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1. Spectral Analysis of Signals

Consider the signal x(n)=sin(2*pi*f1*n/fs)+sin(2*pi*f2*n/fs), where f1=70, f2=80, fs=2000. Get the frequency spectrum of the signal by using 500-point and 2000-point Discrete Fourier Transform (DFT), respectively. Plot your results graphically and check the difference between the two, and then explain the possible reasons why the spectrum of the two cases is different.

Change the f2 to 200 and 200.5, respectively. Calculate the frequency spectrum of the two signals, plot your results and give your solution for improvement.

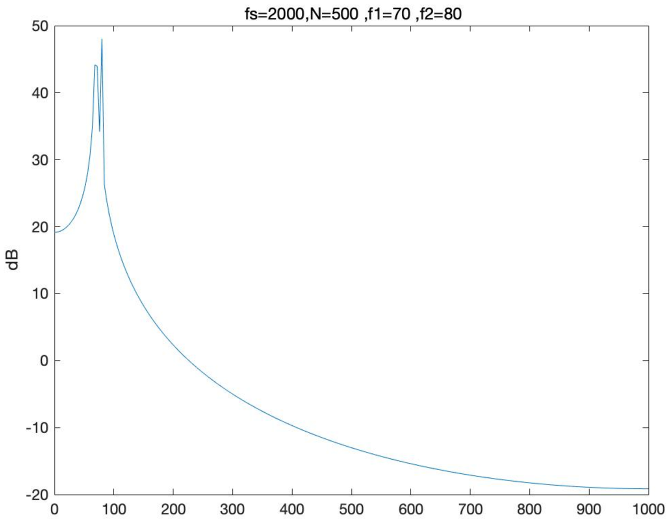

图1 fs=2000, f1=70, f2=80, N=500

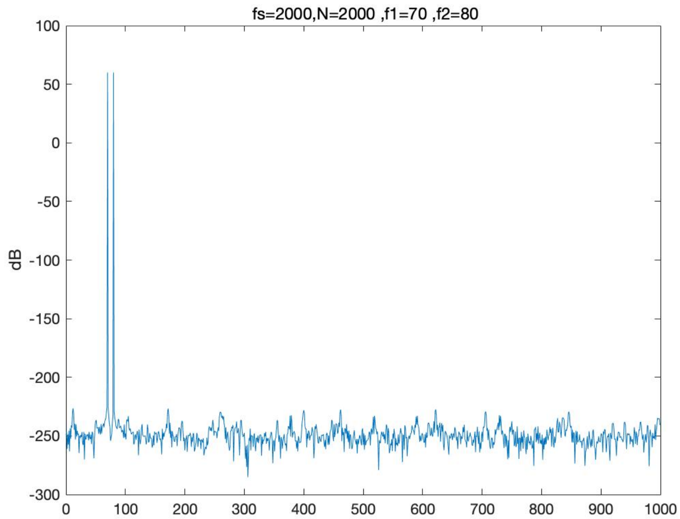

图2 fs=2000, f1=70, f2=80, N=2000

图1、图2分别为500个采样点和2000个采样点的频谱。比较两图可以发现，图1存在频谱泄漏，其原因是：采样点为500个时，采样点之间的间隔为2000/500=4Hz，但是70Hz并不能被4Hz整除，所以500点的频谱并没有包含f1=70Hz，从而导致频谱泄漏。而2000采样点之间的间隔为2000/2000=1Hz，频谱包含了f1=70Hz，f2=80Hz，没有频谱泄漏。

