

Advanced Econometrics Project



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Faculty of Economic Sciences

Analyzing factors affecting employee Job Satisfaction: An ordered Logit/Probit regression approach

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Abstract

This paper investigates the factors influencing job satisfaction among employees, utilizing a comprehensive dataset of 1,470 observations and 34 independent variables. The primary aim is to identify significant predictors of job satisfaction and understand their relative impacts. We employ ordered logit and probit models to analyze the ordinal nature of job satisfaction, adopting a general-to-specific modeling approach to refine the models to include only significant variables.

The methodology involves encoding categorical variables, standardizing continuous variables, and iteratively testing and refining the models. An intermediate model is also explored to examine potential interaction effects between key variables, specifically the interaction between years at the company and gender.

The main findings indicate that attrition, hourly rate, marital status (single), number of companies worked, overtime, and years with the current manager are significant predictors of job satisfaction. Attrition significantly increases the probability of lower job satisfaction, while working overtime decreases it and enhances the probability of very high job satisfaction. Higher hourly rates, contrary to expectations, are associated with higher probabilities of lower job satisfaction, possibly compensating for other negative job aspects.

The analysis provides actionable insights for organizations aiming to enhance employee satisfaction and retention, emphasizing the importance of addressing factors leading to attrition, effectively managing overtime, and understanding the nuanced impacts of compensation on job satisfaction.

Introduction

Job satisfaction remains a pivotal and widely discussed topic in the field of organizational behavior and human resource management. Understanding the factors influencing job satisfaction is crucial, given that these factors can vary significantly from one individual to another. Some employees may feel the need to frequently change jobs to seek better opportunities or environments, while others may find contentment and stability in long-term positions.

While wage is often perceived as a primary determinant of job satisfaction, it is only one of many factors that can influence an employee's overall sense of fulfillment at work. Various other elements, such as work conditions, job roles, organizational culture, and external variables, also play critical roles. For instance, high net income may not suffice to ensure job satisfaction if the job is overly demanding, monotonous, or hazardous.

This study seeks to explore whether external variables like marital status, education level, gender, and age can significantly impact job satisfaction. Additionally, the tenure at a company is another variable of interest—does the length of time spent with a single employer positively or negatively influence job satisfaction?

Given the complexity of job satisfaction, which involves a multitude of influencing factors and subjective evaluations, this project aims to dissect these elements comprehensively. What may be significant for one individual might not hold the same weight for another, making it essential to analyze these variables with a nuanced approach.

The primary aim of this project is to analyze job satisfaction using a dataset comprising 1,470 observations and 34 independent variables, with 'job satisfaction' as the dependent variable. This analysis will be conducted through various ordered choice models, supplemented by rigorous testing to understand the characteristics of the data and the impact of different variables.

Our main hypotheses are as follows:

- **Hypothesis 1:** The relationship between years at the company and job satisfaction is the same for both male and female employees.
- **Hypothesis 2:** Attrition significantly affects job satisfaction levels.
- **Hypothesis 3:** Working overtime significantly affects job satisfaction levels.
- **Hypothesis 4:** Hourly rate significantly affects job satisfaction levels.

The significance of this research lies in its potential to provide actionable insights for organizations aiming to enhance employee satisfaction and retention. By identifying and understanding the key factors that influence job satisfaction, organizations can implement targeted strategies to create more fulfilling and productive work environments.

In the following sections, we will detail the methodology, data, and results of our analysis, ultimately providing a comprehensive understanding of the determinants of job satisfaction and their relative significance.

Literature review

Lakshmi Kanchana and Ruwan Jayathilaka analyze factors impacting employee turnover intentions among professionals in Sri Lankan startups. Their study highlights that employee turnover occurs in stages, with independent factors affecting each stage differently. Critical factors identified include job satisfaction, work-life balance, happiness, management support, career management, innovative work behavior, co-worker support, and leader-member exchange. The results show that job satisfaction and co-worker support negatively impact turnover, while leader-member exchange positively impacts it. Additionally, the study finds that men have higher turnover intentions than women due to higher job satisfaction among female employees.

In "Climbing the Ladders of Job Satisfaction and Employee Organizational Commitment," José A. C. Vieira employs an ordered probit approach using a semi-nonparametric method to analyze job satisfaction and organizational commitment across 36 countries. The study reveals that both employee and job-related attributes influence organizational commitment, mediated by job satisfaction. Management can enhance these factors through controllable instruments. The findings also show differences in job satisfaction across countries, with no direct relationship to geographical location. Additionally, while gender does not directly correlate with job satisfaction levels, it impacts employees' attitudes towards exerting extra effort for organizational success.

Agnieszka Szulc-Obłozka examines job satisfaction among young adults in Poland and the Czech Republic using an ordered logit model. Their study identifies wage satisfaction as the most important factor in both countries. However, significant disparities exist, with educational factors enhancing job satisfaction in Poland, while parents' junior education impacts it in the Czech Republic. Behavioral factors like expense control and discipline positively influence job satisfaction in both countries.

Mahra Essa Mohammed Belarai Alfalasi explores employee attrition and its direct correlation with job satisfaction in her thesis, "Study to Improve the Employee Experiences and Reducing the Employee Attrition." The research predicts employee attrition using machine learning techniques, with the LSVM model demonstrating the highest precision. Key factors influencing attrition include overtime, job role, years in the current role, years with the current manager, stock option level, marital status, total working hours, and years at the company. The study underscores LSVM's potential as a powerful tool for predicting employee attrition, providing valuable insights for workforce management and retention strategies. However, it also notes limitations due to the lack of data ownership and suggests that additional variables may need consideration.

Data

Short Data Description

For our project we used data from kraggle.com with 1470 observations. This dataset contains information about employees of a company, including personal and professional details. The goal is to analyze factors that influence job satisfaction and other work-related aspects. Among the variables included are demographic characteristics such as age and gender, employment details like department, education level, and salary, as well as performance and satisfaction indicators such as work-life balance, relationship satisfaction, and performance ratings. Additionally, the dataset includes information about job mobility, such as the frequency of business travel and company change history. This dataset can be useful for understanding and predicting factors that affect employee satisfaction and retention.

Dependent variable

JobSatisfaction: The dependent variable measuring the level of job satisfaction of an employee: 1 - 'Low', 2 - 'Medium', 3 - 'High', 4 - 'Very High'.

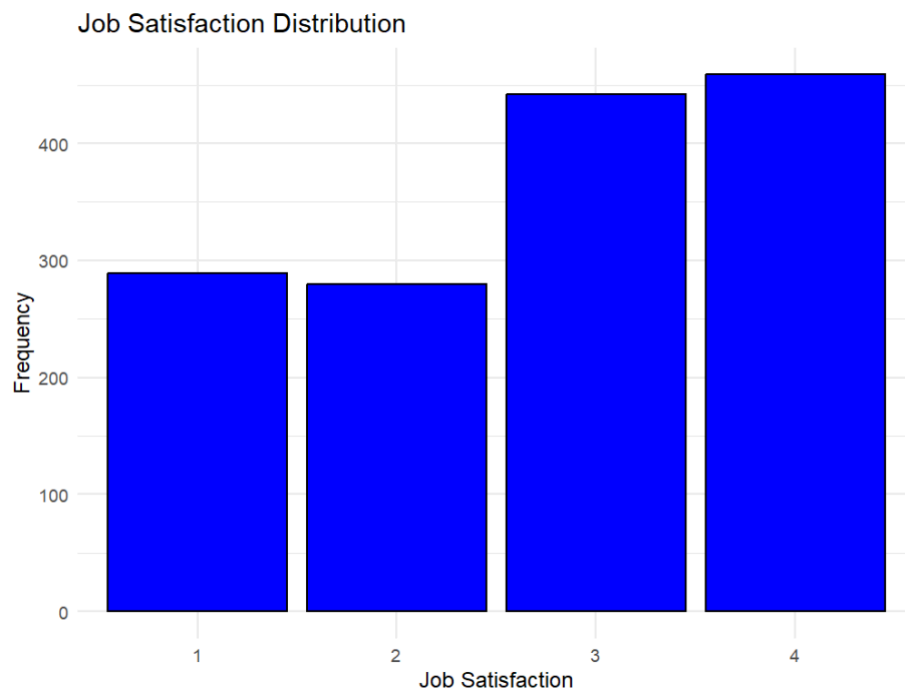


Figure 1 - Job satisfaction distribution

The frequency distribution reveals the prevalence of different job satisfaction levels within the dataset, with certain levels being more predominant than others. Without specifying exact numbers, it can be observed that certain categories have a higher frequency, indicating a greater representation of individuals reporting big particular levels of job

satisfaction. This distribution provides an overview of the distributional pattern of job satisfaction levels and highlights the relative prevalence of each category within the dataset.

Table 1 - Job Satisfaction Distribution

Levels	1	2	3	4
Frequency	289	280	442	459

And if we provide exact numbers:

A total of 289 observations correspond to a job satisfaction level of 1, 280 observations to level 2, 442 observations to level 3, and 459 observations to level 4. These counts provide valuable insight into the prevalence of different levels of job satisfaction among the individuals or cases represented in the dataset, suggesting that a considerable portion of individuals report moderate to high levels of job satisfaction.

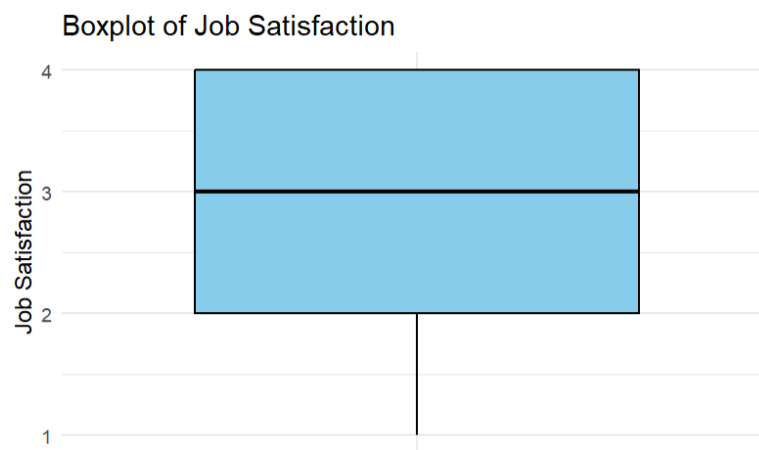


Figure 2 - Job Satisfaction Box Plot

This boxplot indicates that the majority of the data falls within the range of values from 2 to 4 on the y-axis, with the central line (median) positioned at the value of 3.

Independent variables

Age: The age of each worker.

Attrition: Binary variable that reflects if each observation has done an attrition (0-no, 1-yes).

BusinessTravel: A factor variable indicating the frequency of business travel, encoded each one with 0 or 1. ("Non-Travel", "Travel Rarely", "Travel Frequently").

DailyRate: The daily rate of pay for the employee.

Department: A factor variable indicating the department in which the employee works, encoded each one with 0-1. ("Sales", "Research & Development", "Human Resources").

