Introduction to Complex Networks

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Institute of Evolutionary Biology
UPF Barcelona

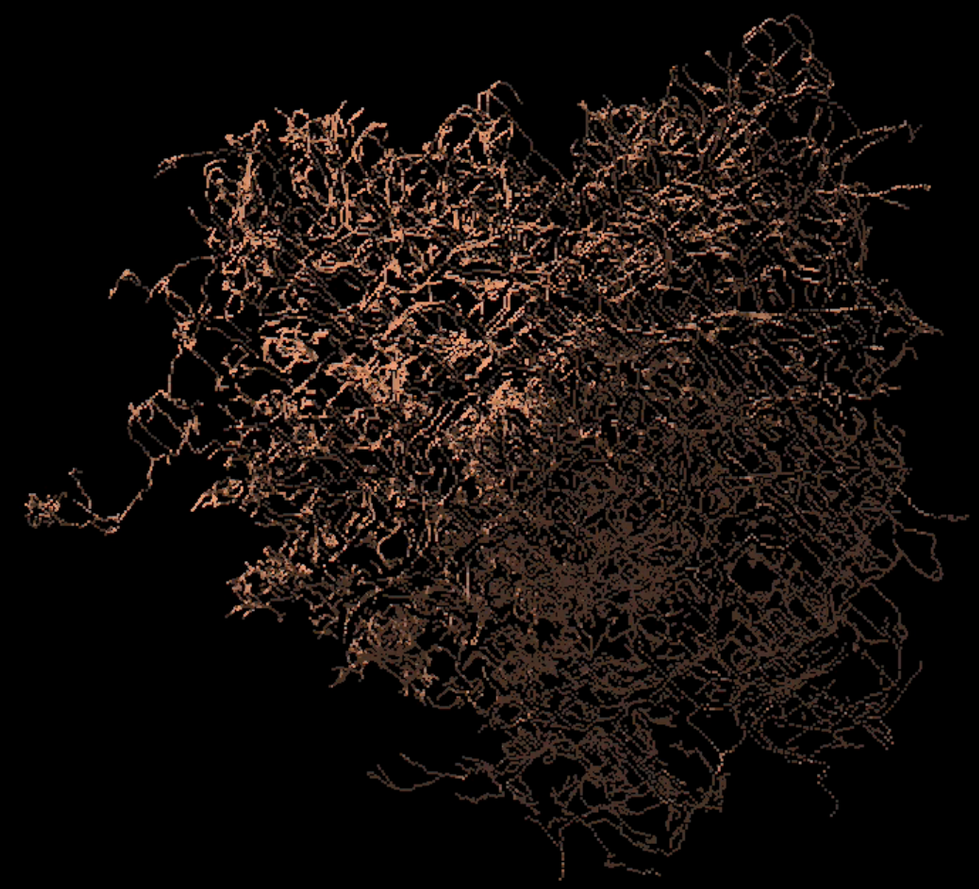
@svalver

Computational Systems Biology
Universitat de València

What is Complexity?

What is Complexity?

Are there Universal Mechanisms?



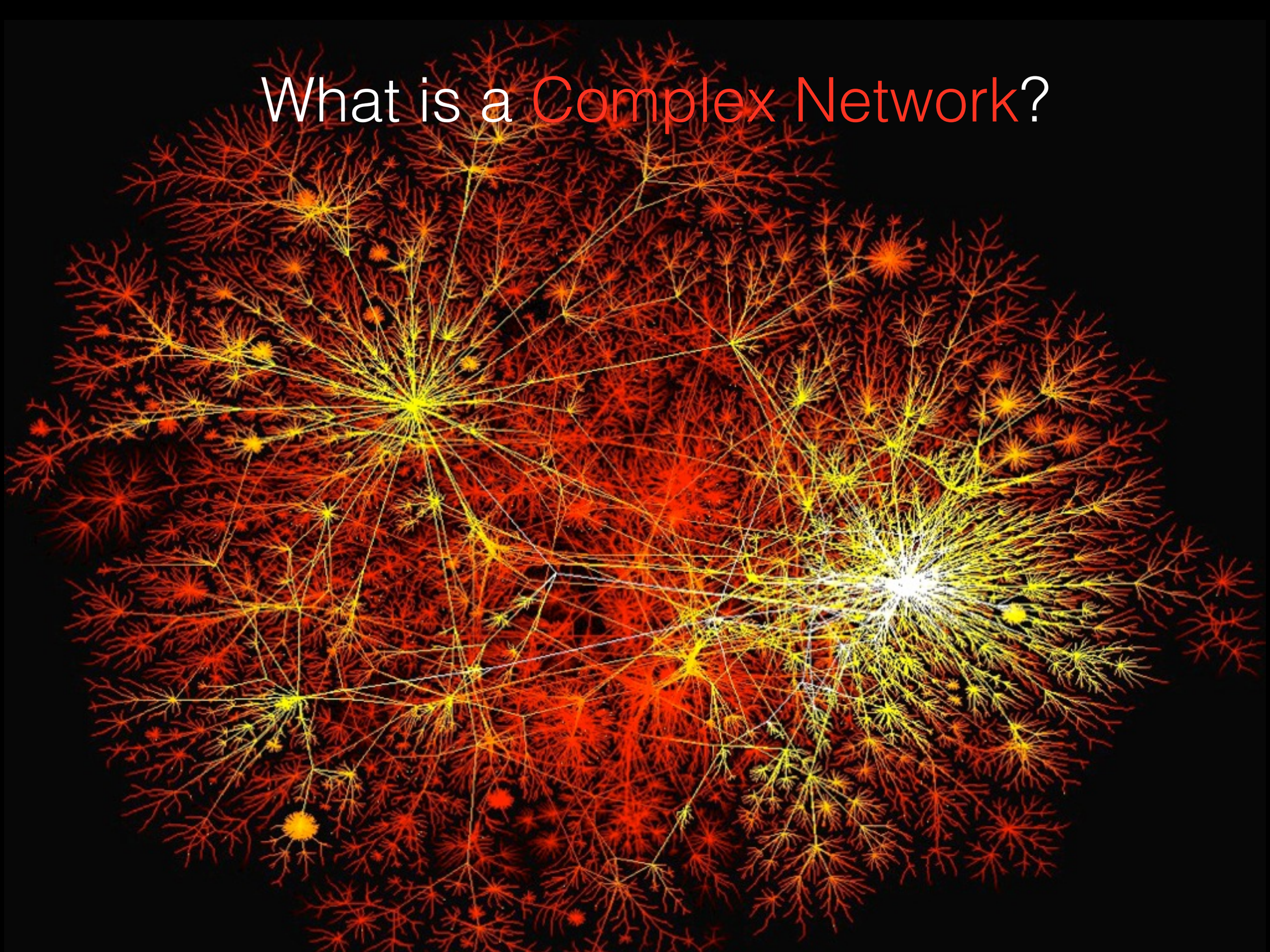
White Matter

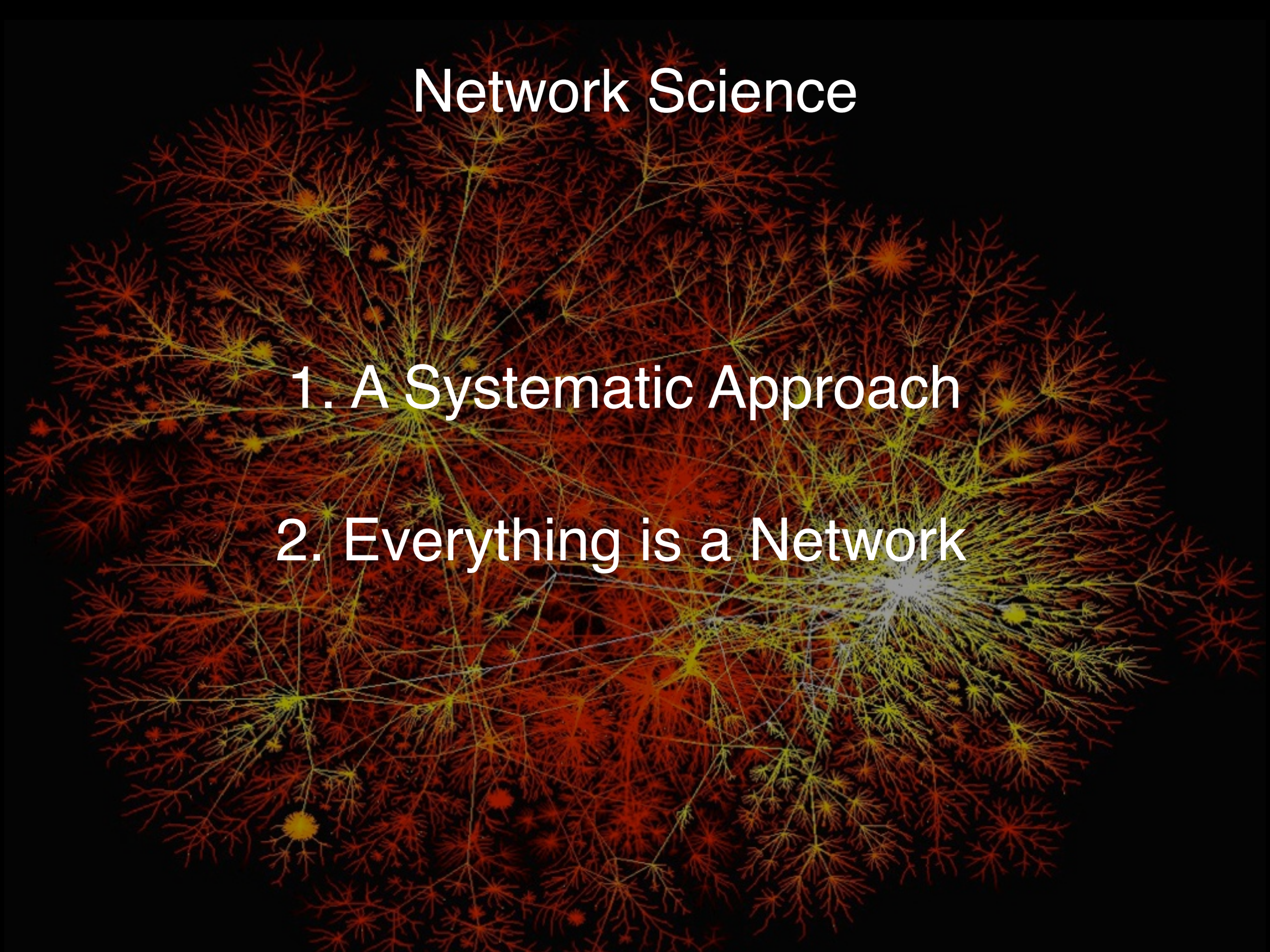
Galleries

Nasutitermes Nest: MeMo057

Hagmann, P. et al., (2007) *PLoS ONE*, e597. doi:10.1371/journal.pone.0000597

What is a Complex Network?





