Homework 2

Question 1: Let us consider a mixture of normal distribution that can be written as:

$$f(x|\Theta) = \sum_{k=1}^{M} a_k \; g(x|\theta_k)$$

where $0 \leq a_k \leq 1$, $\sum_{k=1}^{M} a_k = 1$, and $g(x|\theta_k) = \frac{1}{\sqrt{2\pi}\sigma_k} \exp\left(-\frac{(x-\mu_k)^2}{2\sigma_k^2}\right)$, $\theta_k = (\mu_k, \sigma_k)^t$, and x is a real number.

1. Give the likelihood function
2. Give the formula for the estimation of the parameters

Hints: You need to use the method of Lagrange multipliers and newton method.

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Question 2: The data D={x_1, …, x_n} taken by a random variable x are generated from the pdf defined by:

$$f(x|\mathrm{a}) = \alpha \exp(-a\,x)$$

where x is a non-negative real number and a > 0.

1. Find α so that $f(x|\mathrm{a})$ is a probability density function.
2. Find the expectation and the variance of the random variable. You should provide all the computation details.
3. Let us consider that the prior pdf is a Gamma pdf defined by p(a|α, β)=$\frac{\beta^\alpha}{\Gamma(\alpha)} a^{\alpha-1} \exp(-a\beta)$, give the posterior pdf. You should provide all the computation details.

Question 3: Calculate the following

1. The derivative of $x^a(1-x)^b$
2. The integral of $\int_0^{+\infty} x^{\alpha-1} \exp(-x\beta)dx$.

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