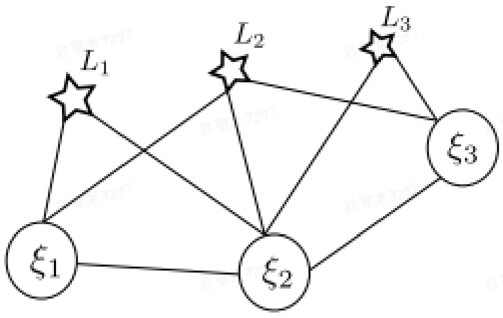
SOUSIC-第 4 章作业

1. 绘制信息矩阵



系统信息记为：

$$\mathbf{x}^\top = [\xi_1, \xi_2, \xi_3, \mathrm{L}_1, \mathrm{L}_2, \mathrm{L}_3]^\top$$

$$\mathbf{J} = \frac{\partial \mathbf{r}}{\partial \mathbf{x}^\top} = \left[\frac{\partial \mathrm{r}(\xi_1,\xi_2)}{\partial \mathbf{x}^\top}, \frac{\partial \mathrm{r}(\xi_1,\mathrm{L}_1)}{\partial \mathbf{x}^\top}, \frac{\partial \mathrm{r}(\xi_1,\mathrm{L}_2)}{\partial \mathbf{x}^\top}, \frac{\partial \mathrm{r}(\xi_2,\xi_3)}{\partial \mathbf{x}^\top}, \frac{\partial \mathrm{r}(\xi_2,\mathrm{L}_1)}{\partial \mathbf{x}^\top}, \frac{\partial \mathrm{r}(\xi_2,\mathrm{L}_2)}{\partial \mathbf{x}^\top}, \frac{\partial \mathrm{r}(\xi_2,\mathrm{L}_3)}{\partial \mathbf{x}^\top}, \frac{\partial \mathrm{r}(\xi_3,\mathrm{L}_2)}{\partial \mathbf{x}^\top}, \frac{\partial \mathrm{r}(\xi_3,\mathrm{L}_3)}{\partial \mathbf{x}^\top} \right]$$

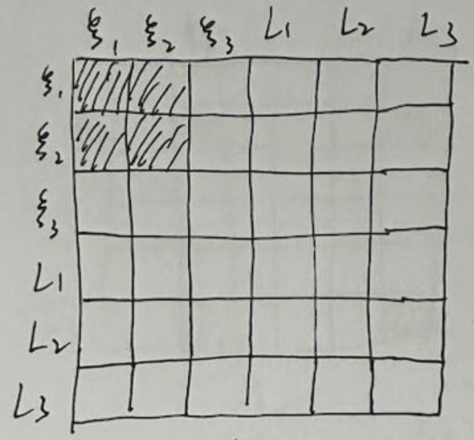
1.1 绘制上述系统的信息矩阵 Λ

以 Λ1 为例：

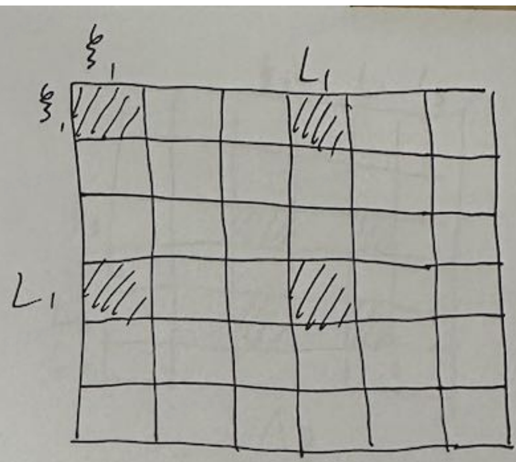
$$\mathbf{J}_1 = \frac{\partial \mathrm{r}(\xi_1,\xi_2)}{\partial \mathbf{x}^\top} = \left[\frac{\partial \mathrm{r}(\xi_1,\xi_2)}{\partial \xi_1}, \frac{\partial \mathrm{r}(\xi_1,\xi_2)}{\partial \xi_2}, \frac{\partial \mathrm{r}(\xi_1,\xi_2)}{\partial \xi_3}, \frac{\partial \mathrm{r}(\xi_1,\xi_2)}{\partial \mathrm{L}_1}, \frac{\partial \mathrm{r}(\xi_1,\xi_2)}{\partial \mathrm{L}_2}, \frac{\partial \mathrm{r}(\xi_1,\xi_2)}{\partial \mathrm{L}_3} \right] = \left[\frac{\partial \mathrm{r}(\xi_1,\xi_2)}{\partial \xi_1}, \frac{\partial \mathrm{r}(\xi_1,\xi_2)}{\partial \xi_2}, \mathbf{0}, \mathbf{0}, \mathbf{0} \right]$$

