Fix $\varepsilon > 0$. Let N be large enough that $\varepsilon > 2/N$, and let $A_i(r) := \{\nu_r^{(i)} \leq N\varepsilon\}$. Following [Möhle and Sagitov, 2003, Proof of Lemma 5.5],

$$\frac{1}{(N)_2}\sum_{i=1}^{N}(\nu_r^{(i)})_2(\mathbb{1}_{A_i(r)} + \mathbb{1}_{A_i(r)^c}) \leq \frac{N\varepsilon}{(N)_2}\sum_{i=1}^{N}\nu_r^{(i)} + \sum_{i=1}^{N}\mathbb{1}_{A_i(r)^c} \leq (1+O(N^{-1}))\varepsilon + \sum_{i=1}^{N}\mathbb{1}_{A_i(r)^c}.$$

Thus,

$$\mathbb{E}[c_N(\tau_N(t))] \leq (1+O(N^{-1}))\varepsilon + \sum_{i=1}^{N}\mathbb{P}(\nu_{\tau_N(t)}^{(i)} > N\varepsilon).$$

By Markov's inequality,

$$\begin{aligned}
\mathbb{E}[c_N(\tau_N(t))] &\leq (1+O(N^{-1}))\varepsilon + \sum_{i=1}^{N}\frac{\mathbb{E}[(\nu_{\tau_N(t)}^{(i)})_3]}{(N\varepsilon)_3} \\
&\leq (1+O(N^{-1}))\varepsilon + \frac{1+O(N^{-1})}{\varepsilon^3}\mathbb{E}\left[\sum_{r=1}^{\tau_N(t)} D_N(r)\right] \to \varepsilon
\end{aligned}$$

as $N \to \infty$.

References

M. Möhle and S. Sagitov. Coalescent patterns in exchangeable diploid population models. *Journal of Mathematical Biology*, 47:337–352, 2003.