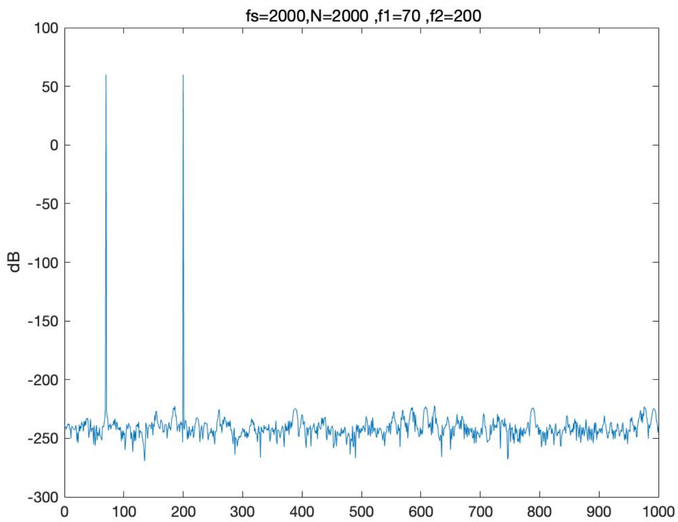

图3 fs=2000, f1=70, f2=200, N=2000

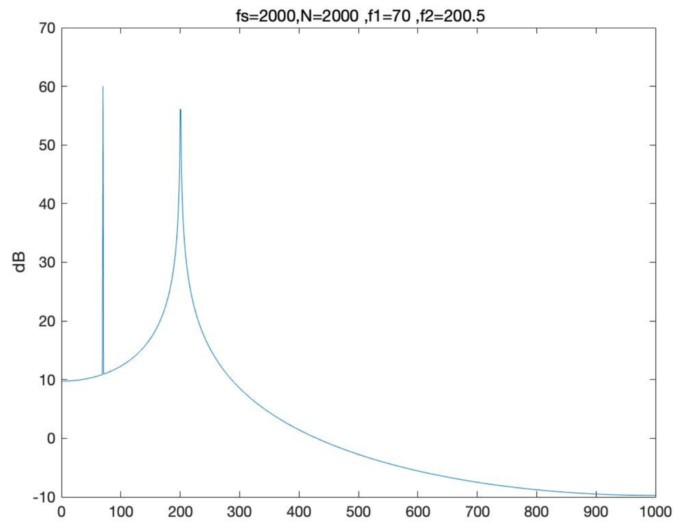

图4 fs=2000, f1=70, f2=200.5, N=2000

图3、图4分别为f2=200和f2=200.5的频谱。如图所示，当采样点为2000个，f2=200Hz时不存在频谱泄漏，f2=200.5时发生了频谱泄漏。

可以通过加窗来减少频谱泄漏。使用blackman后f2=200.5Hz的频谱如图5:

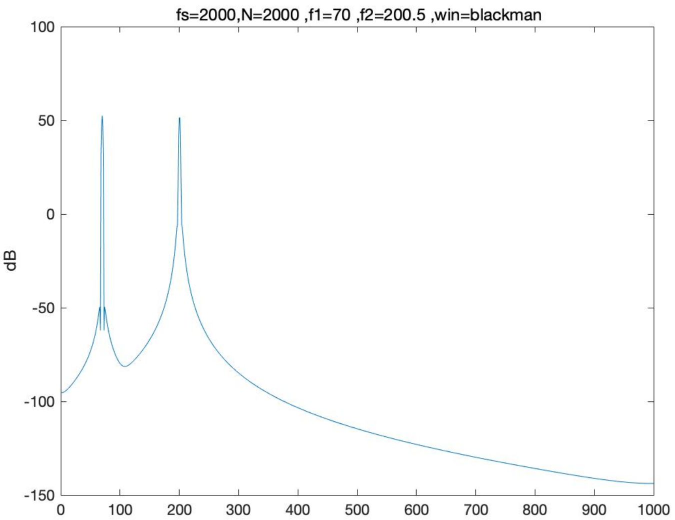

图5 fs=2000, f1=70, f2=200.5, N=2000, 加了blackman窗

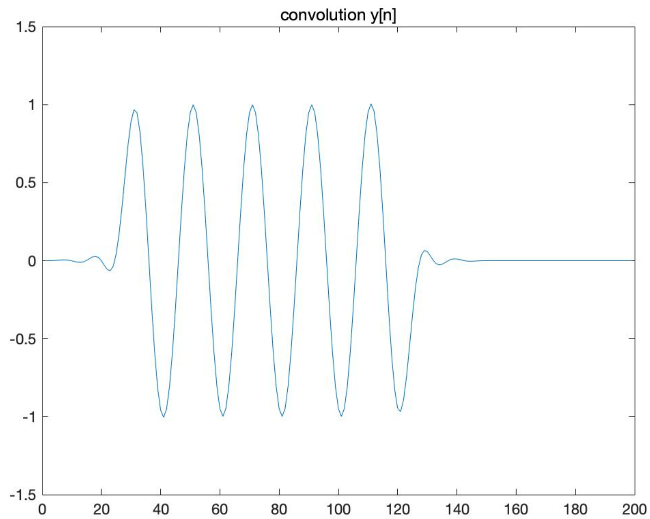

图6 y[n] = x[n] * h[n]

2. DFT and Convolution

For a linear system, its input signal and impulse response signal are given by data files ‘xn_q2.mat’ and ‘hn_q2.mat’, respectively. Use the standard computation method of convolution to find the response y[n] of the system and represent your results graphically.

