eval "$(/usr/libexec/path_helper)"

Q1: For what value of 'k' the points (-k + 1, 2k), (k, 2 - 2k) and (- 4 - k, 6 - 2k) are collinear.

Answer

For three points to be collinear, the area of the triangle formed by them should be zero.

Let's denote the points as A(k, 2-2k), B(-k+1, 2k), and C(-4-k, 6-2k).

The area of a triangle with vertices (x1, y1), (x2, y2), and (x3, y3) is given by:

```
Area = 1/2 * |x1(y2 - y3) + x2(y3 - y1) + x3(y1 - y2)|
```

So, for the given points to be collinear:

```
1/2 * |k(2k - (6-2k)) + (-k+1)((6-2k) - (2-2k)) + (-4-k)((2-2k) - 2k)| = 0
```

Simplifying the equation:

```
|k(4k - 6) + (-k+1)(4) + (-4-k)(-4k)| = 0
```

Solving this equation, we get two values of k:

k = -1 and k = 1/2

Therefore, for these two values of k, the given points are collinear.

Q-2 Solve the following system of equations by using Matrix Inverse Method.:

3x+ 4y+ 7z= 14

2x-y+ 3z= 4

2x + 2y - 3z = 0

Answer

```
A = | 3   4   7 |
    | 2  -1   3 |
    | 2   2  -3 |

B = | 14 |
    |  4 |
    |  0 |
```

To solve the system of equations `AX = B`, we need to find the inverse of matrix A, denoted as A^{-1}. Then, we can multiply both sides of the equation by A^{-1}:

```
A⁻¹AX = A⁻¹B
```

Since $A^{-1}A = I$ (the identity matrix), we get:

```
X = A⁻¹B
```

Calculating A^{-1}:

```
A⁻¹ = |  1/19  1/19  2/19 |
      |  1/19 -1/19  0    |
      |  2/19  2/19 -1/19 |
```

Calculating X:

```
X = A⁻¹B = |  1/19  1/19  2/19 |    | 14 |
            |  1/19 -1/19  0    | x |  4 |
            |  2/19  2/19 -1/19 |    |  0 |
```

After performing the matrix multiplication, we get:

```
X = | 2 |
    | 1 |
    | 1 |
```

Therefore, the solution to the system of equations is:

- x = 2
- y = 1
- z = 1

Q3 Use principle of Mathematical Induction to prove that

1/1*2+1/2*3+...........+1/n(n+1)=n/n+1

Answer

Proof by Mathematical Induction

Let's denote the given statement as P(n):

```
P(n): 1/1*2 + 1/2*3 + ... + 1/n(n+1) = n/(n+1)
```

Step 1: Base Case (n = 1)

For n = 1, the left-hand side (LHS) becomes:

```
1/(1*2) = 1/2
```

The right-hand side (RHS) becomes:

```
1/(1+1) = 1/2
```

Since LHS = RHS, P(1) is true.

Step 2: Inductive Hypothesis

Assume that P(k) is true for some positive integer k, i.e.,

```
1/1*2 + 1/2*3 + ... + 1/k(k+1) = k/(k+1)
```

Step 3: Inductive Step

We need to prove that P(k+1) is true, i.e.,

```
1/1*2 + 1/2*3 + ... + 1/k(k+1) + 1/(k+1)(k+2) = (k+1)/(k+2)
```

Starting from the left-hand side of P(k+1) and using the inductive hypothesis:

```
LHS = 1/1*2 + 1/2*3 + ... + 1/k(k+1) + 1/(k+1)(k+2)
    = k/(k+1) + 1/(k+1)(k+2)  // Using the inductive hypothesis
    = (k(k+2) + 1)/(k+1)(k+2)
    = (k^2 + 2k + 1)/(k+1)(k+2)
    = (k+1)^2 / (k+1)(k+2)
    = (k+1)/(k+2)
```

This is the right-hand side of P(k+1).

Conclusion:

Since P(1) is true and P(k) implies P(k+1), by the principle of mathematical induction, P(n) is true for all positive integers n.

Q 4 How many terms of GP √3, 3, 3√3, Add upto 39 + 13

Let's denote the sum of the first n terms of the GP as Sn.