DistanceFromHome: The distance from the employee's home to the workplace in kilometers.

Education: A numerical measure of Bachelor's Degree GPA of the employee.

EnvironmentSatisfaction: The number of colleagues the employees consider as individuals they feel comfortable collaborating with in the work environment.

Gender: A factor variable indicating the gender of the employee (Male=0, Female=1).

HourlyRate: The hourly rate of pay for the employee.

JobInvolvement: A numerical measure of the employee's involvement in their job.

JobLevel: A numerical measure of the employee's job level.

JobRole: A factor variable indicating the job role of the employee, encoded each one with 0 -1. (Sales Executive, Healthcare Representative, Laboratory Technician, Research Director, Manufacturing Director, Manager, Research Scientist).

MaritalStatus: A factor variable indicating the marital status of the employee, encoded each one with 0-1. ("Single", "Married", "Divorced").

MonthlyIncome: The monthly income of the employee.

MonthlyRate: The monthly rate of pay for the employee.

NumCompaniesWorked: The number of companies the employee has worked for before the current one.

OverTime: A binary factor variable indicating whether the employee works overtime ("Yes", "No").

PercentSalaryHike: The percentage increase in the employee's salary.

PerformanceRating: A numerical measure of the employee's performance.

RelationshipSatisfaction: The number of workplace associates the employees have developed a personal friendship with.

StockOptionLevel: The number of the stock options granted to the employee.

TotalWorkingYears: The total number of years the employee has been working.

TrainingTimesLastYear: The number of training sessions the employee attended in the last year.

WorkLifeBalance: A numerical measure of the employee's work-life balance.

YearsAtCompany: The number of years the employee has been with the current company.

YearsInCurrentRole: The number of years the employee has been in their current role.

YearsSinceLastPromotion: The number of years since the employee's last promotion.

YearsWithCurrManager: The number of years the employee has been working with their current manager.

We have done a presentation for the independent variables that will be the significant variables **after the general to specific approach**. The independent significant variables are the following ones:

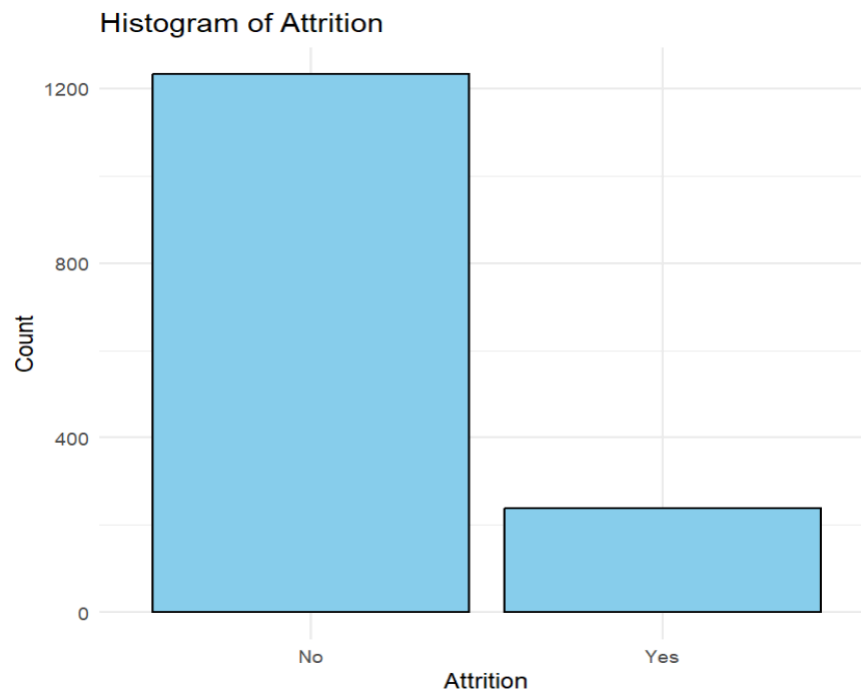


Figure 3 - Attrition Distribution

The histogram for the Attrition variable illustrates a notable disparity in the count of observations between the categories of "Yes" and "No." With over 1200 observations falling under the "No" category and substantially fewer instances in the "Yes" category, the graph underscores a predominant trend of employees not experiencing attrition within the dataset.

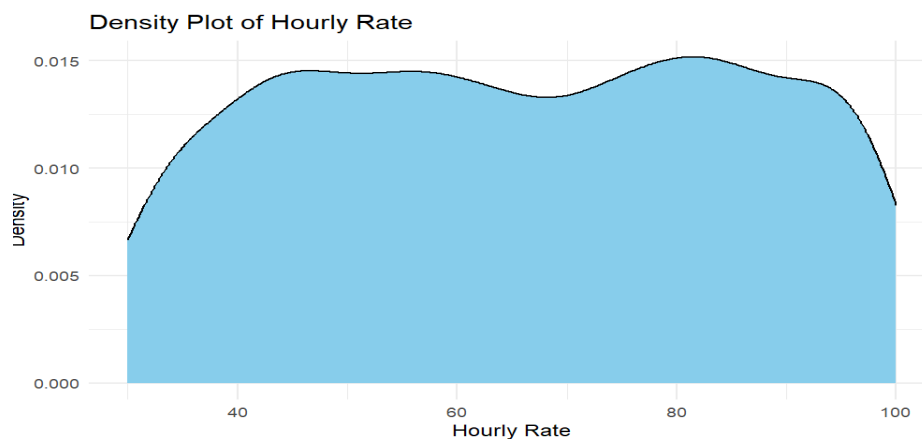


Figure 4 - Hourly RateDistribution

The density plot for the HourlyRate variable reveals a prominent peak in the distribution between the ranges of 40 to 90, indicating that the majority of individuals in the dataset fall within this range. This concentration of observations within the 40 to 90 range implies that a significant portion of the workforce in the dataset earns hourly rates within this band, while hourly rates below 40 or above 90 are less prevalent among the sample population.

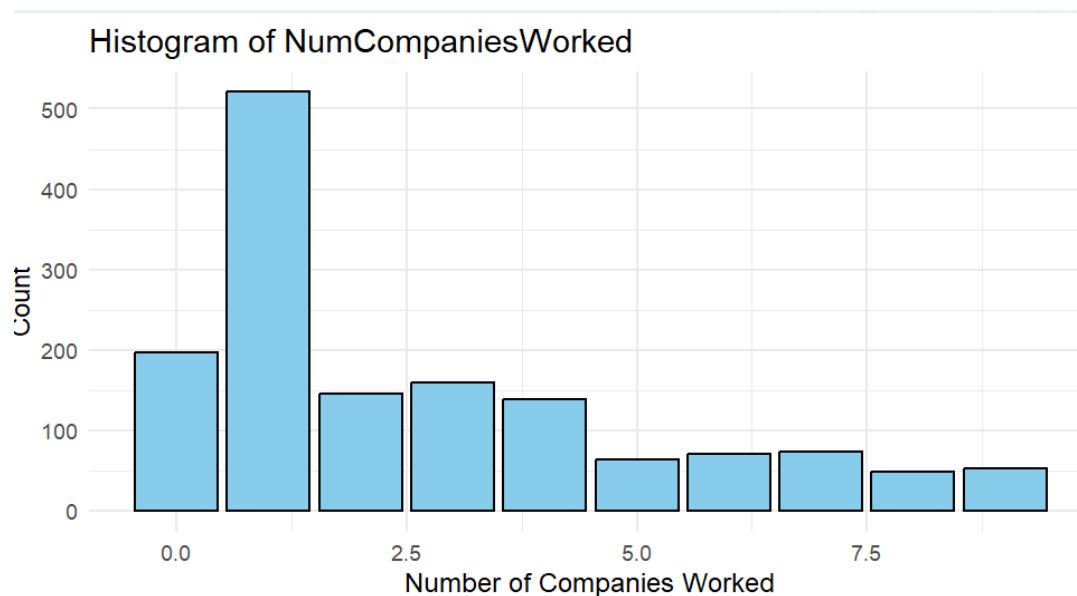


Figure 5 - Number Companies Worked Distribution

The histogram for the NumCompaniesWorked variable highlights a notable concentration of individuals within the category of "1," with the majority of observations falling within this range. In contrast, the frequencies for other values of the variable appear to be relatively consistent and lower compared to the peak observed at "1." This pattern suggests that a significant proportion of individuals in the dataset have worked for a single company.

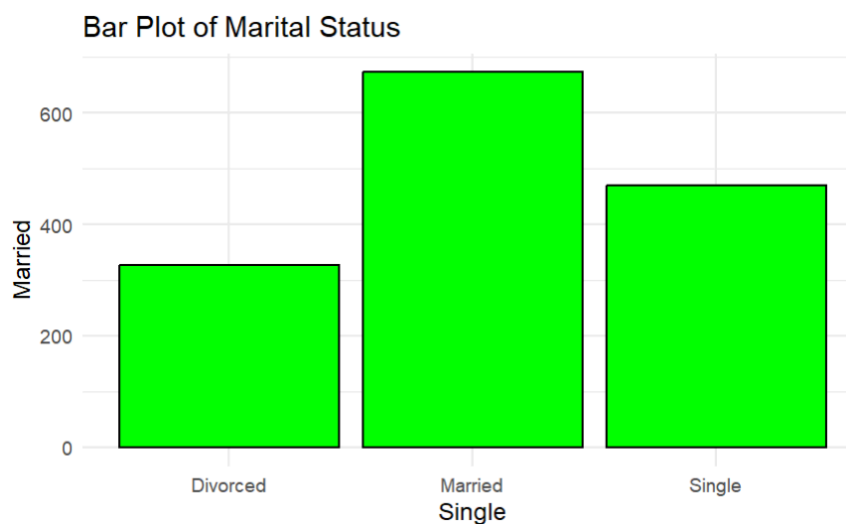


Figure 6 - Marital Status Distribution

The majority of individuals are categorized as "Married," followed by "Single," and then "Divorced." It's important to note that while the exact significant variable name is "Single," we've chosen to present it within the broader context of MaritalStatus to offer a comprehensive overview of the marital status distribution in the dataset.

Pie Chart of OverTime

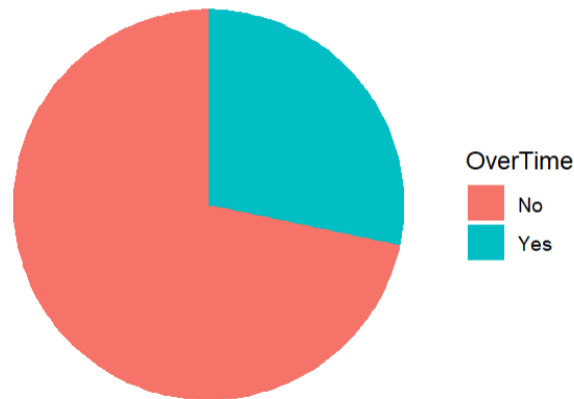


Figure 7 - Pie Chart of OverTime

As we can see on the Pie Chart, there are so many observations that haven't done over time during his worker life against the 26-27% (approx.) of the observations that have suffered this variable.

