

The primary goal of this introduction is to demonstrate how to implement spectral element method in 2d. We seek to produce the displacement field caused by an earthquake in a finite earth model with volume Ω . In order to simplify calculation, we just use stress-free boundary condition. We will talk about artificial absorbing boundary condition in detail later. The displacement field \mathbf{u} is produced by an earthquake is governed by the momentum equation,

$$\rho \partial_t^2 \mathbf{u} = \nabla \cdot \mathbf{T} + \mathbf{f} \quad (1)$$

,where $\mathbf{T} = (\mathbf{T}_1, \mathbf{T}_2, \mathbf{T}_3)^T$, and \mathbf{T}_i is vector function. The weak form of above equation is produced by dotting the momentum equation with an arbitrary test vector $\mathbf{w} = (w_1(\mathbf{x}), w_2(\mathbf{x}), w_3(\mathbf{x}))^T$, integrating by the model volume Ω . Since we use stress-free boundary condition, which is easy to derive the weak form of wave equation, we have $\mathbf{T}|_{\partial\Omega} = 0$

$$\begin{aligned} & \int_{\Omega} \nabla \cdot \mathbf{T} \cdot \mathbf{w}(\mathbf{x}) \, d^3\mathbf{x} \\ &= \sum_{i=1}^3 \int_{\Omega} w_i(\mathbf{x}) \nabla \cdot \mathbf{T}_i \, d^3\mathbf{x} \\ &= \sum_{i=1}^3 \int_{\Omega} \nabla \cdot (w_i(\mathbf{x}) \mathbf{T}_i) - \nabla w_i(\mathbf{x}) \cdot \mathbf{T}_i \, d^3\mathbf{x} \\ &= \sum_{i=1}^3 \left(\int_{\partial\Omega} w_i(\mathbf{x}) \mathbf{T}_i \cdot \mathbf{n} \, d\mathbf{s} - \int_{\Omega} \nabla w_i(\mathbf{x}) \cdot \mathbf{T}_i \, d^3\mathbf{x} \right) \\ &= -\sum_{i=1}^3 \int_{\Omega} \nabla w_i(\mathbf{x}) \cdot \mathbf{T}_i \, d^3\mathbf{x} \\ &= -\int_{\Omega} \nabla \mathbf{w}(\mathbf{x}) : \mathbf{T} \, d^3\mathbf{x} \end{aligned} \quad (2)$$

The Source function is

$$\mathbf{f} = -\mathbf{M} \cdot \nabla \delta(\mathbf{x} - \mathbf{x}_s) S(t) \quad (3)$$

Using the properties of Dirac delta distribution, after integration, it transformed in the following way,

$$\begin{aligned} & \int_{\Omega} \mathbf{f} \cdot \mathbf{w} \, d^3\mathbf{x} \\ &= -S(t) \int_{\Omega} \mathbf{M} \cdot \nabla \delta(\mathbf{x} - \mathbf{x}_s) \cdot \mathbf{w} \, d^3\mathbf{x} \\ &= \mathbf{M} : \nabla \mathbf{w}(\mathbf{x}_s) S(t) \end{aligned} \quad (4)$$

In 2 dimension, We use equation like this ?? I can derive this using Green Formula, But I don't know if it's right.

$$\int_{\Omega} \rho \mathbf{w} \cdot \partial_t^2 \mathbf{u} \, d^2\mathbf{x} = - \int_{\Omega} \nabla \mathbf{w}(\mathbf{x}) : \mathbf{T} \, d^2\mathbf{x} + \mathbf{M} : \nabla \mathbf{w}(\mathbf{x}_s) S(t) \quad (5)$$