The displacement field $\boldsymbol{s}$ is produced by an earthquake is governed by the momentum equation,

$$\rho\, \partial_t^2 \boldsymbol{s} = \boldsymbol{\nabla} \cdot \boldsymbol{T} + \boldsymbol{f} \tag{1}$$

,where $\boldsymbol{T} = (\boldsymbol{T}_1, \boldsymbol{T}_2, \boldsymbol{T}_3)^T$, and $\boldsymbol{T}_i$ is vector function. The weak form of above equation is produced by dotting the momentum equation with an arbitrary test vector $\boldsymbol{w} = (w_1(\boldsymbol{x}), w_2(\boldsymbol{x}), w_3(\boldsymbol{x}))^T$, integrating by the model volume Ω. (We neglect boundary condition for the temporary)

$$\begin{aligned}
&\int_\Omega \boldsymbol{\nabla} \cdot \boldsymbol{T} \cdot w(\boldsymbol{x})\ \mathrm{d}^3\boldsymbol{x} \\
&= \Sigma_{i=1}^3 \int_\Omega w_i(\boldsymbol{x}) \nabla \cdot \boldsymbol{T}_i\ \mathrm{d}^3\boldsymbol{x} \\
&= \Sigma_{i=1}^3 \int_\Omega \nabla \cdot (w_i(\boldsymbol{x}) \boldsymbol{T}_i) - \nabla w_i(\boldsymbol{x}) \cdot \boldsymbol{T}_i\ \mathrm{d}^3\boldsymbol{x} \\
&= -\Sigma_{i=1}^3 \int_\Omega \nabla w_i(\boldsymbol{x}) \cdot \boldsymbol{T}_i\ \mathrm{d}^3\boldsymbol{x} \\
&= -\int_\Omega \boldsymbol{\nabla} \boldsymbol{w}(\boldsymbol{x}) : \boldsymbol{T}\ \mathrm{d}^3\boldsymbol{x}
\end{aligned} \tag{2}$$

The Source function is

$$\boldsymbol{f} = -\boldsymbol{M} \cdot \boldsymbol{\nabla} \delta(\boldsymbol{x} - \boldsymbol{x}_s) S(t) \tag{3}$$

Using the properties fo Dirac delta distribution,after integration, it transformed in the following way,

$$\begin{aligned}
&\int_\Omega \boldsymbol{f} \cdot \boldsymbol{w}\ \mathrm{d}^3\boldsymbol{x} \\
&= -S(t) \int_\Omega \boldsymbol{M} \cdot \boldsymbol{\nabla} \delta(\boldsymbol{x} - \boldsymbol{x}_s) \cdot \boldsymbol{w}\ \mathrm{d}^3\boldsymbol{x} \\
&= \boldsymbol{M} : \boldsymbol{\nabla} \boldsymbol{w}(\boldsymbol{x}_s) S(t)
\end{aligned} \tag{4}$$

In 2 dimension, We use equation like this ?? I can derive this using Green Formula, But I don't know if it's right.

$$\int_\Omega \rho\, \boldsymbol{w} \cdot \partial_t^2 \boldsymbol{s}\ \mathrm{d}^2\boldsymbol{x} = -\int_\Omega \boldsymbol{\nabla} \boldsymbol{w}(\boldsymbol{x}) : \boldsymbol{T}\ \mathrm{d}^2\boldsymbol{x} + \boldsymbol{M} : \boldsymbol{\nabla} \boldsymbol{w}(\boldsymbol{x}_s) S(t) \tag{5}$$