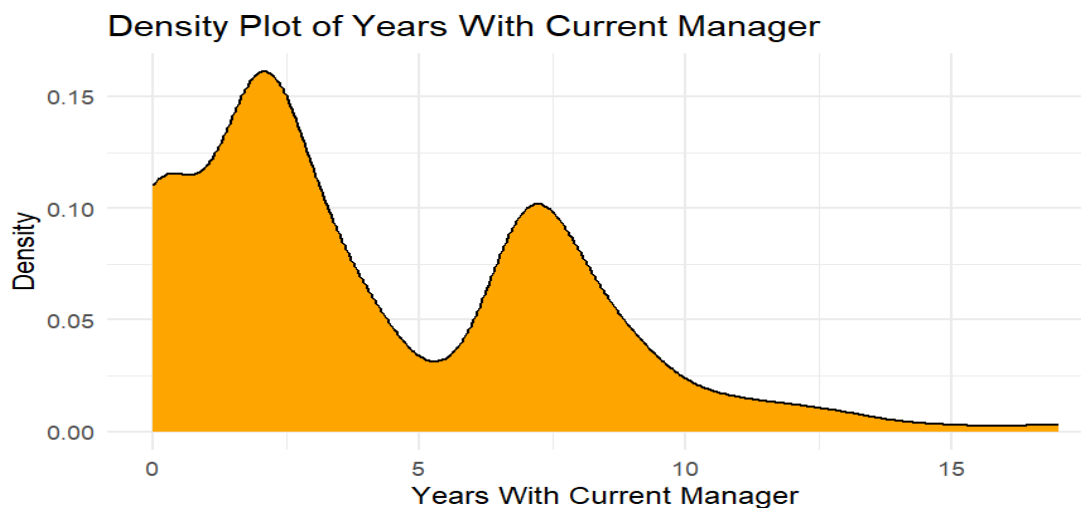


Figure 8 - Years With Current Manager Density

With this density it is clear that the 1, 2, 3 and 7 years are the most current ones for the observations. This pattern suggests that there may be clusters or concentrations of employees who have relatively short durations (1, 2, or 3 years) or medium durations (7 years) with their current managers, with fewer employees falling into the intermediate durations and in the long term (9 or more).

Data Preparation and Transformation

1. Data Inspection

To start with, we used the `str()` and `summary()` functions in R to inspect the structure of the dataset and check for missing values. These functions helped in understanding the data types of each variable and provided summary statistics. It was found that the dataset had no missing values, ensuring that we could proceed with further data transformations without needing imputation.

2. Removal of Irrelevant Variables

Certain variables were deemed irrelevant for the ordered logit model due to their lack of variability or relevance to job satisfaction (Figure 9):

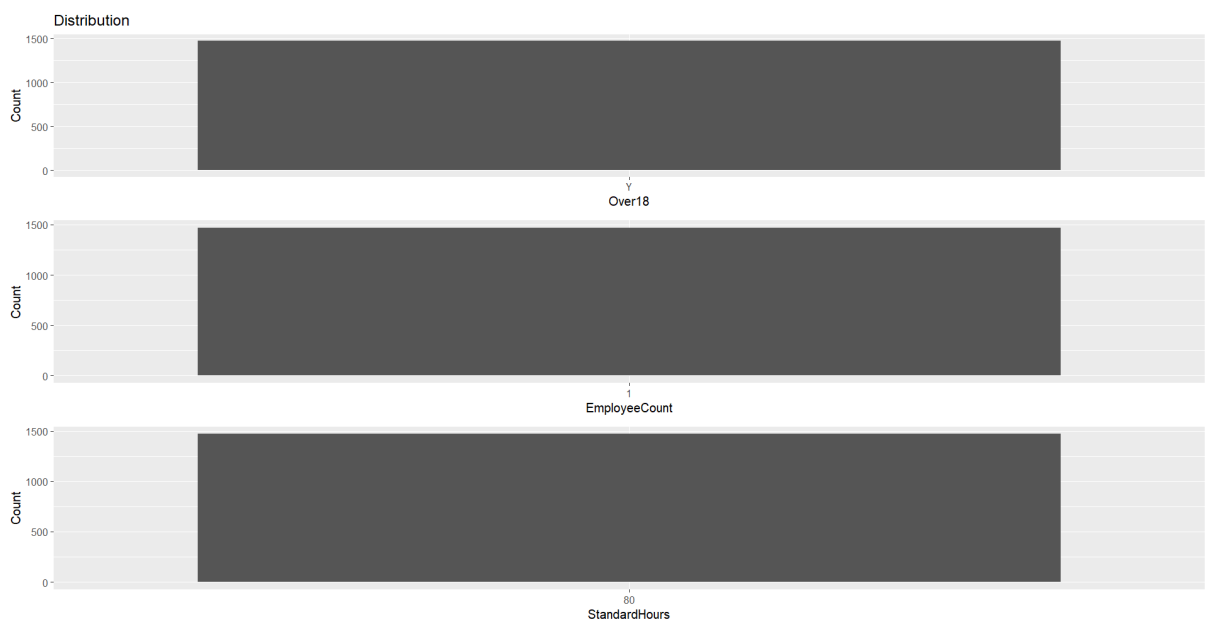


Figure 9 - Uniform Distribution of Irrelevant Variables

- **Over18:** All employees are over 18 years old, making this variable redundant. This variable will have no variability and won't contribute to the model. It's redundant because it's not discriminative in the context of job satisfaction.
- **EmployeeCount:** This variable contains the total number of employees, which is the same for all records. It has no variability and doesn't provide any individual-level information that could impact job satisfaction.
- **StandardHours:** If this variable represents the standard number of hours employees are expected to work and if it is the same for all employees, it won't help in differentiating job satisfaction levels. It's not useful if it doesn't vary across employees.
- **EmployeeNumber:** This is a unique identifier for each employee and does not provide information relevant to job satisfaction.

By removing these variables, we ensure that the model focuses on variables that have meaningful variations and potential impacts on job satisfaction. Including variables with no

variability or irrelevant information can lead to misleading results and reduce the efficiency of the model.

3. Encoding Categorical Variables

Categorical variables are those that represent categories or groups. These need to be converted into a numerical format for use in statistical models. Statistical models, like the ordered logit model, require numerical input. Categorical variables, in their original form, cannot be directly used in these models. Encoding transforms these categories into numerical values, allowing the model to process them effectively.

Table 2 - Encoding Strategies for Categorical Variables

Feature	Binary Encoding	Ordinal Encoding	One-Hot Encoding
Gender	Assign 1 to "Female"; Assign 0 to "Male"		
Attrition	Assign 1 to "Yes"; Assign 0 to "No"		
OverTime	Assign 1 to "Yes"; Assign 0 to "No"		
BusinessTravel		Assign 0 to "Non-Travel"; Assign 1 to "Travel_Rarely"; Assign 2 to "Travel_Frequently"	
Department			Department_HumanResources: 1 if the employee is in Human Resources, otherwise 0; Department_ResearchDevelopment: 1 if the employee is in Research & Development, otherwise 0; Department_Sales: 1 if the employee is in Sales, otherwise 0; etc.
JobRole			JobRole_HealthcareRep: 1 if the role is Healthcare Representative, otherwise 0, etc.
MaritalStatus			MaritalStatus_Divorced: 1 if the status is Divorced, otherwise 0. MaritalStatus_Married: 1 if the status is Married, otherwise 0. MaritalStatus_Single: 1 if the status is Single, otherwise 0.

By encoding the variables in these ways, we transform the categorical data into a format suitable for statistical models, enabling the models to interpret and process the information correctly. This transformation is crucial for accurate model performance and reliable results.

4. Data Transformation

Several transformations were applied to convert character variables into factors and standardize numeric variables to ensure they are on comparable scales (Figure 10).

```
data.frame: 1470 obs. of 30 variables:
 $ Age                : num  0.4462 1.32192 0.00834 -0.42952 -1.08631 ...
 $ Attrition          : Factor w/ 2 levels "0","1": 2 1 2 1 1 1 1 1 1 ...
 $ BusinessTravel     : Factor w/ 3 levels "0","1","2": 2 3 2 3 2 3 2 2 3 2 ...
 $ DailyRate          : num  0.742 -1.297 1.414 1.461 -0.524 ...
 $ Department         : Factor w/ 3 levels "Human Resources",...: 3 2 2 2 2 2 2 2 2 ...
 $ DistanceFromHome   : num  -1.011 -0.147 -0.887 -0.764 -0.887 ...
 $ Education           : num  -0.891 -1.868 -0.891 1.061 -1.868 ...
 $ EnvironmentSatisfaction : num  -0.66 0.255 1.169 1.169 -1.575 ...
 $ Gender             : Factor w/ 2 levels "0","1": 2 1 1 2 1 1 2 1 1 1 ...
 $ HourlyRate         : num  1.383 -0.241 1.284 -0.487 -1.274 ...
 $ JobInvolvement      : num  0.38 -1.03 -1.03 0.38 0.38 ...
 $ JobLevel           : num  -0.0578 -0.0578 -0.9612 -0.9612 -0.9612 ...
 $ JobRole            : Factor w/ 9 levels "Healthcare Representative",...: 8 7 3 7 3 3 3 3 5 1 ...
 $ JobSatisfaction     : Ord.factor w/ 4 levels "1"<"2"<"3"<"4": 4 2 3 3 2 4 1 3 3 3 ...
 $ MaritalStatus       : Factor w/ 3 levels "Divorced","Married",...: 3 2 3 2 2 3 2 1 3 2 ...
 $ MonthlyIncome       : num  -0.108 -0.292 -0.937 -0.763 -0.645 ...
 $ MonthlyRate        : num  0.726 1.488 -1.674 1.243 0.326 ...
 $ NumCompaniesWorked  : num  2.124 -0.678 1.324 -0.678 2.525 ...
 $ OverTime           : Factor w/ 2 levels "0","1": 2 1 2 2 1 1 2 1 1 1 ...
 $ PercentSalaryHike   : num  -1.1502 2.1286 -0.0572 -1.1502 -0.8769 ...
 $ PerformanceRating   : num  -0.426 2.345 -0.426 -0.426 -0.426 ...
 $ RelationshipSatisfaction: num  -1.584 1.191 -0.659 0.266 1.191 ...
 $ StockOptionLevel    : num  -0.932 0.242 -0.932 -0.932 0.242 ...
 $ TotalWorkingYears   : num  -0.421 -0.164 -0.55 -0.421 -0.679 ...
 $ TrainingTimesLastYear : num  -2.171 0.156 0.156 0.156 0.156 ...
 $ WorkLifeBalance     : num  -2.493 0.338 0.338 0.338 0.338 ...
 $ YearsAtCompany      : num  -0.165 0.488 -1.144 0.162 -0.817 ...
 $ YearsInCurrentRole   : num  -0.0633 0.7647 -1.1673 0.7647 -0.6153 ...
 $ YearsSinceLastPromotion : num  -0.6789 -0.3686 -0.6789 0.2521 -0.0583 ...
 $ YearsWithCurrManager : num  0.246 0.806 -1.156 -1.156 -0.595 ...
```

Figure 10 - Summary of Data Frame Structure

Character variables were converted to factors to make them suitable for modeling categorical data. The numerical variables were standardized to have a mean of 0 and a standard deviation of 1 to ensure they contribute equally to the model.

