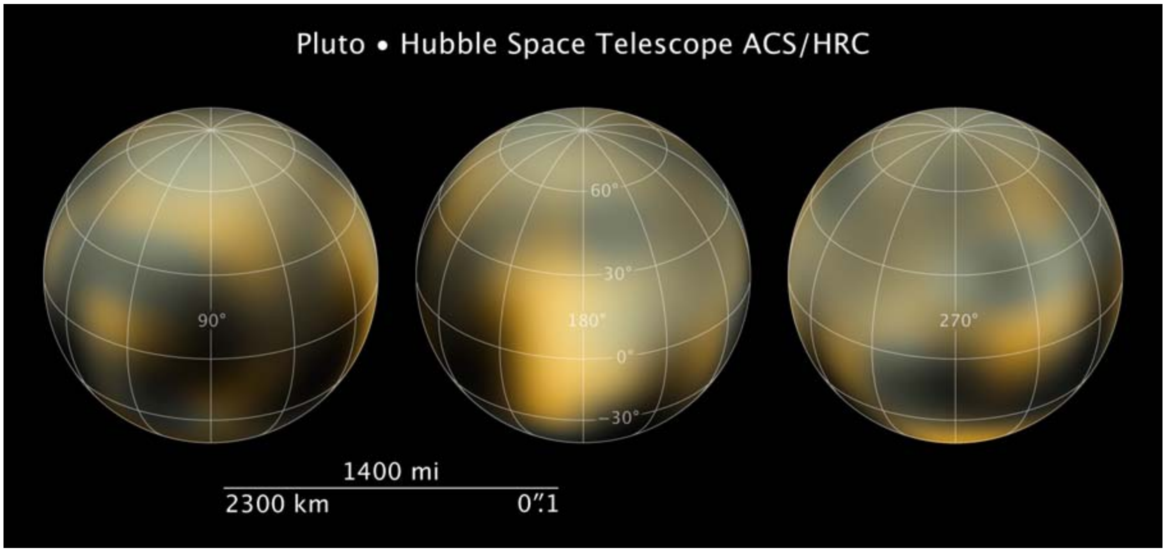
The Changing Atmosphere of Pluto



Recent Hubble Space Telescope studies of Pluto have confirmed that its atmosphere is undergoing considerable change, despite its frigid temperatures. Let's see how this is possible!

Problem 1 - The equation for the orbit of Pluto can be approximated by the formula $2433600=1521x^2+1600y^2$. Determine from this equation, expressed in Standard Form, A) the semi-major axis, a; B) the semi-minor axis, b; C) the ellipticity of the orbit, e; D) the longest distance from a focus called the aphelion; E) the shortest distance from a focus, called the perihelion. (Note: All units will be in terms of Astronomical Units. 1 AU = distance from the Earth to the Sun = 1.5×10^{11} meters).

Problem 2 - The temperature of the methane atmosphere of Pluto is given by the formula

$$T(R)=\left(\frac{L(1-A)}{16\pi\sigma R^2}\right)^{\frac{1}{4}} \qquad \text{degrees Kelvin (K)}$$

where L is the luminosity of the sun ($L=4\times10^{26}$ watts); σ is a constant with a value of 5.67×10^{-8}, R is the distance from the sun to Pluto in meters; and A is the albedo of Pluto. The albedo of Pluto, the ability of its surface to reflect light, is about A = 0.6. From this information, what is the predicted temperature of Pluto at A) perihelion? B) aphelion?

Problem 3 - If the thickness, H, of the atmosphere in kilometers is given by H(T) = 1.2 T with T being the average temperature in degrees K, can you describe what happens to the atmosphere of Pluto between aphelion and perihelion?

