Lecture 011: Examples and Exercises

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0.0.1 Example Problem

Consider the matrix $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$ Compute the RREF of $\left[A|I_2\right]$.

This is basically just

$$\begin{bmatrix} 3 & 5 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{bmatrix}.$$

I don't have enough time to record the row operations, but it turns into

$$\begin{bmatrix} 1 & 0 & 2 & -\frac{5}{2} \\ 0 & 1 & -1 & \frac{3}{2} \end{bmatrix}.$$

You can find the variables in a system of equations with this method.

THEOREM: IF A is an $n \times n$ matrix, then

$$\text{RREF}([A|I_n]) = [I_n|A^{-1}].$$

TAKEAWAY: In order to find A^{-1}, we do not need to record all elementary matrices involved in computing RREF; instead, just carry an identity matrix as you compute the RREF.

0.0.2 Example Problem

Invert the following matrix

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 4 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 2 & | & 1 & 0 & 0 \\ 2 & 3 & 4 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{bmatrix}.$$

$$\rightarrow \begin{bmatrix} 1 & 1 & 2 & | & 1 & 0 & 0 \\ 0 & 1 & 0 & | & -2 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{bmatrix} \rightarrow$$

$$\begin{bmatrix} 1 & 0 & 2 & | & 3 & -1 & 0 \\ 0 & 1 & 0 & | & -2 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & | & 3 & -1 & -2 \\ 0 & 1 & 0 & | & -2 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{bmatrix}.$$

0.0.3 Example Problem

For what values of a is this matrix invertible?

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 2 & a \end{bmatrix}.$$