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Some more things about SMC genealogies...

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SUMMARY

Some key words: Blah; Blah; Blah

1. INTRODUCTION

Notation to introduce at some point: $E_t[...] = E[... \mid \mathcal{F}_{t-1}]$; $(a)_b = a(a-1)...(a-b+1)$;

2. THE UPDATED THEOREM...

THEOREM 1. *Suppose that there exists a deterministic sequence $(b_N)_{N\geq 1}$ such that $b_N \to 0$ and*

$$\frac{1}{(N)_3}\sum_{i=1}^{N} E_t[(\nu_t^{(i)})_3] \leq b_N \frac{1}{(N)_2}\sum_{i=1}^{N} E_t[(\nu_t^{(i)})_2] \tag{1}$$

for all N, uniformly in $t \geq 1$. And the standing assumption... Then something converges to something in some way as $N \to \infty$ ...

Proof. Theorem 1 has the same conclusion as Koskela et al. (2018, Theorem 1), but with much more tractable predicates. We will show that these simpler predicates are sufficient.

The conditions for Koskela et al. (2018, Theorem 1) are the following:

$$E[c_N(t)] \to 0, \tag{2}$$

$$E\left[\sum_{r=\tau_N(s)+1}^{\tau_N(t)} D_N(r)\right] \to 0, \tag{3}$$

$$E\left[\sum_{r=\tau_N(s)+1}^{\tau_N(t)} c_N(r)^2\right] \to 0, \tag{4}$$

$$E[\tau_N(t) - \tau_N(s)] \leq C_{t,s} N; \tag{5}$$

as $N \to \infty$, for some strictly positive constant $C_{t,s}$ that does not depend on N.

The series of Lemmata 1–3 show that the assumptions (2)–(4) all follow from (1). Lemma 4 shows that assumption (5) is not needed. □

LEMMA 1. (3) $\Rightarrow$ (4).

LEMMA 2. (1) $\Rightarrow$ (2).

LEMMA 3. (1) $\Rightarrow$ (3).

LEMMA 4. *write a lemma here...*

Equipped with this simplified statement of the theorem, we can now prove convergence for some more complicated sequential Monte Carlo algorithms.

3. RESAMPLING WITH STOCHASTIC ROUNDINGS

DEFINITION 1. *Let $X \geq 0$. A random variable $Y : \mathbb{R}_+ \to \mathbb{N}$ is a* stochastic rounding *of X if Y takes the values*

$$Y = \begin{cases} \lfloor X \rfloor & \text{with probability } 1 - X + \lfloor X \rfloor \\ \lfloor X \rfloor + 1 & \text{with probability } X - \lfloor X \rfloor \end{cases}$$

4. CONDITIONAL SEQUENTIAL MONTE CARLO UPDATES

REFERENCES

KOSKELA, J., JENKINS, P. A., JOHANSEN, A. M. & SPANÒ, D. (2018). Asymptotic genealogies of interacting particle systems with an application to sequential Monte Carlo. *arXiv preprint arXiv:1804.01811* .

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