

A little calculation

Suzie Brown

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Lemma 1. $D_N(t) \leq c_N(t)$ for all t .

Proof.

$$\begin{aligned} D_N(t) &:= \frac{1}{(N)_2} \sum_{i=1}^N (\nu_t^{(i)})_2 \frac{1}{N} \left\{ \nu_t^{(i)} + \frac{1}{N} \sum_{j \neq i}^N (\nu_t^{(j)})^2 \right\} \leq \frac{1}{(N)_2} \sum_{i=1}^N (\nu_t^{(i)})_2 \frac{1}{N} \left\{ \nu_t^{(i)} + \frac{1}{N} \sum_{j \neq i}^N N \nu_t^{(j)} \right\} \\ &= \frac{1}{(N)_2} \sum_{i=1}^N (\nu_t^{(i)})_2 \frac{1}{N} \left\{ \sum_{j=1}^N \nu_t^{(j)} \right\} \leq \frac{1}{(N)_2} \sum_{i=1}^N (\nu_t^{(i)})_2 = c_N(t). \end{aligned} \tag{1}$$

□