Network Science

1. A Systematic Approach

2. Everything is a Network

Index

Properties of Networks

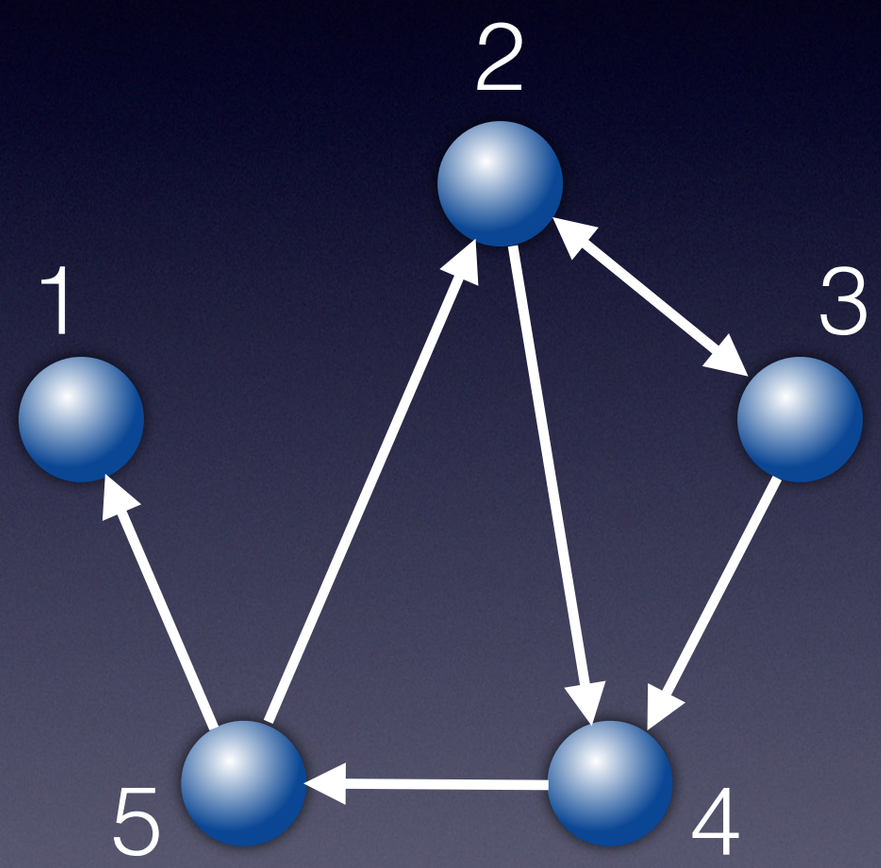
Robustness and Fragility

Hubs, Connectors and Paths

Preferential Attachment

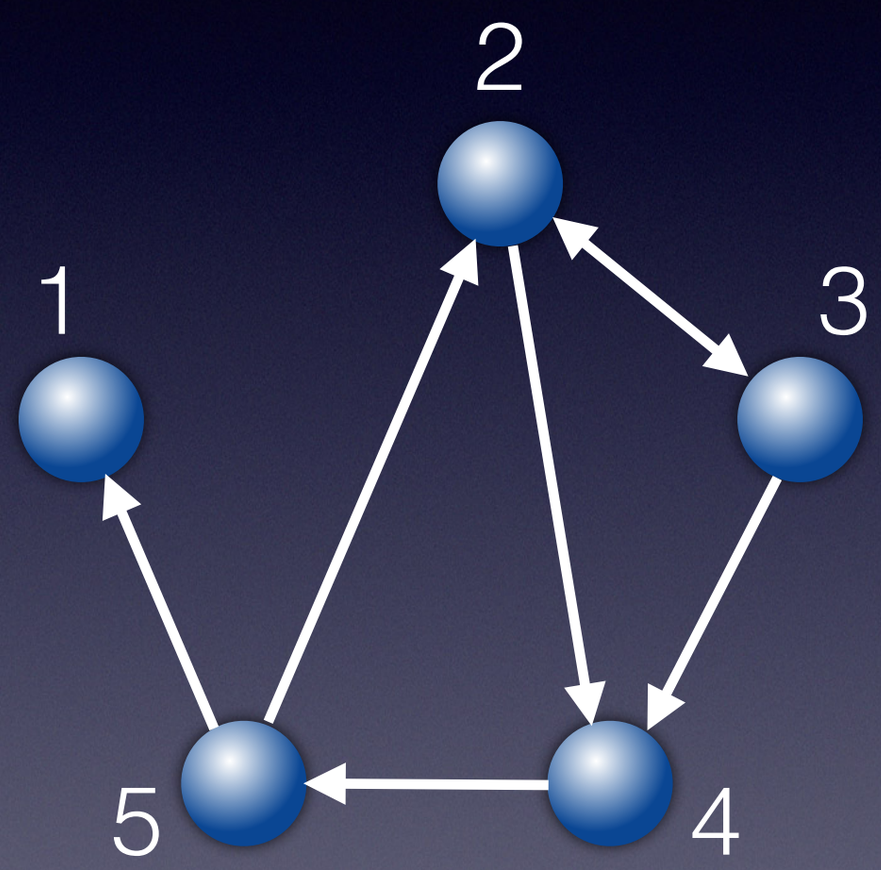
Community Structure

Adjacency Matrix



$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

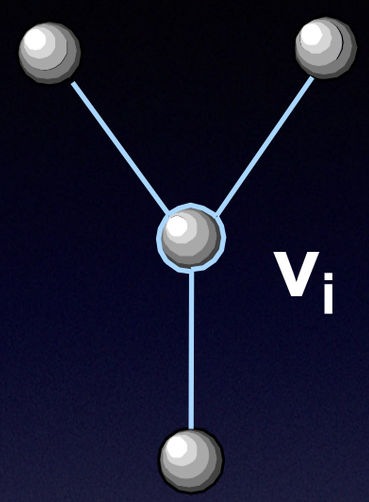
Edge Lists



2	3
3	2
2	4
3	4
4	5
5	2
5	1

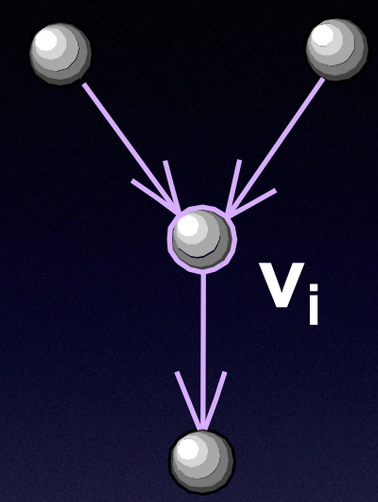
A Visual Language

Degree



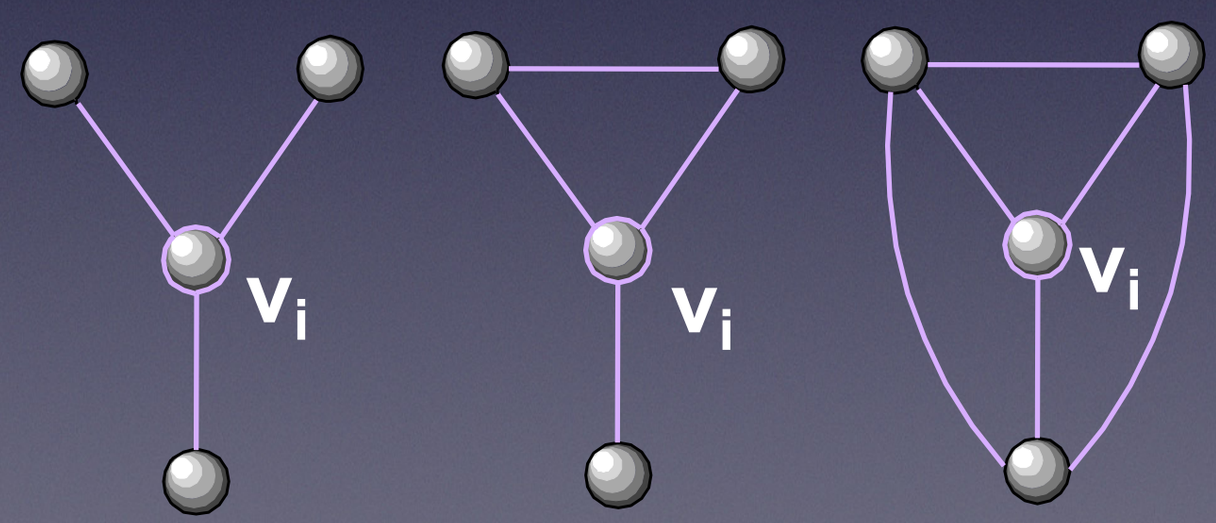
$k_i = 3$

In-degree & Out-degree



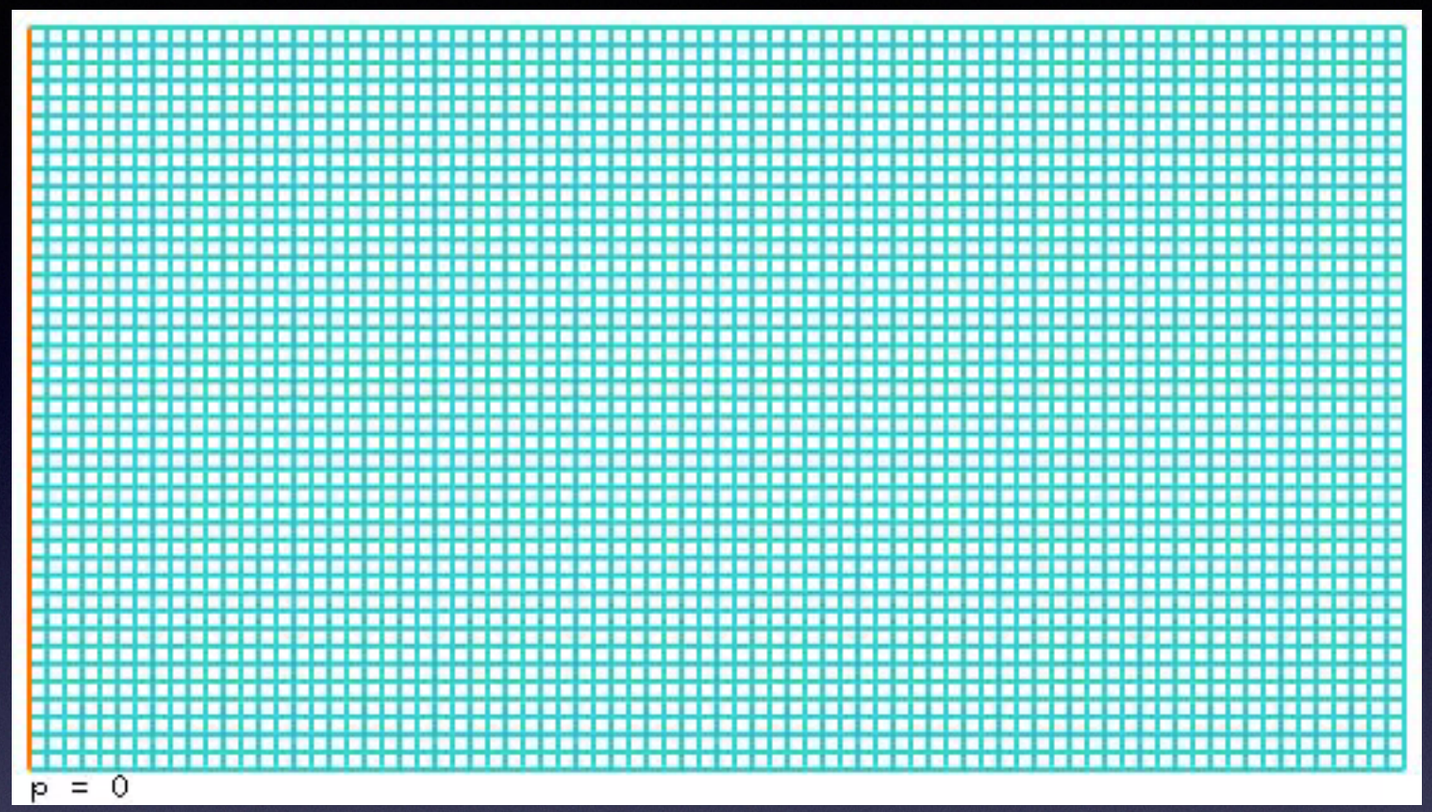
$k_i^{in} = 2$ $k_i^{out} = 1$

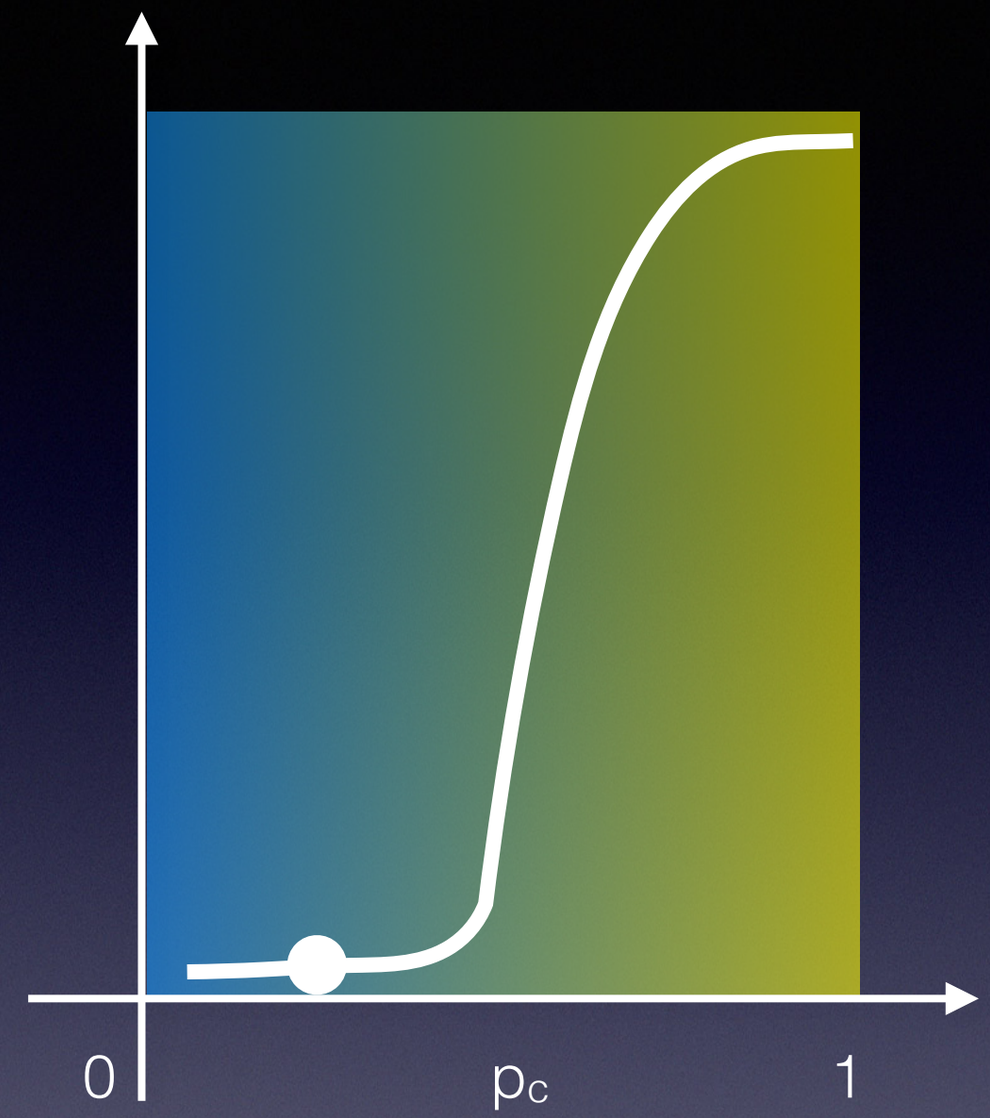
Local Clustering



$C_i = 0$ $C_i = 1/3$ $C_i = 1$

Percolation Transition

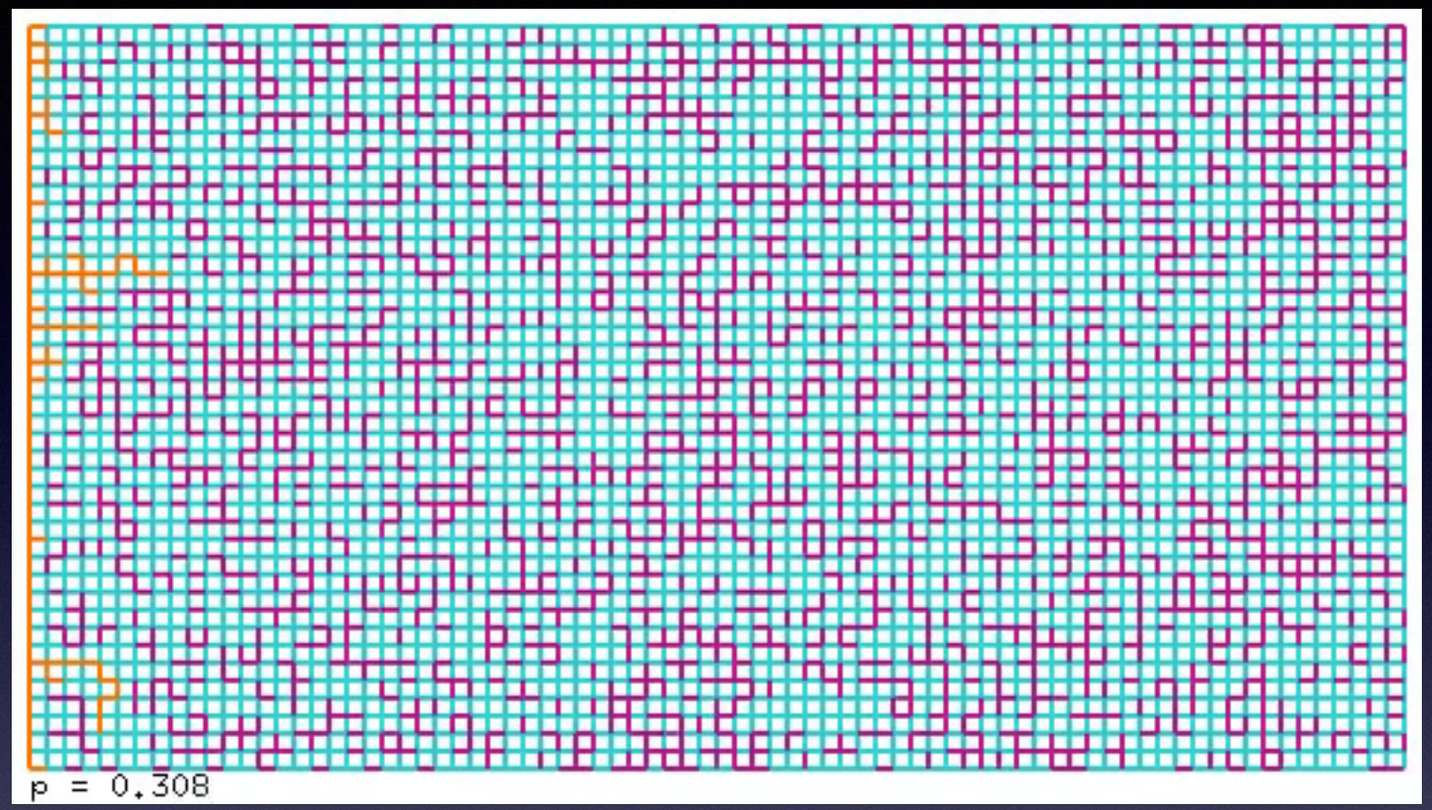


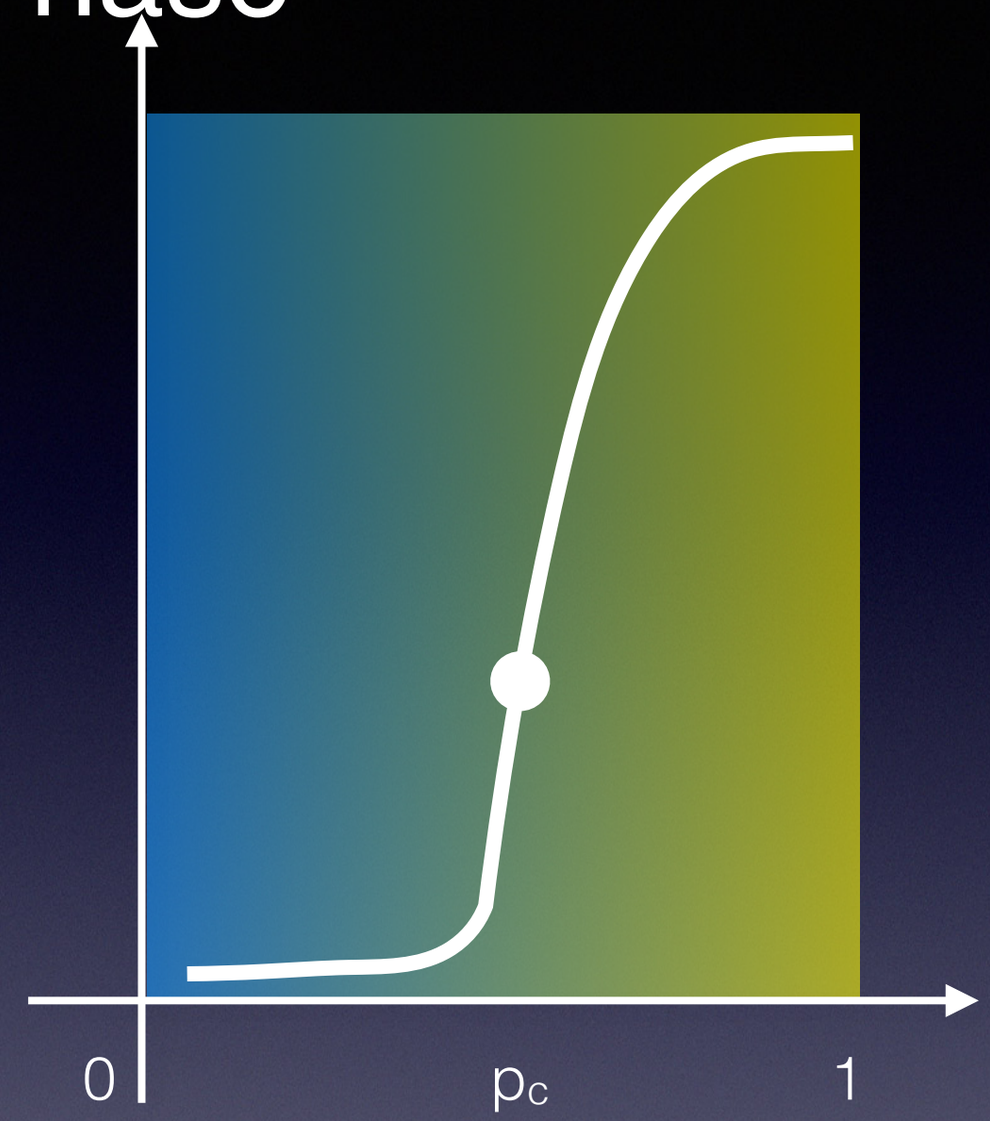


How does connectivity affects behaviour?