Answering Below

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43. The Changing Atmosphere of Pluto

Problem 1.

$$2{,}433{,}600 = 1{,}521 x^2 + 1{,}600 y^2 \qquad \begin{cases} a = 40 \\ b = 39. \end{cases}$$

$$\frac{1}{1{,}600} x^2 + \frac{1}{1521} y^2 = 1.$$

$$\boxed{\frac{1}{a^2} x^2 + \frac{1}{b^2} y^2 = 1}$$

$$e = \begin{cases} \sqrt{\frac{a^2 - b^2}{a^2}} & (b < a) \\ \sqrt{\frac{b^2 - a^2}{b^2}} & (b > a) \end{cases}$$

A) The semi-major axis is 40 AU

B) The semi-minor axis is 39 AU.

C) The elipticity of the orbit e is

$$e = \sqrt{\frac{1{,}600 - 1{,}521}{1600}} = \sqrt{\frac{79}{1600}} \fallingdotseq \underline{0.22}$$

$$\begin{array}{r} 7.8 \\ \times \; 4 \\ \hline 31.2 \end{array}$$

D). The longest distance l from aphelion is

$$l = a(1 + e) = 40(1 + 0.22) = \underline{48.8 \text{ AU}}$$

E) The shortest distance s from perihelion is

$$s = a(1 - e) = 40(1 - 0.22) = \underline{31.2 \text{ AU}}$$

Problem 2.

A) $T(31.2) = 1.39 \times \frac{1}{5.59} \times 10^5 = \underline{2.49 \times 10^4 [\mathrm{k}]}$

B) $T(48.8) = 1.39 \times \frac{1}{6.99} \times 10^5 = \underline{1.99 \times 10^4 [\mathrm{k}]}$

$$T(R) = \left(\frac{L(1-A)}{16 \pi \sigma R^2}\right)^{\frac{1}{4}}$$

$$= \left(\frac{4 \times 10^{25} [\mathrm{watts}] \times 0.4}{16\pi \times 5.67 \times 10^{-8} \times R^2 [\mathrm{AU}] \times 1.5 \times 10^{11} [\mathrm{km}]}\right)^{\frac{1}{4}}$$

$$= \left(\frac{10^2}{5.67 \times 1.5 \pi}\right)^{\frac{1}{4}} \times \frac{1}{\sqrt{R}} \times 10^5$$

$$\fallingdotseq 1.39 \times \frac{1}{\sqrt{R}} \times 10^5$$

Problem 3.

Aphelion: $H(1.99 \times 10^4 [\mathrm{k}]) = 2.39 \times 10^4 [\mathrm{km}]$

Perihelion: $H(2.49 \times 10^4 [\mathrm{k}]) = 2.99 \times 10^4 [\mathrm{km}]$

The nearer to perihelion, the thicker atmosphere.

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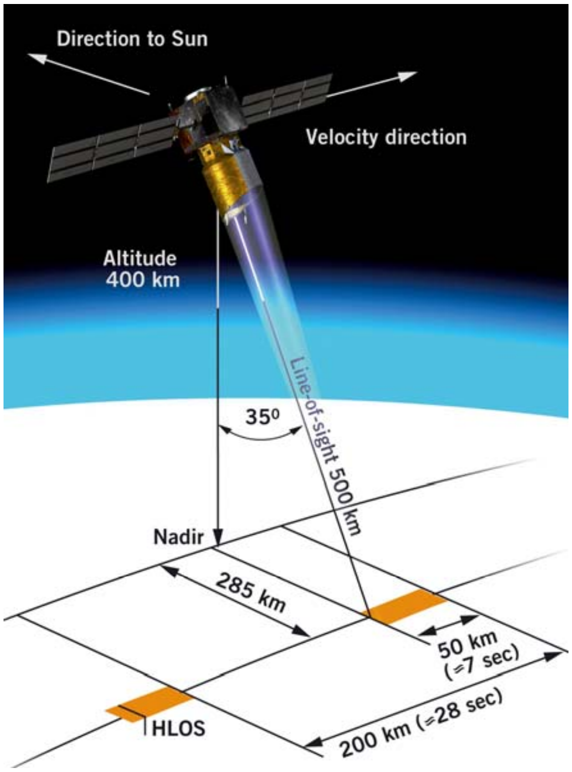

Credits: ESA - AOES Medialab

Most satellite imaging systems do not remain fixed over a target because they are in orbit around Earth or the Moon. For ordinary digital photos we do not want our Subject to move and cause blurring of the image. For satellite photography, it is unavoidable that the satellite in its orbit, or the Target are in motion.

