# Weekly HW 3

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### 0.0.1 Section 1.6, Problem 10

In the following problem, let $f : \mathbb{R}^7 \to \mathbb{R}^3$ for the matrix transformation defined by $f(x) = Ax$, where

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

Determine whether the matrix $w = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is in the range of $f$.

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{cases} x + 2y = 1 \\ y = 1 \\ x + y = 1 \end{cases} \tag{1}$$

Since this system of equations is impossible to solve, $\boxed{\text{the matrix } w \text{ is NOT in the range of } f.}$

### 0.0.2 Section 2.1, Problem 6

Find the reduced row echelon form of each of the given matrices. Record the row operations you perform, using the notation for elementary row operations.

$$\begin{bmatrix} -1 & 2 & -5 \\ 2 & -1 & 6 \\ 2 & -2 & 7 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 & -5 \\ 0 & 3 & -4 \\ 2 & -2 & 7 \end{bmatrix} \to (R_3 + 2R_1 = R_3, -R_1 = R_1) \to \begin{bmatrix} 1 & -2 & 5 \\ 0 & 3 & -4 \\ 0 & 2 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 5 \\ 0 & 3 & -4 \\ 0 & 2 & -3 \end{bmatrix} \to (R_2 - R_3 = R_2) \to \begin{bmatrix} 1 & -2 & 5 \\ 0 & 1 & -1 \\ 0 & 2 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 5 \\ 0 & 1 & -1 \\ 0 & 2 & -3 \end{bmatrix} \to (R_3 - 2R_2 = R_3) \to \begin{bmatrix} 1 & -2 & 5 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 5 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix} \to (R_1 + 2R_2 = R_1) \to \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix} \to (-R_3 = R_3) \to \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix}1 & 0 & 3\\0 & 1 & -1\\0 & 0 & 1\end{bmatrix} \to (R_2 + R_3 = R_2) \to \begin{bmatrix}1 & 0 & 3\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}1 & 0 & 3\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} \to (R_1 - 3R_3 = R_1) \to \boxed{\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = I_3}$$

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$$\begin{bmatrix}1 & 1 & -1\\3 & 4 & -1\\5 & 6 & 3\\-2 & -2 & 2\end{bmatrix}$$

$$\begin{bmatrix}1 & 1 & -1\\3 & 4 & -1\\5 & 6 & 3\\-2 & -2 & 2\end{bmatrix} \to (R_2 - 3R_1 = R_2, R_3 - 5R_1 = R_3, R_4 + 2R_1 = R_4) \to \begin{bmatrix}1 & 1 & -1\\0 & 1 & 2\\0 & 1 & 8\\0 & 0 & 0\end{bmatrix}$$

$$\begin{bmatrix}1 & 1 & -1\\0 & 1 & 2\\0 & 1 & 8\\0 & 0 & 0\end{bmatrix} \to (R_1 - R_2 = R_1, R_3 - R_2 = R_3) \to \begin{bmatrix}1 & 0 & -3\\0 & 1 & 2\\0 & 0 & 6\\0 & 0 & 0\end{bmatrix}$$

$$\begin{bmatrix}1 & 0 & -3\\0 & 1 & 2\\0 & 0 & 6\\0 & 0 & 0\end{bmatrix} \to (R_3 \times \frac{1}{6} = R_3) \to \begin{bmatrix}1 & 0 & -3\\0 & 1 & 2\\0 & 0 & 1\\0 & 0 & 0\end{bmatrix}$$

$$\begin{bmatrix}1 & 0 & -3\\0 & 1 & 2\\0 & 0 & 1\\0 & 0 & 0\end{bmatrix} \to (R_2 - 2R_3 = R_2, R_1 + 3R_3 = R_1) \to \boxed{\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\\0 & 0 & 0\end{bmatrix}}$$

### 0.0.3 Section 2.2, Problem 9

In this exercise, solve the linear system, with the given augmented matrix, if it is consistent.

$$\begin{bmatrix}1 & 2 & 1 & | & 7\\2 & 0 & 1 & | & 4\\1 & 0 & 2 & | & 5\\1 & 2 & 3 & | & 11\\2 & 1 & 4 & | & 12\end{bmatrix}$$

$$\begin{bmatrix}1 & 2 & 1 & | & 7\\2 & 0 & 1 & | & 4\\1 & 0 & 2 & | & 5\\1 & 2 & 3 & | & 11\\2 & 1 & 4 & | & 12\end{bmatrix} \to (R_2 - 2R_1 = R_2, R_3 - R_1 = R_3, R_4 - R_1 = R_4, R_5 - 2R_1 = R_5) \to$$

$$\to \begin{bmatrix} 1 & 2 & 1 & | & 7 \\ 0 & -4 & -1 & | & -10 \\ 0 & -2 & 1 & | & -2 \\ 0 & 0 & 2 & | & 4 \\ 0 & -3 & 2 & | & -2 \end{bmatrix} \to (R_2 - R_5 = R_2) \to \begin{bmatrix} 1 & 2 & 1 & | & 7 \\ 0 & -1 & -3 & | & -8 \\ 0 & -2 & 1 & | & -2 \\ 0 & 0 & 2 & | & 4 \\ 0 & -3 & 2 & | & -2 \end{bmatrix} \to$$

$$\begin{bmatrix} 1 & 2 & 1 & | & 7 \\ 0 & -1 & -3 & | & -8 \\ 0 & -2 & 1 & | & -2 \\ 0 & 0 & 2 & | & 4 \\ 0 & -3 & 2 & | & -2 \end{bmatrix} \to (-R_2 = R_2) \to \begin{bmatrix} 1 & 2 & 1 & | & 7 \\ 0 & 1 & 3 & | & 8 \\ 0 & -2 & 1 & | & -2 \\ 0 & 0 & 2 & | & 4 \\ 0 & -3 & 2 & | & -2 \end{bmatrix}$$

I don't really have to simplify the augmented matrix anymore, now that I know that $2z = 4$. Therefore, $z = 2$. Using the third line, I conclude that $y = 2$. Testing this with the other equations, this substitution of $z = 2$ and $y = 2$ yields no inconsistency. Substituting both of these into the first line yields $x$ as 1.

$$\boxed{x = 1, y = 2, z = 2}$$

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$$\begin{bmatrix} 1 & 2 & 1 & | & 0 \\ 2 & 3 & 0 & | & 0 \\ 0 & 1 & 2 & | & 0 \\ 2 & 1 & 4 & | & 0 \end{bmatrix}$$

There is a trivial solution, but I'll attempt to find another solution using row operations and RREFs.

$$\begin{bmatrix} 1 & 2 & 1 & | & 0 \\ 2 & 3 & 0 & | & 0 \\ 0 & 1 & 2 & | & 0 \\ 2 & 1 & 4 & | & 0 \end{bmatrix} \to (R_2 - 2R_1 = R_2, R_4 - 2R_1 = R_4) \to \begin{bmatrix} 1 & 2 & 1 & | & 0 \\ 0 & -1 & -2 & | & 0 \\ 0 & 1 & 2 & | & 0 \\ 0 & -3 & 2 & | & 0 \end{bmatrix}$$

I don't need to simplify anymore, because I see that there is an inconsistency; both $1y + 2z$ and $-3y + 2z$ are equal to zero, making the system of equations inconsistent. There is only the trivial solution.

$$\boxed{x = 0, y = 0, z = 0}$$