# Possible future research topics

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## Extending the theoretical results

- Relax condition that potentials are bounded below (e.g. see Cappé, Moulines, Rydén "Inference in HMMs", Chapter 9)
- Show that the condition of BJJK Theorem 1 is necessary as well as sufficient (e.g. see Möhle & Sagitov 1998, 2001, 2003?)
- What is the effect of adaptive resampling?
- Corollaries also for residual-multinomial and residual-star resampling (although it might be a negative result in the case of residual-star). For res-star, the finite-time-scale result should be a simple adaptation of the one for stratified resampling.
- Rates of convergence (modifying more of Möhle's work).
- Is it possible to have an SMC system where 3rd moments are non-negligible, leading to a limiting Lambda-coalescent, say?
- Could badly-behaved potentials produce a non-Kingman Lambda-coalescent? For example, use for potentials some heavy-tailed fitnesses like in Schweinsberg model. (This is probably not interesting from an SMC point of view.)
- A way to estimate coalescent rates / time scale a priori, for some specific tractable class of models, say.
- Do our results apply to cloning models, or other continuous-time models?
- See if Jacob & Rubenthaler's ("path storage in the PF") brute-force technique can be adapted for use in Conditional SMC, to UB tree height
- Does CSMC with an "unlikely" immortal line converge to a structured coalescent?

## Resampling

- Derive expressions for the one-step Monte Carlo variance with SSP resampling and residual-systematic resampling, (and systematic resampling?). Is there another ordering result there? In the case of SSP, GCW19 stated that this is an interesting open problem!
- Compare residual-systematic vs. systematic resampling: are they equivalent? What if the weights are sorted? Maybe start by coding up an exploratory experiment.
- Is Li et al 2020 Equation (1) correct? (It was removed between v1 and v2 on arXiv so possibly they found it was wrong.) If so, it could provide a more elegant proof about conditional variance of strat.
- Explore more generally the effect of pre-sorting the weights. How does this link with results of Gerber Chopin Whiteley (where they sort by particle position)? My suspicion is now that sorting the weights only has an effect on the resampling schemes based on inversion-sampling.
- Prove that one-step MC variance is always lower under res-X resampling than under X resampling.

- Compare theoretical computation/storage costs and parallelisability between the different resampling schemes.
- What is the deal with MLJ16's (parallel resampling) rejection-resampler-with-deterministic-first-proposal? Is it biased? We know it reduces the variance compared to multinomial resampling, and less so as the weights' variance increases. Can we be more specific?
- Can the comparison of time-scales for different schemes be wrangled into/ related to a direct comparison of the variances? Is this even useful?
- (See marked notebook 3 page circa Jun/Jul 2020 for some more thoughts/ideas.)
- The difference $\Delta_i$, as defined in `phd/latex/randomised_rounding/randomised_rounding.pdf`, seems to tend to a quadratic in the weight as $N \to \infty$. Prove it?
- (Just for fun:) in stochastic rounding, how many possible ways are there to assign the offspring counts? Consider that each of the N counts takes one of two possible values, but this will overcount a lot because we are also constrained by offspring counts summing to N. It's $\binom{N}{R}$ isn't it? Where $R$ as in residual resampling. Give each parent its minimum number of offspring, and you'll be left with $R$ unassigned offspring that have to be given to $R$ distinct parents among $N$. $\binom{N}{R}$ is maximised if $R \simeq N/2$, in which case $\binom{N}{R} \simeq$??.
- (Conjecture:) pre-sorting of weights reduces resampling variance, but increases the coalescence rate. Intuition: when weights are sorted, small weights that sum to less than 1/N are grouped together so only one of them can have a child. (It may be that this effect is entirely cancelled by the reduction in variance elsewhere.)

## Simulation experiments

- Now we have weak convergence, redo similar experiments to those at end of KJJS, but without having to fudge it.
- Code up all the different sampling schemes (and sorted/unsorted variants etc.) and come up with some useful (if only illustrative) experiments comparing them. (I started doing this by plotting $\tau_N(t)$ against $t$, and got some nice results.)
- Decide which functions would be most interesting to illustrate the performance of resampling schemes, say using ternary diagrams. (See marked notebook 3 page at 8/7/20.)
- Explore pre-asymptotic behaviour of CSMC for example. I did some work on this previously.
- Make edits to my ternary plot "library" (see notebook 3 marked page at 8/7/20).