Once we have determined the resolution that our satellite camera needs to study a Target, we also have to keep track of image and Target motion which can also blur the image.

To avoid blurring, we do not want the scene being photographed to move by more than one pixel during the exposure time.

Problem 1 – The satellite travels at a ground speed of 10 kilometers/sec. The CCD camera will not be designed to mechanically track the Target as it passes-by. What will be the angular speed, W, in pixels/sec, of the ground Target traveling across the CCD image if the satellite is in an orbit 350 km above the ground and has a resolution of 6 arcseconds/pixel?

Problem 2 – What must be the maximum exposure time of the CCD image in order to avoid image blurring?

Answering Below

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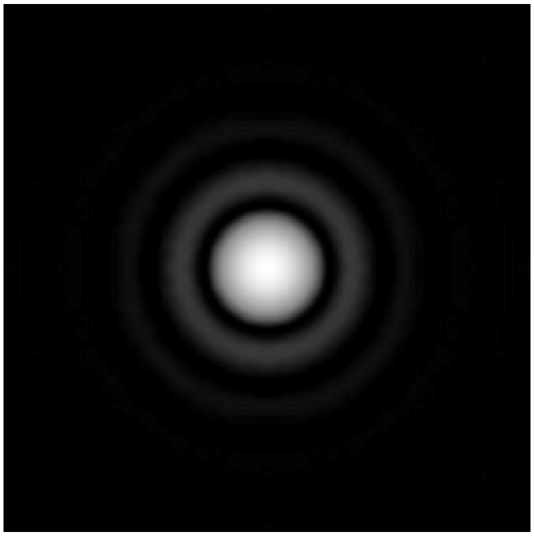

The bright spot is where most of the light energy falls, but it is surrounded by a large number of rings of light, called the diffraction pattern. The angle between the center of the main spot, and the first ring is given by the Airy Disk formula for θ.

Because like is a wave-like phenomenon, it causes interference when it is reflected and concentrated in an optical system. This pattern of interference makes it impossible to clearly see details that are smaller than this interference pattern.

There is a geometric relationship between the resolution of an imaging system and the wavelength at which it operates given by

$$\theta = 1.22\frac{\lambda}{D}$$

where θ is the resolution in units of radians, λ is the wavelength of the radiation in meters, and **D** is the diameter of the camera or telescope lens or mirror in meters.

Problem 1 - If 1 radian = 206265 arcseconds, what is the resolution formula in terms of arcseconds?

Problem 2 - A biologist wants to study deforestation with a satellite camera that has a pixel resolution of 10-meters/pixel, which at the orbit of the satellite corresponds to an angular resolution of 6 arcseconds. To measure the loss of plant matter, she detects the reflection by the ground of chlorophyll, which is the most intense at a wavelength of 700 nanometers (1 nanometer = 10^{-9} meters). What is the diameter of the camera lens that will insure this resolution at the orbit of the satellite?

Problem 3 – Construct a graph that shows the diameter of lens or mirror that is needed to obtain a resolution of 1 arcsecond from far-ultraviolet wavelengths of 200 nanometers to infrared wavelengths of 10 micrometers. From orbit, a human subtends an angle of 1 arcseconds, and emits infrared energy at a wavelength of 10 microns. How large would the camera have to be to resolve a human by his heat emission?

Answering Below

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44. Image Blurring and Motion.

Problem 1.

