Quantum Bernoulli Factory

Suzie Brown

June 22, 2018

Problem statement

Suppose we have access to a black box producing coin flips where the probability of observing heads is p. Roughly speaking, a Bernoulli factory is an algorithm that uses queries of this black box to produce a coin flip where the probability of observing heads is $f(p)$, for some specified function f.

Let us now make this notion more precise.

Definition 1. Let $f : S \to [0,1]$ be a function with domain $S \subseteq [0,1]$, and suppose we have a sequence of Bernoulli random variables $X_1, X_2, \ldots \overset{iid}{\sim} \text{Bernoulli}(p)$ with unknown parameter $p \in [0,1]$. Let $U \in \mathcal{U}$ denote a set of auxiliary random variables with known distributions, independent of p. Let $\tau(U)$ be a stopping time with respect to the natural filtration. A *Bernoulli factory* is a function $\mathcal{A} : \mathcal{U} \times \{0,1\}^T \to \{0,1\}$ such that $\mathcal{A}(U, X_1, \ldots, X_T) \sim \text{Bernoulli}(f(p))$ for all $p \in [0,1]$.

Example 1. Hello world

References

Dale, H., Jennings, D. and Rudolph, T. (2015), 'Provable quantum advantage in randomness processing', *Nature communications* **6**, 8203.