Kesten, Harry (1982), *Percolation theory for mathematicians*, Birkhauser

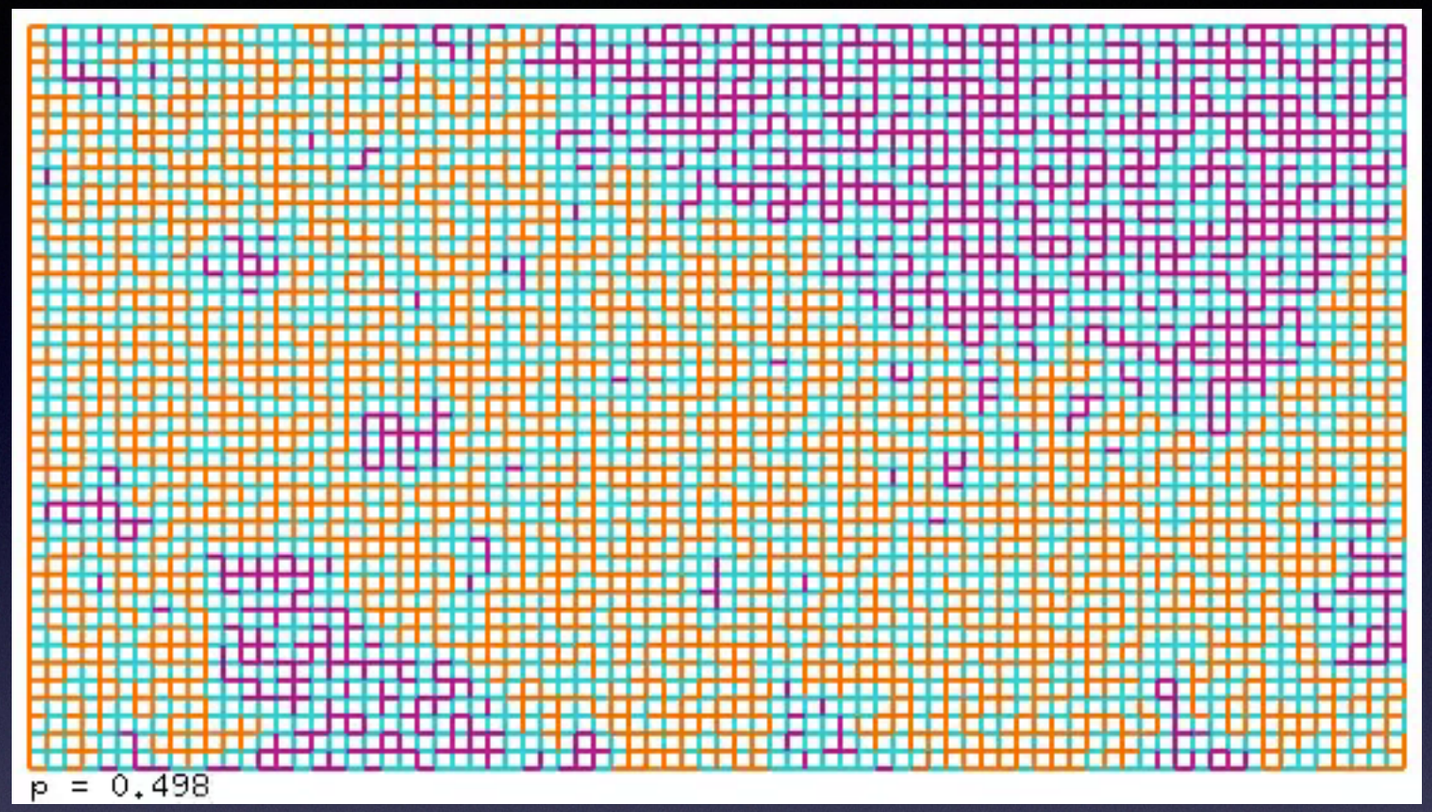
Disconnected Phase

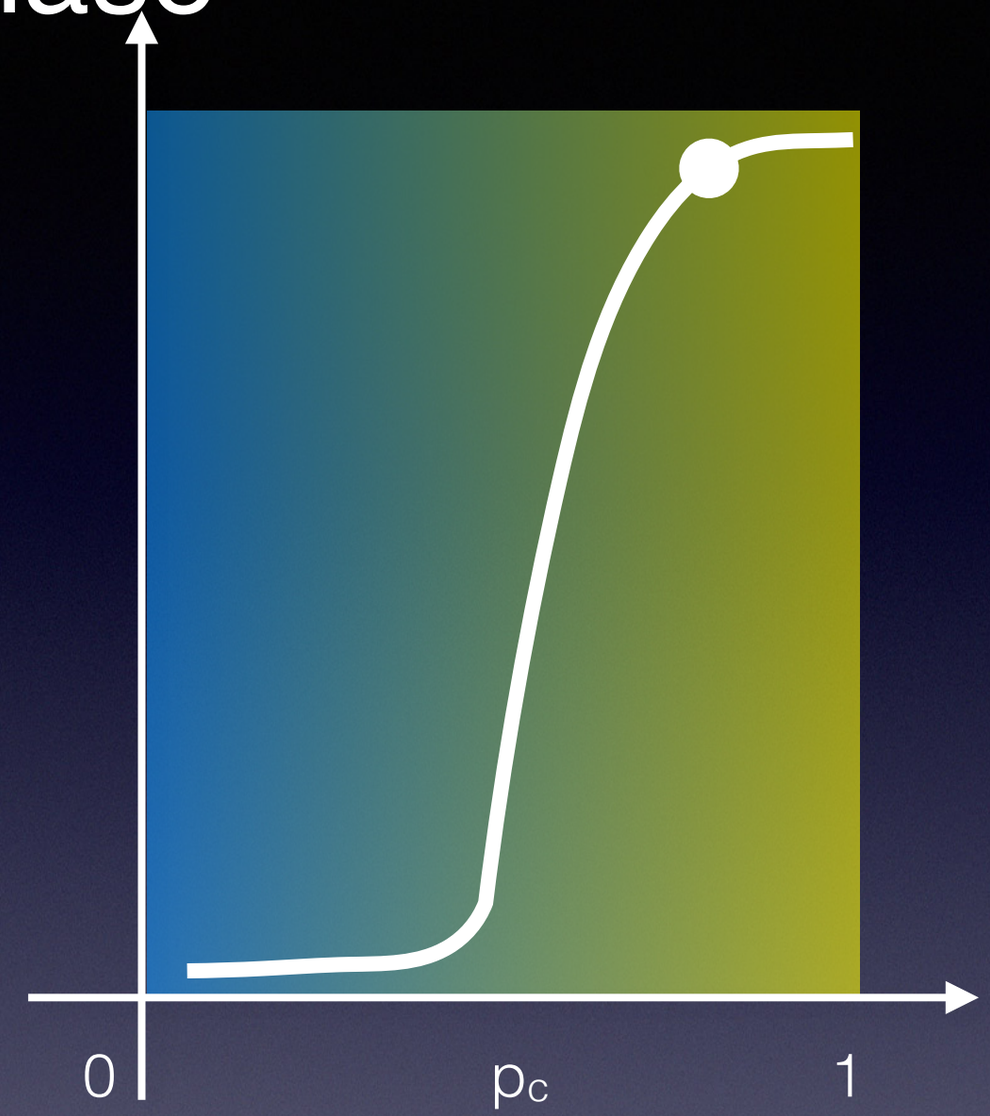


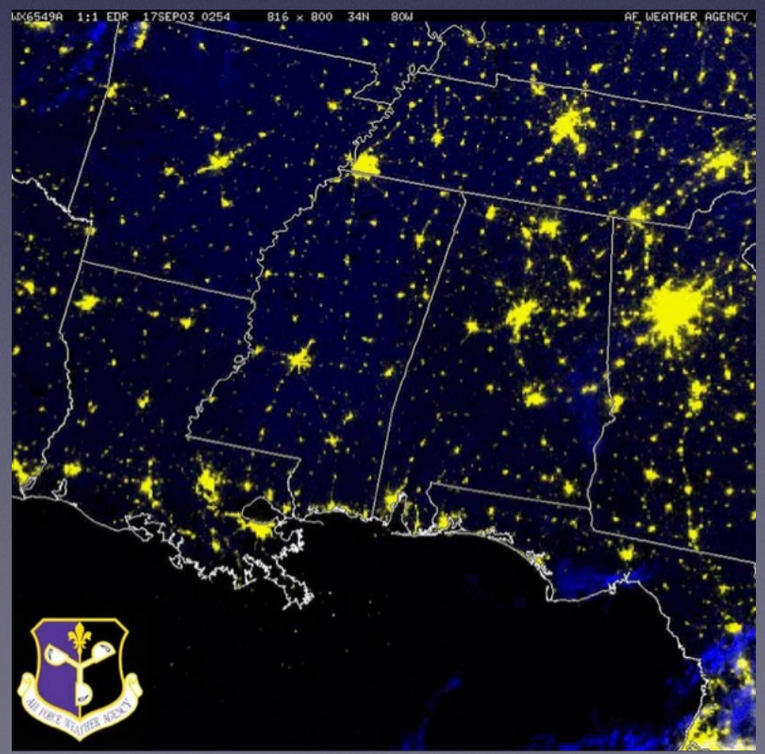


Power outage after Hurricane Katrina hit the Gulf Coast
This image was take Aug 30 and shows the widespread power outages across the Gulf Coast after Hurricane Katrina ravaged the area. U.S. Air Force Image.

Connected Phase







Power grid before the Hurricane Katrina hit the Gulf Coast
This image was taken Sept. 17,2003 and shows the city lights in the Gulf Coast clearly visible. U.S. Air Force Image.

sites.google.com/site/introcomplexnetworks

Introduction to Complex Networks

Search this site

by Sergi Valverde

NetLab

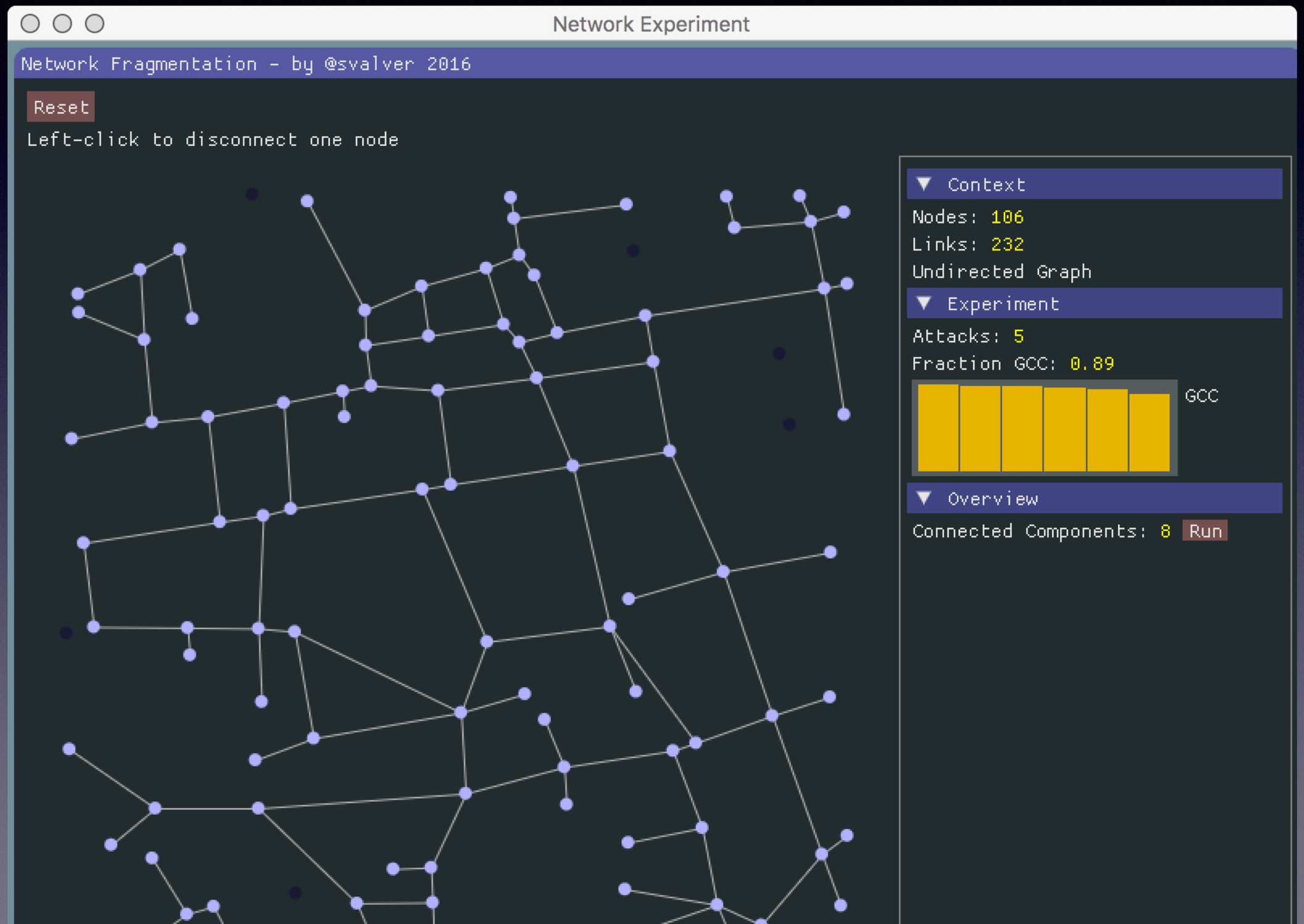
- **Network Attacks** (link): Learn about network fragmentation and the concept of percolation with this interactive website. How many nodes would you have to remove to break up an urban network?
- **Path Length** (link): Interactive visualization of shortest path lengths.
- **Network Layout** (link)
- **Small Worlds** (link): Add 10 shortcuts in order to minimise the average path length in the urban network.
- **Network Modularity** (link): Generates a random modular network and computes its modularity values.
- **Preferential Attachment** (link): Generates a scale-free graph by a process of "preferential attachment", in which new nodes prefer to make a connection to the more popular existing nodes.

The above models have been all developed by Sergi Valverde (Web) with the Netlab system. They require a WebGL-compliant web browser.

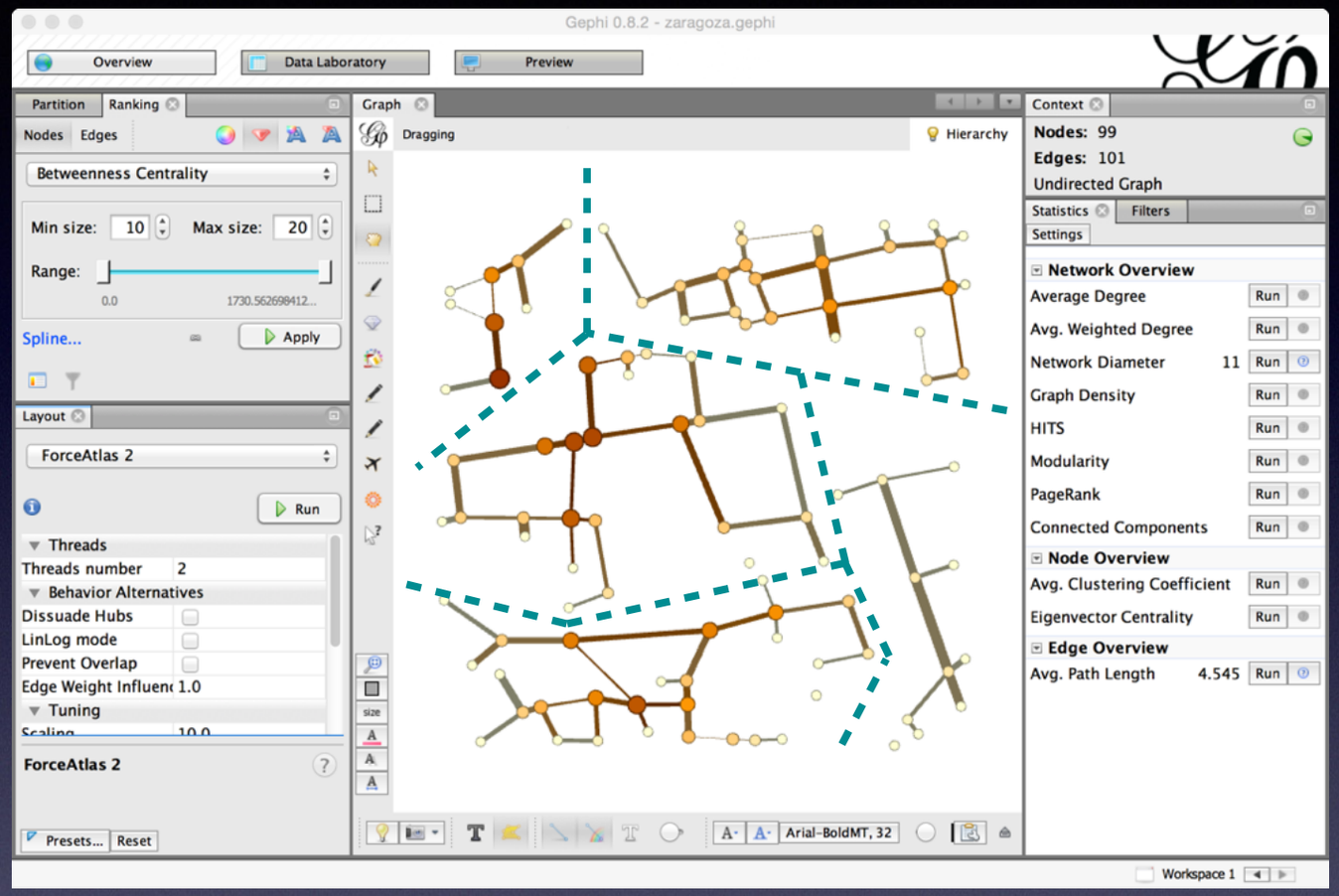
Spatial Networks

Network Attacks

How many nodes would you have to remove to break up the network ?



Fragmentation under Attacks



v93

v87

v34

v20

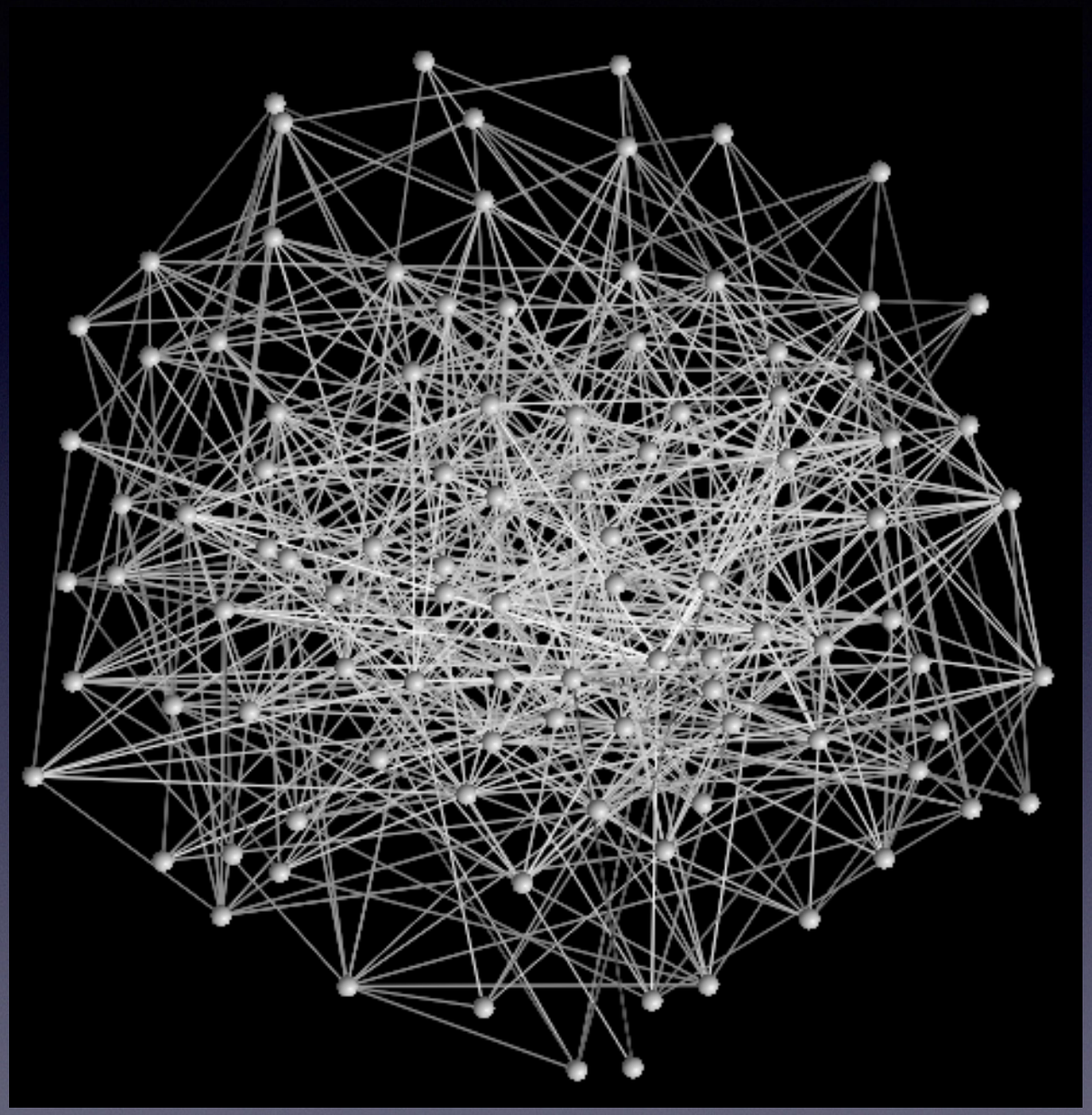
v32

v103

v11

This pattern of fragmentation suggests that the spatial network is composed of different parts, i.e., the neighbourhoods (so-called modules). Nodes within a module are more related among them than with the rest of the group.

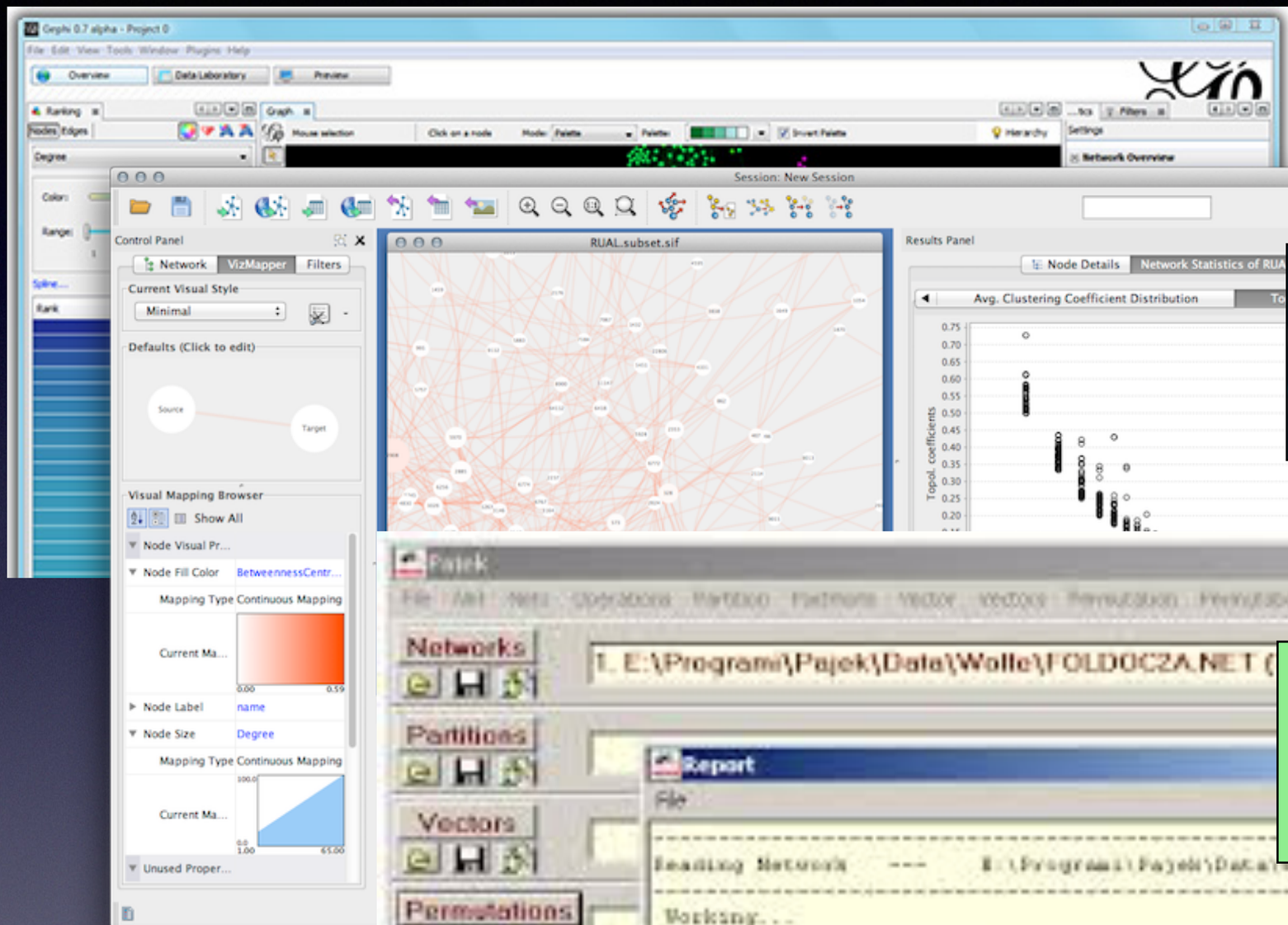
Random Graph: A World without Geography



Pál Erdös
(1913-1996)

