Modern electronic CCD pixels actually count individual photons of light. This is easily revealed in the dramatic short time exposure photos seen to the left, of a scene taken under very low light levels. The human retina is also an organic light imager that can detect individual photons of visible light.

The images were taken at light levels that produce in each pixel of the image, from top to bottom, 0.1 photons/sec, 1 photon/sec, 10 photons/sec and 100 photons/sec.

The number of photons striking a one-square-meter surface illuminated by the noon-day sun for one second is about 500 microMoles, where 1 mole = 6.02×10^{23} photons.

Problem 1 – In scientific notation, how much is 500 microMoles of photons?

3.0×10^{20} photons

500 microMoles = 5.0×10^{-4} Moles = $5.0\times6.02\times10^{19}$ photons

Problem 2 – Suppose a CCD chip were placed in full sunlight without a camera lens. If the chip measures 2 cm square and consists of 4096x4096 pixels, what is the brightness of the sunlight falling on each pixel in photons/sec?

7.1×10^{9} photons/sec

$(2.0\times10^{-2}/4{,}096)^2\times3.0\times10^{20} = 4.88^2\times10^{-12}\times3.0\times10^{20} = 7.14432\times10^{9}$

Problem 3 – If a pixel will completely saturate (turn 'white') if it accumulates more than 10 million photons on a single exposure, what is the maximum exposure time for an image taken in full sunlight?

1.4×10^{-3} [sec]

$10\times10^{6}/7.1\times10^{9} = 1/7.1\times10^{2} = 1.4\times10^{-3}$ [sec]

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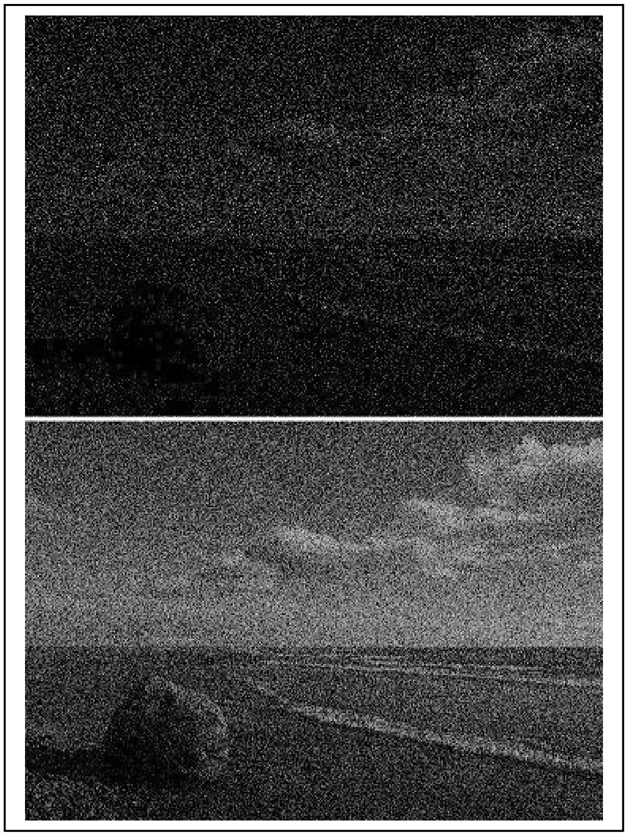

In many astronomical situations, very faint light levels are being measured, such as the light from distant faint stars and nebulae studied by the Hubble Space Telescope. For other situations, such as the Solar Dynamics Observatory studies of the surface of the Sun, huge amounts of light are available that can easily overwhelm sensitive detectors. Although bright sources can be filtered so that they do not 'saturate' the CCD imagers, faint sources must be amplified to register the meager light. In this problem we look at faint sources first.

The image to the left shows a faint image (top) in which individual photons are being registered. In the bottom image, 10-times more photons were detected.

Photon counting follows many of the same statistical rules as conducting surveys. At the end of an exposure for an image, we want to accurately measure the number of photons that were registered in a pixel, which will determine the exact color to assign to the pixel.

Suppose you asked 16 people in Group A the same true/false question as 16 people in Group B. Statistical counting predicts that the responses in each True and False category will differ by as much as $s = (16)^{1/2} = 4$ people in each response between the two groups. If 8 people in Group A answered True, the number of people in Group B that also answered True could be 8 +/- 4 or the specific values of 4, 5, 6, 7, 8, 9, 10, 11 or 12. The survey would be said to have a sampling error of 100% x 4/16 = +/- 25%. If the survey had 1000 people, $s = (1000)^{1/2} = 32$ so the sampling error is +/- 3%.

