Weekly HW 004

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0.0.1 2.2.15

Determine all values of a for which the linear system has no solution, a unique solution, and infinitely many solutions.

$$\begin{cases} x+y+z=2 \\ 2x+3y+2z=5 \\ 2x+3y+(a^2-1)z=a+1 \end{cases}$$

You can see that the bottom 2 equations are the same, so you simply have to match it (or not match it). No solution is $\boxed{a=\sqrt{3},-\sqrt{3}}$, since that makes the bottom two equations have an inconsistency.

$$\begin{bmatrix} 1 & 1 & 1 & | & 2 \\ 2 & 3 & 2 & | & 5 \\ 2 & 3 & a^2-1 & | & a+1 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 & | & 2 \\ 0 & 1 & 0 & | & 1 \\ 2 & 3 & a^2 1 & | & a+1 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 & | & 2 \\ 0 & 1 & 0 & | & 1 \\ 0 & 1 & a^2-3 & | & a-3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & | & 2 \\ 0 & 1 & 0 & | & 1 \\ 0 & 0 & a^2-3 & | & a-4 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 & | & 1 \\ 0 & 1 & 0 & | & 1 \\ 0 & 0 & a^2-3 & | & a-4 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 0 & 1 & | & 1 \\ 0 & 1 & 0 & | & 1 \\ 0 & 0 & 1 & | & \frac{a-4}{a^2-3} \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & | & \frac{a^2-a+1}{a^2-3} \\ 0 & 1 & 0 & | & 1 \\ 0 & 0 & 1 & | & \frac{a-4}{a^2-3} \end{bmatrix}$$

Therefore, for unique solutions,

$$\boxed{x=\frac{a^2-a+1}{a^2-3}, y=1, z=\frac{a-4}{a^2-3}}$$

There are no values to make this system have infinitely many solutions.

0.0.2 2.3.3

Let A be a 3×4 matrix. Find the elementary matrix F that, as poastmultiplier of A – that is, as AF – performs the following elementary column operations on A:

Adds -4 times the first column of A to the second column of A.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \to \boxed{\begin{bmatrix} 1 & -4 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}$$

Interchanges the second and third columns of A.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Multiply the 3rd column of A by 4.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

0.0.3 Problem 2.2.16

If $A^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, find A.

$$\begin{bmatrix} 1 & 1 & 1 & | & 1 & 0 & 0 \\ 1 & 1 & 2 & | & 0 & 1 & 0 \\ 1 & -1 & 1 & | & 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 & | & 1 & 0 & 0 \\ 1 & -1 & 1 & | & 0 & 0 & 1 \\ 1 & 1 & 2 & | & 0 & 1 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 & | & 1 & 0 & 0 \\ 1 & -1 & 1 & | & 0 & 0 & 1 \\ 0 & 0 & 1 & | & -1 & 1 & 0 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 1 & 1 & | & 1 & 0 & 0 \\ 0 & -2 & 0 & | & -1 & 0 & 1 \\ 0 & 0 & 1 & | & -1 & 1 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 & | & 1 & 0 & 0 \\ 0 & 1 & 0 & | & 0.5 & 0 & -0.5 \\ 0 & 0 & 1 & | & -1 & 1 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 & | & 0.5 & 0 & 0.5 \\ 0 & 1 & 0 & | & 0.5 & 0 & -0.5 \\ 0 & 0 & 1 & | & -1 & 1 & 0 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 0 & 1 & | & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 & | & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & | & -1 & 1 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & | & \frac{3}{2} & -1 & \frac{1}{2} \\ 0 & 1 & 0 & | & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & | & -1 & 1 & 0 \end{bmatrix}$$

$$\boxed{A = \begin{bmatrix} \frac{3}{2} & -1 & \frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ -1 & 1 & 0 \end{bmatrix}}$$