Method/Model

Ordered logit and ordered probit models are extensions of logistic regression and probit regression, respectively, designed for use with ordinal dependent variables. These are variables with categories that have a natural order, but the distances between the categories are not necessarily known.

The ordered logit model assumes that the error terms follow a logistic distribution. The ordered probit model assumes that the error terms follow a standard normal distribution. Also, both models have a practical application in real life. Economically, these models are deployed to understand ordered outcomes such as credit ratings or educational attainment levels. In the medical realm, they are valuable for modeling patient health outcomes and disease progression stages. Within marketing contexts, they serve to evaluate customer satisfaction ratings or perceptions of product quality. These are some examples.

Estimating ordered probit and logit models

The ordered logit and ordered probit models are both utilized to analyze the relationship between independent variables and an ordinal dependent variable, such as "JobSatisfaction." They estimate coefficients to quantify the effects of independent variables on the odds of moving to a higher category of the dependent variable.

However, they differ in the underlying distribution functions used for estimation (logistic for ordered logit and standard normal for ordered probit), influencing interpretations and robustness to assumptions.

This model assumes that the levels of job satisfaction ("Low," "Medium," "High," "Very High") follow a specific order, and the probabilities of belonging to each category are influenced by the values of the independent variables.

We have estimated coefficients for each independent variable to quantify their impact on the odds of belonging to a higher category of job satisfaction compared to a lower category, while controlling for other variables in the model. However, it's noted that many of the variables included in the model are not statistically significant, as indicated by their large p-values.

Additionally, the model provides intercepts for each pair of adjacent categories of job satisfaction. These intercepts represent the log-odds of being in the higher category compared to the lower category.

The model's performance is evaluated using the residual deviance and the Akaike Information Criterion (AIC). The residual deviance measures how well the model fits the observed data.

t test of coefficients:

	Estimate	Std. Error	t value	Pr(> t)
Age	0.03227291	0.06801903	0.4745	0.635238
as.factor(Attrition)1	-0.74886300	0.14844553	-5.0447	0.0000005125 ***
as.factor(BusinessTravel)1	-0.02874075	0.15646900	-0.1837	0.854288
as.factor(BusinessTravel)2	0.19666641	0.18476412	1.0644	0.287319
DailyRate	0.04763662	0.04762336	1.0003	0.317345
as.factor(Department)Research & Development	-0.19052587	0.60260328	-0.3162	0.751919
as.factor(Department)Sales	-0.03553326	0.61739146	-0.0576	0.954112
DistanceFromHome	0.00105495	0.00589875	0.1788	0.858087
Education	-0.01486255	0.04783576	-0.3107	0.756074
EnvironmentSatisfaction	-0.04695894	0.04395307	-1.0684	0.285526
as.factor(Gender)1	-0.15354591	0.09776734	-1.5705	0.116515
HourlyRate	-0.13125469	0.04798674	-2.7352	0.006311 **
JobInvolvement	-0.08742997	0.06809792	-1.2839	0.199390
JobLevel	0.06155311	0.16060907	0.3832	0.701593
as.factor(JobRole)Human Resources	-0.42977428	0.68046401	-0.6316	0.527756
as.factor(JobRole)Laboratory Technician	-0.06893245	0.22615990	-0.3048	0.760567
as.factor(JobRole)Manager	-0.03814900	0.38558290	-0.0989	0.921201
as.factor(JobRole)Manufacturing Director	-0.16330562	0.21904146	-0.7455	0.456064
as.factor(JobRole)Research Director	-0.05029494	0.33904599	-0.1483	0.882093
as.factor(JobRole)Research Scientist	-0.01727035	0.22348385	-0.0773	0.938413
as.factor(JobRole)Sales Executive	-0.12559271	0.43757861	-0.2870	0.774141
as.factor(JobRole)Sales Representative	-0.21499125	0.48374399	-0.4444	0.656798
as.factor(MaritalStatus)Married	0.12246143	0.12961996	0.9448	0.344935
as.factor(MaritalStatus)Single	0.42167223	0.17692092	2.3834	0.017284 *
MonthlyIncome	-0.05160728	0.20106017	-0.2567	0.797466
MonthlyRate	0.00015319	0.04738767	0.0032	0.997421
NumCompaniesWorked	-0.02679044	0.02134373	-1.2552	0.209615
as.factor(OverTime)1	0.27143299	0.11271334	2.4082	0.016159 *
PercentSalaryHike	0.01681879	0.02046477	0.8218	0.411304
PerformanceRating	-0.10893226	0.20836813	-0.5228	0.601203
RelationshipSatisfaction	-0.03415387	0.04422473	-0.7723	0.440076
StockOptionLevel	0.12027760	0.07598886	1.5828	0.113681
TotalWorkingYears	-0.01186872	0.01363869	-0.8702	0.384324
TrainingTimesLastYear	-0.02131967	0.03730469	-0.5715	0.567750
WorkLifeBalance	-0.07983870	0.06811535	-1.1721	0.241348
YearsAtCompany	0.01006027	0.01707513	0.5892	0.555836
YearsInCurrentRole	0.00816392	0.02182308	0.3741	0.708389
YearsSinceLastPromotion	-0.00257530	0.01920967	-0.1341	0.893372
YearsWithCurrManager	-0.03525183	0.02224089	-1.5850	0.113188

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Intercepts:

	Value	Std. Error	t value
1 2	-2.4595	0.9720	-2.5305
2 3	-1.4873	0.9703	-1.5328
3 4	-0.2018	0.9698	-0.2081

Residual Deviance: 3943.21

AIC: 4027.21

Figure 11 - Ordered logit of general model

Coefficient Interpretation

Let's interpret the variable "HourlyRate":

The negative and **significant** coefficient for the "HourlyRate" variable (-0.078, p-value = 0.0064) suggests that as the hourly rate increases, individuals are less likely to transition to higher ordinal categories of the dependent variable. This effect holds after accounting for the thresholds represented by the intercepts.

When considered together, these findings suggest that higher hourly rates are associated with lower probabilities of being in higher ordinal categories, even after accounting for the natural thresholds between categories represented by the intercepts.

t test of coefficients:				
	Estimate	Std. Error	t value	Pr(> t)
Age	0.02227997	0.04088089	0.5450	0.585841
as.factor(Attrition)1	-0.45098811	0.08855276	-5.0929	0.0000003997 ***
as.factor(BusinessTravel)1	-0.02604462	0.09570194	-0.2721	0.785551
as.factor(BusinessTravel)2	0.10211367	0.11210602	0.9109	0.362519
DailyRate	0.02751790	0.02862814	0.9612	0.336605
as.factor(Department)Research & Development	-0.14413689	0.36487870	-0.3950	0.692882
as.factor(Department)Sales	-0.00545191	0.37546021	-0.0145	0.988417
DistanceFromHome	0.00040713	0.00354938	0.1147	0.908695
Education	-0.00834237	0.02874266	-0.2902	0.771672
EnvironmentSatisfaction	-0.02662415	0.02647559	-1.0056	0.314773
as.factor(Gender)1	-0.09426786	0.05873165	-1.6051	0.108702
HourlyRate	-0.07821500	0.02868829	-2.7264	0.006482 **
JobInvolvement	-0.05382203	0.04088502	-1.3164	0.188243
JobLevel	0.04133022	0.09601581	0.4305	0.666932
as.factor(JobRole)Human Resources	-0.26406080	0.41211043	-0.6408	0.521786
as.factor(JobRole)Laboratory Technician	-0.03073547	0.13548993	-0.2268	0.820575
as.factor(JobRole)Manager	-0.04657897	0.22898037	-0.2034	0.838836
as.factor(JobRole)Manufacturing Director	-0.08594364	0.13169523	-0.6526	0.514122
as.factor(JobRole)Research Director	-0.02844117	0.20340768	-0.1398	0.888819
as.factor(JobRole)Research Scientist	0.00086506	0.13360071	0.0065	0.994835
as.factor(JobRole)Sales Executive	-0.11898197	0.26126273	-0.4554	0.648883
as.factor(JobRole)Sales Representative	-0.13590910	0.29100715	-0.4670	0.640550
as.factor(MaritalStatus)Married	0.06910646	0.07744568	0.8923	0.372371
as.factor(MaritalStatus)Single	0.25390746	0.10618738	2.3911	0.016926 *
MonthlyIncome	-0.03281045	0.12042696	-0.2725	0.785315
MonthlyRate	-0.00150151	0.02853353	-0.0526	0.958040
NumCompaniesWorked	-0.01655048	0.01286365	-1.2866	0.198439
as.factor(OverTime)1	0.15252765	0.06697949	2.2772	0.022920 *
PercentSalaryHike	0.00967064	0.01236219	0.7823	0.434182
PerformanceRating	-0.05789484	0.12560135	-0.4609	0.644911
RelationshipSatisfaction	-0.01980273	0.02657203	-0.7452	0.456245
StockOptionLevel	0.07474698	0.04582883	1.6310	0.103110
TotalWorkingYears	-0.00716653	0.00813443	-0.8810	0.378459
TrainingTimesLastYear	-0.01173499	0.02239420	-0.5240	0.600347
WorkLifeBalance	-0.04884339	0.04081361	-1.1967	0.231606
YearsAtCompany	0.00856600	0.01005618	0.8518	0.394460
YearsInCurrentRole	0.00372832	0.01304067	0.2859	0.774996
YearsSinceLastPromotion	-0.00231316	0.01151759	-0.2008	0.840855
YearsWithCurrManager	-0.02370949	0.01340204	-1.7691	0.077091 .

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1				

Intercepts:

	Value	Std. Error	t value
1 2	-1.4890	0.5827	-2.5554
2 3	-0.9092	0.5820	-1.5621
3 4	-0.1153	0.5819	-0.1981

Residual Deviance: 3943.01

AIC: 4027.01

Figure 12 - Ordered probit of general model

In the ordered logit and probit models, several independent variables show significant effects on the odds of moving to a higher category of JobSatisfaction. For instance, variables such as Attrition, HourlyRate, OverTime, and MaritalStatus (Single) have significant coefficients, indicating their impact on JobSatisfaction levels. However, like on the ordered logit the majority of the variables are not statistically significant.

