

Most satellites that 'take pictures' of the sun, moon, earth and distant stars use a camera (called an imaging system) based on a Charge-Coupled Device (CCD) chip. These chips, like the one shown to the left, are similar to the ones used inside the common digital camera and consist of millions of individual sensors called 'pixels' in a square format.

CCD cameras are described according to the number of pixels they contain in multiples of one million pixels (1 megapixel). They are also described by their format in rows (M) and columns (N) as containing MxN pixels. The total number of pixels is usually rounded to the nearest power of 2 as is the row and column format.

For example, a CCD with a format of 1024x1024 pixels has a total of 1,048,576 pixels. In the digital camera industry, this is called 1 megapixel. A 4 megapixel CCD has a format of 2048x2048 pixels or 4,194,304 pixels.

Problem 1 - A digital camera created a 1024x2048 pixel image. What was the format of this image and how many megapixels did it contain?

Format: 1,024 rows and 2,048 columns

Pixels: 2 megapixels ($1,024 * 2,048 / 1,000,000 \approx 2.1$)

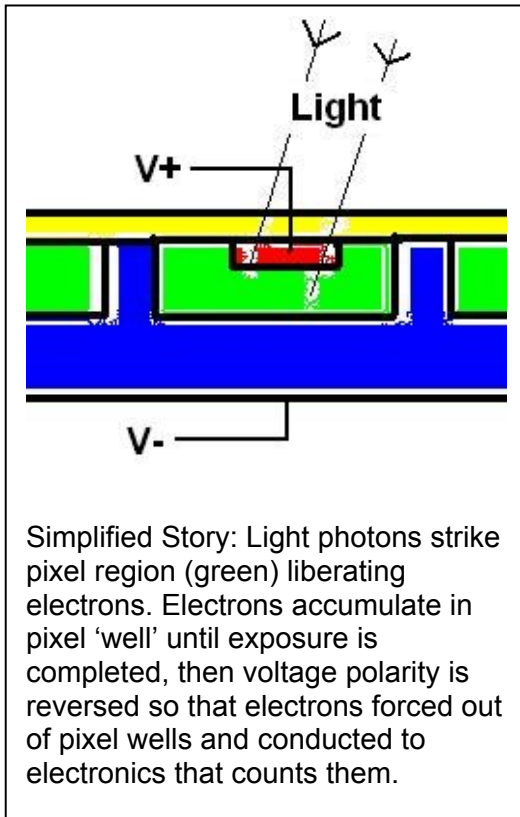
Problem 2 - A new digital camera produces square images containing 16 megapixels. What is the likely format for the image, and the actual number of pixels in the image?

Format: 4,096 rows and 4,096 columns ($16,000,000 = 4,000^2 \approx (2^{12})^2 = 4,096^2$)

Number of pixels: $4,096 * 4,096 = 16,777,216$ pixels

Problem 3 - The CCD and camera optics are designed so that, from an orbiting satellite, the picture will have a resolution of 1 meter per pixel. What are the dimensions of the total area that can be recorded in square 4 megapixel image in kilometers?

4km^2 ($4,000,000 = 2,000 * 2,000$)



Each pixel in a CCD chip is a complicated electronic device called a photodiode. When a photon of light falls on the surface of the pixel, it interacts with the material in the pixel and causes an electron to be freed from its atomic 'prison'. This electron is then stored within the electronic components of the pixel.

After a period of time, called the exposure or readout time, the accumulated electrons within the pixel are counted electronically, and this information is stored.

After all the electrons have been counted, the CCD chip is now ready to accumulate the next batch of electrons for the next image.

CCD camera pixels can accumulate up to 10 million electrons in a pixel 'well' before no more can be generated. The pixel becomes 'saturated' when more than this number of electrons are generated, so the pixel must be read-out before this limit is reached.

Problem 1 - Suppose that a camera was photographing a bright daytime scene that produced 5 billion electrons/sec. If the saturation limit is 10 million electrons per pixel, for how many milliseconds could the camera shutter be left open before the CCD was saturated?