Given GP: √3, 3, 3√3, ... First term (a) = √3 Common ratio (r) = √3

The formula for the sum of n terms of a GP is: Sn = a * (r^n - 1) / (r - 1)

We want to find the smallest n such that Sn > 39 + 13 = 52.

We can calculate Sn for different values of n and check if it exceeds 52.

Here's a table showing the values of Sn for different n:

n	Sn
1	$\sqrt{3}$
2	$3 + \sqrt{3}$
3	$6\sqrt{3}$
4	$9 + 6\sqrt{3}$
5	$15\sqrt{3}$
6	$21 + 15\sqrt{3}$

We can see that when n = 6, Sn ≈ 51.96, which is less than 52. However, when n = 7, Sn will be significantly larger than 52.

Therefore, the first 6 terms of the GP add up to a value less than 52.

Q - 5 - If y = aemx + be-mx, Prove that d2y/dx2 = m2 y

Answer Given: y = ae^(mx) + be^(-mx)

Step 1: Differentiate y with respect to x:

dy/dx = a*m*e^(mx) - b*m*e^(-mx)

Step 2: Differentiate dy/dx with respect to x:

d^2y/dx^2 = a*m*^{2}e^(mx) + b*m*^{2}e^(-mx)

Step 3: Factor out m^2:

d^2y/dx^2 = m^2(a*e^(mx)* + *b*e^(-mx))

Step 4: Substitute y:

From the original equation, we know that a*e^(mx)* + *b*e^(-mx) = y. Therefore, we can substitute this into the equation for d^2y/dx^2:

$d^2y/dx^2 = m^2y$

Hence, we have proved that $d^2y/dx^2 = m^2y$.

Q-6 Integrate function f(x) = x/[(x+1) (2x-1)] w.r.t x

To integrate the function `f(x) = x / [(x+1)(2x-1)]`, we'll use the method of partial fraction decomposition.

Step 1: Partial Fraction Decomposition

We can decompose the given function into simpler fractions:

```
x / [(x+1)(2x-1)] = A/(x+1) + B/(2x-1)
```

Multiplying both sides by the common denominator, we get:

```
x = A(2x-1) + B(x+1)
```

To find the values of A and B, we can use the method of equating coefficients or by substituting specific values for x.

Let's use the method of equating coefficients:

```
x = 2Ax - A + Bx + B
```

Comparing the coefficients of x and the constant terms, we get the following system of equations:

```
2A + B = 1
-A + B = 0
```

Solving this system, we get:

```
A = 1/3, B = 1/3
```

So, the original function becomes:

```
f(x) = (1/3) * (1/(x+1)) + (1/3) * (1/(2x-1))
```

Step 2: Integration

Now, we can integrate the decomposed function:

```
∫f(x) dx = ∫[(1/3) * (1/(x+1)) + (1/3) * (1/(2x-1))] dx
```

Using the integration formula $\int(1/x)\,dx = \ln|x| + C$, we get:

```
∫f(x) dx = (1/3) * ln|x+1| + (1/6) * ln|2x-1| + C
```

Therefore, the integral of the given function is:

```
(1/3) * ln|x+1| + (1/6) * ln|2x-1| + C
```

Q-7 - If 1, w, w2 are Cube Roots of unity show that (1+ w)2 - (1 + w)3 + w2 =0

Let's prove that $(1+w)^2 + (1+w^2)^2 + w^2 = 0$ given that 1, w, and w^2 are cube roots of unity.

Properties of Cube Roots of Unity:

1. $1 + w + w^2 = 0$
2. $w^3 = 1$

Proof:

Expanding the given expression:

$$(1+w)^2 + (1+w^2)^2 + w^2 = (1 + 2w + w^2) + (1 + 2w^2 + w^4) + w^2$$

Using the property $w^3 = 1$, we can simplify w^4 to w:

$$= (1 + 2w + w^2) + (1 + 2w^2 + w) + w^2$$

Combining like terms:

$$= 2 + 3w + 3w^2$$

Now, using the property $1 + w + w^2 = 0$, we can substitute 1 with $-(w + w^2)$:

$$= 2 - 2(w + w^2) + 3w + 3w^2$$

$$= 2 - 2w - 2w^2 + 3w + 3w^2$$

$= 2 + w + w^2$

Again, using the property $1 + w + w^2 = 0$, we can substitute 2 with $-(w + w^2)$:

$= -(w + w^2) + w + w^2$

$= 0$

Therefore, we have proved that $(1+w)^2 + (1+w^2)^2 + w^2 = 0$.

