Comparing expected coalescence rates for multinomial & residual resampling

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Case $N = 2$

We can calculate the expected coalescence rates explicitly. With only $N = 2$ particles, the coalescence rate becomes

$$\mathbb{E}[c_N(t)|\mathcal{F}_{t-1}] = \frac{1}{(N)_2}\sum_{i=1}^{N}\mathbb{E}\left[(v_t^{(i)})_2|\mathcal{F}_{t-1}\right] = \mathbb{P}[v_t^{(1)} = 0] + \mathbb{P}[v_t^{(1)} = 2]$$

For residual resampling,

$$\mathbb{E}[c_2^r(t)|\mathcal{F}_{t-1}] = \mathbb{I}\{w_t^{(1)} \geq 1/2\}(2w_t^{(1)} - 1) + \mathbb{I}\{w_t^{(1)} < 1/2\}(2w_t^{(2)} - 1)$$

And for multinomial resampling,

$$\begin{aligned}
\mathbb{E}[c_2^m(t)|\mathcal{F}_{t-1}] &= (w_t^{(1)})^2 + (w_t^{(2)})^2 \\
&= \mathbb{I}\{w_t^{(1)} \geq 1/2\}((w_t^{(1)})^2 + (w_t^{(2)})^2) + \mathbb{I}\{w_t^{(1)} < 1/2\}((w_t^{(1)})^2 + (w_t^{(2)})^2) \\
&\geq \mathbb{I}\{w_t^{(1)} \geq 1/2\}(w_t^{(1)})^2 + \mathbb{I}\{w_t^{(1)} < 1/2\}(w_t^{(2)})^2
\end{aligned}$$

Then since $(w_t^{(i)} - 1)^2 = (w_t^{(i)})^2 - 2w_t^{(i)} + 1 \geq 0$, we have that $(w_t^{(i)})^2 \geq 2w_t^{(i)} - 1$ and hence we can conclude

$$\mathbb{E}[c_2^m(t)|\mathcal{F}_{t-1}] \geq \mathbb{E}[c_2^r(t)|\mathcal{F}_{t-1}]. \quad \square$$