EE511-F18 Assignment 3: Some Interesting DRV's
Due: Thursday, October 11

1) Sum of Uniform RV's

Define:

$$N = \text{Min}\left\{ n : \sum_{i=1}^{n} U_i > 1 \right\}$$

where $\{U_i\}$ are iid Uniform(0,1) RV's.

Find (by simulation): $\hat{m} = E[N]$ an estimator for the mean.

Can you guess (or derive) the true value for $E[N]$?

2) Minima of Uniform RV's

Define: $N = \text{Min}\left\{ n : U_1 \le U_2 \le \ldots \le U_{n-1} > U_n \right\}$

i.e. the n^{th} term is the first that is less than its predecessor, where $\{U_i\}$ are independent identically distributed (iid) Uniform(0,1) RV's.

Find (by simulation): $\hat{m} = E[N]$ an estimator for the mean.

Can you guess (or derive) the true value for $E[N]$?

3) Maxima of Uniform RV's

Consider the sequence of iid Uniform RV's $\{U_i\}$. If $U_j > \max_{i=1:j-1}\{U_i\}$ we say U_j is a record.

Example: the records are underlined.

$$\{U_i\} = \{\underline{0.2314}, \underline{0.4719}, 0.1133, \underline{0.5676}, 0.4388, \underline{0.9453}, \ldots.\}$$

(note that the U_i are on the real line and we are just showing 4 digits of precision).

Let X_i be an RV for the distance from the $i-1^{\text{st}}$ record to the i^{th} record. Clearly $X_1 = 1$ always. In this example, $X_2 = 1, X_3 = 2, X_4 = 2$.

Distribution of Records: Using simulation, obtain (and graph) a probability histogram for X_2 and X_3 and compute the sample means.

Can you find an analytical expression for $P(X_2 = k)$? (Hint: condition on U_1 and then uncondition.) What does this say about $E[X_2]$?