Lecture 012: Introduction to Determinants

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October 1, 2021

A determinant defines a function det : $M_{n\times n} \to \mathbb{R}$. Recall that in a 2x2 case,

$$det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$$

SIGNED AREA: Suppose you want to integrate the sin function from $-\pi$ to π:

$$\int_{-\pi}^{\pi} \sin(x)$$

The answer to this is zero, since the are aon the left side of the y-axis negates the area on the right side of the y-axis. POINT: It is natural to assign area a sign.

Given vectors $\vec{v_1}$ and $\vec{v_2}$ on $\mathbb{R}$ we denote by $P(v_1, v_2)$ the *parallelogram* defined by v_1 and v_2. Note that the signed area of $P(v_1, v_2)$ is 0 if v_1 and v_2 are parallel/colinear. It is positive if v_1 rotates counterclockwise towards v_2 in $P(v_1, v_2)$, and negative if v_1 rotates clockwise towards v_2.

THEOREM: If A is a 2×2 matrix with row vectors r_1 and r_2, then the determinant of A is the signed area of $P(r_1, r_2)$ For example say you have a matrix

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$$

How do you find the area of the parallelogram? Most poeple use the **SLIDE METHOD**, where