

## Math 475 Homework 4

*Due Wednesday 2nd November at start of lecture*

### Exercises in R

1. Take a random sample (i.i.d.),  $\{X_1, \dots, X_n\}$ , of size  $n = 15$  from an Exponential distribution with mean 7. Compute the sample mean,  $\bar{X}$ . Then repeat this 1000 times, and plot a histogram of the following quantities:
  - (i)  $\bar{X}$
  - (ii)  $\sqrt{n}\bar{X}$
  - (iii)  $\sqrt{n}(\bar{X} - \mu)/\sigma$ , where  $\mu$  is the mean of  $X_1$  and  $\sigma = \sqrt{\text{Var}(X_1)}$
2. Repeat Exercise (1) with  $n = 100$ .
3. Repeat Exercise (1), but use a random sample from a Weibull distribution with shape parameter equal to 5 and scale parameter equal to 1.
4. Repeat Exercise (3) with  $n = 100$ .
5. Look in the documentation of the `statmod` package in R, and learn about the `gauss.quad` function to implement Gaussian quadrature. There is an example there related to the Gamma distribution. Explain what that example is illustrating, and explain why that particular choice of orthogonal polynomials (as given in the example) is or is not appropriate for this problem.
6. Look at the interpolation example from this blog posting:  
<http://blog.revolutionanalytics.com/2015/09/interpolation-and-smoothing-functions-in-base-r.html>

Both splines and loess are used.

- (i) Change the R code to have  $n = 1e2$ , and change the random noise term in the sampling model based on a sine wave. In particular, change the `rnorm` component in line 7 of the code to be an asymmetric distribution, with an appropriately small variance (otherwise there could be more noise than signal). Plot the results.
- (ii) Explain each chunk of the code. That is, explain what each function is doing. Be specific—look through any documentation necessary to say exactly what the default options are for the functions `spline`, `loess`, etc.