For the linear system mentioned above, use DFT method to calculate the output signal y[n] and plot all your results graphically, which includes to respectively get the DFT X[f] and H[f] of x[n] and h[n], to compute the Frequency Domain output Y[f] by Y[f]=X[f]×H[f], and then to take the inverse DFT of Y[f] to get the output signal y(n) in time domain.

Plot the output signals of the two different methods in one figure. Check the figure to see if there was any difference between the output signals, and explain the possible reasons. Finally, give your solutions to obtain the same output y(n) using convolution and DFT.

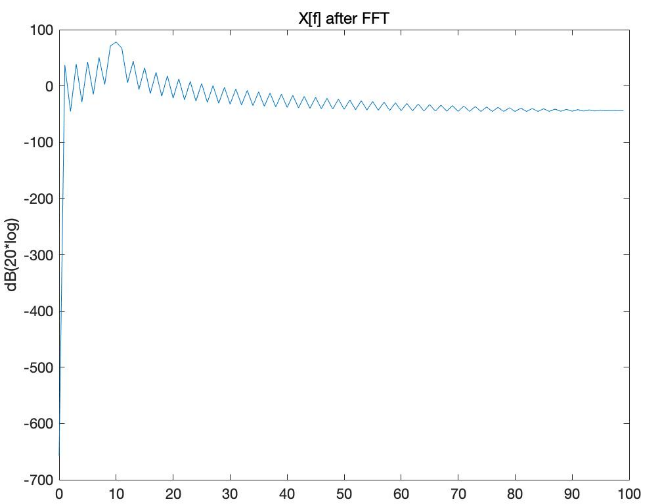

图7 x[n] FFT 得到的X[f]

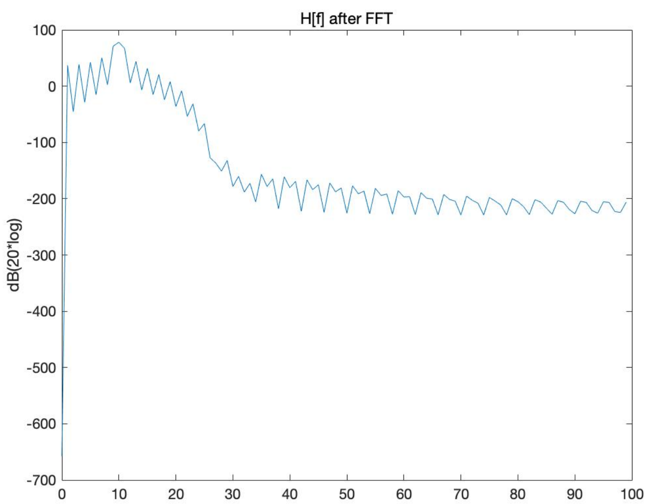

图8 h[n] FFT 得到的H[f]

图6是x[n]与h[n]卷积得到的结果y[n]。

图7和图8分别是x[n] FFT 得到的 X[f] 和 h[n] FFT 得到的 H[f]。

图9是Y[f](= X[f] × H[f])经过IFFT后得到的y[n]。

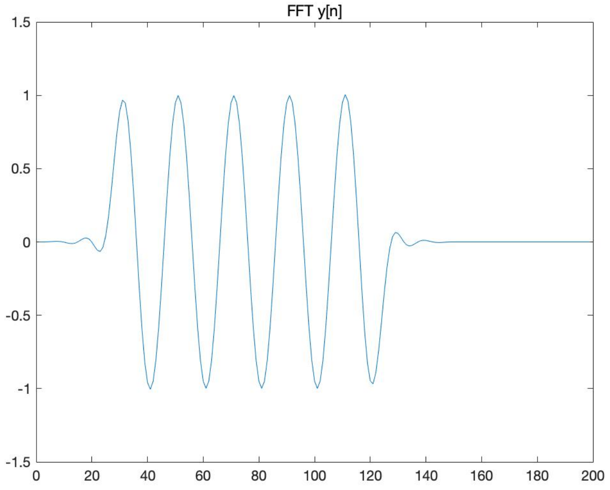

图9 Y[f]经过IFFT后得到的y[n]

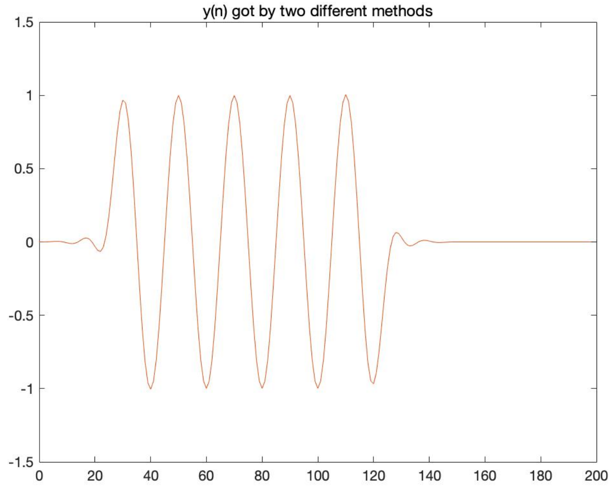

图10 卷积和FFT分别得到的y[n]

