

Exercise 11. Answer Sheet

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Problem 1. (40 points) Consider constructing a random number generator for integers from 1 to 6 using the simplest linear congruential method.

a) What is the equation of this generator? $x_{i+1} = (A * x_i) \bmod 7$

b) Which values of the parameter $A \in [1, 6]$ give the longest sequence?

3 and 5 gives the longest sequence.

The sequence of 3 is “3, 2, 6, 4, 5, 1”

The sequence of 5 is “5, 4, 6, 2, 3, 1”

First, $x_0 = 1$

When $A = 1$:

$$x_1 = (1 * 1) \bmod 7 = 1 \bmod 7 = 1$$

$$x_2 = (1 * 1) \bmod 7 = 1 \bmod 7 = 1$$

$$x_3 = (1 * 1) \bmod 7 = 1 \bmod 7 = 1$$

$$x_4 = (1 * 1) \bmod 7 = 1 \bmod 7 = 1$$

$$x_5 = (1 * 1) \bmod 7 = 1 \bmod 7 = 1$$

$$x_6 = (1 * 1) \bmod 7 = 1 \bmod 7 = 1$$

When $A = 2$:

$$x_1 = (2 * 1) \bmod 7 = 2 \bmod 7 = 2$$

$$x_2 = (2 * 2) \bmod 7 = 4 \bmod 7 = 4$$

$$x_3 = (2 * 4) \bmod 7 = 8 \bmod 7 = 1$$

$$x_4 = (2 * 1) \bmod 7 = 2 \bmod 7 = 2$$

$$x_5 = (2 * 2) \bmod 7 = 4 \bmod 7 = 4$$

$$x_6 = (2 * 4) \bmod 7 = 8 \bmod 7 = 1$$

When $A = 3$:

$$x_1 = (3 * 1) \bmod 7 = 3 \bmod 7 = 3$$

$$x_2 = (3 * 3) \bmod 7 = 9 \bmod 7 = 2$$

$$x_3 = (3 * 2) \bmod 7 = 6 \bmod 7 = 6$$

$$x_4 = (3 * 6) \bmod 7 = 18 \bmod 7 = 4$$

$$x_5 = (3 * 4) \bmod 7 = 12 \bmod 7 = 5$$

$$x_6 = (3 * 5) \bmod 7 = 15 \bmod 7 = 1$$

When $A = 4$:

$$x_1 = (4 * 1) \bmod 7 = 4 \bmod 7 = 4$$

$$x_2 = (4 * 4) \bmod 7 = 16 \bmod 7 = 2$$

$$x_3 = (4 * 2) \bmod 7 = 8 \bmod 7 = 1$$

$$x_4 = (4 * 1) \bmod 7 = 4 \bmod 7 = 4$$

$$x_5 = (4 * 4) \bmod 7 = 16 \bmod 7 = 2$$

$$x_6 = (4 * 2) \bmod 7 = 8 \bmod 7 = 1$$

When $A = 5$:

$$x_1 = (5 * 1) \bmod 7 = 5 \bmod 7 = 5$$

$x_2 = (5 * 5) \bmod 7 = 25 \bmod 7 = 4$
 $x_3 = (5 * 4) \bmod 7 = 20 \bmod 7 = 6$
 $x_4 = (5 * 6) \bmod 7 = 30 \bmod 7 = 2$
 $x_5 = (5 * 2) \bmod 7 = 10 \bmod 7 = 3$
 $x_6 = (5 * 3) \bmod 7 = 15 \bmod 7 = 1$
 When A = 6:
 $x_1 = (6 * 1) \bmod 7 = 6 \bmod 7 = 6$
 $x_2 = (6 * 6) \bmod 7 = 36 \bmod 7 = 1$
 $x_3 = (6 * 1) \bmod 7 = 6 \bmod 7 = 6$
 $x_4 = (6 * 6) \bmod 7 = 36 \bmod 7 = 1$
 $x_5 = (6 * 1) \bmod 7 = 6 \bmod 7 = 6$
 $x_6 = (6 * 6) \bmod 7 = 36 \bmod 7 = 1$

Problem 2. (60 points) Write a program implementing the 3 algorithms from the lecture. Upload your code.

a) (20 points) Fill the following table with the first 5 random numbers generated by each of the algorithms?