General to specific method to variables selection

The general-to-specific (GETS) approach in econometrics involves starting with a comprehensive model that includes all theoretically relevant variables and iteratively refining it by eliminating statistically insignificant or theoretically irrelevant variables.

```
Ordered Logit Regression
Log-Likelihood: -1971.605
No. Iterations: 4
McFadden's R2: 0.01410114
AIC: 4027.21
```

	Estimate	Std. error	t value	Pr(> t)
Age	0.03227883	0.06801899	0.4746	0.635103
as.factor(Attrition)1	-0.74885016	0.14844544	-5.0446	4.544e-07 ***
as.factor(BusinessTravel)1	-0.02874201	0.15646936	-0.1837	0.854256
as.factor(BusinessTravel)2	0.19669271	0.18476449	1.0646	0.287076
DailyRate	0.04763516	0.04762344	1.0002	0.317191
as.factor(Department)Research & Development	-0.19085159	0.60259851	-0.3167	0.751460
as.factor(Department)Sales	-0.03484061	0.61739030	-0.0564	0.954998
DistanceFromHome	0.00105457	0.00589866	0.1788	0.858110
Education	-0.01487519	0.04783576	-0.3110	0.755828
EnvironmentSatisfaction	-0.04696420	0.04395309	-1.0685	0.285292
as.factor(Gender)1	-0.15354010	0.09776746	-1.5705	0.116308
HourlyRate	-0.13125799	0.04798682	-2.7353	0.006232 **
JobInvolvement	-0.08744468	0.06809808	-1.2841	0.199107
JobLevel	0.06152675	0.16060920	0.3831	0.701658
as.factor(JobRole)Human Resources	-0.43031004	0.68045996	-0.6324	0.527138
as.factor(JobRole)Laboratory Technician	-0.06916915	0.22616069	-0.3058	0.759726
as.factor(JobRole)Manager	-0.03889841	0.38558242	-0.1009	0.919644
as.factor(JobRole)Manufacturing Director	-0.16357056	0.21904230	-0.7468	0.455213
as.factor(JobRole)Research Director	-0.05060076	0.33904653	-0.1492	0.881361
as.factor(JobRole)Research Director	-0.05060076	0.33904653	-0.1492	0.881361
as.factor(JobRole)Research Scientist	-0.01749616	0.22348467	-0.0783	0.937599
as.factor(JobRole)Sales Executive	-0.12689053	0.43758388	-0.2900	0.771832
as.factor(JobRole)Sales Representative	-0.21632640	0.48374872	-0.4472	0.654740
as.factor(MaritalStatus)Married	0.12244339	0.12961994	0.9446	0.344846
as.factor(MaritalStatus)Single	0.42164740	0.17692093	2.3833	0.017160 *
MonthlyIncome	-0.05154346	0.20106018	-0.2564	0.797674
MonthlyRate	0.00015487	0.04738770	0.0033	0.997392
NumCompaniesWorked	-0.02679343	0.02134374	-1.2553	0.209359
as.factor(OverTime)1	0.27142688	0.11271336	2.4081	0.016035 *
PercentsSalaryHike	0.01682435	0.02046446	0.8221	0.411006
PerformanceRating	-0.10907068	0.20836864	-0.5235	0.600661
RelationshipSatisfaction	-0.03416198	0.04422480	-0.7725	0.439841
StockOptionLevel	0.12027125	0.07598879	1.5827	0.113478
TotalWorkingYears	-0.01187247	0.01363847	-0.8705	0.384020
TrainingTimesLastYear	-0.02131941	0.03730470	-0.5715	0.567665
WorkLifeBalance	-0.07984901	0.06811545	-1.1723	0.241093
YearsAtCompany	0.01005855	0.01707488	0.5891	0.555805
YearsInCurrentRole	0.00816482	0.02182304	0.3741	0.708302
YearsSinceLastPromotion	-0.00257370	0.01920966	-0.1340	0.893419
YearsWithCurrManager	-0.03525070	0.02224085	-1.5850	0.112977

```
----- Threshold Parameters -----
              Estimate Std. error t value Pr(>|t|)
Threshold (1->2) -2.46073    0.97197 -2.5317  0.01135 *
Threshold (2->3) -1.48849    0.97034 -1.5340  0.12503
Threshold (3->4) -0.20300    0.96980 -0.2093  0.83419
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Figure 13 - Ordered logit of primary model

It begins with a comprehensive model that includes all potential predictors. In this example, the initial ordered logistic regression model (ologit1) includes numerous variables related to job satisfaction. The first step involves evaluating the significance of each variable in the model using statistical tests.

Following this, the process involves iteratively removing the most insignificant variable from the initial model and re-evaluating the model's fit and significance until only statistically significant variables remain, in this case “**MonthlyRate**”.

	#Df	LogLik	Df	Chisq	Pr(>Chisq)
1	42	-1971.6			
2	7	-1984.3	-35	25.44	0.8821

Figure 14 - Likelihood Ratio Test

In addition, we must do the Likelihood Ratio Test. The LRT compares the full model to the reduced model to test if the excluded variables in the full model are jointly insignificant.

Our hypothesis for LRT:

H0: The excluded variables are jointly insignificant.

H1: At least one of the excluded variables is significant.

The p-value from the likelihood-ratio test is 0.8821, which is much greater than the common significance level of 0.05. So, we fail to reject the null hypothesis H0. This means that the excluded variables are jointly insignificant. Therefore, the reduced model is adequate, and the excluded variables do not provide additional explanatory power for job satisfaction.

Let's see the following model without “MonthlyRate”.

Ordered Logit Regression
Log-Likelihood: -1971.605
No. Iterations: 4
McFadden's R2: 0.01410114
AIC: 4025.21

	Estimate	Std. error	t value	Pr(> t)
Age	0.0322807	0.0680165	0.4746	0.635072
as.factor(Attrition)1	-0.7488353	0.1483757	-5.0469	4.491e-07 ***
as.factor(BusinessTravel)1	-0.0287498	0.1564513	-0.1838	0.854200
as.factor(BusinessTravel)2	0.1966842	0.1847464	1.0646	0.287049
DailyRate	0.0476299	0.0475965	1.0007	0.316970
as.factor(Department)Research & Development	-0.1908185	0.6025116	-0.3167	0.751467
as.factor(Department)Sales	-0.0348196	0.6173555	-0.0564	0.955022
DistanceFromHome	0.0010549	0.0058980	0.1789	0.858054
Education	-0.0148837	0.0477654	-0.3116	0.755345
Environmentsatisfaction	-0.0469594	0.0439287	-1.0690	0.285074
as.factor(Gender)1	-0.1535288	0.0977067	-1.5713	0.116107
HourlyRate	-0.1312626	0.0479660	-2.7366	0.006208 **
JobInvolvement	-0.0874419	0.0680928	-1.2842	0.199087
JobLevel	0.0615303	0.1606055	0.3831	0.701635
as.factor(JobRole)Human Resources	-0.4302949	0.6804421	-0.6324	0.527142
as.factor(JobRole)Laboratory Technician	-0.0691874	0.2260918	-0.3060	0.759594
as.factor(JobRole)Manager	-0.0388471	0.3852626	-0.1008	0.919683
as.factor(JobRole)Manufacturing Director	-0.1635694	0.2190422	-0.7467	0.455215
as.factor(JobRole)Research Director	-0.0505556	0.3387651	-0.1492	0.881368
as.factor(JobRole)Research Scientist	-0.0175177	0.2233873	-0.0784	0.937495
as.factor(JobRole)Sales Executive	-0.1268748	0.4375586	-0.2900	0.771846
as.factor(JobRole)Sales Executive	-0.1268748	0.4375586	-0.2900	0.771846
as.factor(JobRole)Sales Representative	-0.2163393	0.4837335	-0.4472	0.654710
as.factor(MaritalStatus)Married	0.1224326	0.1295779	0.9449	0.344732
as.factor(MaritalStatus)Single	0.4216475	0.1769209	2.3833	0.017160 *
MonthlyIncome	-0.0515651	0.2009509	-0.2566	0.797483
NumCompaniesWorked	-0.0267943	0.0213421	-1.2555	0.209311
as.factor(OverTime)1	0.2714276	0.1127131	2.4081	0.016035 *
PercentsSalaryHike	0.0168246	0.0204643	0.8221	0.410996
PerformanceRating	-0.1090779	0.2083573	-0.5235	0.600617
RelationshipSatisfaction	-0.0341624	0.0442246	-0.7725	0.439832
StockOptionLevel	0.1202683	0.0759836	1.5828	0.113463
TotalWorkingYears	-0.0118709	0.0136302	-0.8709	0.383795
TrainingTimesLastYear	-0.0213195	0.0373047	-0.5715	0.567662
WorkLifeBalance	-0.0798491	0.0681155	-1.1723	0.241093
YearsAtCompany	0.0100573	0.0170706	0.5892	0.555754
YearsInCurrentRole	0.0081651	0.0218229	0.3742	0.708291
YearsSinceLastPromotion	-0.0025717	0.0191998	-0.1339	0.893447
YearsWithCurrManager	-0.0352533	0.0222267	-1.5861	0.112721
----- Threshold Parameters -----				
	Estimate	Std. error	t value	Pr(> t)
Threshold (1->2)	-2.46073	0.97197	-2.5317	0.01135 *
Threshold (2->3)	-1.48848	0.97033	-1.5340	0.12503
Threshold (3->4)	-0.20300	0.96980	-0.2093	0.83420

signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1				

Figure 15 - Ordered logit of the model without "MonthlyRate"

After determining that "MonthlyRate" is the most insignificant variable, it is excluded from the model. The revised model (ologit2) is then estimated without this variable. This model still includes a broad set of predictors related to job satisfaction.

Based on the summary of ologit2, "**as.factor(Department)Sales**" is identified as the next most insignificant variable. It will be removed in the subsequent iteration of the model.

Let's see the following model:

Ordered Logit Regression
Log-Likelihood: -1971.605
No. Iterations: 4
McFadden's R2: 0.01410114
AIC: 4025.21

	Estimate	Std. error	t value	Pr(> t)
Age	0.0322807	0.0680165	0.4746	0.635072
as.factor(Attrition)1	-0.7488353	0.1483757	-5.0469	4.491e-07 ***
as.factor(BusinessTravel)1	-0.0287498	0.1564513	-0.1838	0.854200
as.factor(BusinessTravel)2	0.1966842	0.1847464	1.0646	0.287049
DailyRate	0.0476299	0.0475965	1.0007	0.316970
Department_ResearchDevelopment	-0.1559989	0.3905841	-0.3994	0.689599
Department_HumanResources	0.0348196	0.6173555	0.0564	0.955022
DistanceFromHome	0.0010549	0.0058980	0.1789	0.858054
Education	-0.0148837	0.0477654	-0.3116	0.755345
Environmentsatisfaction	-0.0469594	0.0439287	-1.0690	0.285074
as.factor(Gender)1	-0.1535288	0.0977067	-1.5713	0.116107
HourlyRate	-0.1312626	0.0479660	-2.7366	0.006208 **
JobInvolvement	-0.0874419	0.0680928	-1.2842	0.199087
JobLevel	0.0615303	0.1606055	0.3831	0.701635
as.factor(JobRole)Human Resources	-0.4302949	0.6804421	-0.6324	0.527142
as.factor(JobRole)Laboratory Technician	-0.0691874	0.2260918	-0.3060	0.759594
as.factor(JobRole)Manager	-0.0388471	0.3852626	-0.1008	0.919683
as.factor(JobRole)Manufacturing Director	-0.1635694	0.2190422	-0.7467	0.455215
as.factor(JobRole)Research Director	-0.0505556	0.3387651	-0.1492	0.881368
as.factor(JobRole)Research Scientist	-0.0175177	0.2233873	-0.0784	0.937495
as.factor(JobRole)Sales Executive	-0.1268748	0.4375586	-0.2900	0.771846
as.factor(JobRole)Sales Executive	-0.1268748	0.4375586	-0.2900	0.771846
as.factor(JobRole)Sales Representative	-0.2163393	0.4837335	-0.4472	0.654710
as.factor(MaritalStatus)Married	0.1224326	0.1295779	0.9449	0.344732
as.factor(MaritalStatus)Single	0.4216475	0.1769209	2.3833	0.017160 *
MonthlyIncome	-0.0515651	0.2009509	-0.2566	0.797483
NumCompaniesWorked	-0.0267943	0.0213421	-1.2555	0.209311
as.factor(OverTime)1	0.2714276	0.1127131	2.4081	0.016035 *
PercentsalaryHike	0.0168246	0.0204643	0.8221	0.410996
PerformanceRating	-0.1090779	0.2083573	-0.5235	0.600617
Relationshipsatisfaction	-0.0341624	0.0442246	-0.7725	0.439832
StockOptionLevel	0.1202683	0.0759836	1.5828	0.113463
TotalWorkingYears	-0.0118709	0.0136302	-0.8709	0.383795
TrainingTimesLastYear	-0.0213195	0.0373047	-0.5715	0.567662
WorkLifeBalance	-0.0798491	0.0681155	-1.1723	0.241093
YearsAtCompany	0.0100573	0.0170706	0.5892	0.555754
YearsInCurrentRole	0.0081651	0.0218229	0.3742	0.708291
YearsSinceLastPromotion	-0.0025717	0.0191998	-0.1339	0.893447
YearsWithCurrManager	-0.0352533	0.0222267	-1.5861	0.112721
----- Threshold Parameters -----				
	Estimate	Std. error	t value	Pr(> t)
Threshold (1->2)	-2.42591	0.83803	-2.8948	0.003794 **
Threshold (2->3)	-1.45366	0.83614	-1.7386	0.082114 .
Threshold (3->4)	-0.16818	0.83550	-0.2013	0.840472

signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1				

Figure 16 - Ordered logit of the model without "MonthlyRate" and "as.factor(Department)Sales"

After dropping "MonthlyRate" and "as.factor(Department)Sales" in previous steps, the next model (ologit3) is estimated. This model includes all previously retained variables, excluding the two dropped variables.

Let's see the likelihood ratio test:

#Df	LogLik	Df	Chisq	Pr(>Chisq)
42	-1971.6			
41	-1971.6	-1	0	0.9974

Figure 17 - Likelihood ratio test

H0: The simpler model (ologit3) is as good as or better than the more complex model (ologit1). (In this case, the simpler model (ologit3) includes fewer predictors.

H1: The more complex model (ologit1) provides a statistically significant improvement in fit compared to the simpler model (ologit3).

In this case, the p-value is 0.9974, indicating strong evidence against the alternative hypothesis. Since the p-value is much greater than the typical significance level (0.05), we fail to reject the null hypothesis. After confirming that "MonthlyRate" and "as.factor(Department)Sales" are insignificant, the next most insignificant variable in ologit3 is identified, which is "**Department_HumanResources**". So we drop this variable.

We repeat this process (36 more times) until further removal of predictors doesn't substantially affect model fit and until we achieve a final model with only statistically significant predictors. This approach ensures our final model is both parsimonious and optimally predictive.

Diagnostics tests

Hosmer Lemeshow

The Hosmer-Lemeshow test is a goodness-of-fit test commonly used to assess the fit of logistic regression models. It evaluates whether the observed event rates match the expected event rates in subgroups of the model population. This test is particularly useful for assessing binary and ordinal logistic regression models.

Hypotheses:

H0: The model fits the data well (there is no significant difference between observed and expected frequencies).

H1: The model does not fit the data well (i.e., there is a significant difference between observed and expected frequencies).

● Probit

Since the p-value is 0.8197, which is much greater than the common significance level of 0.05, we fail to reject the null hypothesis. This high p-value indicates that there is no

significant difference between the observed and expected frequencies in the groups. Therefore, the Hosmer-Lemeshow test suggests that the model fits the data well.