图10中包含了卷积x[n]*h[n]和FFT X[f]×H[f]分别得到的y[n]，可以看到两种方法得到的y[n]是一样的，在图10中重合了。

3. Frequency Response of a System

The impulse response h(n) of a system is given by “hn_q3.mat”, and the sampling frequency fs is 100Hz. Please answer the following questions:

(1) Please plot the amplitude and phase spectra of this system using Matlab. Observe the spectra and answer: What type of system is it? (low-pass, high-pass or band-pass) What is the cut-off frequency? Is it linear phase or non-linear phase within the passband frequency range?

(2) For an input signal x(n) saved in “xn_q3.mat”, please calculate the output y(n) using Matlab. Plot the time waveforms of both x(n) and y(n), and summarize the changes after the signal pass through the system.

(3) Plot the amplitude spectrum of x(n) and y(n), respectively (please use hamming window prior to FFT). Compare the two spectra and answer: Does it agree with your observations in the time waveform comparison? Does it agree with the frequency response of this system?

(1)图11是h[n]的幅度谱，图12是h[n]的相位谱。可知，h[n]是带通系统(band-pass)，截止频率即通带和过渡带的分界，如果以-3dB来定截止频率的话，截止频率为5.78Hz和19.22Hz。

如图12所示，通带频率范围内是线形相位

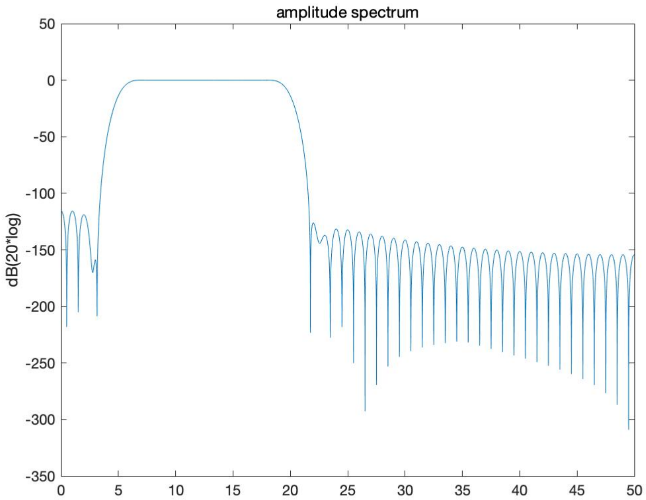

图11 h[n]的幅度谱

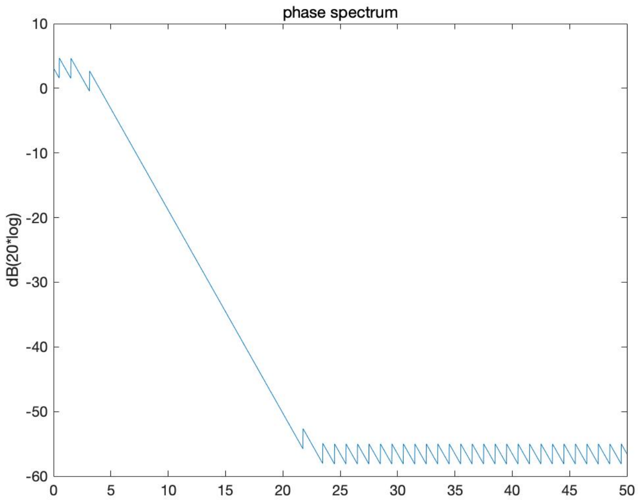

图12 h[n]的相位谱

(2)图13为x[n]的波形图，图14是使用FFT得到的y[n]。

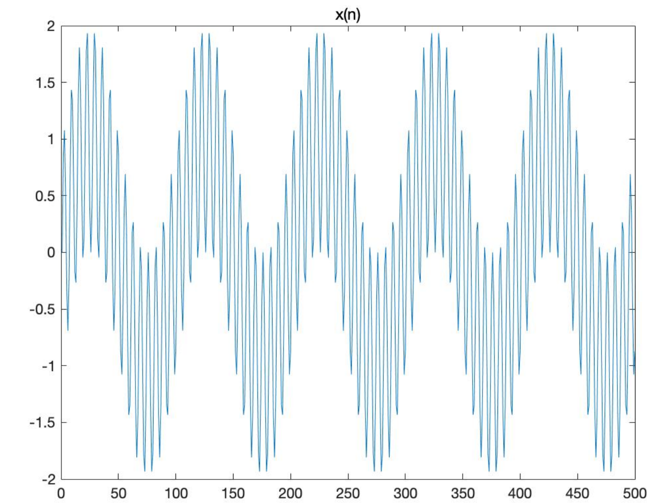

图13 x[n]的波形图

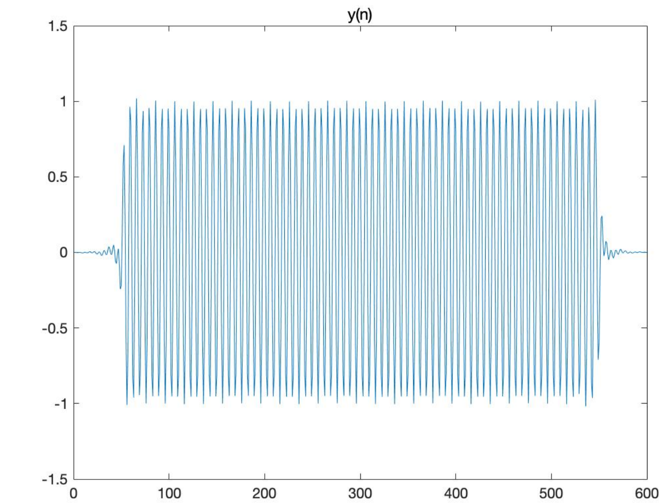

图14 使用FFT得到的y[n]

观察可得：x[n]有两个频率的波动，经过滤波后频率低的波动消失。

(3)图15是x[n]通过加hamming窗后进行了FFT变换的频谱X[f]，图16是y[n]通过加hamming窗后进行了FFT变换的频谱Y[f]。

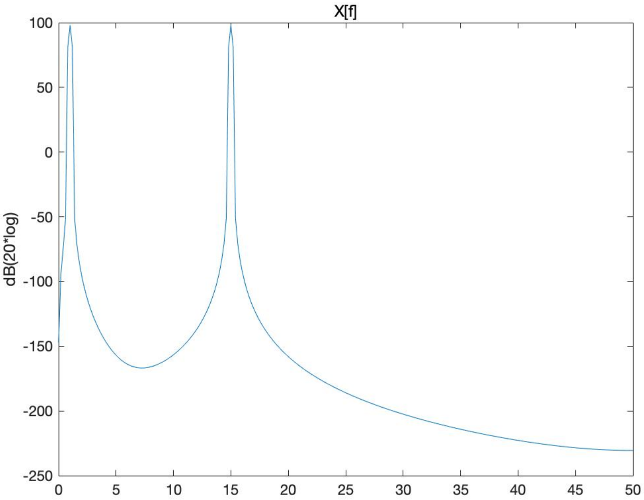

图15 x[n]的频谱X[f]

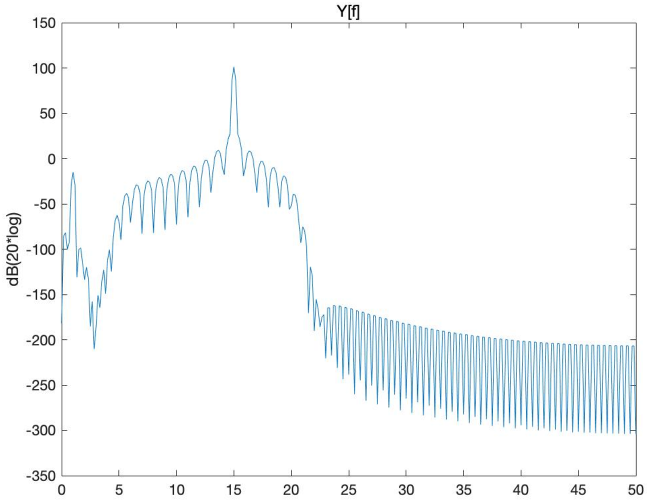

图16 y[n]的频谱Y[f]

经观察可知：x[n]有两个主要频率大约为1Hz和15Hz，经过h[n]滤波之后得到的y[n]主要频率只剩下15Hz，1Hz被滤掉了，这与之前分析的截止频率一致，滤掉了小与5.78Hz和大于19.22Hz的频率。