

Weekly HW 2

1.4

23. Determine a scalar r such that $Ax = rx$, where

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix} \text{ and } x = \begin{bmatrix} \frac{-1}{2} \\ \frac{1}{4} \\ 1 \end{bmatrix}$$

$$Ax = \begin{bmatrix} -0.5 + 0.5 + -1 \\ -0.5 + 0 + 1 \\ -2 - 1 + 5 \end{bmatrix} = \begin{bmatrix} -1 \\ 0.5 \\ 2 \end{bmatrix}$$

$$\boxed{r = 2}$$

1.5

38. The linear system $A^2x = b$ is such that A is nonsingular with

$$A^{-1} = \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix} \text{ and } b = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

Find the solution x .

$$A^{-1^{-1}} = A = \begin{bmatrix} 1/3 & 0 \\ -2/3 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1/3 & 0 \\ -2/3 & 1 \end{bmatrix} \times \begin{bmatrix} 1/3 & 0 \\ -2/3 & 1 \end{bmatrix} = \begin{bmatrix} 1/9 & 0 \\ -8/9 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1/9 & 0 \\ -8/9 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$\frac{x}{9} = -1$$

$$\frac{-8x}{9} + y = -2$$

$$\boxed{x = -9, y = -6}$$

1.6

In the following exercises, let $f : \mathbb{R}^7 \rightarrow \mathbb{R}^3$ for the matrix transformation defined by $f(x) = Ax$, where

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

Determine whether each given vector w is in the range of f .

$$9. w = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$x + 2y = 1$$

$$y = -1$$

$$x + y = 2$$

If $y = -1$, $x = 3$. This works for the first equation too. Therefore, this vector is in the range of f

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