$$\mathbf{\Lambda}_1 = \mathbf{J}_1^\top \mathbf{\Sigma}_1^{-1} \mathbf{J}_1 = \begin{bmatrix} \left(\frac{\partial \mathrm{r}(\xi_1,\xi_2)}{\partial \xi_1}\right)^\top \mathbf{\Sigma}_1^{-1} \frac{\partial \mathrm{r}(\xi_1,\xi_2)}{\partial \xi_1} & \left(\frac{\partial \mathrm{r}(\xi_1,\xi_2)}{\partial \xi_1}\right)^\top \mathbf{\Sigma}_1^{-1} \frac{\partial \mathrm{r}(\xi_1,\xi_2)}{\partial \xi_2} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \left(\frac{\partial \mathrm{r}(\xi_1,\xi_2)}{\partial \xi_2}\right)^\top \mathbf{\Sigma}_1^{-1} \frac{\partial \mathrm{r}(\xi_1,\xi_2)}{\partial \xi_1} & \left(\frac{\partial \mathrm{r}(\xi_1,\xi_2)}{\partial \xi_2}\right)^\top \mathbf{\Sigma}_1^{-1} \frac{\partial \mathrm{r}(\xi_1,\xi_2)}{\partial \xi_2} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

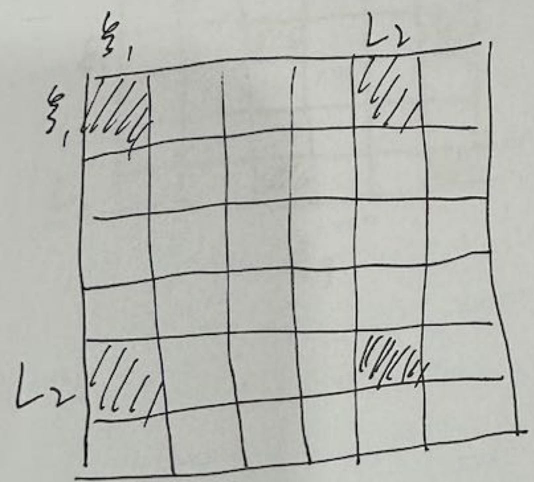
画出信息矩阵：



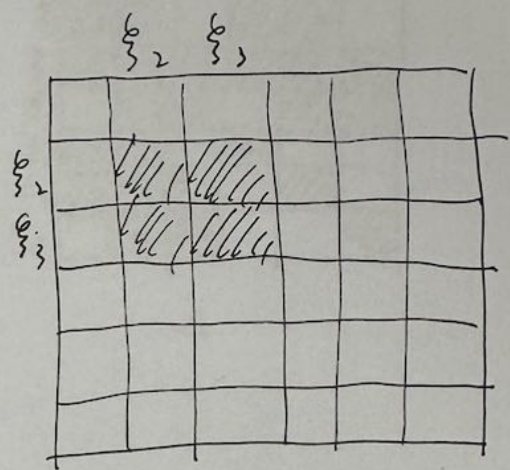
Λ_1



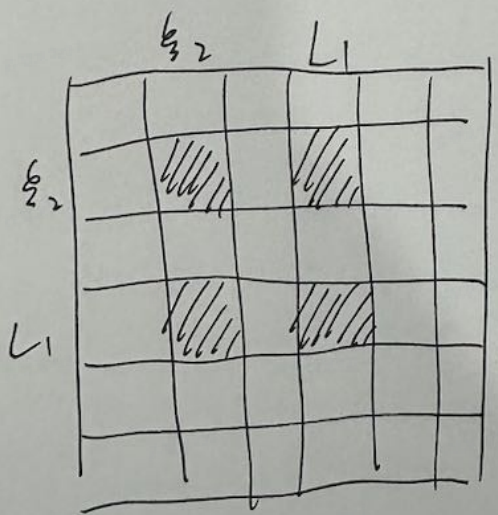
Λ_2



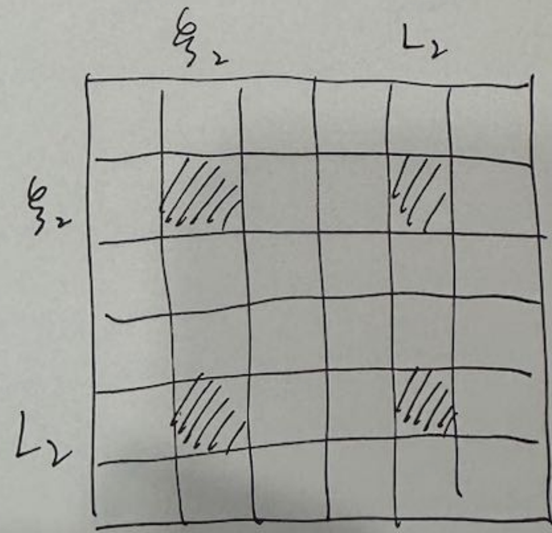
Λ_3



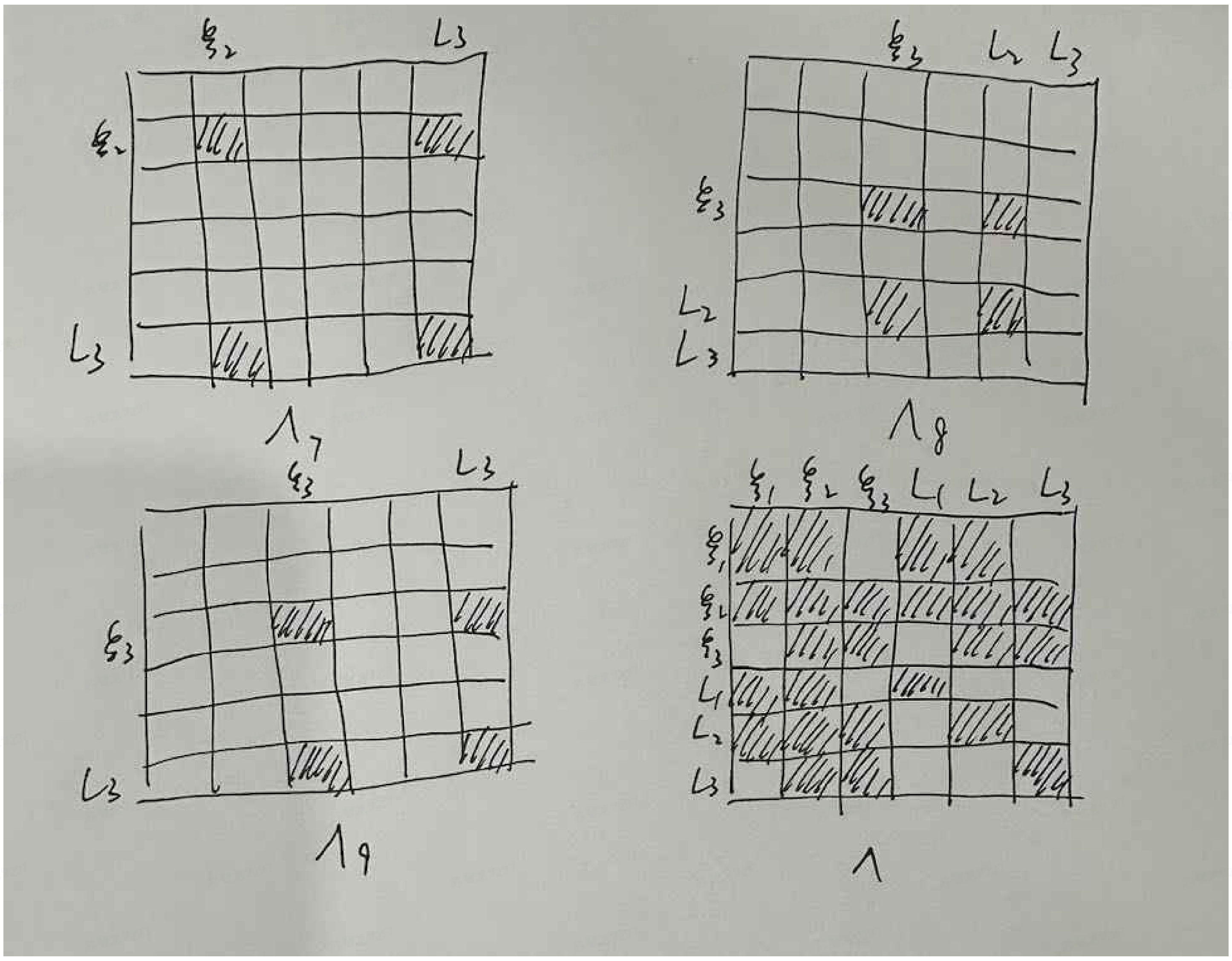
Λ_4



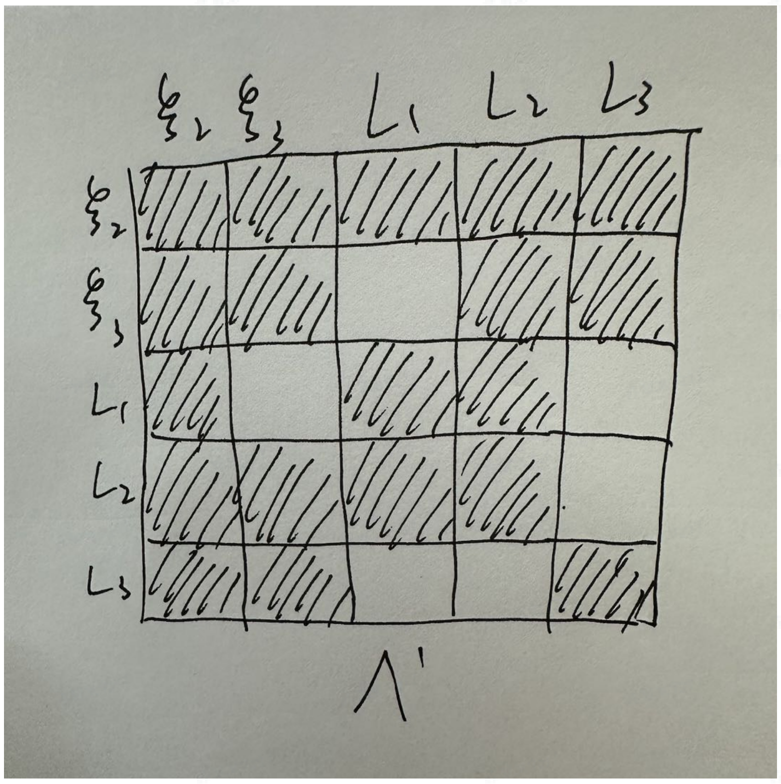
Λ_5



Λ_6



1.2 绘制相机 ξ 1 被 marg 以后的信息矩阵 Λ '



