{CF} ((C*D*E)*F) = 930 + 0 + 3 * 50 * 6 = 1830

Next, by calculating the table of length 5, it gives the matrix below.

	B (10 * 3)	**C** (3 * 12)	**D** (12 * 5)	**E** (5 * 50)	**F** (50 * 6)
A (5 * 10)	150 [A][B]	330 [AB][C]	405 [AB][CD]	1655 [AB][CDE]	
B (10 * 3)	-	360 [B][C]	330 [B][CD]	2430 [B][CDE]	1950 [B][CDEF]
C (3 * 12)	-	-	180 [C][D]	930 [CD][E]	1770 [CD][EF]
D (12 * 5)	-	-	-	3000 [D][E]	1860 [D][EF]
E (5 * 50)	-	-	-	-	1500 [E][F]

M_{AE} (A*(B*C*D*E)) = 0 + 2430 + 5 * 10 * 50 = 4930
M_{AE} ((A*B)*(C*D*E)) = 150 + 930 + 5 * 3 * 50 = 1830
M_{AE} ((A*B*C)*(D*E)) = 330 + 3000 + 5 * 12 * 50 = 3630
M_{AE} ((A*B*C*D)*E) = 405 + 0 + 5 * 5 * 50 = 1655
M_{BF} (B*(C*D*E*F)) = 0 + 1770 + 10 * 3 * 6 = 1950
M_{BF} ((B*C)*(D*E*F)) = 360 + 1860 + 10 * 12 * 6 = 2940
M_{BF} ((B*C*D)*(E*F)) = 330 + 1500 + 10 * 5 * 6 = 2130
M_{BF} ((B*C*D*E)*F) = 2430 + 0 + 10 * 50 * 6 = 5430

Finally, by calculating the table of length 6, it gives the matrix below.
This is the **optimal parenthesization** of a matrix-chain product.

	B (10 * 3)	**C** (3 * 12)	**D** (12 * 5)	**E** (5 * 50)	**F** (50 * 6)
A (5 * 10)	150 [A][B]	330 [AB][C]	405 [AB][CD]	1655 [AB][CDE]	2010 [AB][CDEF]
B (10 * 3)	-	360 [B][C]	330 [B][CD]	2430 [B][CDE]	1950 [B][CDEF]
C (3 * 12)	-	-	180 [C][D]	930 [CD][E]	1770 [CD][EF]
D (12 * 5)	-	-	-	3000 [D][E]	1860 [D][EF]
E (5 * 50)	-	-	-	-	1500 [E][F]

M_{AF} (A*(B*C*D*E*F)) = 0 + 1950 + 5 * 10 * 6 = 2250
M_{AF} ((A*B)*(C*D*E*F)) = 150 + 1770 + 5 * 3 * 6 = 2010
M_{AF} ((A*B*C)*(D*E*F)) = 330 + 1860 + 5 * 12 * 6 = 2550
M_{AF} ((A*B*C*D)*(E*F)) = 405 + 1500 + 5 * 5 * 6 = 2055
M_{AF} ((A*B*C*D*E)*F) = 1655 + 0 + 5 * 50 * 6 = 3155

Problem 2. (60 points) Write a program implementing the algorithms Matrix-Chain-Order and Print-Optimal-Parens given in the lecture. Upload your code. Using your program, find the optimal parenthesization for the following matrix-chain products and show your **m** and **s** matrices.

a) (20 points) p = [30,35,15,5,10,20,25]

Optimal parenthesization is **(A(BC))((DE)F)**

Matrix m is below.

	B	C	D	E	F
A	15750	7875	9375	11875	15125
B	-	2625	4375	7125	10500
C	-	-	750	2500	5375
D	-	-	-	1000	3500
E	-	-	-	-	5000

b) (20 points) p = [10,20,10,15,20,10]

Optimal parenthesization is **(AB)(C(DE))**

Matrix m is below.

	B	C	D	E
A	2000	3500	6500	7500
B	-	3000	7000	6500
C	-	-	3000	4500
D	-	-	-	3000

c) (20 points) p = [100,10,100,1,1000,100]

Optimal parenthesization is **(A(BC))(DE)**

Matrix m is below.

	B	C	D	E
A	100000	2000	102000	112000
B	-	1000	11000	102000
C	-	-	100000	110000
D	-	-	-	100000

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[std6dc24{s1240234}59: javac MatrixChainMultiplication.java
[std6dc24{s1240234}60: java MatrixChainMultiplication
Please enter the sequence of dimensions separated by space(Enter any String to end the Input):30 35 15 5 10 20 25 END
Matrix m:
0       15750   7875    9375    11875   15125
0       0       2625    4375    7125    10500
0       0       0       750     2500    5375
0       0       0       0       1000    3500
0       0       0       0       0       5000
0       0       0       0       0       0
Optimal Parenthesization:((A(AA))((AA)A))
[std6dc24{s1240234}61: java MatrixChainMultiplication
Please enter the sequence of dimensions separated by space(Enter any String to end the Input):10 20 10 15 20 10 END
Matrix m:
0       2000    3500    6500    7500
0       0       3000    7000    6500
0       0       0       3000    4500
0       0       0       0       3000
0       0       0       0       0
Optimal Parenthesization:((AA)(A(AA)))
[std6dc24{s1240234}62: java MatrixChainMultiplication
Please enter the sequence of dimensions separated by space(Enter any String to end the Input):100 10 100 1 1000 100 END
Matrix m:
0       100000  2000    102000  112000
0       0       1000    11000   102000
0       0       0       100000  110000
0       0       0       0       100000
0       0       0       0       0
Optimal Parenthesization:((A(AA))(AA))
std6dc24{s1240234}63:
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