

The difference between Cross Entropy & Logistic Regression Loss

Cross Entropy loss function

(e.g. three-class)

for multi-class classification problem.

Data points

$$\{x_i, y_i\}_{i=1}^m, x_i \in R^n, y_i = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ or }$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \text{ or } \dots \text{ or } \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad y_i = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ or } \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \text{ or } \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\hat{y}_1, \hat{y}_2, \hat{y}_3 = \text{softmax}(z_1, z_2, z_3)$$

$$(\hat{y}_1, \hat{y}_2, \hat{y}_3) = \text{softmax}(z_1, z_2, z_3)$$

$$\hat{y}_i = (\hat{y}_{i1}, \hat{y}_{i2}, \hat{y}_{i3}) = \left(\frac{e^{z_1}}{e^{z_1} + e^{z_2} + e^{z_3}}, \frac{e^{z_2}}{e^{z_1} + e^{z_2} + e^{z_3}}, \frac{e^{z_3}}{e^{z_1} + e^{z_2} + e^{z_3}} \right)$$

$$\hat{y}_{i1} + \hat{y}_{i2} + \hat{y}_{i3} = 1$$

Loss:

$$\frac{1}{m} \sum_{i=1}^m y_{i1} \log \frac{1}{\hat{y}_{i1}} + y_{i2} \log \frac{1}{\hat{y}_{i2}} + y_{i3} \log \frac{1}{\hat{y}_{i3}}$$

only one term is non-zero

Logistic Loss function

for binary classification.

$$\{x_i, y_i\}_{i=1}^m, x_i \in R^n,$$

$$y_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ or } \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Notice: It's not $y_i = 0$ or 1.

$$\hat{y}_i = (\hat{y}_{i1}, \hat{y}_{i2}) \neq$$

$$= (\text{Sigmoid}(\theta^T x_i), 1 - \text{Sigmoid}(\theta^T x_i))$$

$$= \left(\frac{1}{1 + e^{-\theta^T x_i}}, \frac{e^{-\theta^T x_i}}{1 + e^{-\theta^T x_i}} \right)$$

$$\hat{y}_{i1} + \hat{y}_{i2} = 1$$

Loss: only one term is non-zero.

$$\frac{1}{m} \sum_{i=1}^m y_{i1} \log \frac{1}{\hat{y}_{i1}} + (1 - y_{i1}) \log \frac{1}{1 - \hat{y}_{i1}}$$

画出这部分神经网络

