Lecture 010

Svadrut Kukunooru

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EXERCISE: Let A be a generic 3x3 matrix; i.e.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}.$$

Also, let

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}.$$

Compute E_1A, E_2A, E_3A.

$$E_1A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}, E_2A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 3a_{21} & 3a_{22} & 3a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, E_3A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{23} + 3a_{31} & a_{22} + 3a_{32} & a_{23} + a_{33} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}.$$

Note that E_1A is just rows 2 and 3 switched, E_2A is row 2 scaled by 3, and E_3A is just adding 3 copies of row 3 to row 2.

TAKEAWAY: Elementary row operations can be realized by left multiplication by matrices.

Also note that such matrices that realize elementary row operations are called **ELEMENTARY MATRICES**. Recall that an elementary row operation is one of the following:

- Switch two rows
- Scale a row by a nonzero number
- Add (a multiple of) one row to another

0.0.1 EXAMPLE PROBLEM

Suppose I have an elementary row operation T. How do I find a matrix E which realizes T? That is, T applied to a matrix $A = EA$.

$E =$ apply T to the identity matrix I_m so that E is $m \times m$

0.0.2 EXAMPLE PROBLEM

Reduce A to RREF and record the elementary matrices of your steps.

$$A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} \implies \begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix} \implies \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \implies \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

THEOREM: Suppose $E_1, E_2, \ldots E_{12}$ are elementary row operations which reduce a square matrix A to its RREF.

Then (a) A is only invertible if its RREF is the identity matrix. and (b) When this happens, $A{-}1 = E_{Er-1} \ldots E_1$