

Adiabatic process

An adiabatic process is described by a Hamiltonian H that "slowly" transits from an initial H_0 towards a final H_1 :

$$H = (1 - t)H_0 + tH_1, \quad t \in [0, 1] \quad (1)$$

So that ground state remains ground state during the process (in general, the n-th eigenstate remains n-th eigenstate?).

Adiabatic Computing for Constraint Optimization

To solve constraint optimization problem, we provide H_1 in local form (eg. sum of ZZ interactions, or QUBO form) as energy costs, which punishes unwanted solutions, so that the optimal solutions are the ground state of H_1 . And H_0 are usually set up as $\prod_i X_i$'s so that the initial ground state is the uniform superposition. For example:

$$H_1 = \sum_{ij} c_{ij} Z_i Z_j + \sum_k d_k Z_k \quad (2)$$

$$H_0 = \prod_i X_i \quad (3)$$

$$\varphi_0 = \left(\prod_i H_i \right) |00..0\rangle \quad (4)$$

Then the process will be realized on some Ising model or other physical devices, and measuring the final state will likely produce a solution, guaranteed by the adiabatic theorem.

Trotterization and Circuit simulation

Trotterization can be used to simulate the adiabatic process on a circuit model. The time evolution corresponds to (1) is:

$$e^{-iH\Delta t} = e^{-i((1-t)H_0+tH_1)\Delta t} \quad (5)$$

And by trotterization (verify non-commutativity?):

$$e^{A+B} = \lim_{n \rightarrow \infty} (e^{A/n} e^{B/n})^n \quad (6)$$

$$\approx (e^{A/N} e^{B/N})^N \quad (7)$$

with large enough N (how large exactly?). Notice if H_0 and H_1 are given as (2) and (3), then the first exponential will be the RX gates and the second RZ and RZZ gates, in QAOA algorithms they are called cost layers and mixer layers.