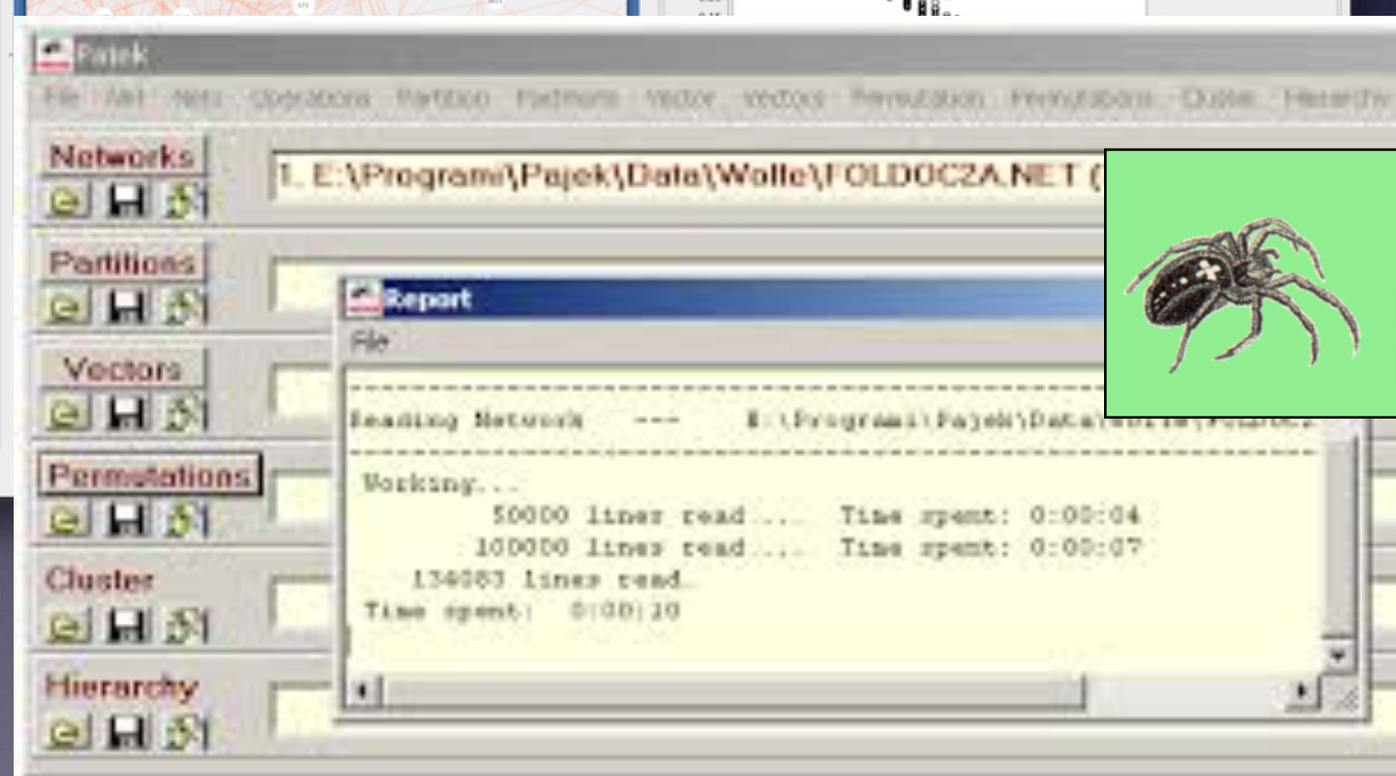
$$P_{>}(k) = N \exp(-k/k_c)$$

Network Tools



Gephi
makes graphs handy

Cytoscape



Networks / Pajek

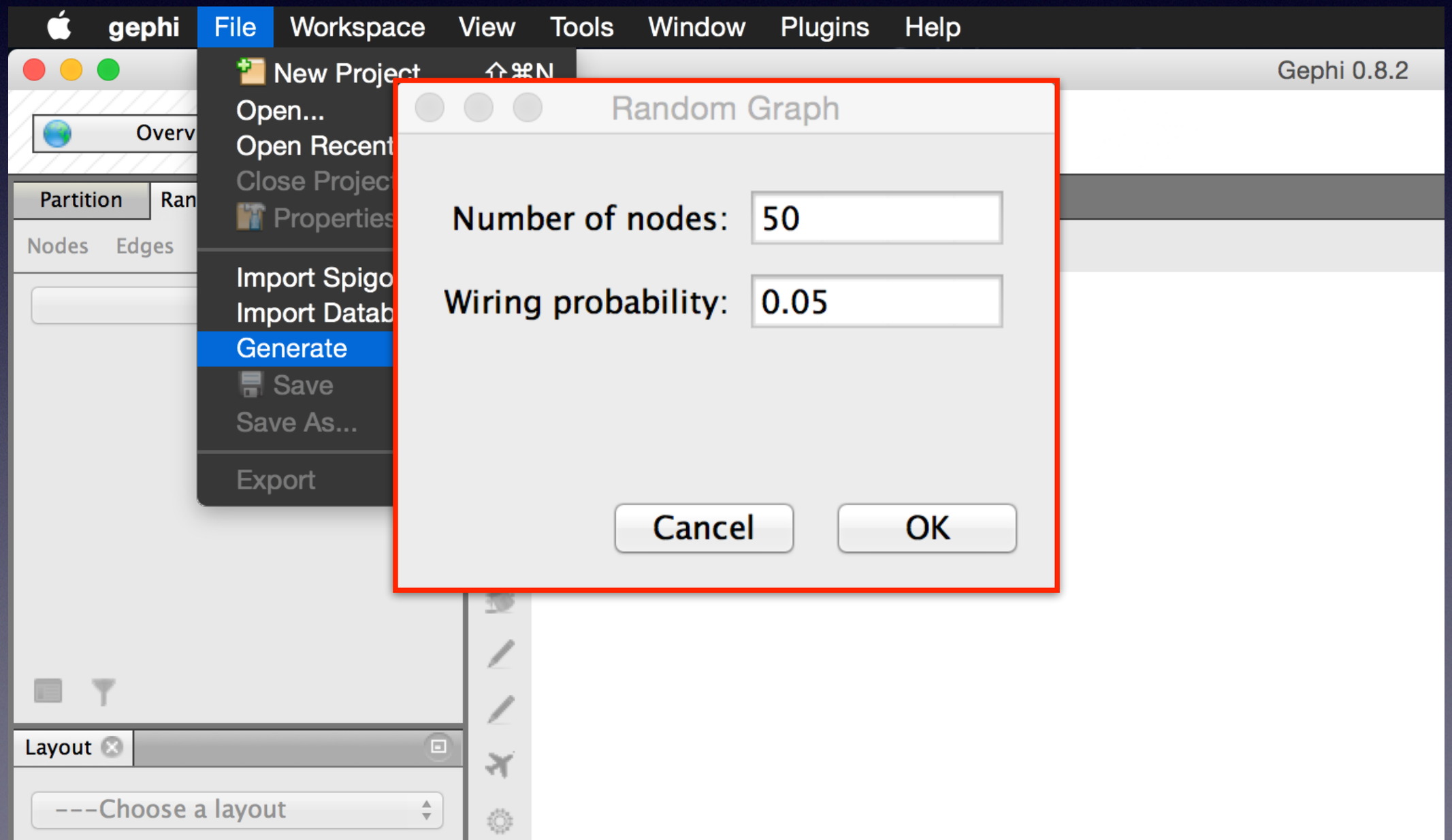
Program for Large Network Analysis

Gephi: Random Graphs

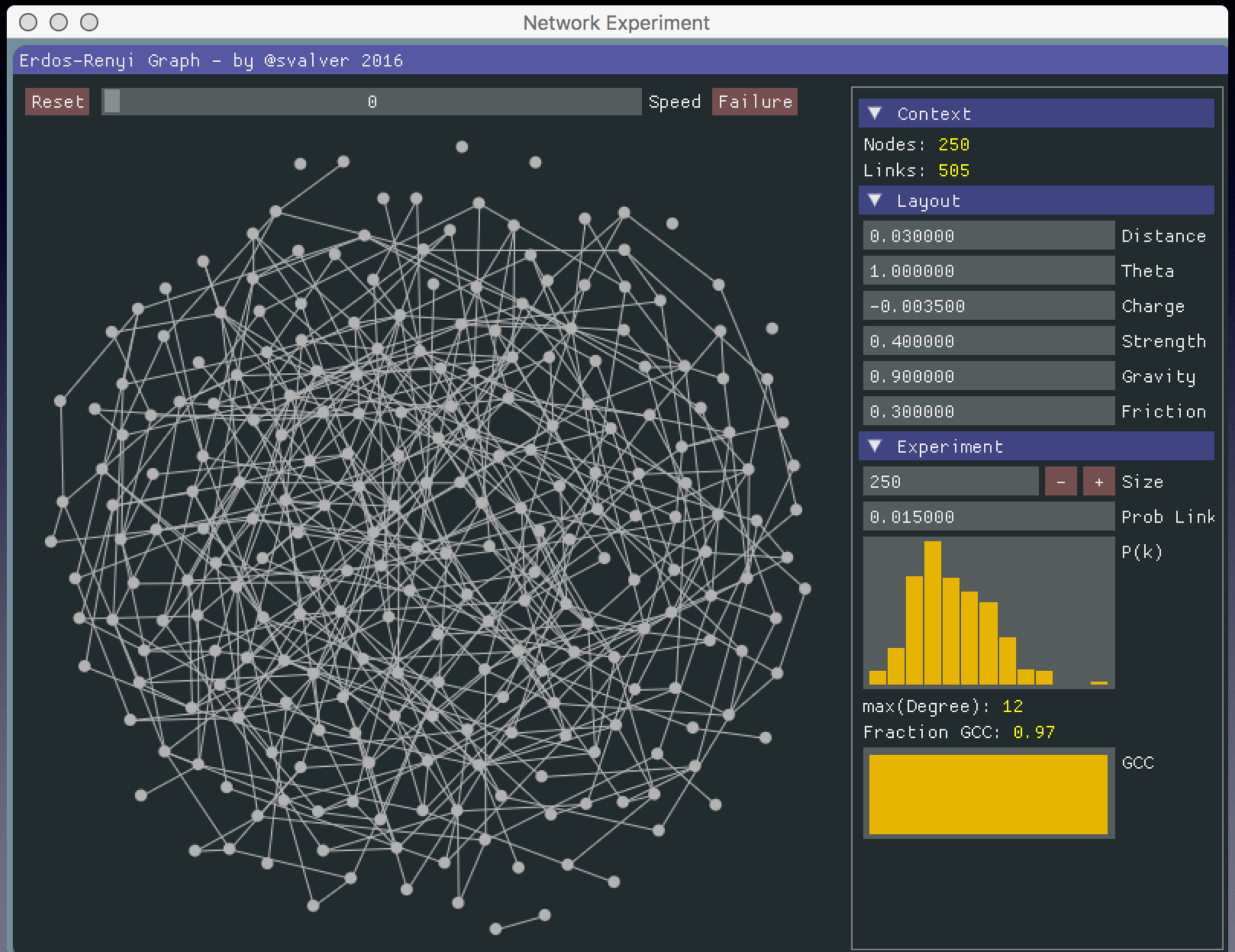
(N, p)-model For each pair of nodes:

with probability p we add the link

with probability (1-p) we don't

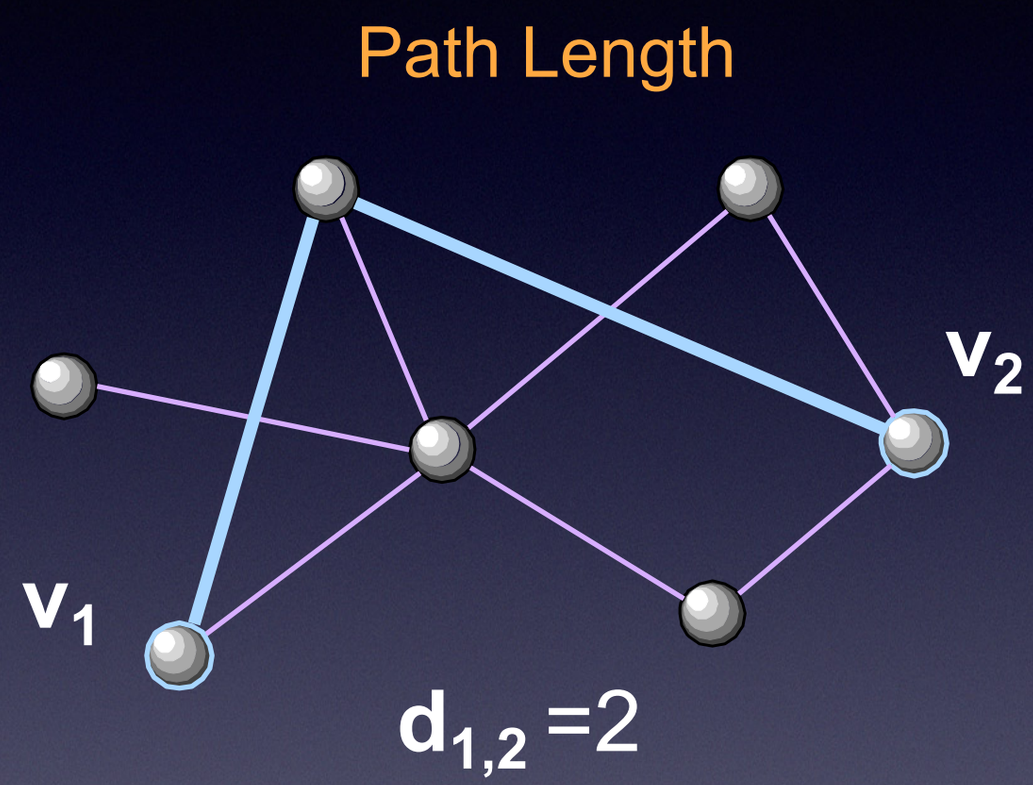


Random Graph



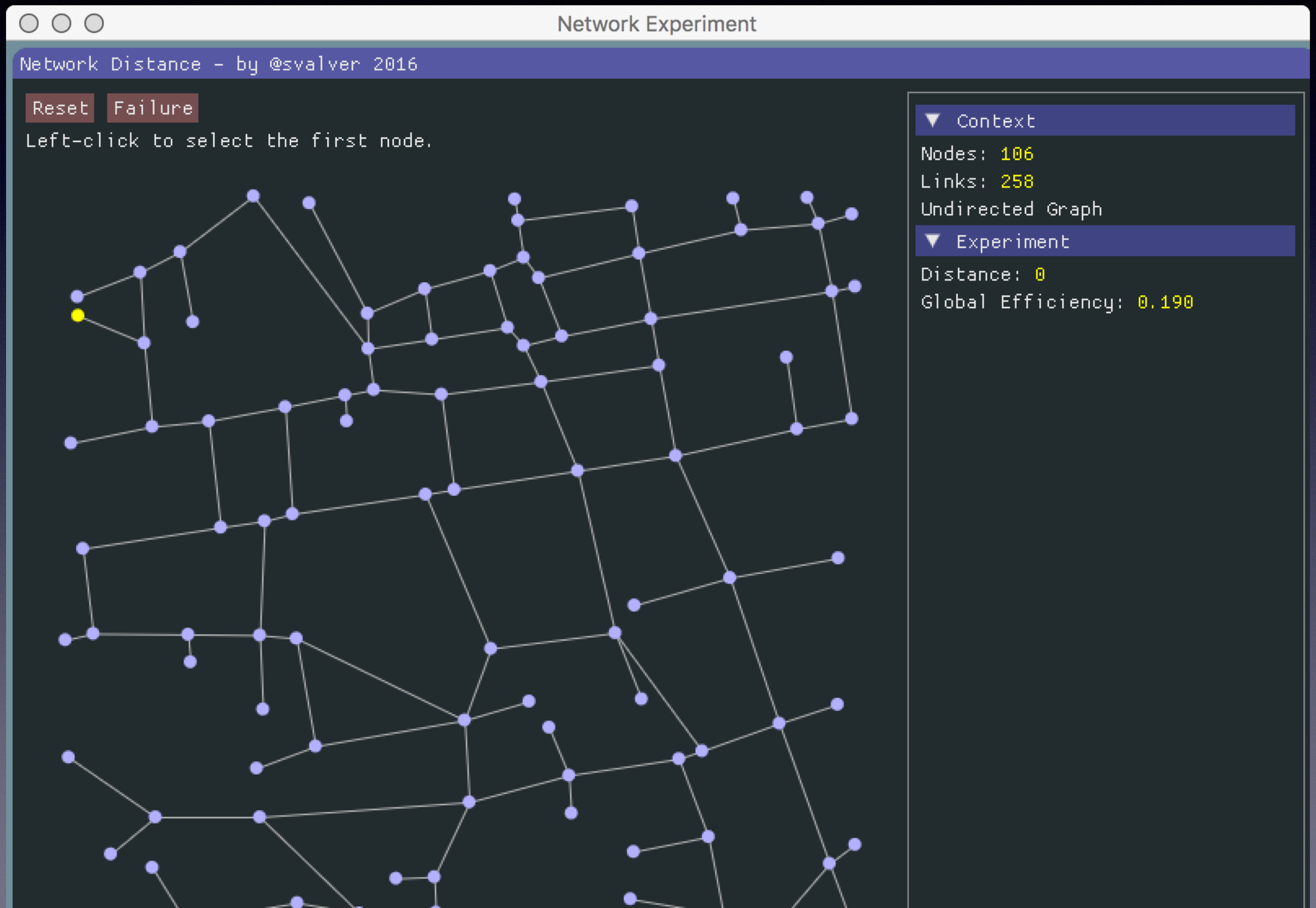
Hubs, Connectors and Paths

- Path Length
- Power of Matrices
- Geodesic Path
- Diameter
- Components
- Global Efficiency



Path Length

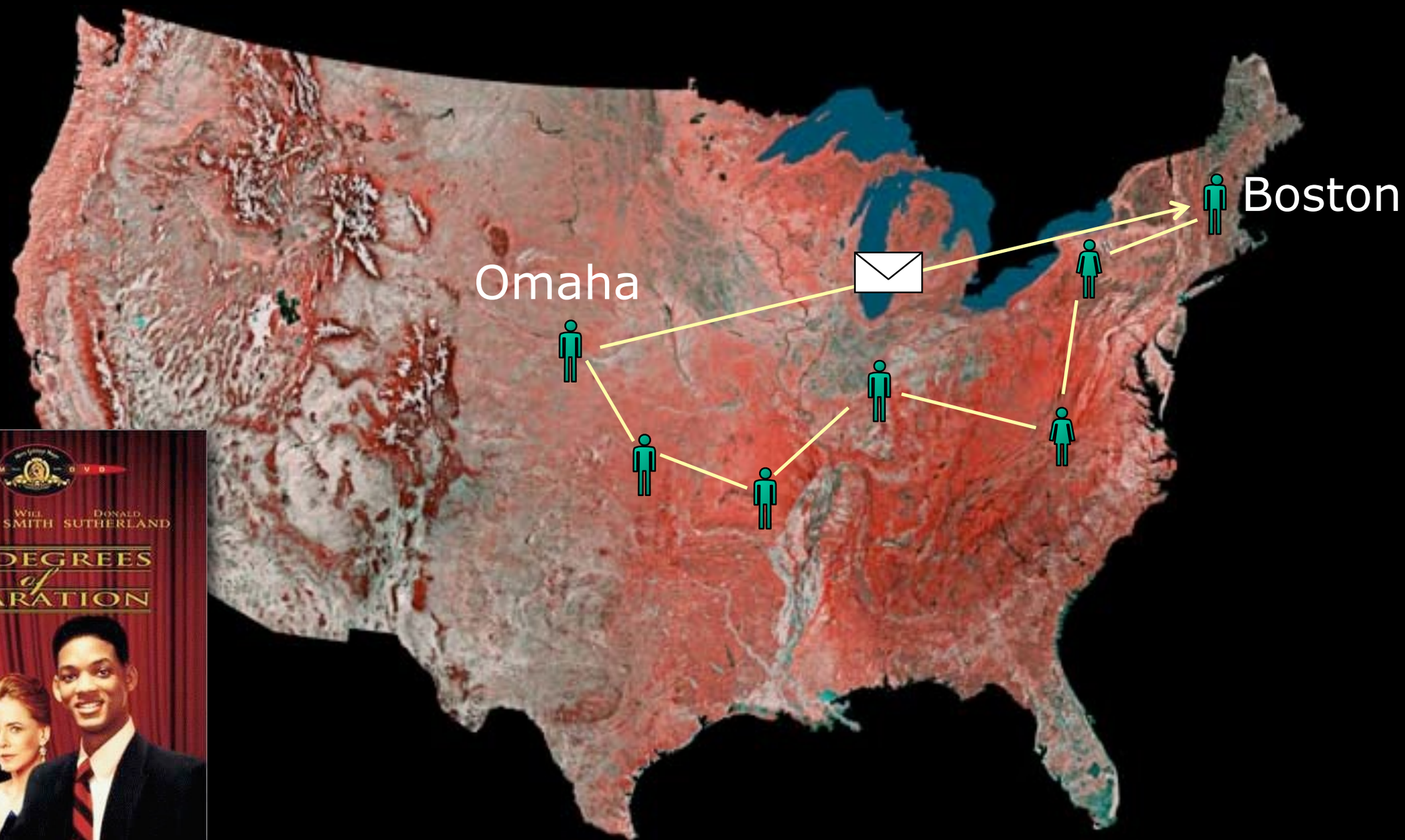
Visualisation of the shortest path between any pair of nodes