2. 阅读《Relationship between the Hessian and Covariance Matrix for Gaussian Random Variables》. 证明信息矩阵和协方差的逆之间的关系

信息矩阵的定义：

$$[\mathcal{I}(\theta)]_{i,j} = \mathrm{E}\left[\left(\frac{\partial}{\partial \theta_i}\log f(X;\theta)\right)\left(\frac{\partial}{\partial \theta_j}\log f(X;\theta)\right)\middle|\theta\right]$$

论文中多元高斯分布负对数似然估计的Hessian等于协方差的逆的证明，该过程与信息矩阵定义一致：

Consider a Gaussian random vector $\boldsymbol{\theta}$ with mean $\boldsymbol{\theta}^\star$ and covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{\theta}}$ so its joint probability density function (PDF) is given by:

$$p(\boldsymbol{\theta}) = (2\pi)^{-\frac{N_\theta}{2}} |\boldsymbol{\Sigma}_{\boldsymbol{\theta}}|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}(\boldsymbol{\theta}-\boldsymbol{\theta}^\star)^T \boldsymbol{\Sigma}_{\boldsymbol{\theta}}^{-1}(\boldsymbol{\theta}-\boldsymbol{\theta}^\star)\right] \tag{A.1}$$

The objective function can be defined as its negative logarithm:

$$J(\boldsymbol{\theta}) \equiv -\ln p(\boldsymbol{\theta}) = \frac{N_\theta}{2}\ln 2\pi + \frac{1}{2}\ln|\boldsymbol{\Sigma}_{\boldsymbol{\theta}}| + \frac{1}{2}(\boldsymbol{\theta}-\boldsymbol{\theta}^\star)^T \boldsymbol{\Sigma}_{\boldsymbol{\theta}}^{-1}(\boldsymbol{\theta}-\boldsymbol{\theta}^\star) \tag{A.2}$$

which is a quadratic function of the components in $\boldsymbol{\theta}$. By taking partial differentiations with respect to θ_l and $\theta_{l'}$, the (l, l') component of the Hessian matrix can be obtained:

$$\mathcal{H}^{(l,l')}(\boldsymbol{\theta}^\star) = \left.\frac{\partial^2 J(\boldsymbol{\theta})}{\partial\theta_l \partial\theta_{l'}}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}^\star} = (\boldsymbol{\Sigma}_{\boldsymbol{\theta}}^{-1})^{(l,l')} \tag{A.3}$$

so the Hessian matrix is equal to the inverse of the covariance matrix:

$$\mathcal{H}(\boldsymbol{\theta}^\star) = \boldsymbol{\Sigma}_{\boldsymbol{\theta}}^{-1} \tag{A.4}$$

3. 补充作业代码中单目 Bundle Adjustment 信息矩阵的计算

```cpp
// zhilong > Documents > vio_homework > VIO_Hw > ch4 > nullspace_test > hessian_nullspace_test.cpp > main()
            /// 请补充完整作业信息矩阵块的计算
            // H.block(?,?,?,?) += ?;
            // H.block(?,?,?,?) += ?;
            // H.block(?,?,?,?) += ?;
            H.block(j*3 + 6*poseNums, j*3 + 6*poseNums, 3, 3) += jacobian_Pj.transpose() * jacobian_Pj;
            H.block(i*6, j*3 + 6*poseNums, 6, 3) += jacobian_Ti.transpose() * jacobian_Pj;
            H.block(j*3 + 6*poseNums, i*6 , 3, 6) += jacobian_Pj.transpose() * jacobian_Ti;
```

- 奇异值最后 7 维接近于 0，表明零空间的维度为 7。

```
  0.00351651
  0.00302963
  0.00253459
  0.00230246
  0.00172459
0.000422374
3.21708e-17
2.06732e-17
1.43188e-17
7.66992e-18
6.08423e-18
6.05715e-18
3.94363e-18
root@zhilong-u
```