Problem 1 – A CCD camera operates under low light level conditions and in one photograph, one pixel registers N= 25 photons. If a second photograph is taken, what are the possible numbers of photons detected in this pixel in the second image? 20, 21, 22, 23, 24, 25, 26, 27, 28, 29 and 30

$25^{1/2} = 5$; 25 +/- 5 = {20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30}

Problem 2 – Three images are compared and the same pixel registers 245, 230 and 252 photons. A) What is the average number of photons registered? B) What is the sampling error of the average number of photons? C) What is the possible of range of values relative to the average value?

A) (245+230+252) / 3 = 242.33...
B) $242.3^{1/2} \approx 15.56$; 15.56/242.3x100 ≈ +/- 6.42 [%]
C) 242 +/- 15

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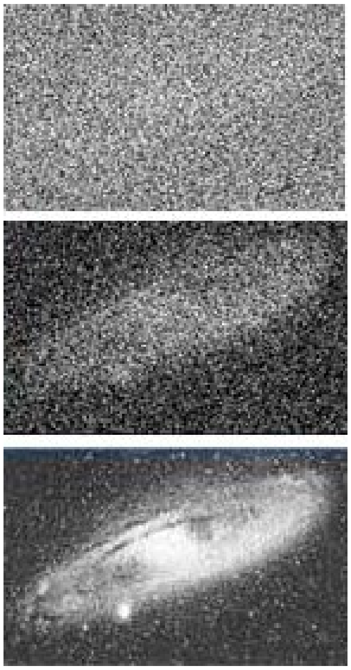

Courtesy: Ulmer and Wessels, NorthwesternUniversity: m-ulmer2@northwestern.edu;.

A recognizable image of a surface detail or an astronomical object requires that each pixel's measurement be done with high accuracy. Otherwise, if the measurements are imprecise, the image becomes indistinct. What this means is that we have to make the standard deviation of the final image as small as possible so that we have an image with a small percentage of error in its pixel values. Fortunately, there is a very simple process that guarantees this outcome. By averaging a large number of images together, pixel by pixel, we can greatly increase the accuracy of each pixel measurement. This fundamental process is called image stacking or image coadding. The three images to the left dramatically shows what happens as the number of coadded images is increased from top to bottom.

If the standard deviation of the pixel value in any one image is given by s, then the standard deviation of N coadded images is given by $$S = \frac{s}{\sqrt{N}}$$

Problem 1 – A CCD camera is taking images of a dark region of the sky to search for faint stars not visible to the human eye. The camera is set to make one exposure every second, and for each exposure the standard deviation per pixel is +/- 50 photons. A) How many exposures must be combined to lower the standard deviation to +/- 1 photon per pixel so that the faint star can be imaged? B) How long will it take to accumulate these images if it takes one second of exposure for each image, and 0.01 second to store and process each image?

A) $N = (50/1)^2 = 2500$
B) (1 + 0.01)[s] x 2500[images] = 2525[seconds]

Problem 2 – The Hubble Space Telescope obtained the Southern Deep Field image (HDF-South) and detected thousands of distant galaxies more than 5 billion light years from Earth. Each WFPC-2 image lasted about 1000 seconds and the total exposure time was about 39 hours. A) How many images were stacked to get the final image? B) By what factor, B, is the uncertainty in the stacked image smaller than a single image of the same field? C) Astronomers measure star brightness in terms of a magnitude scale, m, defined by $B = 10^{-0.4m}$. Example, If Star X is 5 magnitudes fainter than Star Y, its brightness is 1/100 of Star Y. How many magnitudes fainter are the faintest galaxies in the stacked image than in the original image?

A) 39x60x60/1000=140.4[images]
B) The number of coadded images
C) $\sqrt{140} = 10^{-0.4m}$; $m = -\log\sqrt{140} / 0.4 = -\log 11.83/0.4 = -1.073/0.4 \approx -2.68$[magnitudes]

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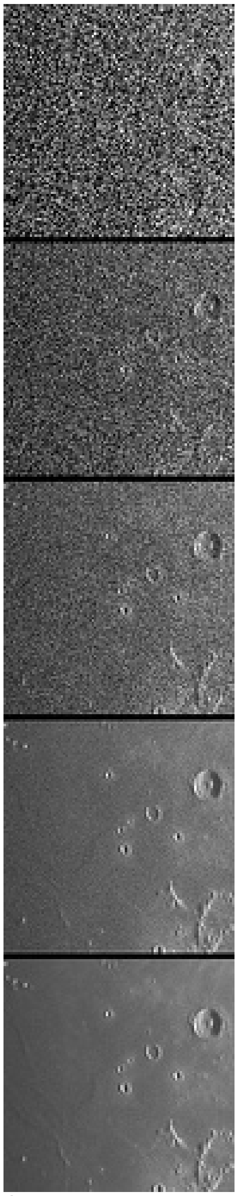

Courtesy Joe Zawodny.