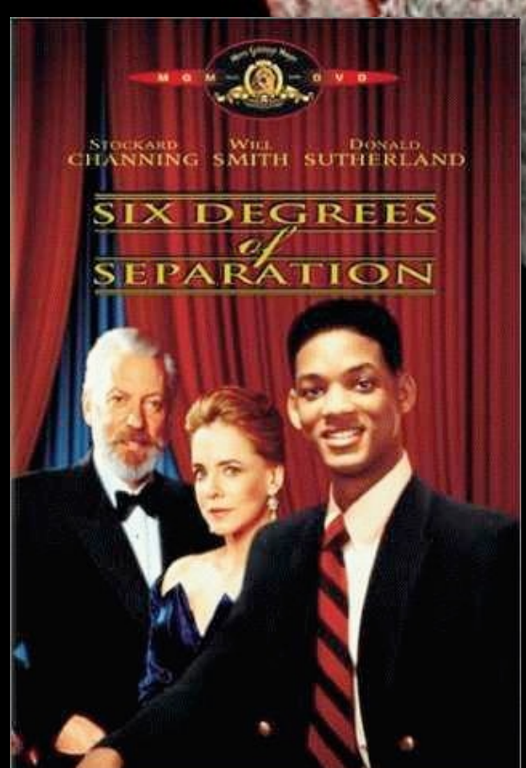


Small-Worlds

Social Distance : Six Degrees of Separation

- Stanley Milgram (1967)

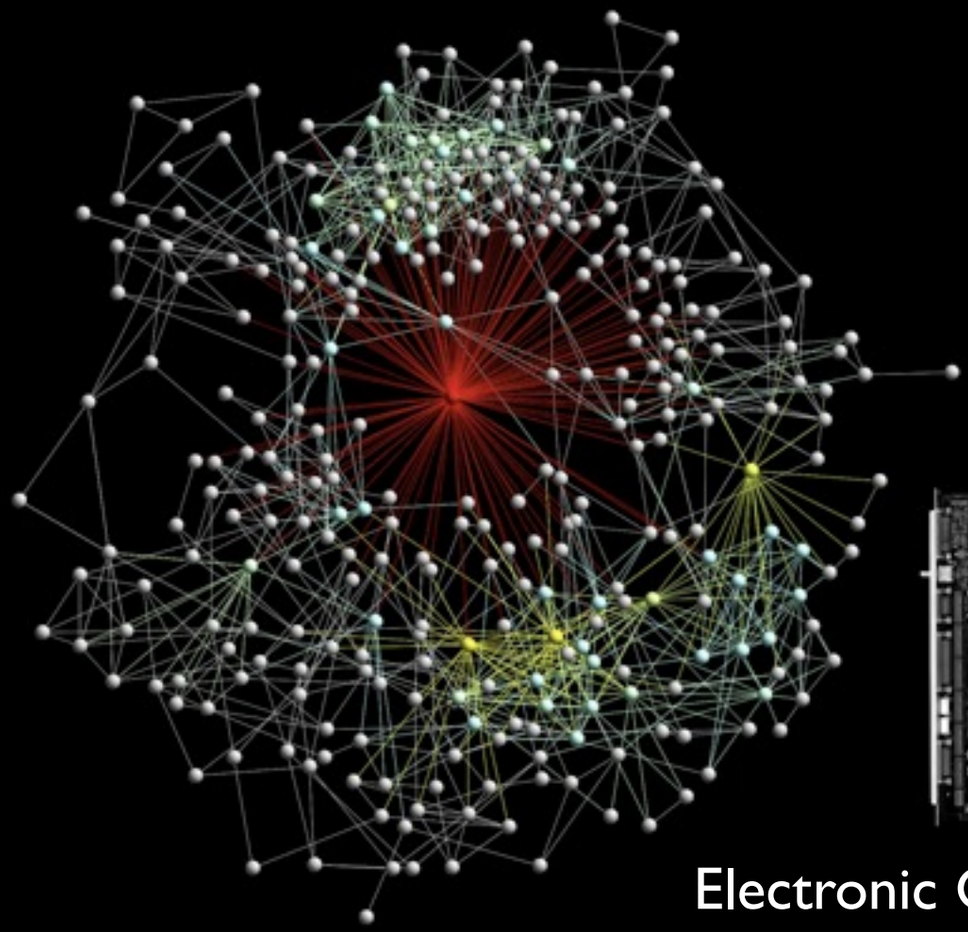


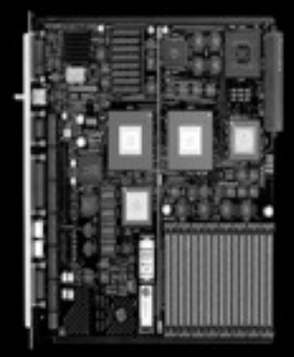


Why Many Networks are Small-Worlds?

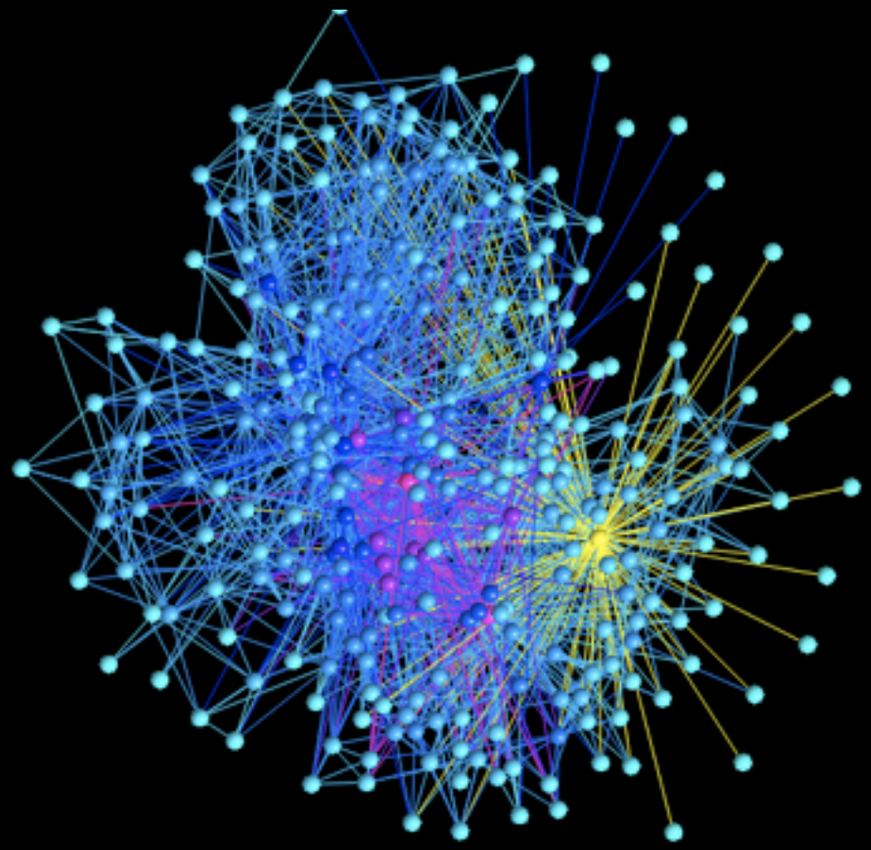
Small-Worlds Everywhere

Linguistic Networks

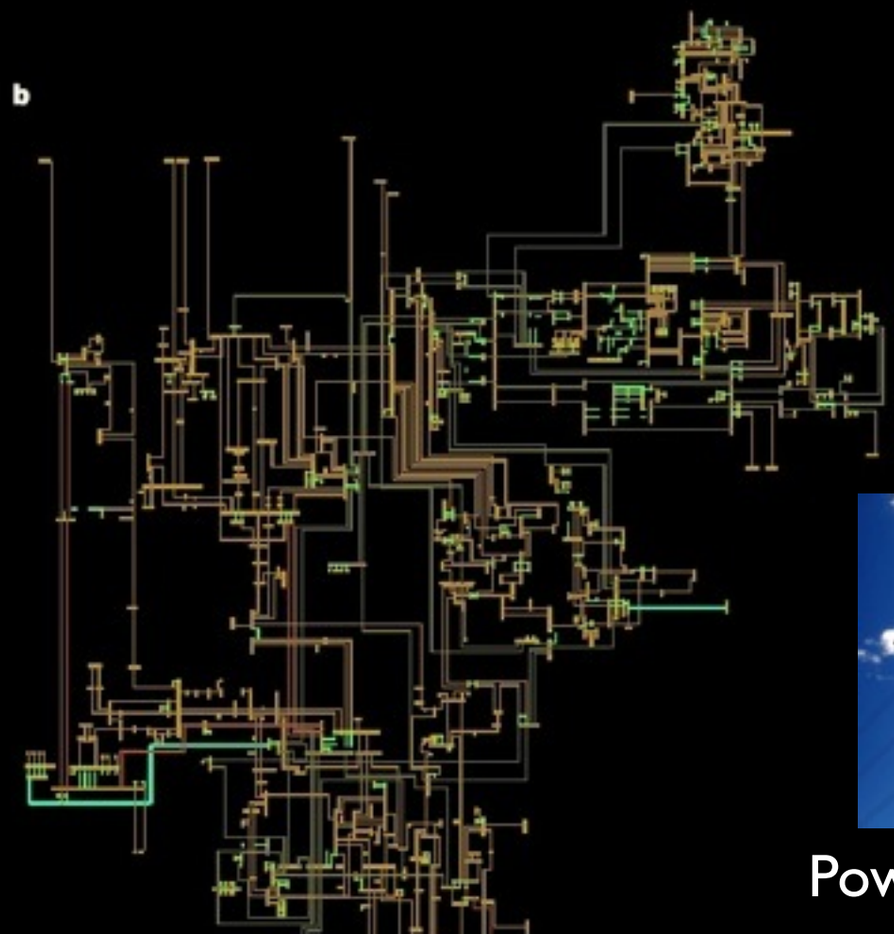




Electronic Circuits



Brain of a worm (*C. Elegans*)



Power grids

Average Path Length

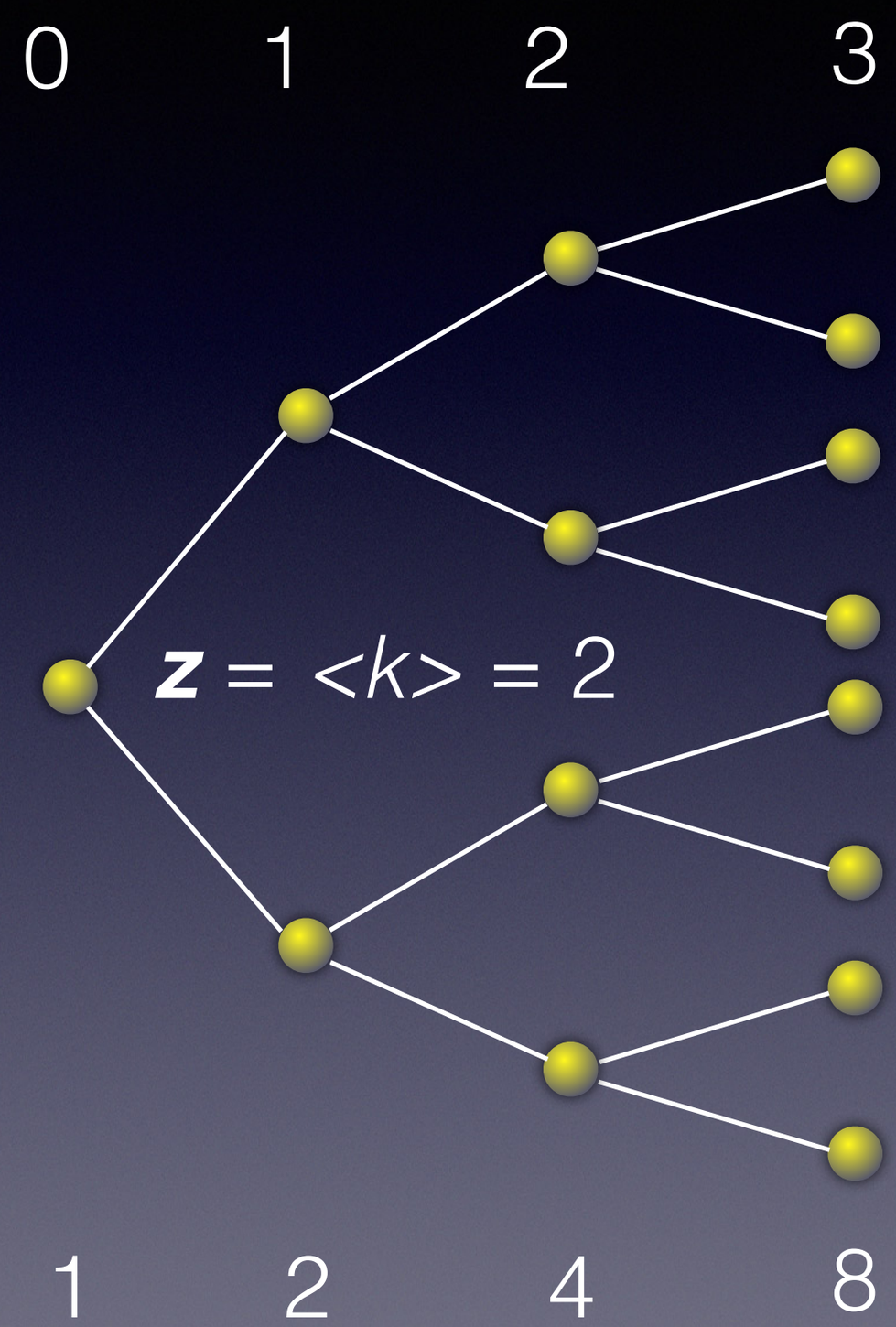
How many hops on average between each pair of nodes ?

Each of your friends has $\boldsymbol{z} = \langle k \rangle$ friends besides you

