Thesis Outline at 30 months

Suzie Brown

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1 Introduction

2 Background (done)

2.1 Interacting Particle Systems

General description of what an IPS is, what they are useful for, and how to simulate one. SMC as a specific class of IPS. Relevant background about SMC.

2.2 Coalescent Theory

Review of literature from population genetics, including the relevant population models and results about the corresponding coalescent processes.

2.3 SMC Genealogies

Description of how genealogies are induced by SMC algorithms and how this is related to the performance of the algorithms. Existing results characterising these genealogies.

2.4 Resampling

Tour of resampling schemes and other ways to possibly improve performance by varying the resampling step, e.g. adaptive resampling. Existing results and conjectures comparing the performance of different schemes. Introduction of stochastic rounding as a class of resampling schemes. Implementation and usage in practice.

3 Limiting Coalescents for SMC Genealogies (done)

3.1 General Result for IPSs

A refinement of Koskela et al. (2018, Theorem 1) with more tractable conditions. Proof of the theroem.

3.2 Application to Multinomial Resampling

Corollary for multinomial resampling, as the simplest case.

3.3 Application to Stochastic Rounding-based Resampling

Proof that the theorem holds for SMC with stochastic rounding-based resampling, and interpretation of that result including comparison on time scale versus multinomial resampling.

3.4 Application to Conditional SMC

[Possibly move the introduction to CSMC into the Background chapter.] Introduction of conditional SMC as a key component of particle MCMC, and what type of problem this is useful for. Proof that the theorem holds for conditional SMC and interpretation of this result. Discussion of behaviour in the pre-limiting regime, supported by simulation studies.

4 Ancestor Sampling (in progress)

Several adaptations to standard SMC have been proposed to mitigate for ancestral degeneracy. An important contribution in the case of particle Gibbs is ancestor sampling, whereby a new parent for the immortal particle is chosen after each forward transition. This substantially complicates the genealogical process compared to the basic particle Gibbs algorithm. I will attempt to describe this within a similar framework to other SMC genealogies, with the aim of comparing and quantifying the extent to which ancestor sampling can eliminate ancestral degeneracy.

5 Stronger Mode of Convergence (future work)

So far I only proved convergence in the sense of finite-dimensional distributions. In SMC applications we are typically interested in expectations of test functions, so an upgrade to weak convergence is desirable. The required tightness argument has been set out in the case of neutral IPSs (Möhle, 1999), but an extension to the non-neutral case required for application to SMC is non-trivial. I will attempt to extend the techniques of Möhle (1999) and Gerber et al. (2019) to prove weak convergence for these SMC algorithms.

6 Conclusions / Discussion

References

Gerber, M., Chopin, N. and Whiteley, N. (2019), 'Negative association, ordering and convergence of resampling methods', *The Annals of Statistics* **47**(4), 2236–2260.

Koskela, J., Jenkins, P. A., Johansen, A. M. and Spanò, D. (2018), 'Asymptotic genealogies of interacting particle systems with an application to sequential Monte Carlo', *arXiv preprint arXiv:1804.01811* .

Lindsten, F., Schön, T. B. et al. (2013), 'Backward simulation methods for Monte Carlo statistical inference', *Foundations and Trends® in Machine Learning* **6**(1), 1–143.

Möhle, M. (1999), 'Weak convergence to the coalescent in neutral population models', *Journal of Applied Probability* **36**(2), 446–460.