Ordinary digital photography consists of relying on a camera to automatically determine the focus, exposure speed and f/stop in order to produce one 'perfect' image.

In scientific photography, especially astronomical imaging, hundreds or even thousands of images may be taken of the same target. These images are then carefully sorted to eliminate poor quality images, then the remaining images are combined together to produce the final image. Although a single image may have an exposure of only a few seconds, it may be impossible to prolong the exposure for minutes or hours to detect the faintest objects or details. By taking hundreds of short-exposure images, and 'stacking' them together, exposures in the final summed image can exceed hours, or even days.

The images to the left show the dramatic effect of summing or 'coadding' many images together. The top image is the original '1-second' exposure. The second image combines 4 of these 1-second images. The next image combines 16, followed by 256 and 4096 images.

The gaininess in each summed image represents the statistical noise. This noise is caused by a combination of instrumental measuring errors in each pixel, and the quantum aspects of light photons when only a few photons are present. By combining more images, the statistical noise in the final image is greatly reduced, allowing progressively fainter features to be discerned.

The fundamental mathematical formula relating the statistical noise in one image, s, to the final statistical noise in N coadded images, S, is

$$S = \frac{s}{\sqrt{N}}$$

In the 2MASS all-sky infrared survey, astronomers use a ground-based telescope to photograph the sky. The basic digital image lasts 1.6 seconds and the noise is measured to be s = +/- 2.5 DN. The faintest star detectable in these images has a brightness of 0.0004 Jy.

Problem 1 - In a graph, plot the final noise level in DNs after coadding up to 10,000 images. Use Log(N) as the horizontal axis and S in units of DN as the vertical axis.

In the next page

Problem 2 – If the brightness of the faintest star scales linearly with the noise in the image, what is the brightness of the faintest star visible after coadding 10,000 images?

From the graph in the next page, S = 0.25 when N = 10,000

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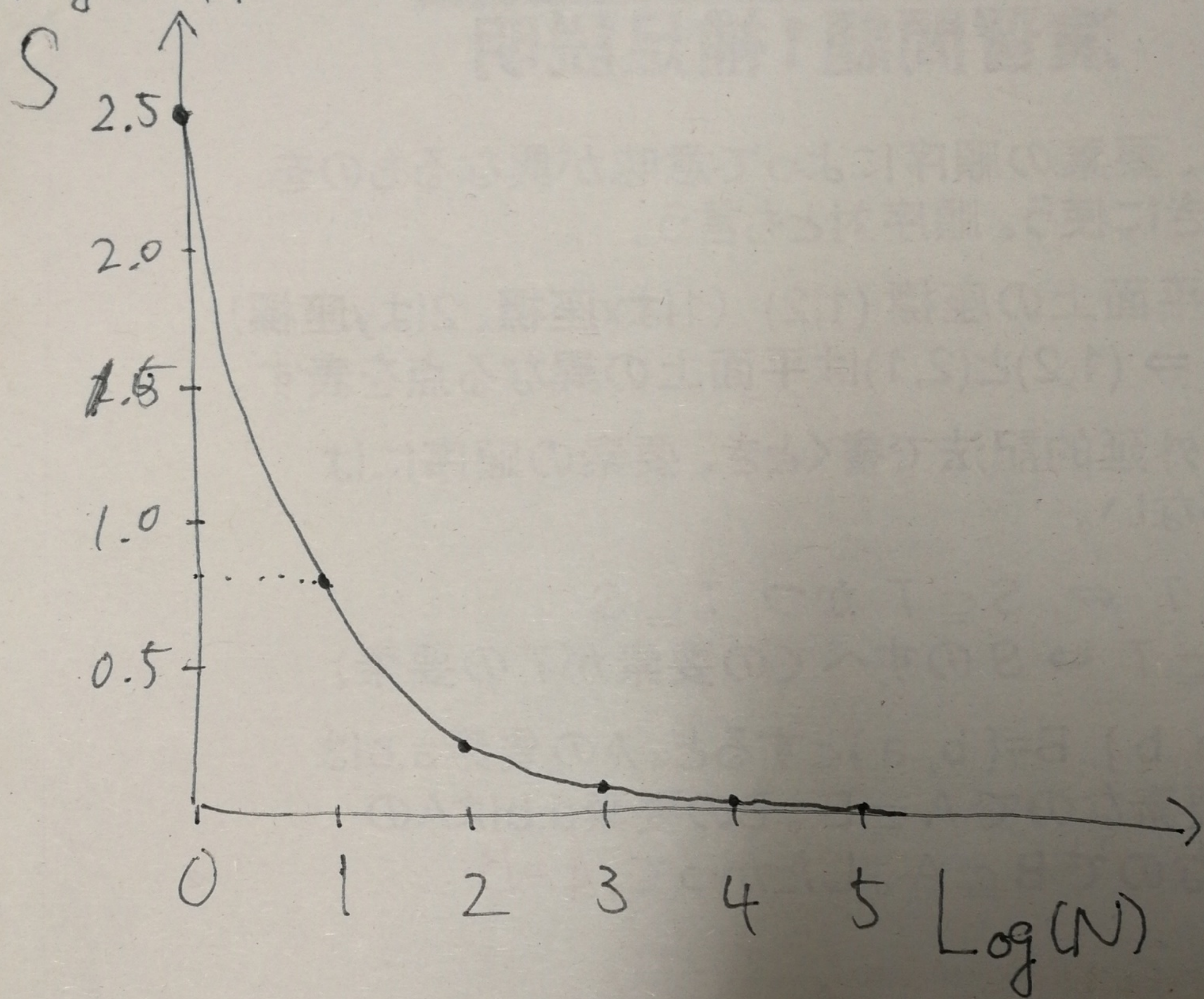
Page 21 Problem 1



