Singular Values of Trained Networks,  $\sigma_k(\hat{f}; \rho)$ r = 1, n = 64r = 1, n = 128r = 1, n = 256 $10^{1}$  $10^{-1}$  $10^{-3}$  $10^{-5}$  $10^{-7}$  $10^{-9}$ r = 1, n = 512r = 1, n = 1024r = 1, n = 2048 $10^{1}$  $10^{-1}$  $10^{-3}$  $10^{-5}$  $10^{-7}$  $10^{-9}$ r = 2, n = 64r = 2, n = 128r = 2, n = 256 $10^{1}$  $10^{-1}$  $10^{-3}$  $10^{-5}$  $10^{-7}$  $10^{-9}$ r=2, n=512r = 2, n = 1024r=2, n=2048 $10^{1}$  $10^{-1}$  $10^{-3}$  $10^{-5}$  $10^{-7}$ 10<sup>-9</sup> 5 10 15 20 5 10 15 20 5 10 15 20 Index, k Index, k Index, k  $\sigma = 0$ without linear layers  $\sigma = 0.25$ with linear layers  $\sigma = 0.5$ effective rank tolerance,  $\varepsilon = 10^{-3}$ 

 $\sigma = 1$