Conditional SMC definition

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Aim: to settle on a definition of conditional SMC with multinomial resampling from which to base results, preferably a definition that is compatible with the standing assumption of Koskela et al. (2018).

The quantities we know upfront are described below. The setting of this algorithm within particle MCMC (Andrieu et al., 2010) is as a single run of the particle filter within one step of the MCMC algorithm: the 'immortal trajectory' on which we are conditioning is sampled from the preceding step.

N	number of particles
T	number of SMC iterations
$\{K_t\}_{t=1,\dots,T}$	Markov kernels
$\{g_t\}_{t=0,\dots,T}$	potentials
μ	initial distribution
$x^*_{0:T}$	positions of immortal trajectory
$a^*_{0:T}$	indices of immortal trajectory

Require: $N, T, \mu, \{K_t\}, \{g_t\}, x^*_{0:T}, a^*_{0:T}$

1: **for** $i \in \{1, \dots, N\}$ **do**
2:     Sample $X_0^{(i)} \sim \mu$
3: **end for**
4: $X_0^{(a_0^*)} \leftarrow x_0^*$
5: **for** $i \in \{1, \dots, N\}$ **do**
6:     $w_0^{(i)} \leftarrow \frac{g_0(X_0^{(i)})}{\sum_{j=1}^N g_0(X_0^{(j)})}$
7: **end for**
8: **for** $t \in \{0, \dots, T-1\}$ **do**
9:     Sample $a_t^{(1:N)} \sim \text{Categorical}(\{1, \dots, N\}, w_t^{(1:N)})$
10:     $a_t^{(a^*_{t+1})} \leftarrow a_t^*$
11:     **for** $i \in \{1, \dots, N\}$ **do**
12:         Sample $X_{t+1}^{(i)} \sim K_{t+1}(X_t^{(a_t^{(i)})}, \cdot)$
13:     **end for**
14:     $X_{t+1}^{(a^*_{t+1})} \leftarrow X^*_{t+1}$
15:     **for** $i \in \{1, \dots, N\}$ **do**
16:         $w_{t+1}^{(i)} \leftarrow \frac{g_{t+1}(X_t^{(a_t^{(i)})}, X_{t+1}^{(i)})}{\sum_{j=1}^N g_{t+1}(X_t^{(a_t^{(j)})}, X_{t+1}^{(j)})}$
17:     **end for**
18: **end for**

- Lines 9–10 okay since marginal of Categorical distribution is still Categorical (so we can sample all N indices and just overwrite the immortal particle's parent index).
- This algorithm assumes the indices of the immortal line are given, as well as its positions. This is the situation described in Andrieu et al. (2010).

- For our purposes we can therefore suppose that the immortal line comprises particle 1 in each generation.
- This can be achieved even in a real scenario by simply relabelling the particles after resampling so that the immortal particle takes the label 1.

However, this method for the resampling is not consistent with the standing assumption because, given the immortal indices, the assignment of offspring is biased towards the (fixed) immortal parent. We can overcome this problem by considering a slightly different framework (which can just as well be applied in practice) where the immortal index is not pre-specified, but is instead sampled uniformly at each generation.

Require: $N, T, \mu, \{K_t\}, \{g_t\}, x^*_{0:T}$

for $i \in \{1, \ldots, N\}$ **do**
    Sample $X_0^{(i)} \sim \mu$
end for
Sample $a_0^* \sim \text{Uniform}(\{1, \ldots, N\})$
$X_0^{(a_0^*)} \leftarrow x_0^*$
for $i \in \{1, \ldots, N\}$ **do**
    $w_0^{(i)} \leftarrow \frac{g_0(X_0^{(i)})}{\sum_{j=1}^N g_0(X_0^{(j)})}$
end for
for $t \in \{0, \ldots, T-1\}$ **do**
    Sample $a_t^{(1:N)} \sim \text{Categorical}(\{1, \ldots, N\}, w_t^{(1:N)})$
    Sample $a_{t+1}^* \sim \text{Uniform}(\{1, \ldots, N\})$
    $a_t^{(a_{t+1}^*)} \leftarrow a_t^*$
    for $i \in \{1, \ldots, N\}$ **do**
        Sample $X_{t+1}^{(i)} \sim K_{t+1}(X_t^{(a_t^{(i)})}, \cdot)$
    end for
    $X_{t+1}^{(a_{t+1}^*)} \leftarrow X_{t+1}^*$
    for $i \in \{1, \ldots, N\}$ **do**
        $w_{t+1}^{(i)} \leftarrow \frac{g_{t+1}(X_t^{(a_t^{(i)})}, X_{t+1}^{(i)})}{\sum_{j=1}^N g_{t+1}(X_t^{(a_t^{(j)})}, X_{t+1}^{(j)})}$
    end for
end for

References

Andrieu, C., Doucet, A. and Holenstein, R. (2010), 'Particle markov chain monte carlo methods', *Journal of the Royal Statistical Society: Series B (Statistical Methodology)* **72**(3), 269–342.

Koskela, J., Jenkins, P. A., Johansen, A. M. and Spano, D. (2018), 'Asymptotic genealogies of interacting particle systems with an application to sequential monte carlo', *arXiv preprint arXiv:1804.01811* .