$\mathrm{Log}(N)=0 \cdots N=10^0=1, \quad S=\frac{2.5}{\sqrt{1}}=2.5$

$\mathrm{Log}(N)=1 \cdots N=10^1=10, \quad S=\frac{2.5}{\sqrt{10}}\fallingdotseq 0.79$

$\mathrm{Log}(N)=2 \cdots N=10^2=100, \quad S=\frac{2.5}{\sqrt{100}}=0.25$

$\mathrm{Log}(N)=3 \cdots N=10^3=1000, \quad S=\frac{2.5}{\sqrt{1000}}=0.079$

$\mathrm{Log}(N)=4 \cdots N=10^4=10000, \quad S=\frac{2.5}{100}=0.025$

$\mathrm{Log}(N)=5 \cdots N=10^5=100{,}000, \quad S=0.0079\ldots$

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Nikon d3000 image of moon, 1/25-sec, 200mm lens, F/32.

When we take ordinary digital photos for our family album, we are not concerned about the actual quantity of light that fell on the CCD to create the final image. Only the clarity and color balance of the final image matters.

In scientific 'imaging' it is almost always the case that the quantity of light, and its variation from pixel-to-pixel, matter greatly in the study at hand.

The process of working backwards from the digital numbers that code the pixel brightness (DNs) to physical quantities such as watts, watts/meter^{2} or more complex units, is called calibration.

Method 1: If you know exactly how your entire imager (CCD+optical system and filters) responds to light, you can work from the data unit values in DNs to actual brightness units using mathematical steps. This approach is commonly used when you know nothing about the 'unknown' object you are imaging.

Method 2: Alternately, you can image a few different types of known objects (such as stars) whose brightness you know exactly, and measure their DNs. You can then establish a relationship that '1 DN equals x number of brightness units'.

Problem 1 – The bright star Vega has an intensity of I = 3.7×10^{-8} watts/meter^{2}/arcsecond^{2}/micron and its light is concentrated into one CCD pixel. The processing of this image indicates a data word for Vega of 150,000 DNs. What is the calibration constant C, that relates the computer DNs to the actual, physical intensity, I?

$C=3.7\mathrm{x}10^{-8}/1.5\mathrm{x}10^{5} \approx 2.467\mathrm{x}10^{-13}$[watts/meter^{2}/arcsecond^{2}/micron/DNs]

Problem 2 - What is the intensity of the faintest star that registers on the CCD as 25 DNs per pixel?

$I = 2.467\mathrm{x}10^{-13}\mathrm{x}25 \approx 6.17 \mathrm{~x~} 10^{-12}$[watts/meter^{2} /arcsecond^{2} /micron]

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