Put the angular speed as A, then

$A = 206,265 \times \frac{10}{350} \fallingdotseq 5893.29$ [arcsec/sec]

Because the resolution is 6 [arcsec/pixel], the speed S is

$S = 5893.29/6 \fallingdotseq 982.21$ [pixels/sec]

Problem 2.

Put the exposure time as T, then.

$T = \frac{1\,[\text{sec}]}{982\,[\text{frames}]} \fallingdotseq \underline{1.02 \times 10^{-3}\,[\text{sec}]}$

45. Angular Resolution and Wavelength

Problem 1.

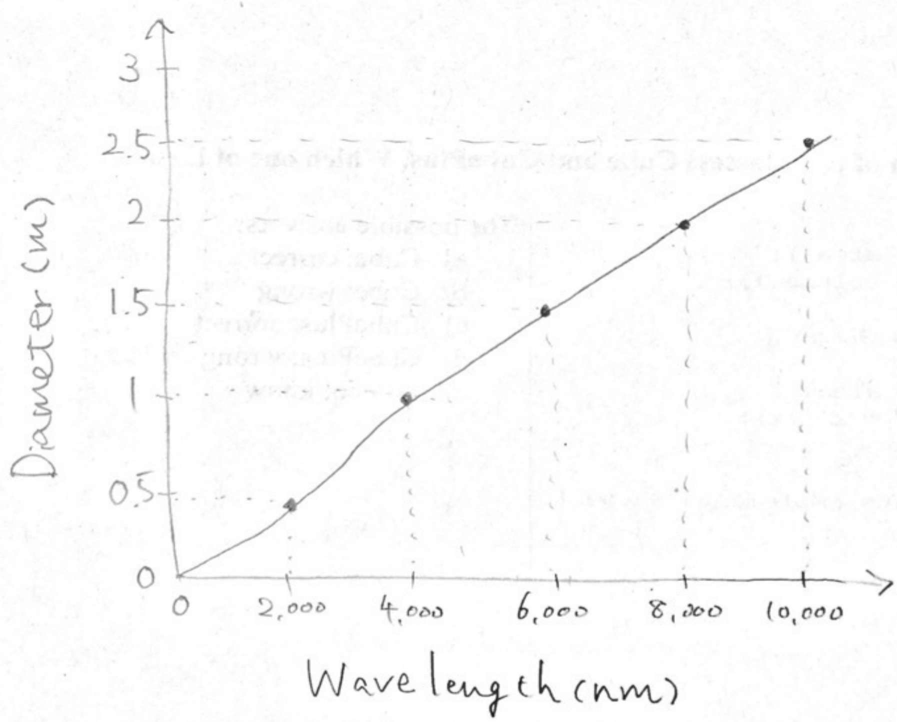
$\theta = 1.22 \times 206,265 \times \frac{\lambda}{D} = \underline{251,643.3 \frac{\lambda}{D}}$

Problem 2.

Put the diameter as D, then

$D = 251,643.3 \times \frac{7.0 \times 10^{-7}\,[\text{m}]}{6\,[\text{arcsec}]} \fallingdotseq \underline{2.94 \times 10^{-2}\,[\text{m}]}$

Problem 3.



