Weekly HW 006

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0.0.1 4.2.15

Consider the differential equation $y'' - y' + 2y = 0$. A solution is a real-valued function f satisfying the equation. Let V be the set of all solutions to the given differential equation; define $+$ and $\cdot$ as in Exercise 14. Prove that V is a vector space.

To prove that V is a vector space, we can use scalar addition and scalar multiplication to prove it.
Say that we have two solutions to this equation, y_1 and y_2.

$$(y_1 + y_2)'' - (y_1 + y_2)' + 2(y_1 + y_2) = 0$$
$$(y_1'' - y_1' + 2y_1) + (y_2'' - y_2' + 2y_2)$$

Since both of these mini-equations are in the same format as this equation,

$$0 + 0 = 0$$

Using scalar multiplication,

$$c(y'' - y' + 2y) = 0$$

Since the differentiation equation already equals zero, no matter what c is, the solution will always equal zero. Therefore, V is a subspace and therefore a $\boxed{\text{vector space}}$.

0.0.2 4.3.17

Which of the following subsets of the vector space M_{nn} are subspaces?

(a) The set of all $n \times n$ symmetric matrices.

If $A = A^T$ and $B = B^T$ (since both are symmetric)

$$A^T + B^T = M_{nn}$$

because a transposed symmetric matrix is identical to the original matrix.

As for multiplication, multiplying a symmetric matrix by a constant will always result in the same dimensions.

(b) The set of all $n \times n$ diagonal matrices.

All diagonal matrices are a subspace of symmetric matrices, which in turn are a subspace of the vector space $M_n n$.

(c) The set of all $n \times n$ nonsingular matrices.
A vector space is required to have a zero vector in it. As a zero vector can never be nonsingular, this set is not a subspace of M_{nn}.

0.0.3 4.4.4b

Determine whether the given vector A in M_{22} belongs to span $[A_1, A_2, A_3]$, where

$$A_1 = \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, A_3 = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$$

and

$$A = \begin{bmatrix} -3 & -1 \\ 3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -3 & -1 \\ 3 & 2 \end{bmatrix} + 3A_3 = \begin{bmatrix} 3 & 5 \\ 0 & 5 \end{bmatrix} - 2A_2 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} + A_1 = \begin{bmatrix} 2 & 2 \\ 0 & 4 \end{bmatrix} \cdot \frac{1}{2} = \boxed{\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}}$$

Therefore, A is in span $[A_1, A_2, A_3]$.