## Comparing expected coalescence rates for multinomial & residual resampling

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## Case N=2

We can calculate the expected coalescence rates explicitly. With only N=2 particles, the coalescence rate becomes

$$\mathbb{E}[c_N(t)|\mathcal{F}_{t-1}] = \frac{1}{(N)_2} \sum_{i=1}^N \mathbb{E}\left[(v_t^{(i)})_2 | \mathcal{F}_{t-1}\right] = \mathbb{P}[v_t^{(1)} = 0] + \mathbb{P}[v_t^{(1)} = 2]$$

For residual resampling,

$$\mathbb{E}[c_2^r(t)|\mathcal{F}_{t-1}] = \mathbb{I}\{w_t^{(1)} > 1/2\}(2w_t^{(1)} - 1) + \mathbb{I}\{w_t^{(1)} < 1/2\}(2w_t^{(2)} - 1)$$

And for multinomial resampling,

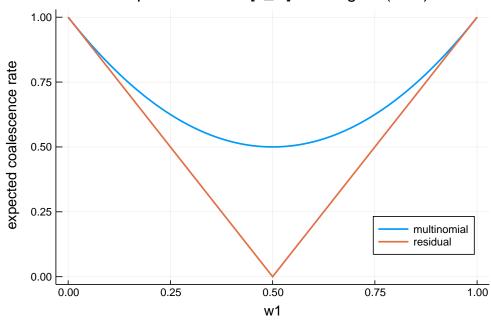
$$\mathbb{E}[c_2^m(t)|\mathcal{F}_{t-1}] = (w_t^{(1)})^2 + (w_t^{(2)})^2$$

$$= \mathbb{I}\{w_t^{(1)} > 1/2\}((w_t^{(1)})^2 + (w_t^{(2)})^2) + \mathbb{I}\{w_t^{(1)} < 1/2\}((w_t^{(1)})^2 + (w_t^{(2)})^2)$$

$$\geq \mathbb{I}\{w_t^{(1)} > 1/2\}(w_t^{(1)})^2 + \mathbb{I}\{w_t^{(1)} < 1/2\}(w_t^{(2)})^2$$

Then since  $(w_t^{(i)} - 1)^2 = (w_t^{(i)})^2 - 2w_t^{(i)} + 1 \ge 0$ , we have that  $(w_t^{(i)})^2 \ge 2w_t^{(i)} - 1$  and hence we can conclude  $\mathbb{E}[c_2^m(t)|\mathcal{F}_{t-1}] \ge \mathbb{E}[c_2^r(t)|\mathcal{F}_{t-1}]$ .

## dependence of E[c\_N] on weights (N=2)



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