Lec 009: RREFs, Continued

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RECAP: An RREF looks like this:

$$\begin{bmatrix} 1 & x & x \\ 0 & 1 & x \\ 0 & 0 & 1 \end{bmatrix}.$$

The diagonal of ones is called an **ECHELON**. Note that even though the diagonal of ones might be interrupted, the matrix will still be in RREF, such as:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

0.0.1 EXAMPLE PROBLEM

Solve this system of equations:

$$\begin{cases} x + 2y + 3z = 9 \\ 2x - y + z = 8 \\ 3x - z = 3 \end{cases} \tag{1}$$

$$\begin{cases} x + 2y + 3z = 9 \\ -5y - 5z = -10 \\ -6y - 10z = -24 \end{cases} \tag{2}$$

$$\begin{cases} x + 2y + 3z = 9 \\ y + z = 2 \\ z = 3 \end{cases} \tag{3}$$

$$\boxed{x = 2, y = -1, z = 3}$$

Next, convert the following matrix to RREF:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} 9 \\ 10 \\ -24 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -5 \\ 0 & -6 & -10 \end{bmatrix} \begin{bmatrix} 9 \\ 10 \\ -24 \end{bmatrix}.$$

$$\Longrightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 9 \\ 2 \\ 3 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 9 \\ -1 \\ 3 \end{bmatrix} \Longrightarrow \boxed{\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix}}.$$

You can see that solving a system of linear equations in the form $A\vec{x} = \vec{b}$ is the same thing as reducing $[A|\vec{b}]$ to its RREF.

0.0.2 EXAMPLE PROBLEM

$$\begin{cases} x+y+z+w=0 \\ x+w=0 \\ x+2y+z=0 \end{cases} \tag{4}$$

What are the solutions to this system? HINT: The RREF is

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Convert the RREF to a system of equations:

$$\begin{cases} x+w=0 \\ y-w=0 \\ z+w=0 \end{cases} \tag{5}$$

A variable is called a **FREE VARIABLE** if the corresponding column in RREF does not have a leading one. Other wise, it is a **LEADING VARIABLE**. In the above example w is the free variable and x, y, z are the leading variables. Set parameters for free variables, then express the leading variables in terms of these parameters.