Weekly Homework 1

Section 1.1

22. Is there a value of r so that $x = 1$, $y = 2$, $z = r$ is a solution to the following linear system? If there is, find it.

$$\begin{aligned} 2x + 3y - z &= 11 \\ x - y + 2z &= -7 \\ 4x + y - 2z &= 12 \end{aligned}$$

$\boxed{r \text{ can be equal to -3 for the system of equations to be consistent.}}$

Section 1.2

11. Is the matrix $\begin{bmatrix} 4 & 1 \\ 0 & -3 \end{bmatrix}$ a linear combination of the matrices $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$? Justify your answer.

A linear combination can be obtained by multiplying none, one, or both of the matrices by a scalar and adding them together. Note that for the top right value of the solution matrix, 1, it is impossible to multiply 0 by a scalar to get a non-zero number. Therefore,

$$\boxed{\text{The matrix } \begin{bmatrix} 4 & 1 \\ 0 & -3 \end{bmatrix} \text{ is not a linear combination of matrices } A \text{ and } B}$$

Section 1.3

43. If $A = [a_{ij}]$ is an $n \times n$ matrix, then the **trace** of A, $\mathrm{Tr}(A)$, is defined as the sum of all elements on the main diagonal of A, $\mathrm{Tr}(A) = \sum_{i=1}^{n} a_{ii}$. Show each of the following:

1. $\mathrm{Tr}(cA) = c\mathrm{Tr}(A)$, where c is a real number

$$\mathrm{Tr}(cA) = \sum_{i=1}^{n} ca_{ii} = c\sum_{i=1}^{n} a_{ii} = cTr(A)$$

$$\boxed{\text{QED}}$$

2. $\mathrm{Tr}(A + B) = \mathrm{Tr}(A) + \mathrm{Tr}(B)$

$$\mathrm{Tr}(A + B) = \sum_{i=1}^{n}(a_{ii} + b_{ii}) = \sum_{i=1}^{n} a_{ii} + \sum_{i=1}^{n} b_{ii} = \mathrm{Tr}(A) + \mathrm{Tr}(B)$$

$$\boxed{\text{QED}}$$

3. $\mathrm{Tr}(AB) = \mathrm{Tr}(BA)$

$$\mathrm{Tr}(AB) = \mathrm{Tr}(C) = \sum_{j=1}^{n} c_{ii} = \sum_{i=1}^{n}\sum_{k=1}^{n}(a_{ii} \times b_{ii})$$

$$\mathrm{Tr}(BA) = \sum_{k=1}^{n}\sum_{i=1}^{n}(b_{ii} \times a_{ii}) \Rightarrow \sum_{i=1}^{n}\sum_{k=1}^{n}(a_{ii} \times b_{ii})$$

$$\mathrm{Tr}(AB) = \mathrm{Tr}(BA)$$

$$\boxed{\text{QED}}$$

4. $\text{Tr}(A^T) = \text{Tr}(A)$

$$A = \begin{bmatrix} a_{ii} & k & k \\ j & a_{ii} & k \\ j & j & a_{ii} \end{bmatrix}$$

$$A^T = \begin{bmatrix} a_{ii} & j & j \\ k & a_{ii} & j \\ k & k & a_{ii} \end{bmatrix}$$

$$a_{ii} + a_{ii} + \ldots + a_{ii} = a_{ii} + a_{ii} + \ldots + a_{ii}$$

$$\text{Tr}(A^T) = \text{Tr}(A)$$

$$\boxed{\text{QED}}$$

5. $\text{Tr}(A^T A) \geq 0$

   Transposing does not change the main diagonal's sign. Therefore, if you multiply two of the same diagonals together, each of the numbers will result in a zero or a positive, which results in the sum being either zero or positive. $\boxed{\text{QED}}$.