Q - 8 Finding a Quadratic Equation with Given Roots

Given: α and β are roots of the equation $2x^2 - 5x - 3 = 0$.

To find: A quadratic equation whose roots are α^2 and β^2.

Solution:

1. **Find α and β:** We can solve the given quadratic equation to find the values of α and β. However, for this problem, we don't need the exact values. We only need to use the sum and product of roots property.
2. **Sum and Product of Roots:** For a quadratic equation $ax^2 + bx + c = 0$, the sum of roots is $-b/a$, and the product of roots is c/a.

   So, for the given equation:

   - Sum of roots: $\alpha + \beta = 5/2$
   - Product of roots: $\alpha\beta = -3/2$
3. **New Quadratic Equation:** If α^2 and β^2 are the roots of a new quadratic equation, then:

   - Sum of new roots: $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
   - Product of new roots: $\alpha^2\beta^2 = (\alpha\beta)^2$

   Substituting the values from the given equation:

   - Sum of new roots: $(5/2)^2 - 2*(-3/2) = 25/4 + 3 = 37/4$
   - Product of new roots: $(-3/2)^2 = 9/4$

   Therefore, the new quadratic equation is: $x^2 - (\text{sum of roots})x + \text{product of roots} = 0$ $x^2 - (37/4)x + 9/4 = 0$ Or, multiplying through by 4: $\mathbf{4x^2 - 37x + 9 = 0}$

Q9 - Solving the Inequality and Graphing the Solution Set

Given: $(3/5)x^2 - 2 \leq (3/5)x + 1$

Solution:

1. **Rearrange the Inequality:** $(3/5)x^2 - (3/5)x - 3 \leq 0$
2. **Multiply by 5/3 to simplify:** $x^2 - x - 5 \leq 0$
3. **Factor the Quadratic:** $(x - 5)(x + 1) \leq 0$
4. **Find the Critical Points:** The critical points are $x = 5$ and $x = -1$.
5. **Test Intervals:** We can test the intervals $(-\infty, -1)$, $(-1, 5)$, and $(5, \infty)$ to determine where the inequality holds.
6. **Solution Set:** The inequality holds for $x \in [-1, 5]$.

Graphing the Solution Set: To graph the solution set, draw a number line and mark the points -1 and 5. Shade the region between these two points to represent the solution set.

[Image of a number line with the interval [-1, 5] shaded]

Q10 - If a positive number exceeds its positive square root by 12, then find the number

Let the positive number be x.

According to the problem, we have:

$$x = \sqrt{x} + 12$$

To solve this equation, we can square both sides:

$x^2 = (\sqrt{x} + 12)^2$

Expanding the right side:

$x^2 = x + 24\sqrt{x} + 144$

Rearranging the equation:

$x^2 - x - 24\sqrt{x} - 144 = 0$

This equation is not easily solvable directly. However, we can make a substitution to simplify it.

Let $y = \sqrt{x}$. Then, $x = y^2$. Substituting this into the equation:

$y^4 - y^2 - 24y - 144 = 0$

This is a quadratic equation in terms of y^2. We can factor it:

$(y^2 - 16)(y^2 + 9) = 0$

This gives us two possible values for y^2:

1. $y^2 = 16 \Rightarrow y = \pm 4$
2. $y^2 = -9$ (no real solutions)

Since we're looking for a positive number, we take $y = 4$.

Therefore, $x = y^2 = 4^2$.

So, the number is 16.

Q11: Find the area bounded by the curves x2 = y and y=x.

To find the area bounded by the curves `x = y^2` and `y = x`, we first need to determine the points of intersection of these curves.

Finding the Points of Intersection:

Setting the two equations equal to each other, we get:

```
y^2 = y
```

This equation has two solutions: `y = 0` and `y = 1`.

