Constraints on Corrections to General Relativity using the Cassini Probe

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Abstract

The theory of general relativity has been verified experimentally to a high degree of accuracy; therefore, any corrections must be within certain bounds. In this project, we consider a type of well defined correction to general relativity that causes different polarizations of light to travel at different speeds, breaking the equivalence principle. We test this against data from the Cassini probe to place a bound on the microscopic parameter in the theory.

Background

All of known physics arises from the following action:

$$S = \int d^4x\mathcal{L} \tag{1}$$

which is an integral over spacetime of $\mathcal{L}$, the Lagrangian density. Minimizing this action derives the laws of classical physics, whereas those of quantum physics arise from forming the amplitude $A = \sum e^{iS/\hbar}$, with $\hbar$ as Planck's constant, and the probability of events P equalling $P \propto |A|^2$.

The Langrangian of Standard physics takes the form

$$\mathcal{L} = \sqrt{-g}\left[\frac{R}{16\pi G_N} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right] + \mathcal{L}_{other} \tag{2}$$

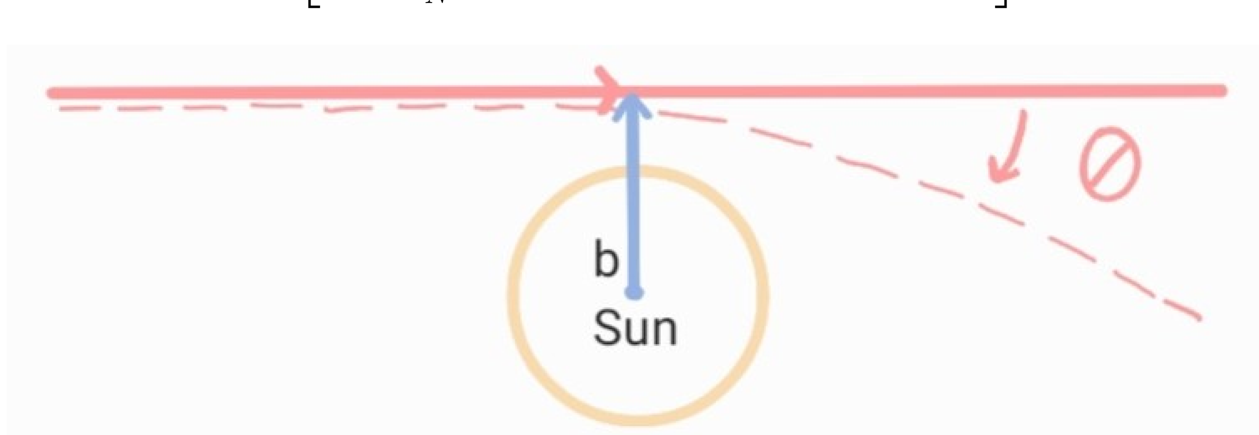
where $g_{\mu\nu}$ is the metric of spacetime matrix, R is the contraction of the Riemann Curvature, and $F_{\mu\nu}$ is the matrix of the electromagnetic field strength. The first term accounts for the effects of gravity with G_N being Newton's gravitational constant, the second term accounts for photons and electromagnetic theory, and the third accounts for all other particles, such as electrons and quarks.

Minimizing this equation provides the equation for classical gravity, $G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$, and the following geodesic equation, which is of our concern: $\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda} = 0$. This equation predicts the equivalence principle, meaning that all particles will fall in the same way. This is particularly impactful when we consider how light travels in a gravitational field: according to this principle, both polarizations will follow the same path.

Introducing Corrections

It is nearly impossible to add corrections to this action due to the underlying principles of relativity and quantum mechanics, and since general relativity has been successfully proved numerous times. Nevertheless, there is a well-defined set of possible corrections, and there is only one concerning light, which is bolded below.

$$\mathcal{L} = \sqrt{-g}\left[\frac{R}{16\pi G_N} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathbf{aR}_{\mu\nu\alpha\beta}\mathbf{F}^{\mu\nu}\mathbf{F}^{\alpha\beta}\right] + \mathcal{L}_{other} \tag{3}$$



The angle of the bending of light around the sun is given by the equation $\theta_{GR} = \frac{4G_N M_s un}{bc^2}$, with b being the impact parameter and c being the speed of light.