If there are no loops, the number of people you have at distance d is

$$\boldsymbol{z}^d$$

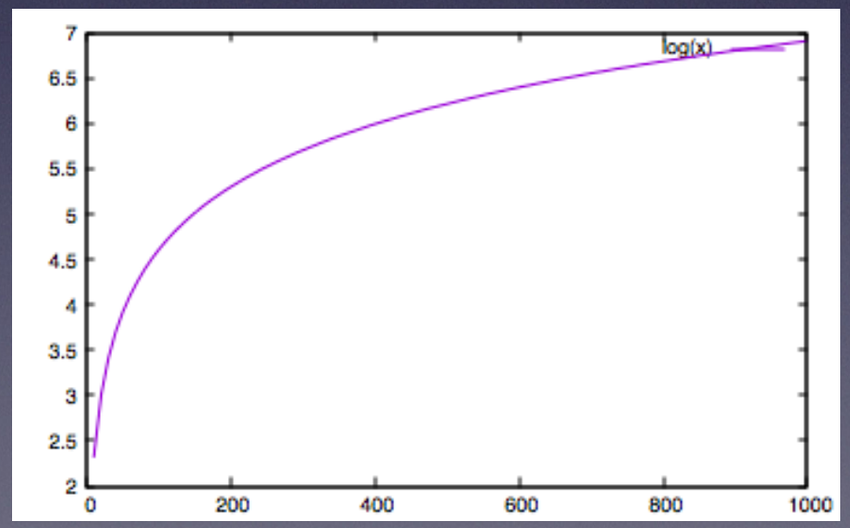
Average Path Length



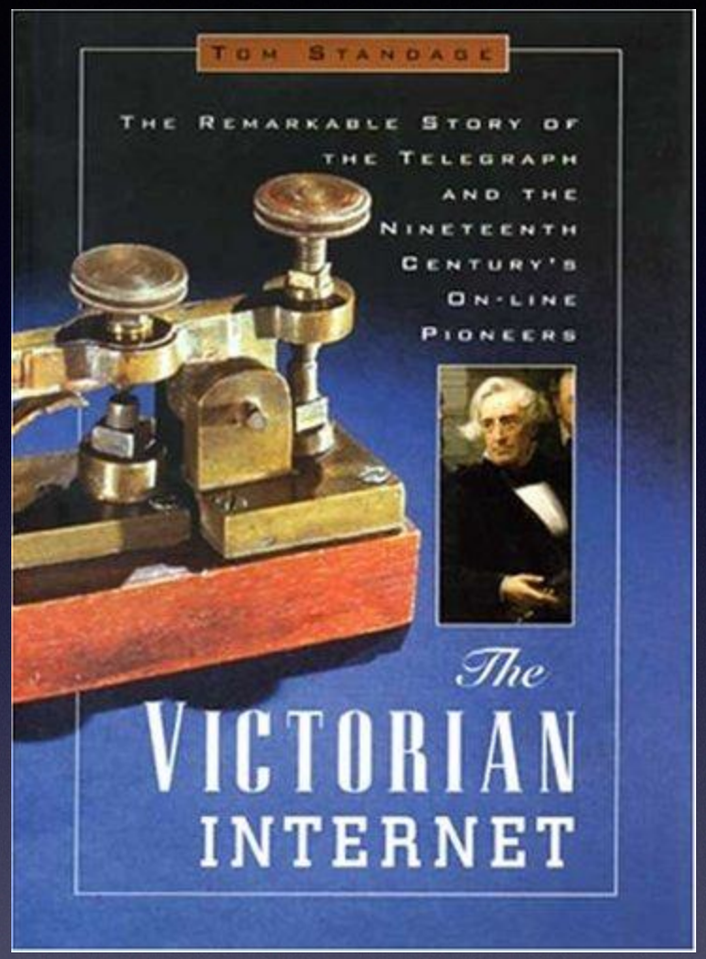
$$N_d = \boldsymbol{z}^d$$

$$\log N = d \log \boldsymbol{z}$$

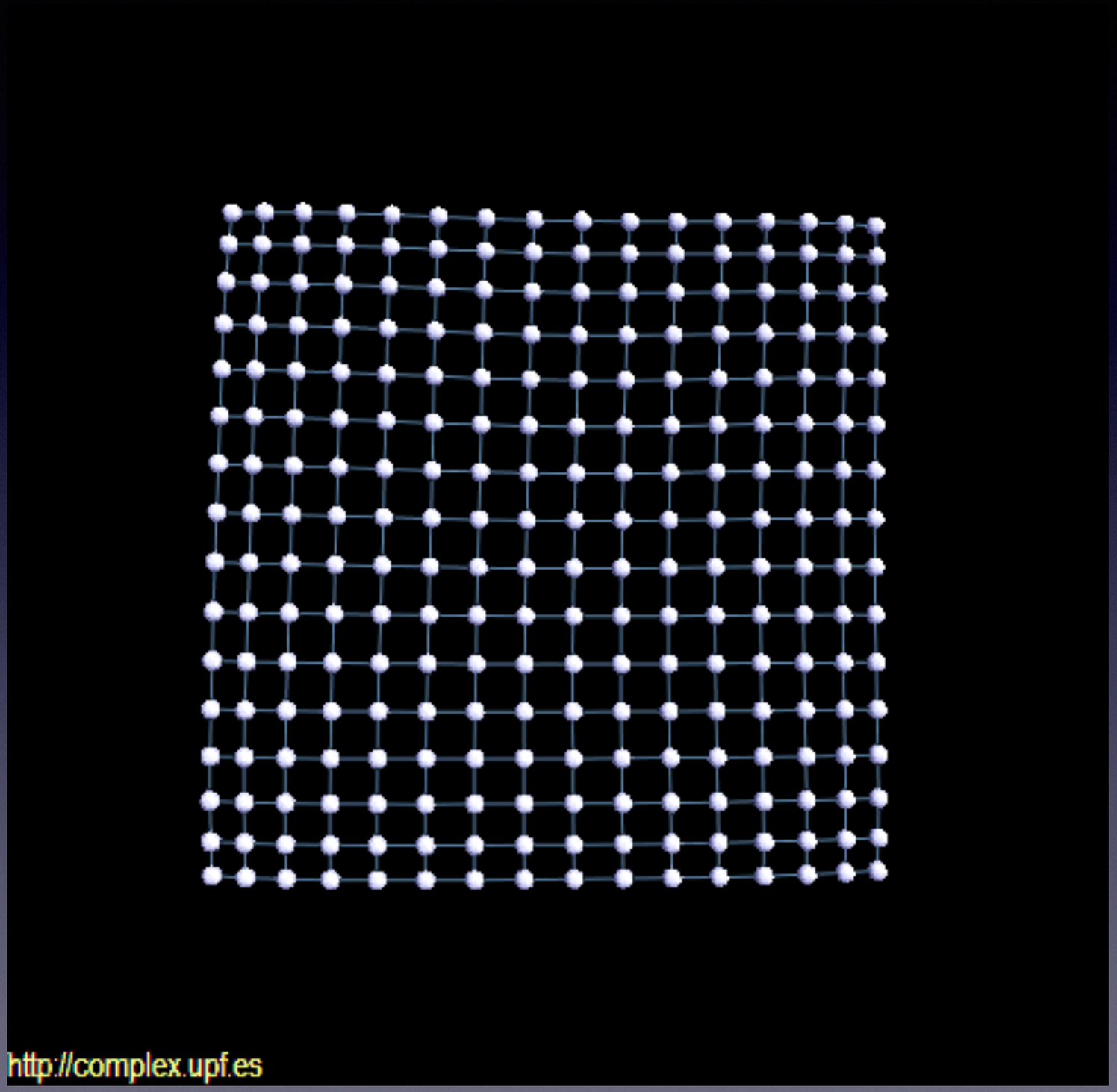
$$\langle d \rangle \simeq \frac{\log N}{\log \boldsymbol{z}}$$



The End of Geography

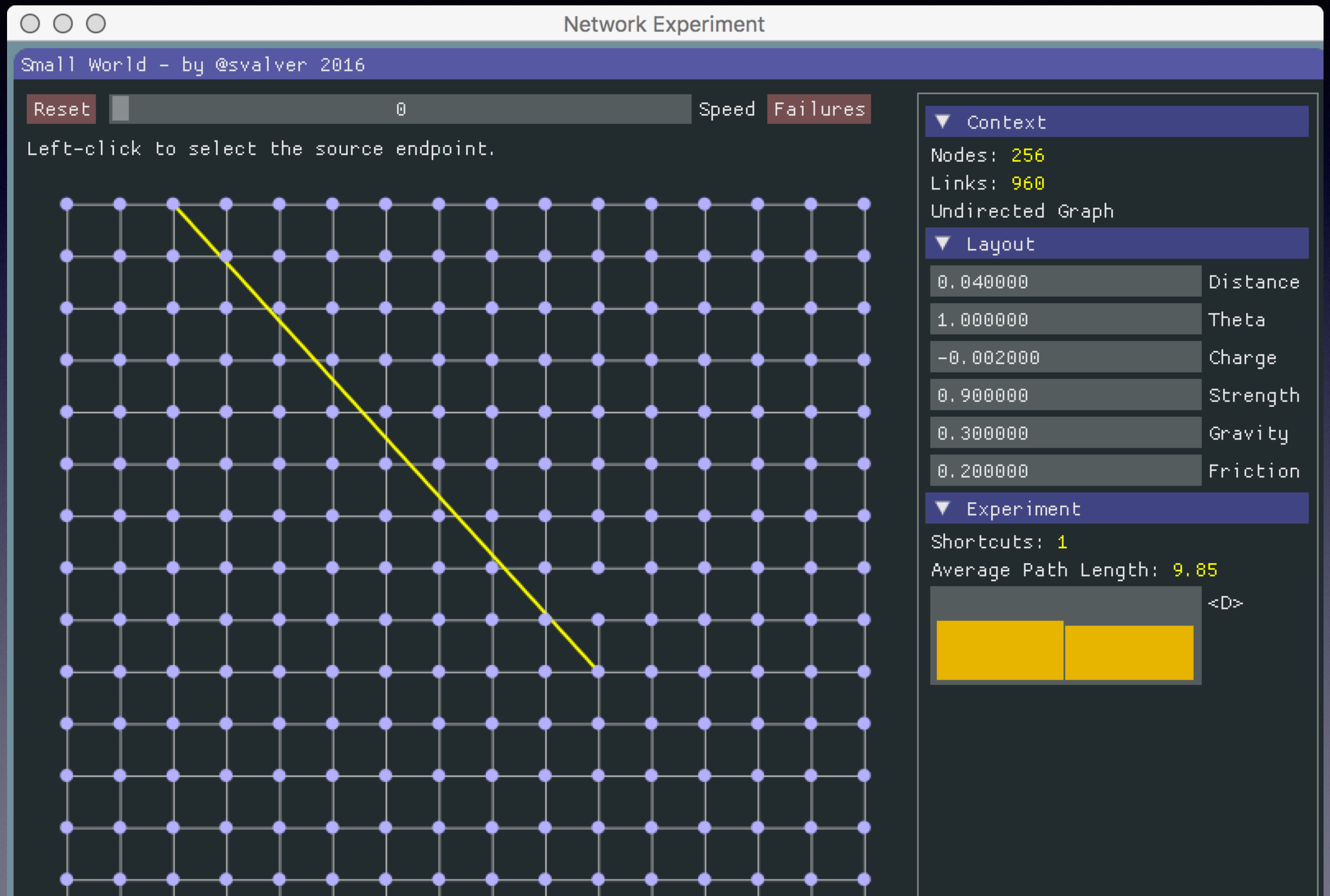


Steven Strogatz



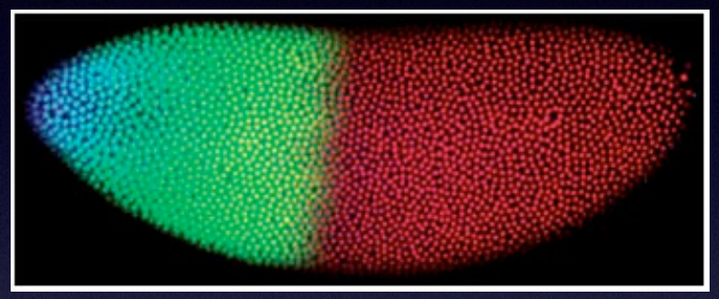
Small-Worlds

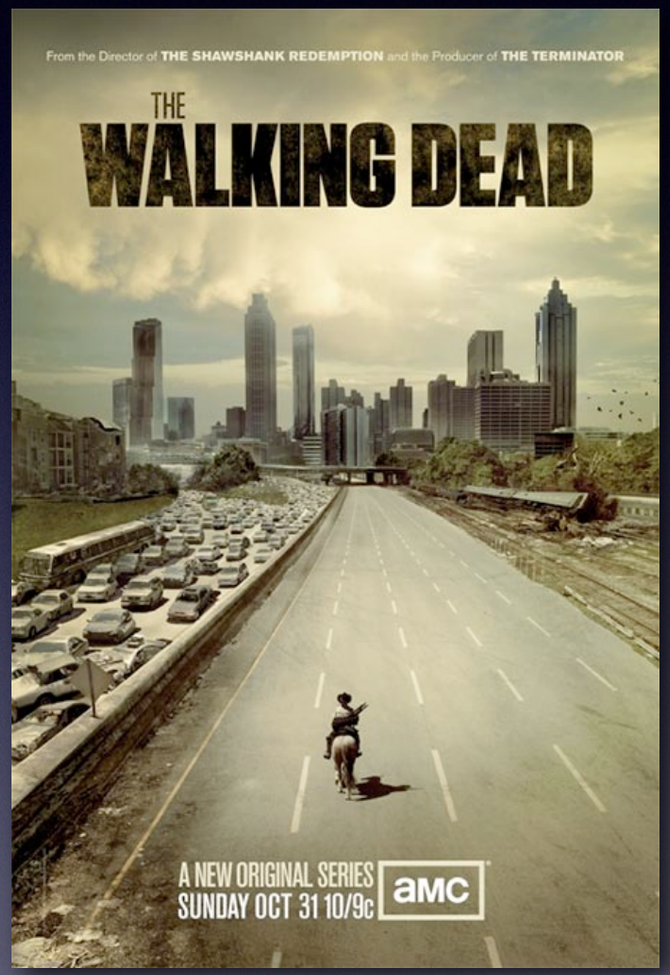
What shortcuts minimise the average path length ?



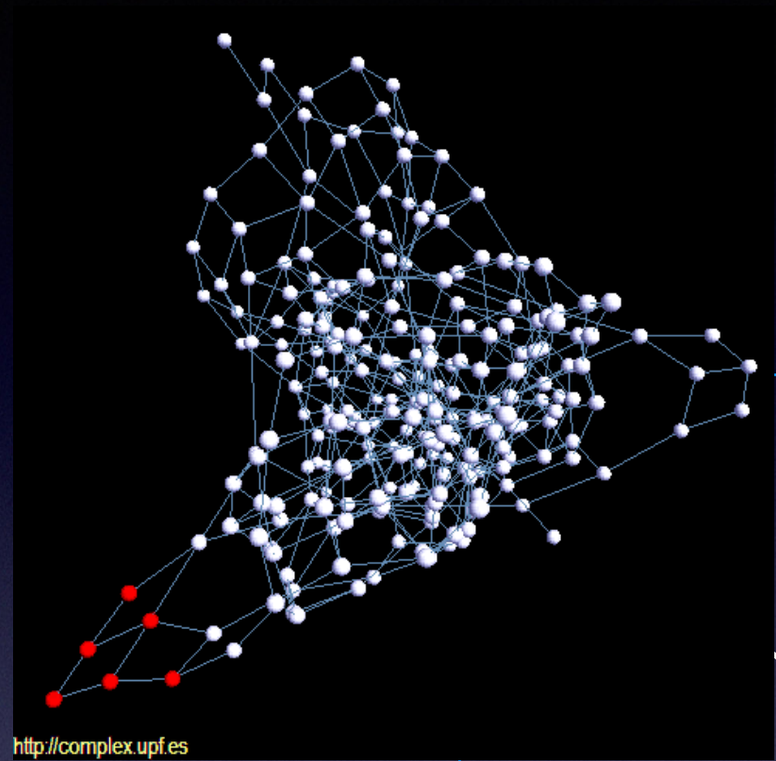
Diffusion Processes

The dynamical implications of small-world architectures

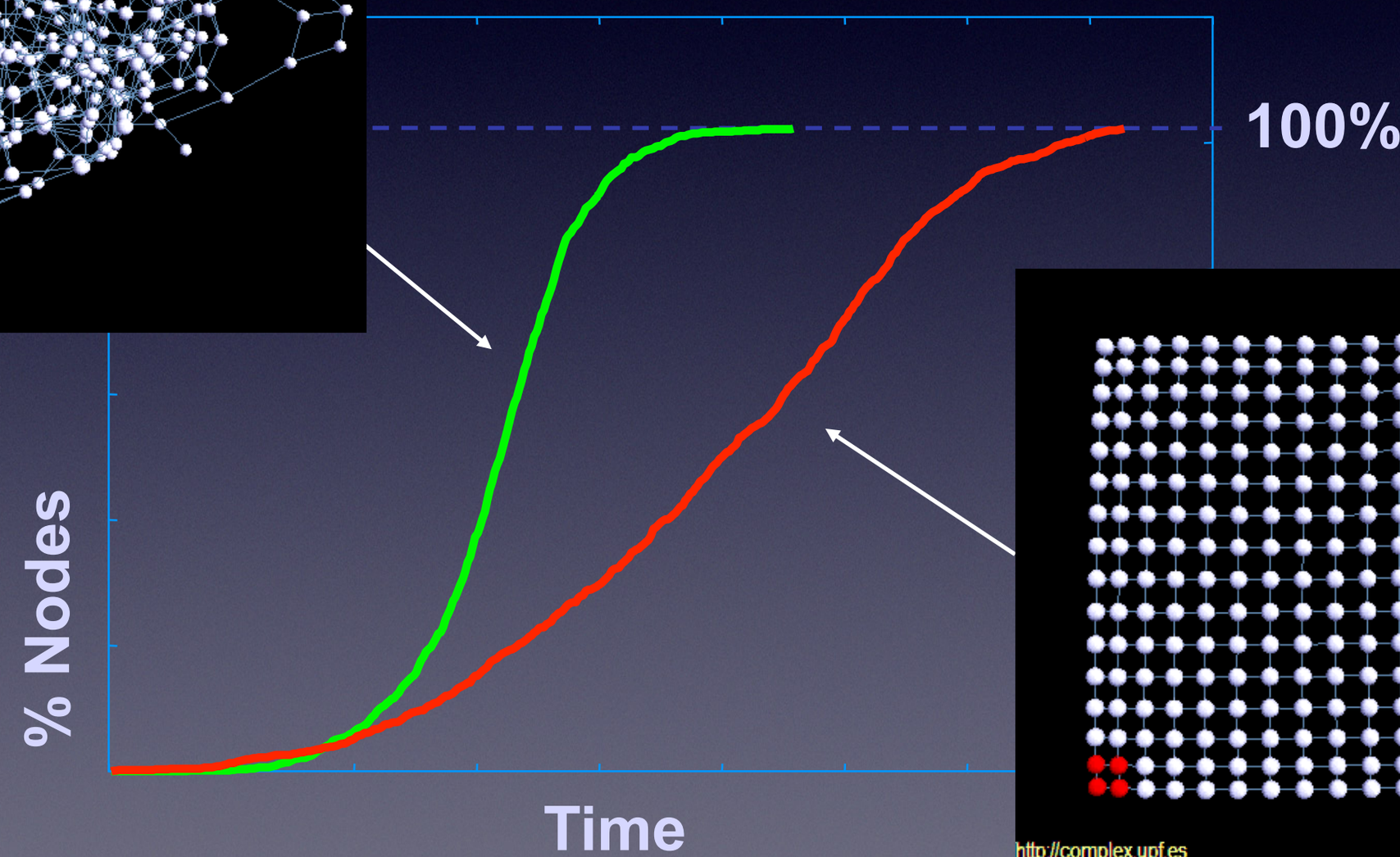




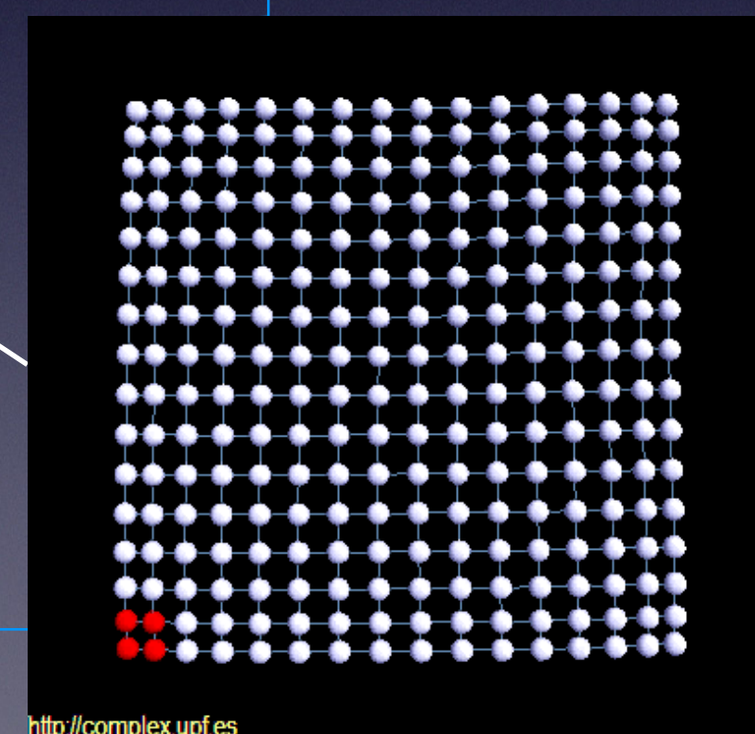
Diffusion Processes on Small-Worlds



Small-World



Lattice

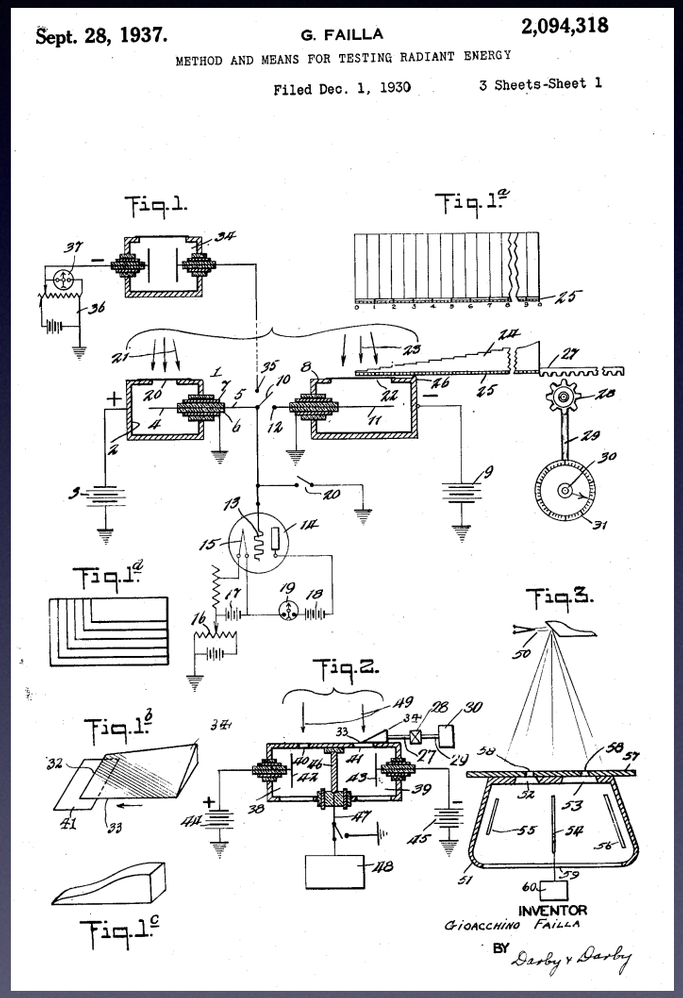


Evolution

Patent Citations

1930 - 1937

Gioacchino Failla



Sept. 28, 1937. G. FAILLA 2,094,318

METHOD AND MEANS FOR TESTING RADIANT ENERGY

Filed Dec. 1, 1930 3 Sheets-Sheet 1

Fig. 1. Fig. 1a. Fig. 1d. Fig. 1b. Fig. 1c. Fig. 2. Fig. 3.