2 milliseconds ($10 \text{ million} / 5 \text{ billion} * 1000 = 2$)

Problem 2 - An astronomer wants to photograph a faint nebula whose light produces 1000 electrons/sec in each CCD pixel. If the saturation limit is 10 million electrons, to the nearest tenth of an hour, what is the longest exposure that the astronomer can use before image saturation occurs?

2.8 hours ($10,000,000 / 1,000 = 10,000 \text{ sec} = 2.77777... \text{ hours} \approx 2.8 \text{ hours}$)

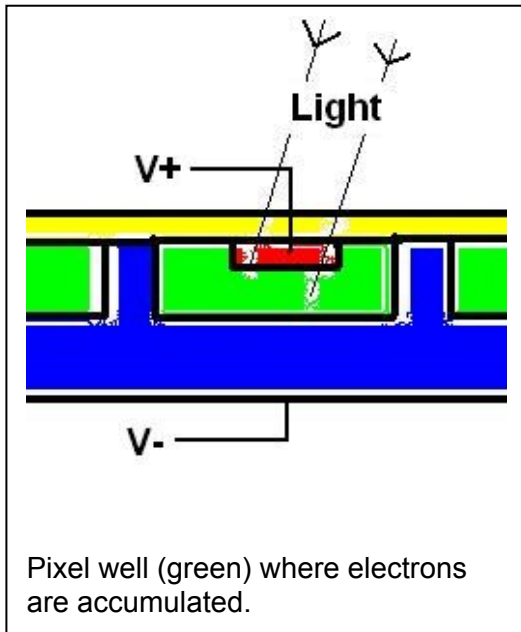
Problem 3 - An astronaut wants to photograph the Space Shuttle as it approaches the International Space Station, but wants to also see stars in the sky. The white paint on the Space Shuttle has a brightness equivalent to 3 million electrons/sec and the stars have a brightness equivalent to 30,000 electrons/sec. If the camera takes an exposure of 1/30 second, how many electrons will be accumulated for the pixels covering the Space Shuttle, and how many will be accumulated for the stars?

Space Shuttle pixels: $3,000,000 / 30 = 100,000 \text{ electrons}$

Stars pixels: $30,000 / 30 = 1,000 \text{ electrones}$

Storing Pixel Data

3



After the electrons that have accumulated in a pixel 'well' are counted at the conclusion of each exposure, the numbers counted in each pixel have to be stored and processed by a computer.

The number of electrons counted in a pixel is stored as a binary number based on powers of 2. This number is called a digital or 'data' word (DN), and its size determines how many megabytes of memory are needed to store the information in each image after each exposure.

For example, an 8-bit data word can store numbers with values up to $2^8 = 256$. A 10-bit data word can store numbers up to $2^{10} = 1024$, and so on.

Problem 1 - An engineer designs a CCD that can count up to 10 million electrons in each pixel. What is the minimum number of bits needed in each data word in order to store the electron number counts?

24-bit data word is needed

$$10,000,000 = 10 * 1000 * 1000 \approx 10 * 2^{10} * 2^{10}$$

$$2^{23} = 8,388,608$$

$$2^{24} = 16,777,216$$

Problem 2 - An astronomer wants to take a picture of a distant galaxy for which the brightness ratio between the faintest and the brightest features is 1/100,000. What is the minimum data word size, in bits, that can accommodate this 'dynamic range' of 100,000?

17-bit data word is the minimum ($2^{16} < 100,000 < 2^{17}$)

Bits	Value	Bits	Value	Bits	Value
1	2	11	2048	21	2,097,152
2	4	12	4096	22	4,194,304
3	8	13	8192	23	8,388,608
4	16	14	16,384	24	16,777,216
5	32	15	32,768	25	33,554,432
6	64	16	65,536	26	67,108,864
7	128	17	131,072	27	134,217,728
8	256	18	262,144	28	268,435,456
9	512	19	524,288	29	536,870,912
10	1024	20	1,048,576	30	1,073,741,824

$$I = \begin{pmatrix} 10387 & 29876 & 40987 \\ 20345 & 30987 & 50328 \\ 40923 & 50876 & 70987 \end{pmatrix}$$

$$I = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix}$$

When a digital image is generated by a CCD chip, it consists of an array of numbers. Each number gives the number of electrons that were counted in each specific pixel.

