Thoughts on residual-multinomial resampling

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- $R := N - \sum \lfloor N w_t^{(i)} \rfloor$
- $r_i := \frac{1}{R}(N w_t^{(i)} - \lfloor N w_t^{(i)} \rfloor)$
- Parent i is deterministically assigned $\lfloor N w_t^{(i)} \rfloor$ offspring, for each i, and the remaining R offspring are assigned to parents chosen independently $\sim$ Categorical$(r_{1:N})$
- Let $\mathcal{I} \subseteq [N]$ denote the index set of offspring that are assigned to the "deterministic slots"
- $|\mathcal{I}| = N - R = \sum \lfloor N w_t^{(i)} \rfloor$
- $\mathcal{I} \mid w_t^{(1:N)}$ is uniform over the $\binom{N}{R}$ possible subsets of size $N - R$, due to the Standing Assumption
- $a_t^{\mathcal{I}}$ and $a_t^{\mathcal{I}^c}$ are conditionally independent given $\mathcal{I}$, due to the Standing Assumption
- The assumed bounds on g_t imply almost surely $w_t^{(i)} \in [\frac{1}{a^2 N}, \frac{a^2}{N}]$, hence $\lfloor N w_t^{(i)} \rfloor \in [a^{-2}, a^2]$ and $|\mathcal{I}| = O(N)$

So...

$$\mathbb{P}[a_t^{(1:N)} = a_{1:N} \mid \mathcal{H}_t] = \sum_{\mathcal{I} \subseteq [N]} \mathbb{P}[\mathcal{I} \mid \mathcal{H}_t]\, \mathbb{P}[a_t^{\mathcal{I}} = a_{\mathcal{I}} \mid \mathcal{I}, \mathcal{H}_t]\, \mathbb{P}[a_t^{\mathcal{I}^c} = a_{\mathcal{I}^c} \mid \mathcal{I}, \mathcal{H}_t] \tag{1}$$

$\mathbb{P}[\mathcal{I} \mid \mathcal{H}_t]$ is not tractable, but will sum to one if the other terms can be bounded independently of $\mathcal{I}$.

$$\mathbb{P}[a_t^{\mathcal{I}} = a_{\mathcal{I}} \mid \mathcal{I}, \mathcal{H}_t] \propto \left(\prod_{i=1}^{N} \mathbb{1}_{\{|\{j \in \mathcal{I} : a_j = i\}| = \lfloor N w_t^{(i)} \rfloor\}} \right) \left(\prod_{i \in \mathcal{I}} q_{t-1}(X_t^{(a_i)}, X_{t-1}^{(i)}) \right) \tag{2}$$

Indicators ensure correct number of deterministic slots for each parent, q's incorporate probability of particular parent-offspring assignment.

$$\mathbb{P}[a_t^{\mathcal{I}^c} = a_{\mathcal{I}^c} \mid \mathcal{I}, \mathcal{H}_t] \propto \prod_{i \in \mathcal{I}^c} r_{a_i} q_{t-1}(X_t^{(a_i)}, X_{t-1}^{(i)}) \tag{3}$$

r's are the probabilities from the Categorical sampling of parents, q's as above.