Corrections to Light Bending

This new term breaks the equivalence principle, allowing different polarizations of light to travel at different angles and speeds. The coefficient a is used to quantify how much impact these corrections have on the overall action. The angle changes with this correction and the change is bolded below, with the plus or minus representing the differing polarizations.

$$\theta = \theta_{GR}(1 \pm \frac{\mathbf{a}}{\mathbf{b^2}}) = \frac{4GM}{bc^2}(1 \pm \frac{\mathbf{a}}{\mathbf{b^2}}) \tag{4}$$

Corrections to Light Travel Time

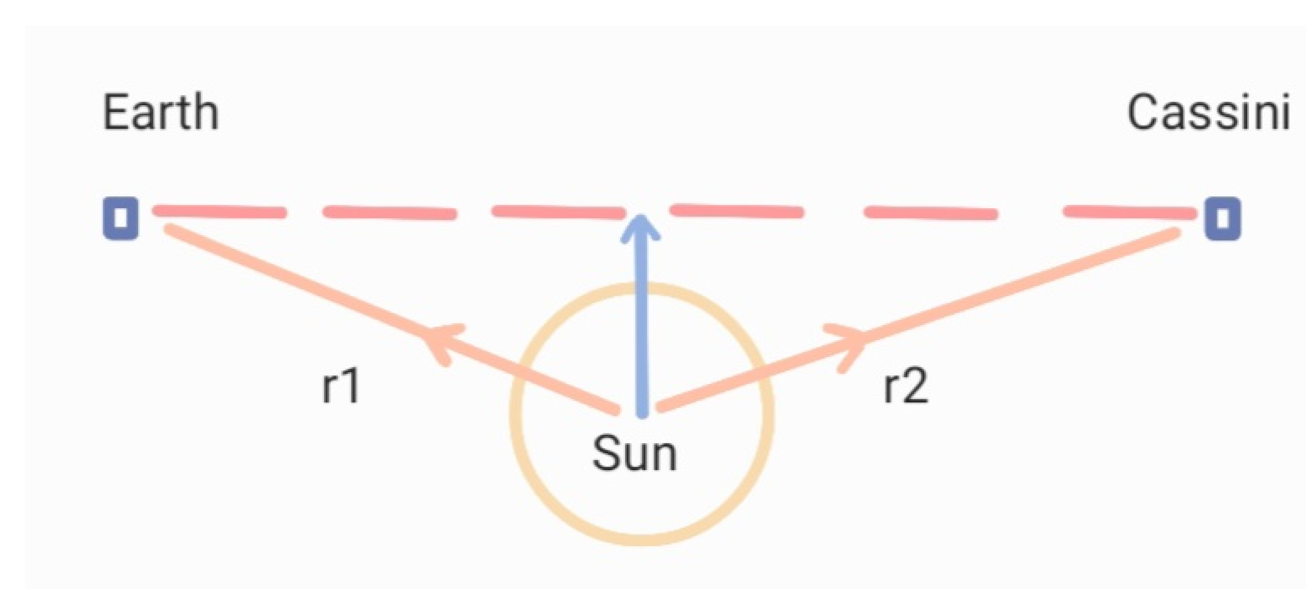
The Shapiro Time Delay predicts that $\Delta T_{GR} = \frac{4GM_sun}{c^3}ln\frac{4r_1r_2}{b^2}$, with $b << r_1, r_2$. Our corrections, ΔT_{new}, subtract $2\int dx\Delta v/c^2$.

$$\frac{\Delta v}{c^2} = \pm\frac{12GM}{c^3r(x)^3}a \tag{5}$$

$$\Delta T = \Delta T_{GR} \mp \frac{24GMa}{c^3}\int\frac{dx}{(b^2+x^2)^{3/2}} \tag{6}$$