```
Hosmer and Lemeshow test (ordinal model)

data:  hr$JobSatisfaction, fitted(final_model_prob)
X-squared = 19.386, df = 26, p-value = 0.8197
```

Figure 18 - Hosmer Lemeshow test (probit)

•Logit

Since the p-value is 0.3416, which is greater than the common significance level of 0.05, we fail to reject the null hypothesis. This indicates that there is no significant difference between the observed and expected frequencies in the groups. Therefore, the Hosmer-Lemeshow test suggests that the ordered logistic regression model fits the data well.

```
Hosmer and Lemeshow test (ordinal model)

data:  hr$JobSatisfaction, fitted(final_model_log)
X-squared = 28.346, df = 26, p-value = 0.3416
```

Figure 19 - Hosmer Lemeshow test (logit)

Lipsitz test

The Lipsitz test is a statistical test used to assess the homogeneity of odds ratios or relative risks across multiple groups in categorical data analysis.

H0: The odds ratios are homogeneous across all groups

H1: At least one odds ratio or relative risk differs significantly from the others.

•Probit

The p-value associated with the test statistic is 0.4667268. This indicates that there is insufficient evidence to reject the null hypothesis at the conventional significance level of 0.05. Therefore, we do not have significant evidence to suggest that the odds ratios differ significantly across groups based on the Lipsitz test.

•Logit

The p-value associated with the test statistic is 0.4575435. This indicates that there is insufficient evidence to reject the null hypothesis at the conventional significance level of 0.05. Therefore, we do not have significant evidence to suggest that the odds ratios or relative risks differ significantly across deciles based on the Lipsitz test.

Pulkstenis-Robinson test

It evaluates how well the model fits the data by comparing observed and expected frequencies in contingency tables.

H0: The model fits the data well (the proportional odds assumption holds and there is no significant difference between observed and expected frequencies).

H1: The model does not fit the data well (the proportional odds assumption does not hold and there is a significant difference between observed and expected frequencies).

●Probit

Since the p-value is 0.3356, which is greater than the common significance level of 0.05, we fail to reject the null hypothesis.

This indicates that there is no significant difference between the observed and expected frequencies in the contingency tables.

Therefore, the Pulkstenis-Robinson test suggests that the ordered probit model fits the data well.

Pulkstenis-Robinson chi-squared test

```
data: formula: as.factor(JobSatisfaction) ~ Attrition + HourlyRate + NumCompaniesWork
ed + formula:      MaritalStatus_Single + OverTime + YearsWithCurrManager
X-squared = 7.9662, df = 7, p-value = 0.3356
```

Figure 20 - Pulkstenis-Robinson test (probit)

●Logit

Since the p-value is 0.3386 which is greater than the common significance level 0.05, we don't reject the null hypothesis. So we can say that the Pulkstenis-Robinson test suggests that the ordered logit model fits the data well. Also, this means that there are not significant differences between the observed and expected frequencies in the contingency tables.

Pulkstenis-Robinson chi-squared test

