Counter-example to multinomial dominating residual coalescence rate

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THIS DOCUMENT IS OBSOLETE. After all, it asserts one example which turns out to be incorrect. For a collection of other failed attempts, see `resmn_roundup_210526`.

We have conjectured that the expected coalescence rate should be higher for multinomial resampling than for residual resampling. This is definitely true asymptotically (equation 9 in the report), but here is a counter-example in the finite case.

Take the scenario with $N = 3$ particles, and weights $w_t^{(1:3)} = (0.3, 0.3, 0.4)$. The possible outcomes of resampling are listed in the table below, along with their probabilities under multinomial and residual resampling.

Outcome	No. of cases	$\sum(v_t^{(i)})_2$	Probability (multinomial)	Probability (residual)
$(3,0,0)$ or $(0,3,0)$	2	6	27/1000	0
$(0,0,3)$	1	6	64/1000	4/64
$(2,1,0)$ or $(1,2,0)$	2	2	81/1000	0
$(2,0,1)$ or $(0,2,1)$	2	2	108/1000	9/64
$(1,0,2)$ or $(0,1,2)$	2	2	144/1000	12/64
$(1,1,1)$	1	0	216/1000	18/64

Actually not really, I just added it up wrong the first time.

$\mathbb{E}[c_N^m(t)|\mathcal{F}_{t-1}] = 0.34$
$\mathbb{E}[c_N^r(t)|\mathcal{F}_{t-1}] = 18/64 \simeq 0.28$