Corresponding x-values:

- For $y = 0, x = 0$
- For $y = 1, x = 1$

So, the points of intersection are (0,0) and (1,1).

Setting up the Integral:

To find the area between the curves, we'll integrate the difference between the upper curve (y = x) and the lower curve (x = y^2) with respect to y, from y = 0 to y = 1:

```
Area = ∫[0, 1] (x - y²) dy
```

Substituting x = y:

```
Area = ∫[0, 1] (y - y²) dy
```

Integrating:

```
Area = [y²/2 - y³/3] from 0 to 1
```

Evaluating the Definite Integral:

```
Area = [(1/2 - 1/3) - (0 - 0)]
```

```
Area = 1/6
```

Therefore, the area bounded by the curves $x = y^2$ and $y = x$ is **1/6 square units**.

Q12: Finding the Inverse of Matrix A

Given:

```
A = | 1   6   4 |
    | 2  -1   5 |
    | 1   -1   2 |
```

To find the inverse of A, we'll use the adjoint method:

1. **Find the determinant of A:** |A| = 1(-12 - 5(-1)) - 6(22 - 51) + 4(2*(-1) - (-1)*1) = -1
2. **Find the adjoint of A:**

```
adj(A) = | -1  -2  11 |
         | -1   -2   7  |
         | -1   -1   -7 |
```

3. **Find the inverse of A:** A^{-1} = adj(A) / |A| = (-1/(-1)) * adj(A)

```
A^-1 = |  1   2 -11  |
       |  1   2  -7  |
       |  1   1   7  |
```

Q13: mth and nth terms of an AP

Given: m times the mth term of an AP is equal to n times its nth term.

Let the first term of the AP be 'a' and the common difference be 'd'.

So, m(a + (m-1)d) = n(a + (n-1)d)

Expanding and simplifying:

$m^2d - nd = n^2d - md$

$(m^2 - n^2)d = (n - m)d$

(m+n)(m-n)d = (n-m)d

Since m ≠ n, we can cancel out (m-n) from both sides:

(m+n)d = d

Therefore, (m+n)th term of the AP is:

a + (m+n-1)d = a + d = the first term of the AP.

Hence, the (m+n)th term of the AP is zero.

Q14: Limits and Continuity

i) $\lim_{x\to 0} |x|/x$:

As x approaches 0 from the positive side, $|x|/x = 1$. As x approaches 0 from the negative side, $|x|/x = -1$.

Since the left-hand limit and the right-hand limit are different, the limit does not exist.

ii) $f(x) = |x|$ is continuous at $x = 0$:

A function is continuous at a point 'a' if:

1. $f(a)$ exists.
2. $\lim_{x\to a} f(x)$ exists.
3. $\lim_{x\to a} f(x) = f(a)$

For $f(x) = |x|$, at $x = 0$:

1. $f(0) = 0$
2. $\lim_{x\to 0} |x| = 0$ (both left-hand and right-hand limits are 0)
3. $\lim_{x\to 0} |x| = f(0)$

Therefore, $f(x)$ is continuous at $x = 0$.

Q15: Investment Problem

This is a linear programming problem. We can set up the following inequalities:

- $x + y \leq 12000$ (total investment)
- $x \geq 2000$ (investment in saving certificates)
- $y \geq 4000$ (investment in national saving bonds)

The objective function to maximize is the total interest:

$$Z = 0.08x + 0.10y$$

To solve this linear programming problem, we can use graphical methods or simplex method. The optimal solution will occur at one of the corner points of the feasible region.

Q16: Related Rates Problem

We are given: $dV/dt = 900$ cm³/sec. We need to find dr/dt when $r = 15$ cm.

The volume of a sphere is given by $V = (4/3)\pi r^3$.

Differentiating both sides with respect to time t, we get:

$$dV/dt = 4\pi r^2 * dr/dt$$

Substituting the given values:

$$900 = 4\pi(15)^2 * dr/dt$$

Solving for dr/dt:

$$dr/dt = 900 / (4\pi * 225) = 1/(\pi) \text{ cm/sec.}$$

Therefore, the radius is increasing at a rate of $1/\pi$ cm/sec when the radius is 15 cm.