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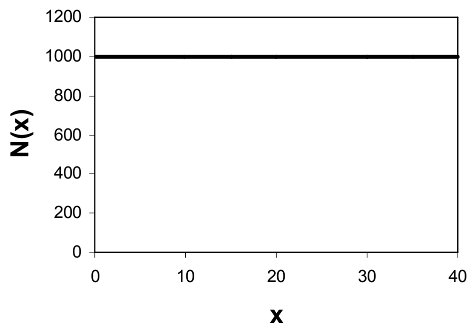
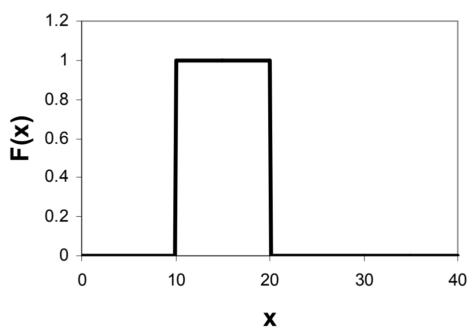
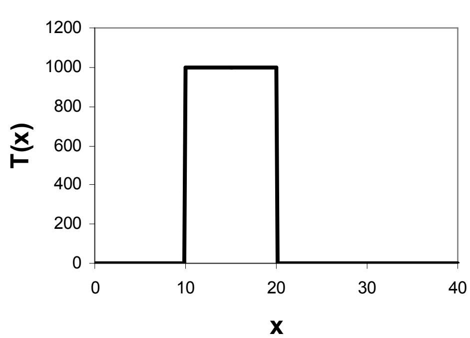
Graph of photon emission N(x)



Graph of filter transmission F(x)



Graph of photon transmission T(x)



Sunglasses are one of the most common, every-day filters that we use. They work in much the same way as the far more sophisticated filters used in professional and scientific photography and digital imaging.

A light source creates huge numbers of photons all across the electromagnetic spectrum. A filter blocks out all of the photons and passes only a narrow range of photons with the desired wavelengths. This process can be described mathematically.

Suppose a light source emits N photons according to the function N(x)=1000 photons shown in the graph to the left (Top). Suppose a filter can be defined according to the piecewise function F(x) = 1.0 for $10 < x < 20$, and F(x)=0 for all other values of x (middle graph). The number of photons passed by this filter is given by T(x) = N(x)F(x). It is easy to see in the bottom graph for T(x) that only photons between $10 < x < 20$ will be passed. The number of photons passed is just P = T(x)x(dx) where the base length is defined by the dx=20-10 = 10-unit width of the filter between x=10 and x=20, and the height is just 1000, so P = 1000x10 = 10,000 photons.

Problem 1 – Suppose that N(x)=1000 and the filter is designed to match the table below:

x	F(x)
0 to 20	0
21 to 25	0.5
26 to 30	1.0
31 to 40	0.5
41 to infinity	0

A) Graph N(x) and F(x). B) What is the total number of photons passed? (Hint: create a table for each wavelength interval) and list N(x), F(x),T(x) and P)

Answering Below

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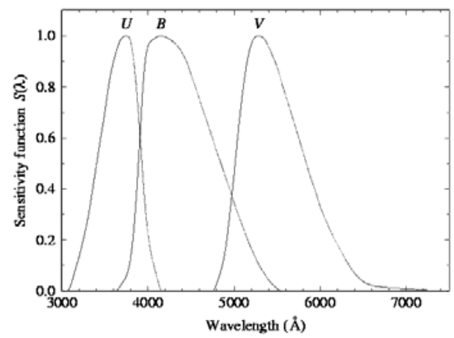

The 'UBV' set of filters are used in astronomy to classify the light from distant stars and galaxies. The U, B and V brightness of a star can be used to determine the star's temperature. For example, a hot star is brighter in the U-band than in the B or V bands. A cold star is brighter in the V-band than in the U or B bands.

To handle more complex filters with realistic functions, one needs to use calculus to compute the total number of photons passed. The general formula is

$$P = \int_0^{+\infty} S(\lambda)F(\lambda)d\lambda$$

where $S(\lambda)$ is the Source Function that defines how the source emits radiation at each wavelength, and $F(\lambda)$ is the filter function which defines the transmission of the filter over the wavelength range.

$S(\lambda)$ is a physical function whose units are photons per square meter wavelength interval (example, photons/meter^2/nanometer). $F(\lambda)$ is a function that gives the filter transmission at each wavelength as a number from 1.0 to 0.0.