The information to the left gives the number of electrons counted in a CCD image that consists of 3 rows and 3 columns of pixels. Each pixel is labeled by its row and column number starting in the upper left cell element. For example 'P₂₃' is name of the pixel located in row 2, column 3 of the CCD array.

Problem 1 - How many electrons were counted in pixel P₃₂?

50,876

Problem 2 - Which pixel counted the smallest number of electrons?

P₁₁

Problem 3 - In which direction is the brightness of the image increasing?

Increasing to the right side

Problem 4 - What is the average brightness of the pixels in this image?

38410.67

$(10,387 + 29,876 + 40,987 + 20,345 + 30,987 + 50,328 + 40,923 + 50,876 + 70,987) / 9 = 38410.666...$

Problem 5 - The image was exposed for 0.001 seconds. To two significant figures, what was the brightness of the brightest portion of scene being photographed in electrons/sec?

$70,987 * 1,000 = 70,987,000$ electrons/sec

$$I = \begin{pmatrix} 23 & 22 & 24 & 23 & 24 & 23 & 22 & 24 \\ 24 & 83 & 24 & 25 & 22 & 25 & 23 & 23 \\ 22 & 22 & 23 & 23 & 24 & 22 & 24 & 23 \\ 22 & 24 & 23 & 24 & 79 & 25 & 23 & 24 \\ 23 & 22 & 22 & 23 & 24 & 23 & 25 & 22 \\ 24 & 23 & 24 & 22 & 25 & 22 & 23 & 23 \\ 23 & 24 & 23 & 65 & 22 & 24 & 24 & 22 \\ 99 & 24 & 23 & 25 & 22 & 22 & 24 & 25 \end{pmatrix}$$

When a CCD imager creates an image, it is unavoidable that defects in the CCD chip are also recorded in the data. The first of these defects are 'hot pixels' in which the counting data has been corrupted by cosmic ray 'hits' or because of manufacturing problems when the electronics for a specific pixel were fabricated.

Hot pixels are easily identified because their count values are very different than those in neighboring pixels, or because the counts change erratically between exposures.

In the problems below, use the pixel naming method I_{mn} where m = row and n = column. For example, in the array above, I_{45} , is the value found at row=4, column=5, which is the value 79.

Problem 1 - The above array of pixel values shows a portion of an image after a single readout of the CCD array. Identify all of the hot pixels in the field.

I_{22} , I_{45} , I_{74} , I_{81}

Problem 2 - Based on the average values of surrounding pixels, what may have been the actual counts that should have appeared at each hot pixel?

$$I_{22} = 23 \quad (23 + 22 + 24 + 24 + 24 + 22 + 22 + 23) / 8 = 23 \quad I_{74} = 23 \quad (24 + 22 + 25 + 23 + 22 + 23 + 25 + 22) / 8 = 23.25$$

$$I_{45} = 24 \quad (23 + 24 + 22 + 24 + 25 + 23 + 24 + 23) / 8 = 23.5 \quad I_{81} = 24 \quad (23 + 24 + 24) / 3 = 23.666\dots$$

Problem 3 - An astronomer wants to clean-up his images by eliminating hot pixels. What approach would you suggest using that preserves the over-all image quality?

Use the average value of surrounding pixels of the hot pixel like the Problem 2 instead of the broken value.

Problem 4 - Approximately what is the average number of electrons in the pixels across this image?

23

Use the row 3 as the sample: $(22 + 22 + 23 + 23 + 24 + 22 + 24 + 23) / 8 = 22.875$

Space Math

<http://spacemath.gsfc.nasa.gov>