```
data: formula: as.factor(JobSatisfaction) ~ Attrition + HourlyRate + NumCompaniesWork
ed + formula:      MaritalStatus_Single + OverTime + YearsWithCurrManager
X-squared = 7.9326, df = 7, p-value = 0.3386
```

Figure 21 - Pulkstenis-Robinson test (probit)

Proportional odds assumption

It states that the relationship between the predictor variables and the logits of cumulative odds is consistent across all levels (or categories) of the ordinal response variable. Ensuring that the proportional odds assumption holds is crucial for the validity and interpretability of the ordinal logistic regression model.

The Brant test is used to assess the parallel regression assumption in ordinal logistic regression models.

H0: The effects of the predictor variables are consistent across all levels of the ordinal response variable.

H1: The effects of the predictor variables differ across levels of the ordinal response variable.

Table 3 - Results of Brant test

Test for ability	X2	df	p-value
Omnibus	7.79	12	0.8
Attrition1	1.92	2	0.38
HourlyRate	1.65	2	0.44
NumCompaniesWorked	0.1	2	0.95
MaritalStatus_Single	0.95	2	0.62
OverTime1	2.22	2	0.33
YearsWithcurrentManager	1.29	2	0.53

Results:

The omnibus test evaluates the overall parallelism of the model. In this case, the test statistic (X2) is 7.79 with 12 degrees of freedom, resulting in a p-value of 0.80. Since the p-value is greater than the significance level (commonly set at 0.05), we fail to reject the null hypothesis. This suggests that, overall, the model satisfies the parallel regression assumption.

The individual tests assess the parallelism of each predictor variable separately. For each predictor, the test provides a test statistic (X2), degrees of freedom (df), and probability (p-value). For example, the test for "Attrition1" yields a test statistic of 1.92 with 2 degrees of freedom and a p-value of 0.38. Similarly, for "HourlyRate," the test statistic is 1.65 with a p-value of 0.44. In all cases, the p-values are greater than 0.05, indicating that we fail to reject the null hypothesis for each individual predictor. Thus, there is no evidence to suggest that any specific predictor violates the parallel regression assumption.

This implies that the effects of the predictor variables are consistent across all levels of the ordinal response variable, validating the reliability of the model's parameter estimates and predictions.

Analysis of AIC/DIC for models

AIC is a measure used to compare the relative quality of statistical models for a given dataset. It balances the goodness of fit of the model with the complexity of the model (number of parameters). A lower AIC indicates a better model.

BIC is similar to AIC but introduces a stronger penalty for models with more parameters. It is used to compare models and select the one that best balances fit and complexity. A lower BIC indicates a better model.

Table 4 - AIC/BIC for ordered logit and probit

	Ordered logit model	Ordered probit model
AIC	3979.946	3979.620
BIC	4027.583	4027.257

The ordered probit model has a slightly lower AIC (3979.620) compared to the ordered logit model (3979.946). This suggests that, based on AIC, the ordered probit model has a slightly better fit to the data.

The ordered probit model also has a lower BIC (4027.257) compared to the ordered logit model (4027.583). This further suggests that the ordered probit model is preferred when penalizing for the number of parameters.

We choose the **Ordered Logit model** based on its performance in diagnostic tests and its comparative metrics. Specifically, the Ordered Logit model successfully passed all diagnostic tests and satisfied the underlying assumptions necessary for reliable analysis. Although the Ordered Probit model exhibited slightly smaller AIC and BIC values, the differences between these indicators for the two models were not substantial. Given the robustness of the Ordered Logit model in passing diagnostic evaluations and the minimal disparity in information criteria compared to the Ordered Probit model, we opted to use the Ordered Logit model for further diagnosing our hypotheses. This decision ensures the integrity and reliability of our analytical outcomes while maintaining consistency with the diagnostic validation results.

Results

In this paper, we formulate several hypotheses to understand the factors influencing job satisfaction. Using ordered logit and probit models, we test these hypotheses based on the available data. The results from the general models and the final models (after applying the general-to-specific approach) are presented in the Table 5. The general models included a comprehensive set of variables to identify potential predictors of job satisfaction. Through iterative testing and refinement, we arrived at the final models, which contain only the significant variables.

There are the explanation of final model estimators for ordered logit and probit. The final models include the following significant variables:

- **Attrition:**

Ordered Logit: -0.659

Ordered Probit: -0.397

Employees who exhibit attrition demonstrate an increased probability of experiencing lower job satisfaction. Employees who exhibit attrition demonstrate a decreased probability of experiencing very high job satisfaction. This finding aligns with theoretical expectations as dissatisfaction often leads to attrition.

- **Hourly Rate:**

Ordered Logit: -0.123

Ordered Probit: -0.075

Elevated hourly wage rates are associated with a higher probability of diminished job satisfaction (first alternative), while elevated hourly wage rates will decrease the probability of very high job satisfaction.

This counterintuitive result may suggest that higher pay is compensating for other negative job aspects, such as stress or long hours.

- **Marital Status (Single):**

Ordered Logit: 0.175

Ordered Probit: 0.106

Single employees exhibit a reduced likelihood of experiencing low job satisfaction compared to their married counterparts. In other words, being single is associated with an increased probability of attaining very high job satisfaction. This could be due to fewer family-related stressors affecting work-life balance.

Table 5 - Ordered Logit and Probit Model Results: General and Final Models for Job Satisfaction

	Dependent variable:			
	JobSatisfaction			
	ordered logistic General (1)	ordered probit General (2)	ordered logistic Final (3)	ordered probit Final (4)
Age	0.032 (0.068)	0.022 (0.041)		
as.factor(Attrition)1	-0.749*** (0.148)	-0.451*** (0.089)	-0.659*** (0.137)	-0.397*** (0.082)
as.factor(BusinessTravel)1	-0.029 (0.156)	-0.026 (0.096)		
as.factor(BusinessTravel)2	0.197 (0.185)	0.102 (0.112)		
DailyRate	0.048 (0.048)	0.028 (0.029)		
as.factor(Department)Research Development	-0.191 (0.603)	-0.144 (0.365)		
as.factor(Department)Sales	-0.036 (0.617)	-0.005 (0.375)		
DistanceFromHome	0.001 (0.006)	0.0004 (0.004)		
Education	-0.015 (0.048)	-0.008 (0.029)		
EnvironmentSatisfaction	-0.047 (0.044)	-0.027 (0.026)		
as.factor(Gender)1	-0.154 (0.098)	-0.094 (0.059)		
HourlyRate	-0.131*** (0.048)	-0.078*** (0.029)	-0.123*** (0.047)	-0.075*** (0.028)
JobInvolvement	-0.087 (0.068)	-0.054 (0.041)		
JobLevel	0.062 (0.161)	0.041 (0.096)		
as.factor(JobRole)Human Resources	-0.430 (0.680)	-0.264 (0.412)		
as.factor(JobRole)Laboratory Technician	-0.069 (0.226)	-0.031 (0.135)		

as.factor(JobRole)Manager	-0.038 (0.386)	-0.047 (0.229)		
as.factor(JobRole)Manufacturing Director	-0.163 (0.219)	-0.086 (0.132)		
as.factor(JobRole)Research Director	-0.050 (0.339)	-0.028 (0.203)		
as.factor(JobRole)Research Scientist	-0.017 (0.223)	0.001 (0.134)		
as.factor(JobRole)Sales Executive	-0.126 (0.438)	-0.119 (0.261)		
as.factor(JobRole)Sales Representative	-0.215 (0.484)	-0.136 (0.291)		
as.factor(MaritalStatus)Married	0.122 (0.130)	0.069 (0.077)		
as.factor(MaritalStatus)Single	0.422** (0.177)	0.254** (0.106)	0.175* (0.103)	0.106* (0.062)
MonthlyIncome	-0.052 (0.201)	-0.033 (0.120)		
MonthlyRate	0.0002 (0.047)	-0.002 (0.029)		
NumCompaniesWorked	-0.027 (0.021)	-0.017 (0.013)	-0.038** (0.019)	-0.023** (0.011)
as.factor(OverTime)1	0.271** (0.113)	0.153** (0.067)	0.244** (0.110)	0.137** (0.065)
PercentSalaryHike	0.017 (0.020)	0.010 (0.012)		
PerformanceRating	-0.109 (0.208)	-0.058 (0.126)		
RelationshipSatisfaction	-0.034 (0.044)	-0.020 (0.027)		
StockOptionLevel	0.120 (0.076)	0.075 (0.046)		
TotalWorkingYears	-0.012 (0.014)	-0.007 (0.008)		
TrainingTimesLastYear	-0.021 (0.037)	-0.012 (0.022)		
WorkLifeBalance	-0.080 (0.068)	-0.049 (0.041)		

YearsAtCompany	0.010 (0.017)	0.009 (0.010)		
YearsInCurrentRole	0.008 (0.022)	0.004 (0.013)		
YearsSinceLastPromotion	-0.003 (0.019)	-0.002 (0.012)		
YearsWithCurrManager	-0.035 (0.022)	-0.024* (0.013)	-0.024* (0.013)	-0.015* (0.008)
Observations	1,470	1,470	1,470	1,470
Note:	*p<0.1; **p<0.05; ***p<0.01			

●**Number of Companies Worked:**

Ordered Logit: -0.038

Ordered Probit: -0.023

The employees who worked in more companies are associated with an increased probability of getting low job satisfaction. In other words, more companies negatively correlated with very high job satisfaction. Frequent job changes might indicate instability or a history of dissatisfaction.

●**OverTime:**

Ordered Logit: 0.244

Ordered Probit: 0.137

Engaging in overtime work is associated with a decreased likelihood of experiencing low job satisfaction. Consequently, working overtime correlates with an increased probability of achieving very high job satisfaction. This may reflect a sense of accomplishment or financial rewards for overtime work.

●**Years with Current Manager:**

Ordered Logit: -0.024

Ordered Probit: -0.015

Longer tenure with the same manager is linked to the higher probability of experiencing low job satisfaction, possibly due to stagnation or interpersonal issues.

The results of the ordered logit and probit models provide valuable insights into the factors influencing job satisfaction. The final models indicate that attrition, hourly rate, marital status, number of companies worked, overtime, and years with the current manager

are significant predictors. These findings are consistent with theoretical expectations and provide actionable insights for improving employee satisfaction.

Our next step is intermediate model analysis (Figure 22). The intermediate model was explored to examine potential interactions between key variables and their combined effect on job satisfaction. Interaction terms help identify whether the relationship between a predictor and the outcome variable changes depending on the level of another predictor. This allows for a deeper understanding of the dynamics between multiple variables and job satisfaction.

The intermediate model includes the variables of final model and an **interaction** term “YearsAtCompany * Gender”.

There are the following hypotheses that we would like to check:

- **H0:** The relationship between years at the company and job satisfaction is the same for both male and female employees.
- **H1:** As years at the company increase, the impact on job satisfaction becomes more favorable for male employees compared to female employees.

```
Ordered Logit Regression
Log-Likelihood: -1977.278
No. Iterations: 4
McFadden's R2: 0.01126432
AIC: 3978.556

              Estimate Std. error t value Pr(>|t|)
Attrition1    -0.6750819  0.1373010 -4.9168 0.0000008797 ***
HourlyRate    -0.1206732  0.0474604 -2.5426  0.011003 *
NumCompaniesWorked -0.0368698  0.0189862 -1.9419  0.052147 .
MaritalStatus_Single 0.1945543  0.1034712  1.8803  0.060071 .
OverTime1     0.2561970  0.1101580  2.3257  0.020033 *
YearsWithCurrManager -0.0364283  0.0208686 -1.7456  0.080879 .
YearsAtCompany -0.0049956  0.0130785 -0.3820  0.702482
as.factor(Gender)1 -0.3916906  0.1484748 -2.6381  0.008337 **
YearsAtCompany:as.factor(Gender)1 0.0358293  0.0159649  2.2443  0.024816 *
----- Threshold Parameters -----
              Estimate Std. error t value Pr(>|t|)
Threshold (1->2) -1.74892  0.13052 -13.3999 < 2.2e-16 ***
Threshold (2->3) -0.78211  0.12320  -6.3483 2.176e-10 ***
Threshold (3->4)  0.49588  0.12222  4.0574 4.963e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Figure 22 - Ordered Logit Regression Results with Interaction Term

The interaction term included in the intermediate model is significant ($p_value < 0.05$) and the estimator is equal to 0.036. The significant interaction term suggests rejecting the null hypothesis. This indicates that the positive coefficient suggests that as years at the company increase, the impact on job satisfaction becomes more favorable for male employees compared to female employees.

The intermediate model provides valuable insights by revealing interaction effects that were not captured in the final model. Understanding these interactions helps in better comprehending the multifaceted nature of job satisfaction and tailoring interventions more effectively. While the final model simplifies the predictors to the most significant ones, the intermediate model underscores the importance of considering combined effects of multiple factors.

Marginal effects analysis for the final model

The calculation and interpretation of marginal effects provide insights into how changes in predictor variables influence the probability of different levels of job satisfaction. These marginal effects are calculated at the mean values of the predictor variables (atmeans=TRUE) for four outcomes of job satisfaction (Figure 13).

The variables (MaritalStatus_Single, YearsWithCurrManager, and NumCompaniesWorked) often show p-values greater than 0.05, indicating that their effects are not statistically significant across most outcomes. Even when some effects are statistically significant ($p < 0.1$), the effect sizes for these variables are often small. This means that their practical significance in terms of influencing job satisfaction is limited. Therefore, for clarity and to emphasize actionable insights, it is beneficial to focus on variables with strong and consistent effects. Attrition, OverTime, and HourlyRate show significant and meaningful impacts on job satisfaction, making them more relevant for detailed interpretation.

```
> margins.oglmx(ologit, atmeans = TRUE)
Marginal Effects on Pr(Outcome==1)
      Marg. Eff Std. error t value Pr(>|t|)
as.factor(Attrition)1  0.1165615  0.0271073  4.3000 0.00001708 ***
MaritalStatus_Single -0.0267072  0.0154536 -1.7282  0.083949 .
as.factor(OverTime)1 -0.0368078  0.0159888 -2.3021  0.021330 *
HourlyRate           0.0191071  0.0074056  2.5801  0.009877 **
NumCompaniesWorked   0.0059229  0.0029466  2.0100  0.044427 *
YearsWithCurrManager  0.0038024  0.0021043  1.8070  0.070770 .
-----
Marginal Effects on Pr(Outcome==2)
      Marg. Eff Std. error t value Pr(>|t|)
as.factor(Attrition)1  0.0443342  0.0078092  5.6772 0.00000001369 ***
MaritalStatus_Single -0.0143858  0.0086354 -1.6659  0.09573 .
as.factor(OverTime)1 -0.0202532  0.0093547 -2.1650  0.03039 *
HourlyRate           0.0099582  0.0039226  2.5387  0.01113 *
NumCompaniesWorked   0.0030869  0.0015514  1.9898  0.04662 *
YearsWithCurrManager  0.0019817  0.0011050  1.7935  0.07290 .
-----
Marginal Effects on Pr(Outcome==3)
      Marg. Eff Std. error t value Pr(>|t|)
as.factor(Attrition)1 -0.03347475  0.01104447 -3.0309 0.002438 **
MaritalStatus_Single  0.00333007  0.00191033  1.7432 0.081300 .
as.factor(OverTime)1  0.00392341  0.00188211  2.0846 0.037107 *
HourlyRate           -0.00287002  0.00139157 -2.0624 0.039167 *
NumCompaniesWorked   -0.00088966  0.00051165 -1.7388 0.082070 .
YearsWithCurrManager -0.00057115  0.00035749 -1.5977 0.110111
-----
Marginal Effects on Pr(Outcome==4)
      Marg. Eff Std. error t value Pr(>|t|)
as.factor(Attrition)1 -0.1274209  0.0236976 -5.3770 0.00000007576 ***
MaritalStatus_Single  0.0377630  0.0225070  1.6778  0.093379 .
as.factor(OverTime)1  0.0531376  0.0242771  2.1888  0.028612 *
HourlyRate           -0.0261952  0.0101270 -2.5867  0.009691 **
NumCompaniesWorked   -0.0081201  0.0040354 -2.0122  0.044197 *
YearsWithCurrManager -0.0052130  0.0028805 -1.8098  0.070331 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> |
```