Since the integral equals 2 when we take the limits $-\infty \to \infty$, we have that

$$\Delta T = \Delta T_{GR} \mp \frac{48GM_{sun}}{c^3}\frac{a}{b^2} = \frac{4GM_{sun}}{c^3}[\ln\frac{4r_1r_2}{b^2} \mp \frac{12a}{b^2}] \tag{7}$$

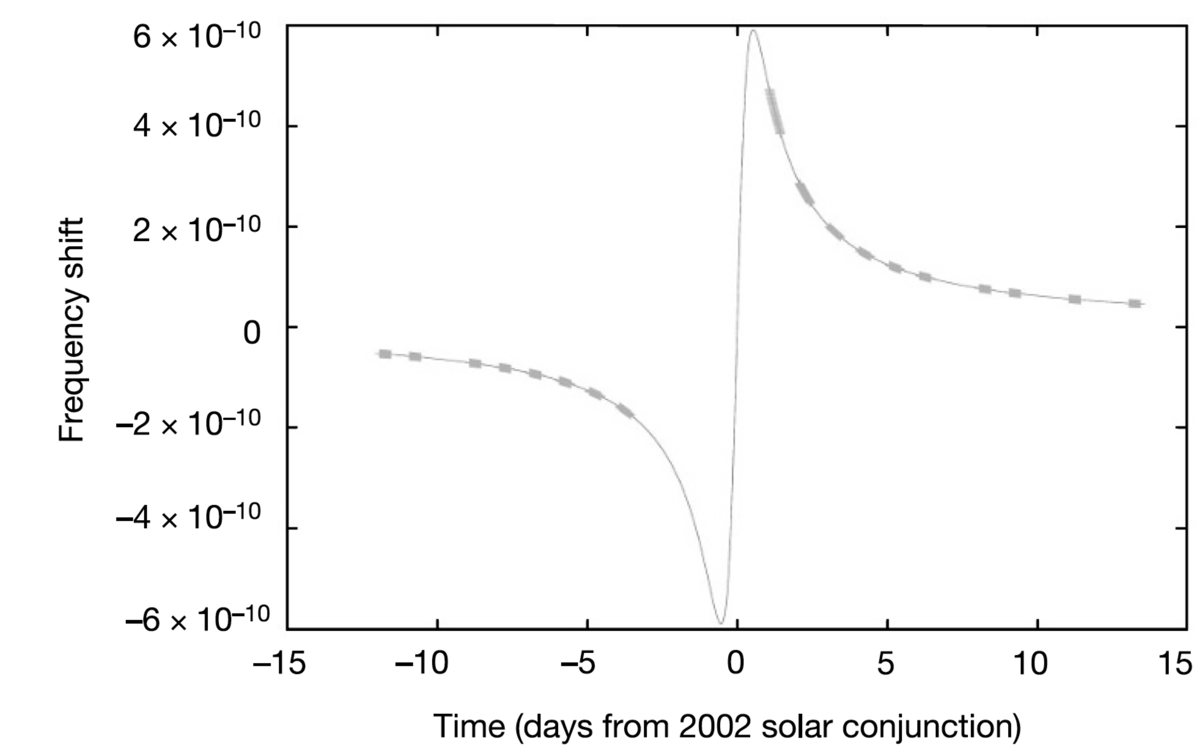


We define the change in the impact parameter, b, to be $r(x) = \sqrt{b^2+x^2}$. The Doppler Effect shows the altered frequency of a light as b changes with time with the equation $y = \frac{\nu_R - \nu_T}{\nu_T} = -\frac{\Delta T}{dt}$, where ν_R is the received frequency and ν_T is transmitted.