An astronomer is studying the distant quasar 3C273 using the Very Large Array radio telescope in Socorro, New Mexico. The quasar has an emission spectrum represented by the power function

$S(\lambda) = 100\lambda^{-3/4}$ Jansky/cm

where the wavelength, λ, is given in centimeters. Suppose that the radio telescope uses a filter at a wavelength of $\lambda = 3.0$ cm that has a parabolic shape defined by the piecewise function:

$F(\lambda) = -4(\lambda-2.5)(\lambda-3.5)$ for $2.5 < \lambda < 3.5$ and
$F(\lambda) = 0$ for all other λ

Problem 1 – Graph the functions $F(\lambda)$ and $S(\lambda)$.

Problem 2 – Over what domain will you need to perform the integration?

Problem 3 - How bright, in Janskys, will quasar 3C273 appear at the wavelength being studied?

Answering Below

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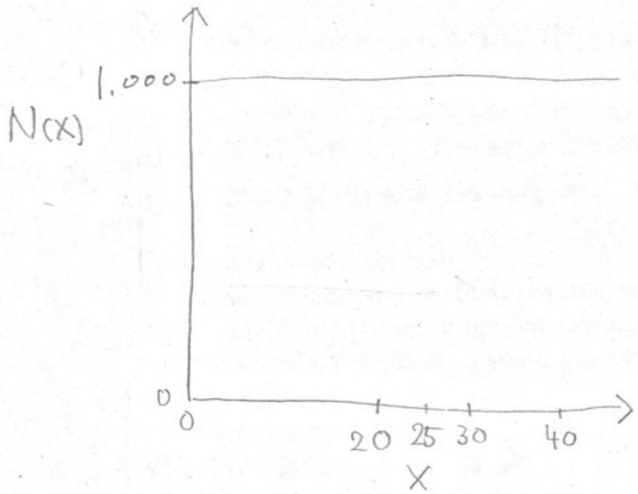
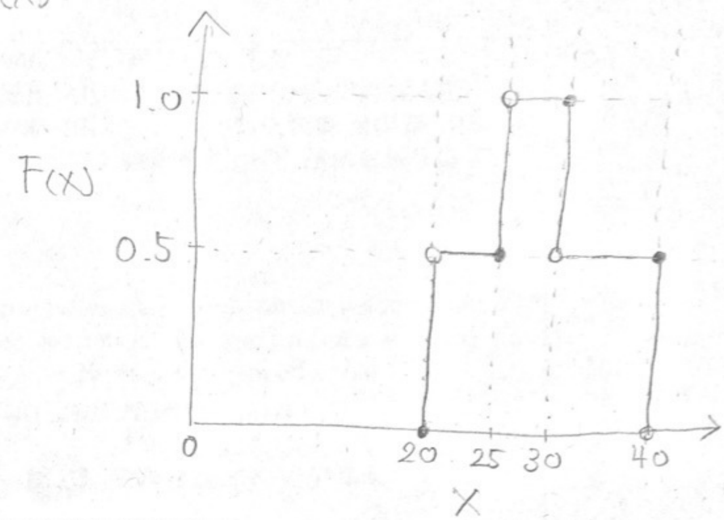
52. Digital Imaging and Filters

Problem 1.

A). N(x):



F(X):



B)