INVENTOR GIOACCHINO FAILLA

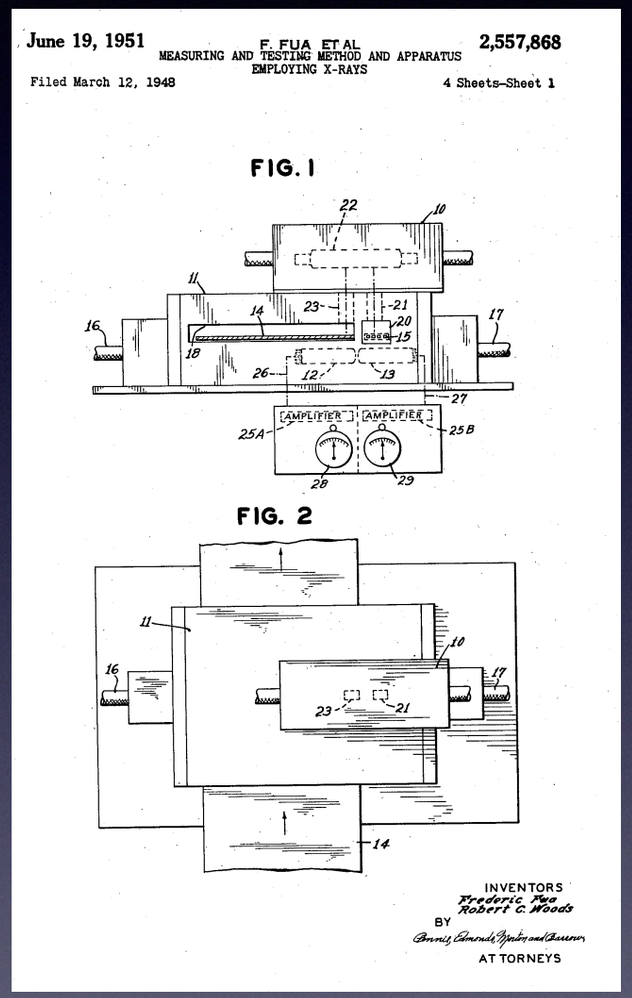
BY Darby & Darby

2,094,318

A method of measuring the quality of electromagnetic radiations such as X-rays for testing metal foil and the like

1944 - 1951

Frederic Fua & Robert C. Woods



June 19, 1951 F. FUA ET AL 2,557,868

MEASURING AND TESTING METHOD AND APPARATUS EMPLOYING X-RAYS

Filed March 12, 1948 4 Sheets-Sheet 1

FIG. 1

FIG. 2

INVENTORS Frederic Fua Robert C. Woods

BY

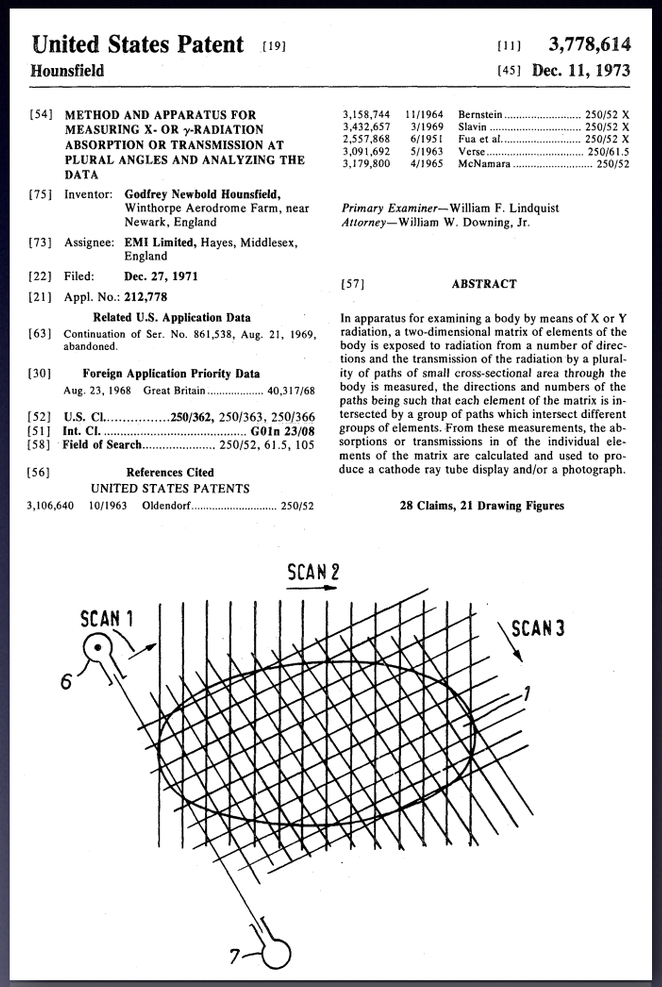
ATTORNEYS

2,557,868

Accurately testing thickness of materials such as metal by X-rays in the fast moving product and without touching it

1969 - 1973

Godfrey Hounsfield



United States Patent [19]

Hounsfield

[11] **3,778,614**

[45] **Dec. 11, 1973**

[54] **METHOD AND APPARATUS FOR MEASURING X- OR γ-RADIATION ABSORPTION OR TRANSMISSION AT PLURAL ANGLES AND ANALYZING THE DATA**

[75] Inventor: **Godfrey Newbold Hounsfield,** Winthorpe Aerodrome Farm, near Newark, England

[73] Assignee: **EMI Limited,** Hayes, Middlesex, England

[22] Filed: **Dec. 27, 1971**

[21] Appl. No.: **212,778**

Related U.S. Application Data

[63] Continuation of Ser. No. 861,538, Aug. 21, 1969, abandoned.

[30] **Foreign Application Priority Data**

Aug. 23, 1968 Great Britain 40,317/68

[52] **U.S. Cl.** **250/362,** 250/363, 250/366

[51] **Int. Cl.** **G01n 23/08**

[58] **Field of Search** 250/52, 61.5, 105

[56] **References Cited**

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2,557,868	6/1951	Fua et al.	250/52 X
3,091,692	5/1963	Verse	250/61.5
3,179,800	4/1965	McNamara	250/52

Primary Examiner—William F. Lindquist
Attorney—William W. Downing, Jr.

[57] **ABSTRACT**

In apparatus for examining a body by means of X or Y radiation, a two-dimensional matrix of elements of the body is exposed to radiation from a number of directions and the transmission of the radiation by a plurality of paths of small cross-sectional area through the body is measured, the directions and numbers of the paths being such that each element of the matrix is intersected by a group of paths which intersect different groups of elements. From these measurements, the absorptions or transmissions in of the individual elements of the matrix are calculated and used to produce a cathode ray tube display and/or a photograph.

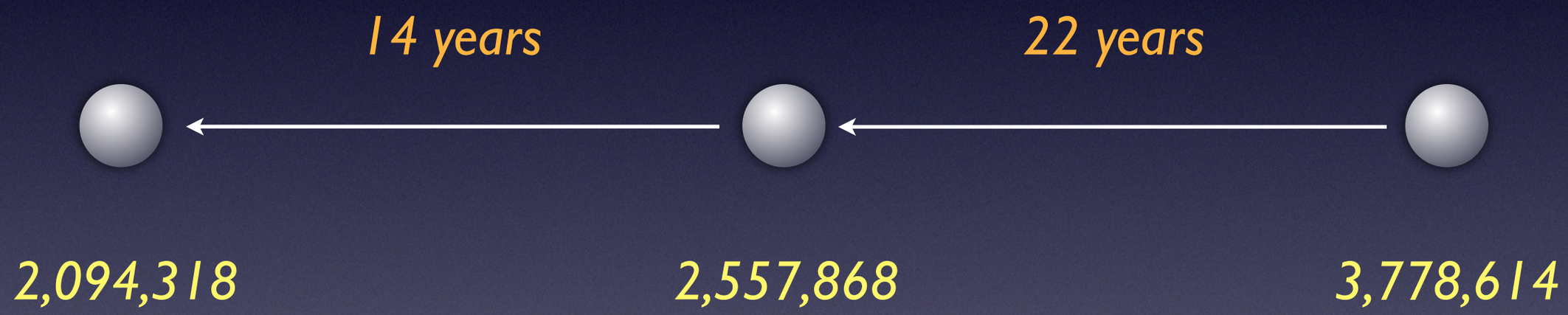
28 Claims, 21 Drawing Figures

SCAN 1 SCAN 2 SCAN 3

3,778,614

Computer assisted Tomography

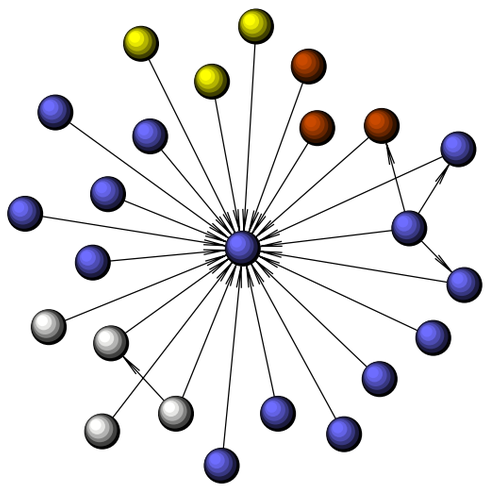
Patent Citation Networks



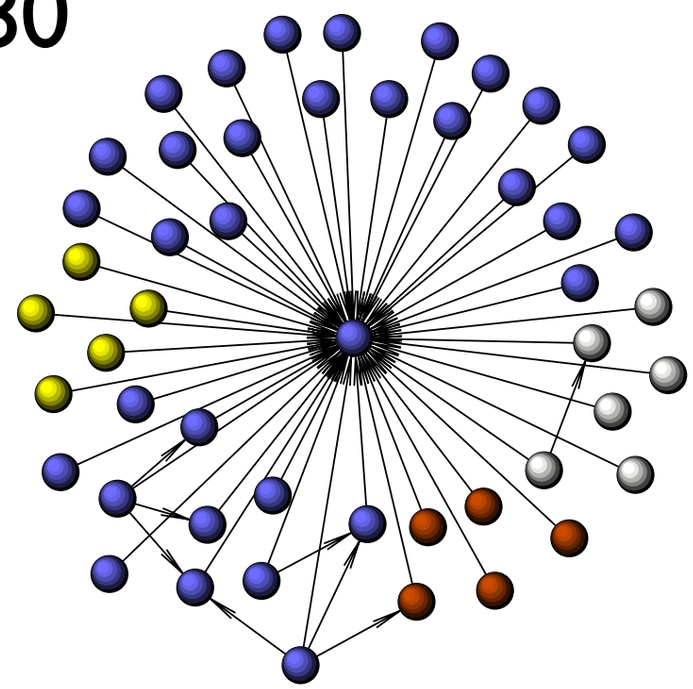
Valverde S, Solé RV, Bedau MA, Packard N (2007)
Topology and evolution of technology innovation networks.
Phys Rev E 76:056118.