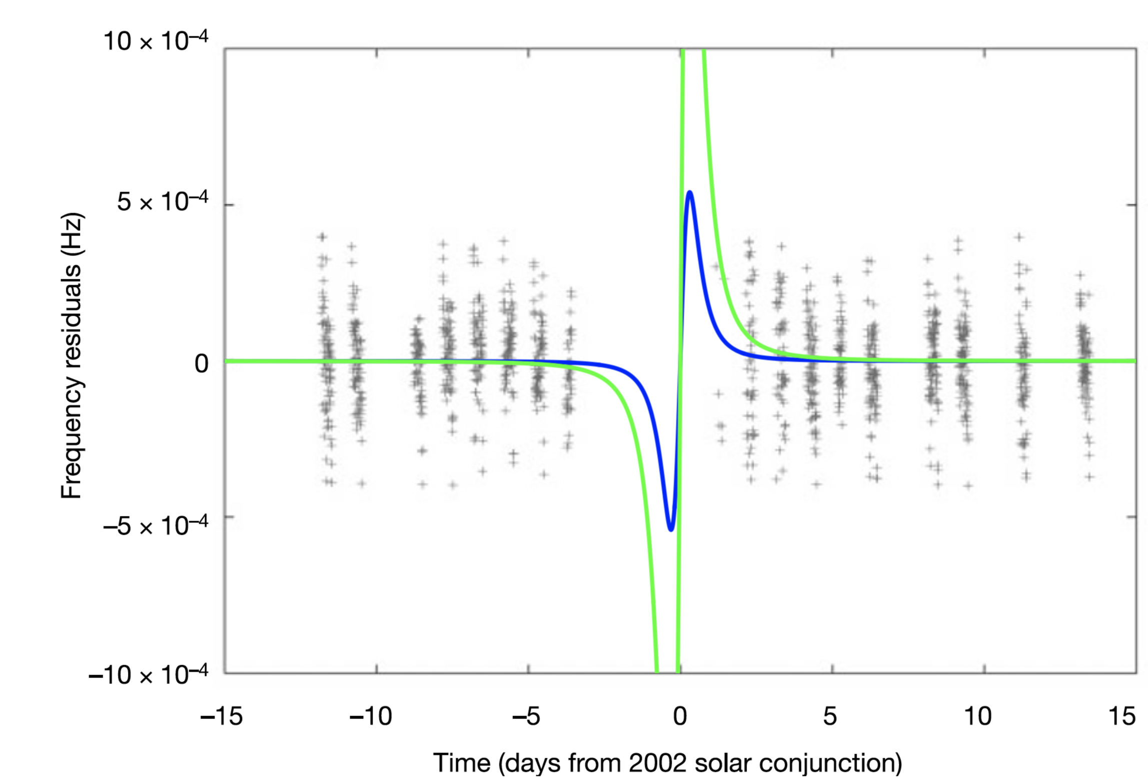
$$y = -\frac{d}{dt}\{\frac{4GM_{sun}}{c^3}\left[\ln\frac{4r_1r_2}{b^2} \mp \frac{12a}{b^2}\right]\} = y_{GR}[1 \mp \frac{12a}{b^2}] \tag{8}$$

Cassini Solar Conjunction Experiment

During the cruise phase of the Cassini probe's mission to Saturn, the spacecraft transmitted radio signals at 2.3 GHz (S-band), 8.4 GHz (X-band), and 32 GHz (Ka-band). The Solar Conjunction Experiment measured 15 days before and after an observed alignment of the spacecraft, the Sun, and the Earth (as observed from Earth) on June 21, 2002. Below is a graph of the two-way relativistic frequency shift (the gravitational signal).



The figure below is a graph of the two-way frequency residuals as a function of time, where each black cross is a data point. The blue and green lines are the mean value predicted by our new equation if we change the value of the parameter a from equation 8.



The green line shows what happens when a is $(8,000km)^2$ whereas the blue curve is when we set it to $(4,000km)^2$. We set a to these values to show why we know this parameter must be very small; clearly, there is a low statistical probability that the blue or green curve accurately represents the data.

Next Steps

A further analysis of the existing data is required to effectively constrain the parameter a. Currently, the raw data related to the Cassini mission is either unreadable or inaccessible. The equations we have derived thus far will be a starting point for this analysis. Additionally, it is necessary to acquire the polarization data from the Cassini Probe to determine whether our equations should be positive or negative.

Future research could include newer data that includes the middle sections (as 3 days of data were lost to system failure), or it could focus more on the angle of light bending instead of the time delay as we have. Results are thus far mixed.

References

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