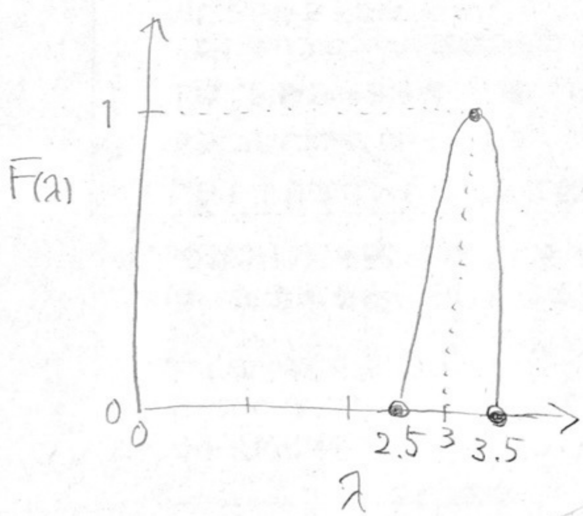
X	F(X)	N(X)	T(X)	dx	P = T(x) × dx
0 to 20	0	1,000	0	20	0
21 to 25	0.5	1,000	500	4	2,000
26 to 30	1.0	1,000	1,000	4	4,000
31 to 40	0.5	1,000	500	9	4,500
41 to Infinity	0	1,000	0	∞	0

$P = 2{,}000 + 4{,}000 + 4{,}500$
$= \underline{10{,}500}$

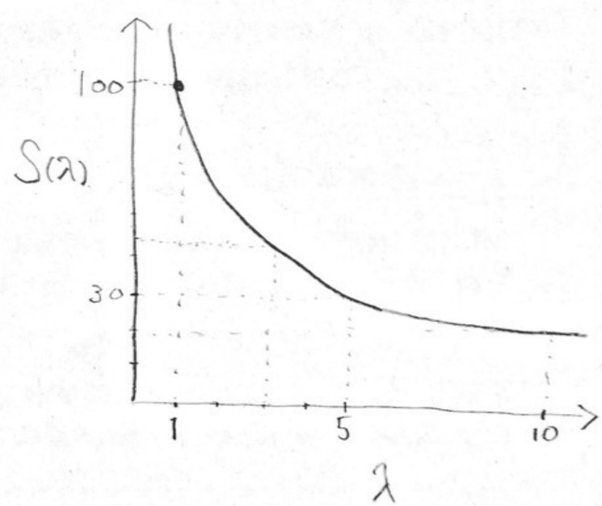
53. Advanced Filter Math using Calculus

Problem 1.

$F(\lambda)$:



$S(\lambda)$:



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Problem2.

$F(\lambda)=0$ for all λ except the range of $2.5<\lambda<3.5$, then It needs to perform the integration just for $\underline{2.5<\lambda<3.5}$

Problem3.

$$P=\int_{2.5}^{3.5} 100\frac{1}{\lambda^{\frac{3}{4}}}(-4)(\lambda-2.5)(\lambda-3.5)\,d\lambda$$

$$=-400\int_{2.5}^{3.5}\frac{(\lambda^2-6\lambda+8.75)}{\lambda^{\frac{3}{4}}}\,d\lambda$$

$$=-400\int_{2.5}^{3.5}\left(\lambda^{\frac{5}{4}}-6\lambda^{\frac{1}{4}}+8.75\frac{1}{\lambda^{\frac{3}{4}}}\right)d\lambda$$

$$=-400\int_{2.5}^{3.5}\left(\frac{4}{9}\lambda^{\frac{9}{4}}\right)'d\lambda+2{,}400\int_{2.5}^{3.5}\left(\frac{4}{5}\lambda^{\frac{5}{4}}\right)'d\lambda-3{,}500\int_{2.5}^{3.5}\left(4\lambda^{\frac{1}{4}}\right)'d\lambda$$

$$=-\frac{1{,}600}{9}\left[\lambda^{\frac{9}{4}}\right]_{2.5}^{3.5}+1{,}920\left[\lambda^{\frac{5}{4}}\right]_{2.5}^{3.5}-14{,}000\left[\lambda^{\frac{1}{4}}\right]_{2.5}^{3.5}$$

$$=\underbrace{-\frac{1{,}600}{9}\cdot 8.9}_{1{,}581.58}+\underbrace{1{,}920\cdot 1.64}_{3{,}155.82}-\underbrace{14{,}000\cdot 0.11}_{1{,}544.89}$$

$$=\underline{3119.13}\ \text{[Janskys]}$$

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