Figure 23 - Marginal Effects of Predictors on Job Satisfaction Levels

The table below presents the marginal effects for the four outcomes of job satisfaction:

Table 6 - Marginal Effects of Key Predictors on Job Satisfaction Outcomes

Variables	Outcome = 1 (Low)	Outcome = 2 (Medium)	Outcome = 3 (High)	Outcome = 4 (Very high)
Attrition	Employees who exhibit attrition have an increased probability (by 11.7 p.p) of experiencing the lowest job satisfaction.	Employees who exhibit attrition have an increased probability (by 4.4 p.p) of experiencing medium job satisfaction.	Employees who exhibit attrition have a decreased probability (by 3.3 p.p) of experiencing high job satisfaction.	Employees who exhibit attrition have a decreased probability (by 12.7 p.p) of experiencing the highest job satisfaction.
OverTime	Working overtime decreases the probability (by 3.6 p.p) of experiencing the lowest job satisfaction.	Working overtime decreases the probability (by 2.0 p.p) of experiencing medium job satisfaction.	Working overtime slightly increases the probability (by 0.4 p.p) of experiencing high job satisfaction, but this effect is not statistically significant.	Working overtime increases the probability (by 5.3 p.p) of experiencing the highest job satisfaction.
HourlyRate	Higher hourly rates slightly increase the probability (1.91 p.p) of experiencing the lowest job satisfaction.	Higher hourly rates slightly increase the probability (by 0.9 p.p) of experiencing medium job satisfaction.	Higher hourly rates slightly decrease the probability (by 0.28 p.p) of experiencing high job satisfaction, but this effect is not statistically significant.	Higher hourly rates slightly decrease the probability (by 2.6 p.p) of experiencing the highest job satisfaction.
NumCompaniesWorked	The number of companies worked increases the probability (by 0.6 p.p) of experiencing the lowest job satisfaction.	The number of companies worked increases the probability (by 0.3 p.p) of experiencing medium job satisfaction.	Marg. Eff. = -0.0008 p_value = 0.08 non-significant impact on the probability of experiencing the high job satisfaction	Marg. Eff. = -0.008 p_value = 0.07 The number of companies worked has a negligible and non-significant impact on the probability of experiencing the highest job satisfaction.

Based on the marginal effects analysis, we verify the following important hypotheses:

- H0: Attrition has no significant effect on job satisfaction levels.
- H1: Attrition significantly affects job satisfaction levels.

The results indicate significant marginal effects for attrition across all four outcomes, supporting the alternative hypothesis. Employees who exhibit attrition have an increased probability of experiencing lower job satisfaction and a decreased probability of experiencing higher job satisfaction.

- H0: Working overtime has no significant effect on job satisfaction levels.

- H1: Working overtime significantly affects job satisfaction levels.

The results show significant marginal effects for overtime, particularly in increasing the probability of the highest job satisfaction and decreasing the probability of the lowest job satisfaction. This supports the alternative hypothesis that working overtime positively impacts job satisfaction.

- H0: Hourly rate has no significant effect on job satisfaction levels.
- H1: Hourly rate significantly affects job satisfaction levels.

The results indicate that higher hourly rates significantly increase the probability of the lowest job satisfaction and slightly decrease the probability of the highest job satisfaction. This supports the alternative hypothesis that hourly rate impacts job satisfaction.

It is important to carefully analyze the contradiction between the impacts of Hourly Rate and OverTime on job satisfaction. We find that higher hourly rates are associated with a higher probability of lower job satisfaction and a lower probability of very high job satisfaction. This may suggest that higher hourly rates are often provided to compensate for more demanding or less desirable job conditions. Employees in high-paying hourly jobs might face higher stress or less favorable work environments, which negatively impacts their overall job satisfaction despite the higher pay. Working overtime significantly decreases the probability of experiencing lower job satisfaction and increases the probability of achieving very high job satisfaction. This indicates that overtime work, often voluntary and leading to higher total earnings, can enhance job satisfaction. Employees who work overtime might feel a sense of accomplishment and appreciate the additional income, which positively influences their job satisfaction.

How to harmonize this contradiction between the impacts of Hourly Rate and OverTime on job satisfaction? The negative impact of higher hourly rates on job satisfaction likely reflects specific job contexts where high pay compensates for undesirable aspects of the job. This context may not apply to overtime work, which employees might choose voluntarily. Overtime work is often a choice made by employees seeking additional income or driven by a sense of dedication and achievement. This voluntary nature can lead to positive feelings about their job and increased satisfaction. Employees working overtime may perceive a direct link between their effort and the reward they receive, leading to higher satisfaction. In contrast, higher hourly rates might not have the same positive perception if they are seen as compensation for negative job aspects.

Interpretation of R² Statistics

The interpretation of R² statistics helps assess the goodness-of-fit and explanatory power of the ordered logit for the final model. We focus on three key R² statistics: McKelvey-Zavoina R², Count R², and Adjusted Count R².

- R² mz = 0.1623566

McKelvey-Zavoina R^2 of 0.162 indicates that approximately 16.2% of the variance in the latent variable underlying job satisfaction can be explained by the predictors in the model. This value suggests a moderate explanatory power.

●Count $R^2 = 0.3272109$

A Count R^2 of 0.327 implies that the model correctly predicts the job satisfaction category for about 32.7% of the observations. While this may seem modest, it reflects the challenge of predicting ordinal outcomes and indicates that the model performs better than random guessing.

●Adjusted Count $R^2 = 0.02176063$

An Adjusted Count R^2 of 0.0218 suggests that the model's predictive accuracy is only slightly better than what would be expected by random chance. Specifically, the model explains about 2.2% of the variation in job satisfaction beyond what could be achieved by always predicting the most frequent category. This highlights the inherent difficulty in accurately predicting individual job satisfaction levels.

Findings

The analysis conducted in this paper provides significant insights into the factors influencing job satisfaction among employees. We tested several hypotheses using ordered logit model, focusing on the following key aspects: gender differences in job satisfaction relative to tenure, the impact of attrition, the effect of working overtime, and the influence of hourly rates on job satisfaction.

1. Gender differences in job satisfaction relative to tenure.

H0: The relationship between years at the company and job satisfaction is the same for both male and female employees.

H1: As years at the company increase, the impact on job satisfaction becomes more favorable for male employees compared to female employees.

Our findings:

The intermediate model, which included an interaction term between years at the company and gender, revealed a significant positive interaction coefficient. This indicates that as years at the company increase, job satisfaction improves more for male employees compared to female employees. This finding suggests that male employees might experience greater benefits or have fewer issues with prolonged tenure at a company than female employees.

Based on this information the company can:

- Implement mentorship programs tailored to female employees to address potential stagnation or interpersonal issues over time.
- Create career development plans that ensure both male and female employees have equal opportunities for advancement and job satisfaction improvement.
- Promote an inclusive work environment that supports long-term career growth for all employees, focusing on addressing any gender-specific challenges.

2. The impact of attrition on job satisfaction.

H0: Attrition has no significant effect on job satisfaction levels.

H1: Attrition significantly affects job satisfaction levels.

Our findings:

The results indicate that attrition significantly impacts job satisfaction. Employees who exhibit attrition are more likely to experience lower job satisfaction. Marginal effects analysis also showed that attrition increases the probability of experiencing low job satisfaction by 11.7 percentage points and decreases the probability of very high job satisfaction by 12.7 percentage points.

Based on this information the company can:

- Develop targeted retention strategies to reduce attrition rates, such as improving work conditions and enhancing employee engagement.
- Regularly collect and act on employee feedback to identify and address factors leading to dissatisfaction and attrition.
- Offer clear career growth opportunities and professional development to retain talented employees and reduce turnover.

3. The effect of working overtime on job satisfaction.

H0: Working overtime has no significant effect on job satisfaction levels.

H1: Working overtime significantly affects job satisfaction levels.

Our findings:

Working overtime has a significant positive effect on job satisfaction. Engaging in overtime work is associated with a decreased likelihood of experiencing low job satisfaction. Marginal effects analysis showed that overtime decreases the probability of low job satisfaction by 3.6 percentage points and increases the probability of very high job satisfaction by 5.3 percentage points.

Based on this information the company can:

- Ensure that overtime work is voluntary and provide appropriate incentives and compensation.
- Monitor work-life balance to prevent burnout, ensuring that employees do not feel pressured to work overtime excessively.
- Implement recognition programs that reward employees for their extra efforts, thereby enhancing job satisfaction.

4. The influence of hourly rate on job satisfaction.

H0: Hourly rate has no significant effect on job satisfaction levels.

H1: Hourly rate significantly affects job satisfaction levels.

Our findings:

Higher hourly rates are associated with a higher probability of lower job satisfaction. Marginal effects analysis indicated that higher hourly rates increase the probability of low job satisfaction by 1.91 percentage points and decrease the probability of very high job satisfaction by 2.6 percentage points. This suggests that higher pay may compensate for negative job aspects, such as stress or long hours.

Based on this information the company can:

- Develop holistic compensation packages that include non-monetary benefits such as flexible working hours, wellness programs, and career development opportunities.

- Consider redesigning high-stress jobs to improve working conditions and job satisfaction, ensuring that higher pay is not solely compensating for undesirable job aspects.
- Implement support programs that address job-related stress and improve the overall work environment, enhancing job satisfaction beyond financial compensation.

These findings underscore the complexity of job satisfaction and highlight the importance of considering multiple factors, including gender, attrition, overtime, and pay. By addressing these areas, companies can improve employee satisfaction, reduce turnover, and foster a more positive and productive work environment.

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