The displacement field $\boldsymbol{s}$ is produced by an earthquake is governed by the momentum equation,

$$\rho\,\partial_t^2 \boldsymbol{s} = \boldsymbol{\nabla}\cdot\boldsymbol{T} + \boldsymbol{f} \tag{1}$$

The weak form of above equation is produced by dotting the momentum equation with an arbitrary test vector $\boldsymbol{w}$, integrating by the model volume Ω. (We neglect boundary condition for the temporary)

$$\begin{aligned}
&\int_\Omega \boldsymbol{\nabla}\cdot\boldsymbol{T}\cdot w(\boldsymbol{x})\ \mathrm{d}^3\boldsymbol{x} \\
&= \Sigma_{i=1}^3 \int_\Omega w_i(\boldsymbol{x})\nabla\cdot\boldsymbol{T}_i\ \mathrm{d}^3\boldsymbol{x} \\
&= \Sigma_{i=1}^3 \int_\Omega \nabla\cdot(w_i(\boldsymbol{x})\boldsymbol{T}_i) - \nabla w_i(\boldsymbol{x})\cdot\boldsymbol{T}_i\ \mathrm{d}^3\boldsymbol{x} \\
&= -\Sigma_{i=1}^3 \int_\Omega \nabla w_i(\boldsymbol{x})\cdot\boldsymbol{T}_i\ \mathrm{d}^3\boldsymbol{x} \\
&= -\int_\Omega \boldsymbol{\nabla}\boldsymbol{w}(\boldsymbol{x}) : \boldsymbol{T}\ \mathrm{d}^3\boldsymbol{x}
\end{aligned}$$

The Source function is

$$\boldsymbol{f} = -\boldsymbol{M}\cdot\boldsymbol{\nabla}\delta(\boldsymbol{x}-\boldsymbol{x}_s)S(t) \tag{2}$$

Using the properties fo Dirac delta distribution,after integration, it transformed in the following way,

$$\begin{aligned}
&\int_\Omega \boldsymbol{f}\cdot\boldsymbol{w}\ \mathrm{d}^3\boldsymbol{x} \\
&= -S(t)\int_\Omega \boldsymbol{M}\cdot\boldsymbol{\nabla}\delta(\boldsymbol{x}-\boldsymbol{x}_s)\cdot\boldsymbol{w}\ \mathrm{d}^3\boldsymbol{x} \\
&= \boldsymbol{M} : \boldsymbol{\nabla}\boldsymbol{w}(\boldsymbol{x}_s)S(t)
\end{aligned}$$

In 2 dimension, We use equation like this ?? I can derive this using Green Formula, But I don't know if it's right.

$$\int_\Omega \rho\,\boldsymbol{w}\cdot\partial_t^2\boldsymbol{s}\ \mathrm{d}^2\boldsymbol{x} = -\int_\Omega \boldsymbol{\nabla}\boldsymbol{w}(\boldsymbol{x}) : \boldsymbol{T}\ \mathrm{d}^2\boldsymbol{x} + \boldsymbol{M} : \boldsymbol{\nabla}\boldsymbol{w}(\boldsymbol{x}_s)S(t) \tag{3}$$