Fall 2018 EE511 Simulation

Project #7: due Thursday December 6

(absolute deadline Weds Dec 12, 5pm)

If Y is uniformly distributed over (0,1); then $X = -\ln(1-Y)$ or $X = -\ln(Y)$ will be exponentially distributed with parameter λ.

a) Why is this so?

b) Use this to generate 1000 samples X_i with $\lambda = 1$ and plot the empirical distribution for these samples and compare to an exponentially distributed RV.

c) Define the random variable $S_n = \sum_{i=1}^{n} X_i$. The pdf for S_n can be shown to be:

$$f_{S_n}(x) = \frac{(\lambda x)^{n-1}}{(n-1)!} \lambda e^{-\lambda x}$$

And the CDF can be expressed as:

$$F_{S_n}(x) = 1 - \sum_{j=0}^{n-1} \frac{1}{j!} (\lambda x)^j e^{-\lambda x} \quad \text{(defining } 0! = 1 \text{ for convenience)}$$

This distribution is the Erlang-*n* distribution. So, we can generate an RV that has an Erlang-*n* distribution by summing *n* samples of an RV which is exponentially distributed. Use this approach to generate 1000 samples from an Erlang-3 distribution.

d) A more (computationally) efficient way to generate samples for S_n uses:

$$S_n = \frac{-\ln\left(\prod_{i=1}^{n} U_i\right)}{\lambda}$$

Where $U_i \sim Uniform(0,1)$. (Why does this work?) Use this approach to generate a series of samples for S_n (for $n = 3$), compute mean and sample variance, and compare to part c).

e) Use the rejection method to generate samples from an Erlang-3 distribution directly. Compute mean and variance and compare to c) and d). (See notes for Lecture 10/11 on rejection method for CRVs))

f) Compare the time taken to generate N samples of Erlang-3 distributions using the three approaches. (Use a value of N that causes the slowest method to take ~30 seconds.)