**Exercise 9. Answer Sheet**

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***Problem 1.***  (40 points) Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions is <5, 10, 3, 12, 5, 50, 6>. Show your work (costs matrix **m,** number of multiplications).

Table of length 2:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **B**  **(10 \* 3)** | **C**  **(3 \* 12)** | **D**  **(12 \* 5)** | **E**  **(5 \* 50)** | **F**  **(50 \* 6)** |
| **A**  **(5 \* 10)** | 150 |  |  |  |  |
| **B**  **(10 \* 3)** | - | 360 |  |  |  |
| **C**  **(3 \* 12)** | - | - | 180 |  |  |
| **D**  **(12 \* 5)** | - | - | - | 3000 |  |
| **E**  **(5 \* 50)** | - | - | - | - | 1500 |

MAB = 5 \* 10 \* 3 = 150

MBC = 10 \* 3 \* 12 = 360

MCD = 3 \* 12 \* 5 = 180

MDE = 12 \* 5 \* 50 = 3000

MEF = 5 \* 50 \* 6 = 1500

Next, by calculating the table of length 3, it gives the matrix below.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **B**  **(10 \* 3)** | **C**  **(3 \* 12)** | **D**  **(12 \* 5)** | **E**  **(5 \* 50)** | **F**  **(50 \* 6)** |
| **A**  **(5 \* 10)** | 150  [A][B] | 330  [AB][C] |  |  |  |
| **B**  **(10 \* 3)** | - | 360  [B][C] | 330  [B][CD] |  |  |
| **C**  **(3 \* 12)** | - | - | 180  [C][D] | 930  [CD][E] |  |
| **D**  **(12 \* 5)** | - | - | - | 3000  [D][E] | 1860  [D][EF] |
| **E**  **(5 \* 50)** | - | - | - | - | 1500  [E][F] |

MAC (A\*(B\*C)) = 0 + 360 + 5 \* 10 \* 12 = 960

MAC ((A\*B)\*C) = 150 + 0 + 5 \* 3 \* 12 = 330

MBD (B\*(C\*D)) = 0 + 180 + 10 \* 3 \* 5 = 330

MBD ((B\*C)\*D) = 360 + 0 + 10 \* 12 \* 5 = 960

MCE (C\*(D\*E)) = 0 + 3000 + 3 \* 12 \* 50 = 4800

MCE ((C\*D)\*E) = 180 + 0 + 3 \* 5 \* 50 = 930

MDF (D\*(E\*F)) = 0 + 1500 + 12 \* 5 \* 6 = 1860

MDF ((D\*E)\*F) = 3000 + 0 + 12 \* 50 \* 6 = 3360

Next, by calculating the table of length 4, it gives the matrix below.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **B**  **(10 \* 3)** | **C**  **(3 \* 12)** | **D**  **(12 \* 5)** | **E**  **(5 \* 50)** | **F**  **(50 \* 6)** |
| **A**  **(5 \* 10)** | 150  [A][B] | 330  [AB][C] | 405  [AB][CD] |  |  |
| **B**  **(10 \* 3)** | - | 360  [B][C] | 330  [B][CD] | 2430  [B][CDE] |  |
| **C**  **(3 \* 12)** | - | - | 180  [C][D] | 930  [CD][E] | 1770  [CD][EF] |
| **D**  **(12 \* 5)** | - | - | - | 3000  [D][E] | 1860  [D][EF] |
| **E**  **(5 \* 50)** | - | - | - | - | 1500  [E][F] |

MAD (A\*(B\*C\*D)) = 0 + 330 + 5 \* 10 \* 5 = 580

MAD ((A\*B)\*(C\*D)) = 150 + 180 + 5 \* 3 \* 5 = 405

MAD ((A\*B\*C)\*D) = 330 + 0 + 5 \* 12 \* 5 = 630

MBE (B\*(C\*D\*E)) = 0 + 930 + 10 \* 3 \* 50 = 2430

MBE ((B\*C)\*(D\*E)) = 360 + 3000 + 10 \* 12 \* 50 = 3960

MBE ((B\*C\*D)\*E) = 330 + 0 + 10 \* 5 \* 50 = 2830

MCF (C\*(D\*E\*F)) = 0 + 1860 + 3 \* 12 \* 6 = 2076

MCF ((C\*D)\*(E\*F)) = 180 + 1500 + 3 \* 5 \* 6 = 1770

MCF ((C\*D\*E)\*F) = 930 + 0 + 3 \* 50 \* 6 = 1830

Next, by calculating the table of length 5, it gives the matrix below.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **B**  **(10 \* 3)** | **C**  **(3 \* 12)** | **D**  **(12 \* 5)** | **E**  **(5 \* 50)** | **F**  **(50 \* 6)** |
| **A**  **(5 \* 10)** | 150  [A][B] | 330  [AB][C] | 405  [AB][CD] | 1655  [AB][CDE] |  |
| **B**  **(10 \* 3)** | - | 360  [B][C] | 330  [B][CD] | 2430  [B][CDE] | 1950  [B][CDEF] |
| **C**  **(3 \* 12)** | - | - | 180  [C][D] | 930  [CD][E] | 1770  [CD][EF] |
| **D**  **(12 \* 5)** | - | - | - | 3000  [D][E] | 1860  [D][EF] |
| **E**  **(5 \* 50)** | - | - | - | - | 1500  [E][F] |