	1	2	3	4	5
Rand1	1688960045	707878749	1576208621	583533981	1844902829
Rand2	48271	182605794	1291394886	1914720637	2078669041
Rand2	890394181	479799993	1961812856	1081941947	1764863974

b) (40 points) Generate $N = \{10, 1000, 1000000\}$ real random numbers in the interval (0.0, 1.0) using each algorithm. Calculate the histogram of the number distribution (**in %**) for 10 intervals and fill the table:

N	0.0-0.1	0.1-0.2	0.2-0.3	0.3-0.4	0.4-0.5	0.5-0.6	0.6-0.7	0.7-0.8	0.8-0.9	0.9-1.0
Rand1										
10	0	10	10	30	0	10	10	20	10	0
1000	8.8	8.4	11.1	10.4	11.9	11.0	9.1	11.0	9.8	8.5
1000000	10.01	9.97	10.03	9.95	9.96	10.08	10.03	10.01	9.98	9.98
Rand2										
10	20	10	10	10	0	10	10	10	10	10
1000	9.9	10.7	9.4	10.0	9.4	9.8	10.6	10.9	9.0	10.3
1000000	10.00	10.02	10.00	9.98	10.04	10.04	9.96	10.01	9.98	9.98
Rand3										
10	10	10	10	0	20	10	0	0	30	10
1000	11.7	9.6	9.6	11.0	10.8	9.6	10.4	8.2	9.4	9.7
1000000	9.96	10.00	9.98	10.02	10.04	10.02	9.93	9.98	10.02	10.04

<How to compile/run>

Command:

javac LinearCongruentialGenerator.java

java LinearCongruentialGenerator

Then it automatically output the answer of this problem 2.

```
[std6dc35{s1240234}52: javac LinearCongruentialGenerator.java
```

```
[std6dc35{s1240234}53: java LinearCongruentialGenerator
```

```
<Problem 2-a, 5 random numbers>
```

```
Rand1: 1688960045 707878749 1576208621 583533981 1844902829
```

```
Rand2: 48271 182605794 1291394886 1914720637 2078669041
```

```
Rand3: 890394181 479799993 1961812856 1081941947 1764863974
```

```
<Problem 2-b, Histogram of each random method>
```

```
N      0.0-0.1 0.1-0.2 0.2-0.3 0.3-0.4 0.4-0.5 0.5-0.6 0.6-0.7 0.7-0.8 0.8-0.9 0.9-1.0
```

```
Rand1
```

```
10      0.00%   10.00%   10.00%   30.00%   0.00%   10.00%   10.00%   20.00%   10.00%   0.00%
```

```
1000    8.80%   8.40%   11.10%   10.40%   11.90%   11.00%   9.10%   11.00%   9.80%   8.50%
```

```
1000000 10.01%   9.97%   10.03%   9.95%   9.96%   10.08%   10.03%   10.01%   9.98%   9.98%
```

```
Rand2
```

```
10      20.00%   10.00%   10.00%   10.00%   0.00%   10.00%   10.00%   10.00%   10.00%   10.00%
```

```
1000    9.90%   10.70%   9.40%   10.00%   9.40%   9.80%   10.60%   10.90%   9.00%   10.30%
```

```
1000000 10.00%   10.02%   10.00%   9.98%   10.04%   10.04%   9.96%   10.01%   9.98%   9.98%
```

```
Rand3
```

```
10      10.00%   10.00%   10.00%   0.00%   20.00%   10.00%   0.00%   0.00%   30.00%   10.00%
```

```
1000    11.70%   9.60%   9.60%   11.00%   10.80%   9.60%   10.40%   8.20%   9.40%   9.70%
```

```
1000000 9.96%   10.00%   9.98%   10.02%   10.04%   10.02%   9.93%   9.98%   10.02%   10.04%
```

```
std6dc35{s1240234}54: █
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