MAE (A\*(B\*C\*D\*E)) = 0 + 2430 + 5 \* 10 \* 50 = 4930

MAE ((A\*B)\*(C\*D\*E)) = 150 + 930 + 5 \* 3 \* 50 = 1830

MAE ((A\*B\*C)\*(D\*E)) = 330 + 3000 + 5 \* 12 \* 50 = 3630

MAE ((A\*B\*C\*D)\*E) = 405 + 0 + 5 \* 5 \* 50 = 1655

MBF (B\*(C\*D\*E\*F)) = 0 + 1770 + 10 \* 3 \* 6 = 1950

MBF ((B\*C)\*(D\*E\*F)) = 360 + 1860 + 10 \* 12 \* 6 = 2940

MBF ((B\*C\*D)\*(E\*F)) = 330 + 1500 + 10 \* 5 \* 6 = 2130

MBF ((B\*C\*D\*E)\*F) = 2430 + 0 + 10 \* 50 \* 6 = 5430

Finally, by calculating the table of length 6, it gives the matrix below.

This is the o**ptimal parenthesization** of a matrix-chain product.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **B**  **(10 \* 3)** | **C**  **(3 \* 12)** | **D**  **(12 \* 5)** | **E**  **(5 \* 50)** | **F**  **(50 \* 6)** |
| **A**  **(5 \* 10)** | 150  [A][B] | 330  [AB][C] | 405  [AB][CD] | 1655  [AB][CDE] | 2010  [AB][CDEF] |
| **B**  **(10 \* 3)** | - | 360  [B][C] | 330  [B][CD] | 2430  [B][CDE] | 1950  [B][CDEF] |
| **C**  **(3 \* 12)** | - | - | 180  [C][D] | 930  [CD][E] | 1770  [CD][EF] |
| **D**  **(12 \* 5)** | - | - | - | 3000  [D][E] | 1860  [D][EF] |
| **E**  **(5 \* 50)** | - | - | - | - | 1500  [E][F] |

MAF (A\*(B\*C\*D\*E\*F)) = 0 + 1950 + 5 \* 10 \* 6 = 2250

MAF ((A\*B)\*(C\*D\*E\*F)) = 150 + 1770 + 5 \* 3 \* 6 = 2010

MAF ((A\*B\*C)\*(D\*E\*F)) = 330 + 1860 + 5 \* 12 \* 6 = 2550

MAF ((A\*B\*C\*D)\*(E\*F)) = 405 + 1500 + 5 \* 5 \* 6 = 2055

MAF ((A\*B\*C\*D\*E)\*F) = 1655 + 0 + 5 \* 50 \* 6 = 3155

Then the best order is [AB][CDEF] → **[AB][[CD][EF]]**

***Problem 2.*** (60 points)Write a program implementing the algorithms Matrix-Chain-Order and Print-Optimal-Parens given in the lecture. Upload your code. Using your program, find the optimal parenthesization for the following matrix-chain products and show your **m** and **s** matrices.

a) (20 points) p = [30,35,15,5,10,20,25]

Optimal parenthesization is **(A(BC))((DE)F)**

Matrix m is below.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **B** | **C** | **D** | **E** | **F** |
| **A** | 15750 | 7875 | 9375 | 11875 | 15125 |
| **B** | - | 2625 | 4375 | 7125 | 10500 |
| **C** | - | - | 750 | 2500 | 5375 |
| **D** | - | - | - | 1000 | 3500 |
| **E** | - | - | - | - | 5000 |

b) (20 points) p = [10,20,10,15,20,10]

Optimal parenthesization is **(AB)(C(DE))**

Matrix m is below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **B** | **C** | **D** | **E** |
| **A** | 2000 | 3500 | 6500 | 7500 |
| **B** | - | 3000 | 7000 | 6500 |
| **C** | - | - | 3000 | 4500 |
| **D** | - | - | - | 3000 |

c) (20 points) p = [100,10,100,1,1000,100]

Optimal parenthesization is **(A(BC))(DE)**

Matrix m is below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **B** | **C** | **D** | **E** |
| **A** | 100000 | 2000 | 102000 | 112000 |
| **B** | - | 1000 | 11000 | 102000 |
| **C** | - | - | 100000 | 110000 |
| **D** | - | - | - | 100000 |

**<How to compile/run>**

Command:

javac MatrixChainMultiplication.java

java MatrixChainMultiplication

After it’s started, please enter the matrix-chain products and enter some non-number character to end the input. Example is shown in below.