Evolution of the CT Patent Citation Network

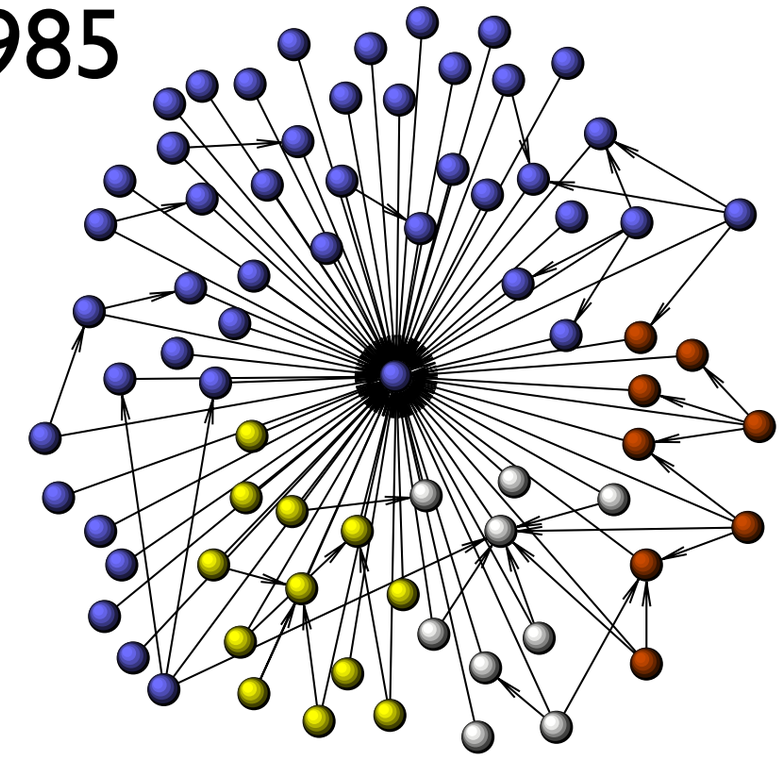
1973



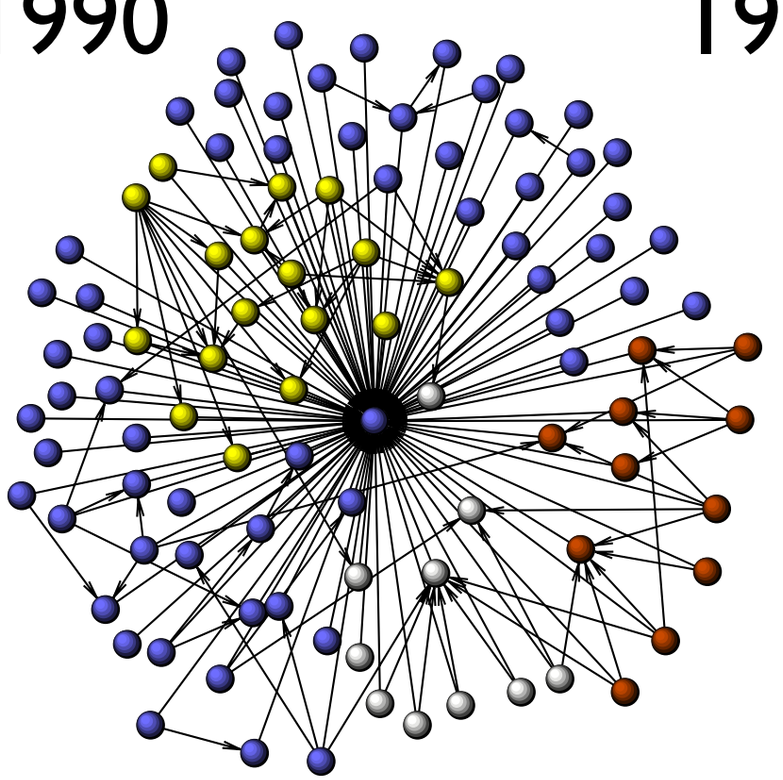
1980



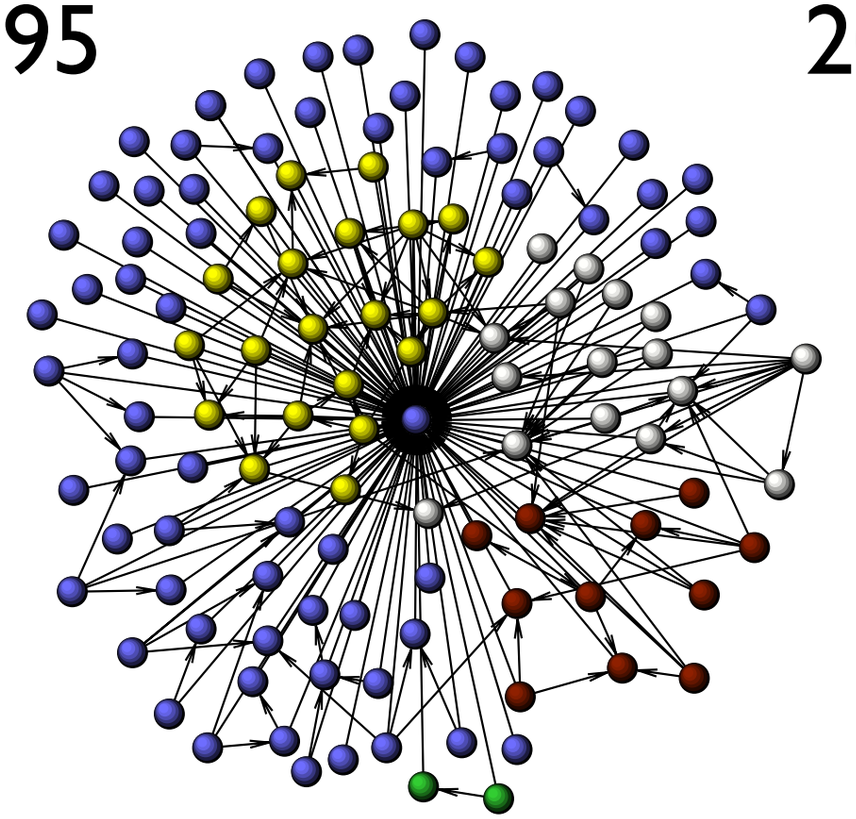
1985



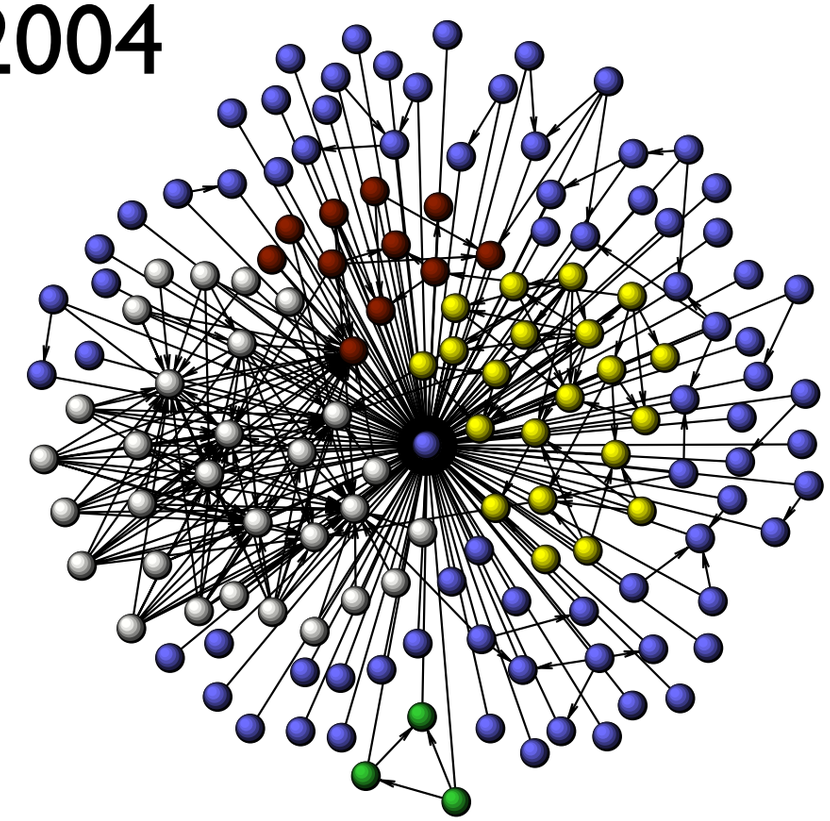
1990



1995

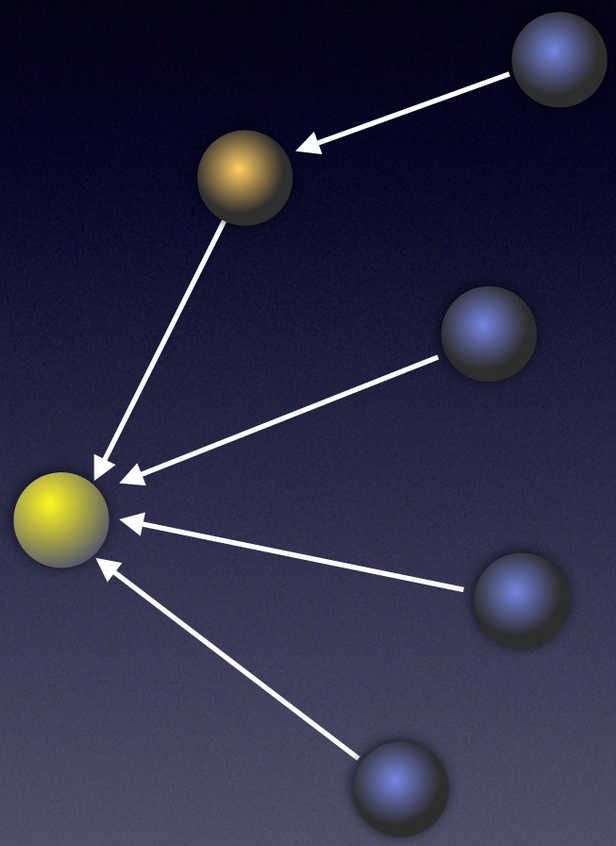


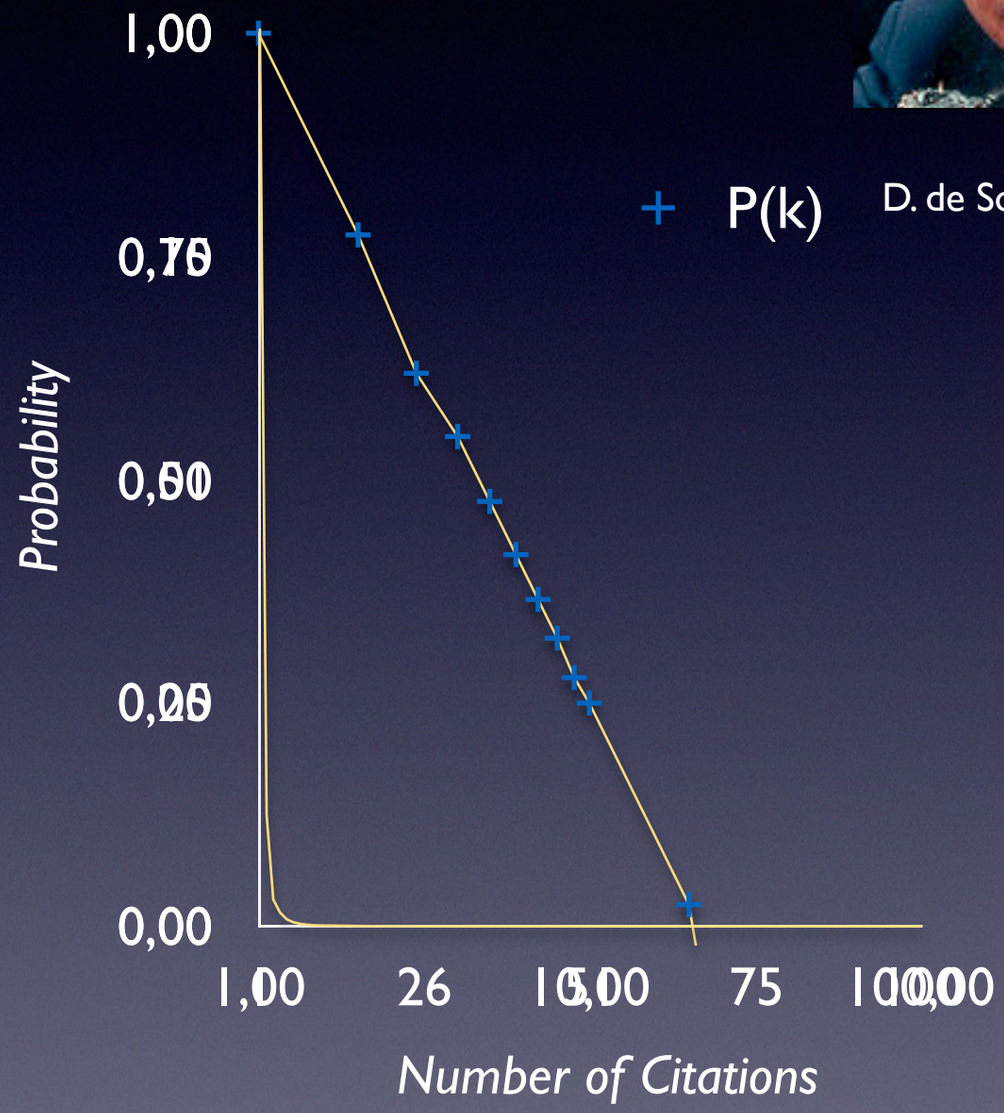
2004



Preferential Attachment: Models of Network Growth

$$\Pi(k) \sim k^{\beta} \longrightarrow p(k) \sim k^{-\gamma}$$





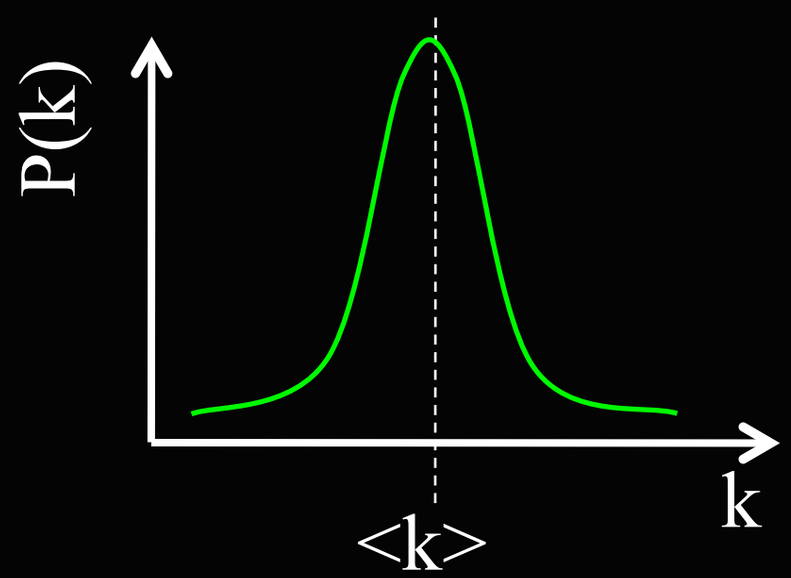
D. de Solla Price

(Price, 1965) & (Price, 1976)

Network Models: What they look like

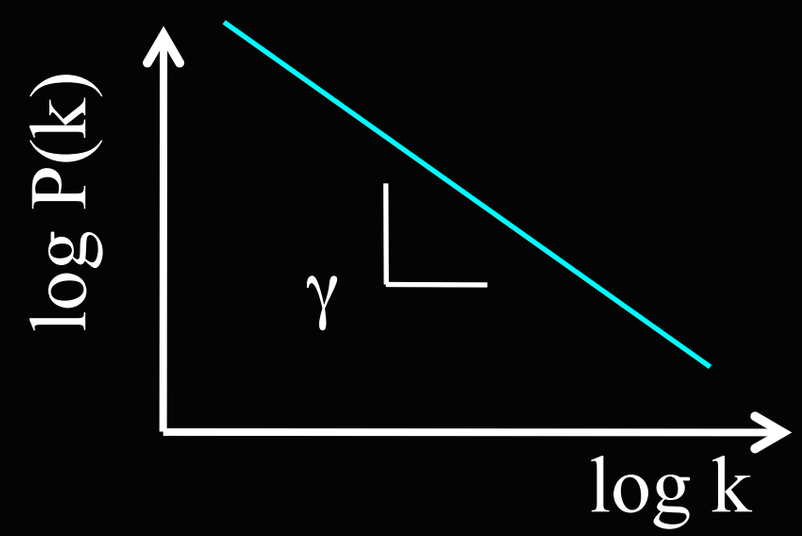
P. Erdös

Random



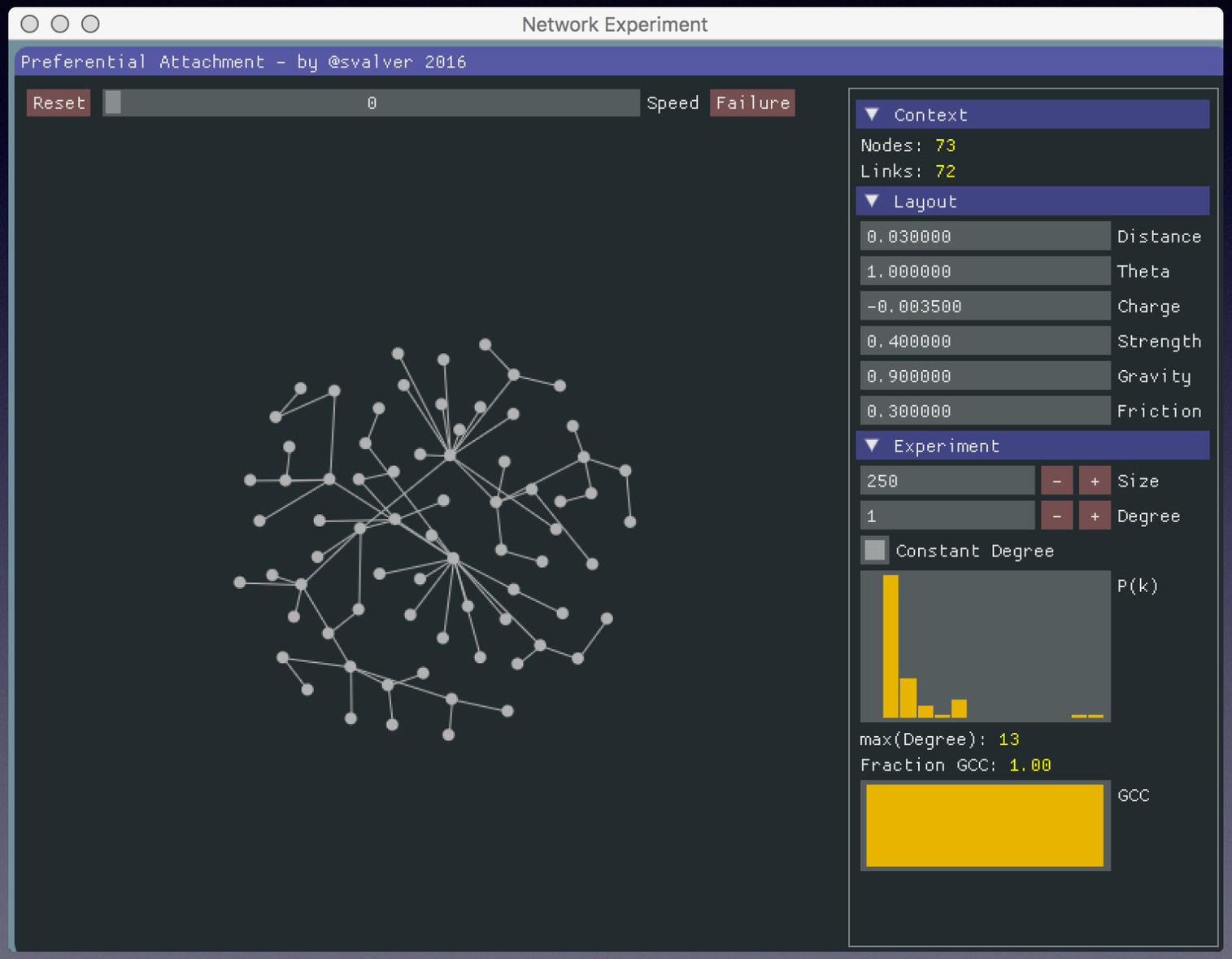
Scale-Free

V. Pareto



Preferential Attachment

Exploring the role of history and amplification effects in the emergence of hubs



How many nodes are “hubs”?

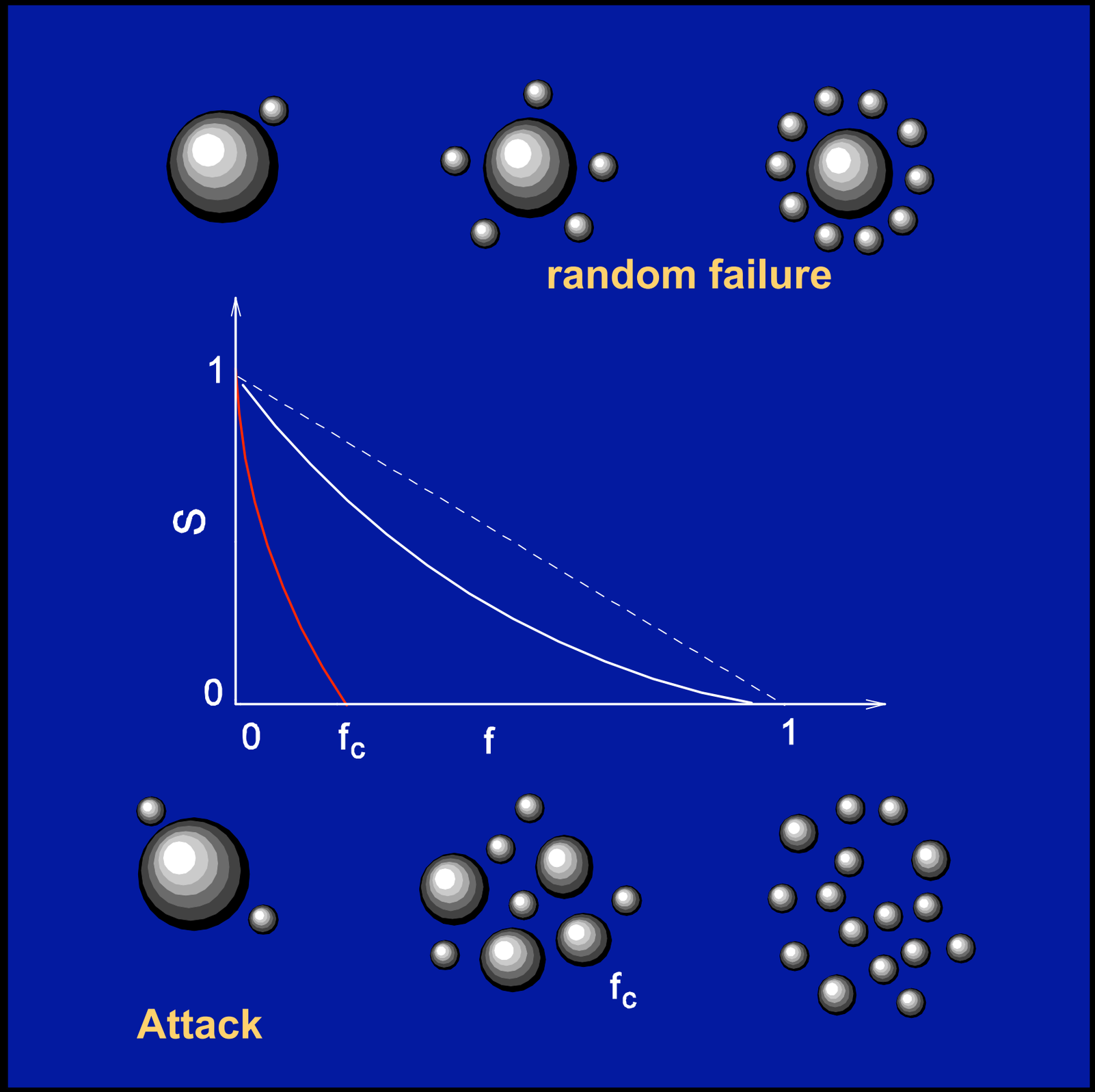
How many have only a few links?

Does some low k node ever become a hub? How often?

Complexity and Fragility in Scale-Free Networks

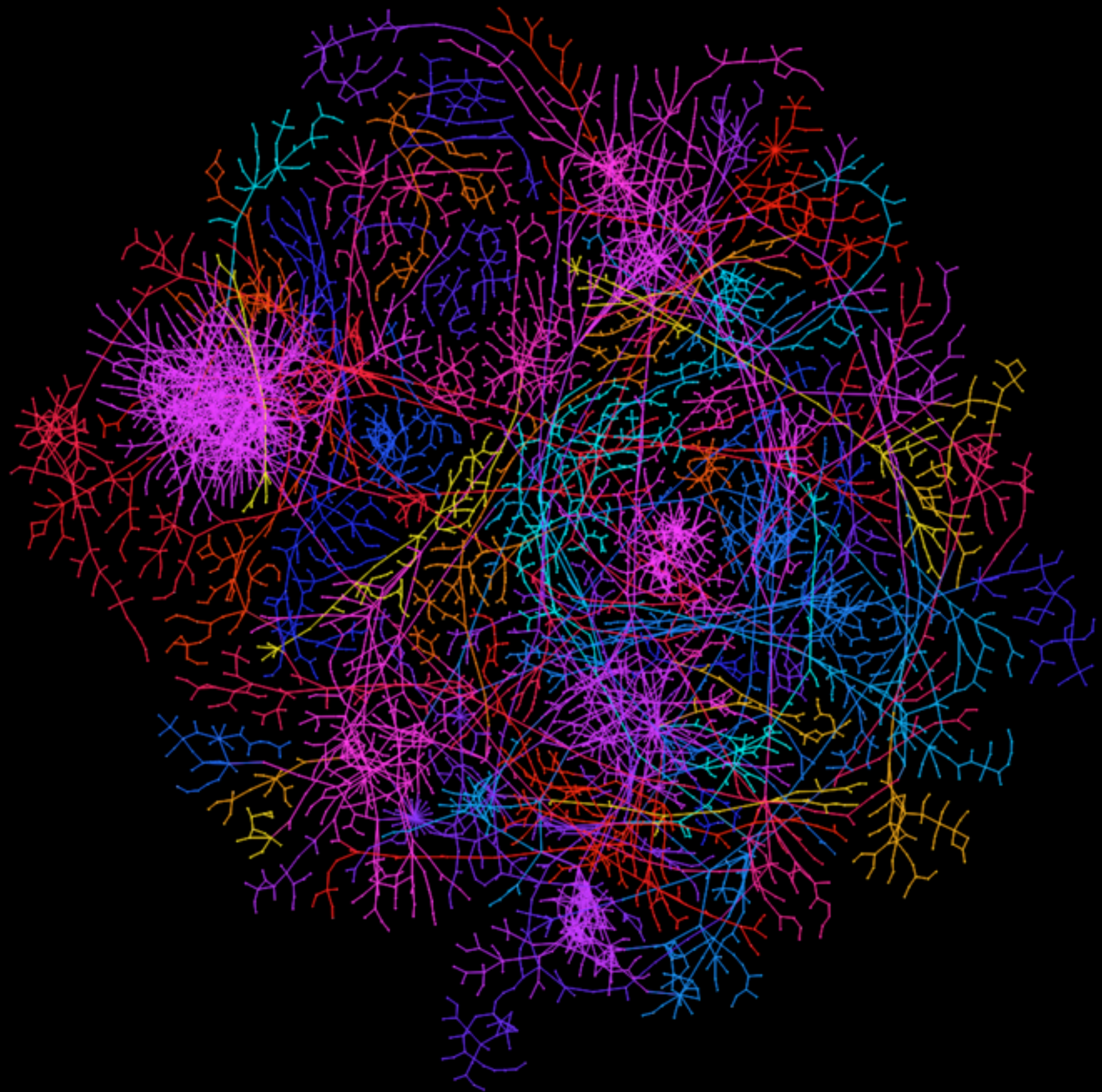
"Error and attack tolerance of complex networks"
R. Albert, H. Jeong & L-A Barabási
Nature **406** (2000) 378-382

Les xarxes reals són molt homeostàtiques, però també molt fràgils enfront d'un atac als elements més connectats



Modularity

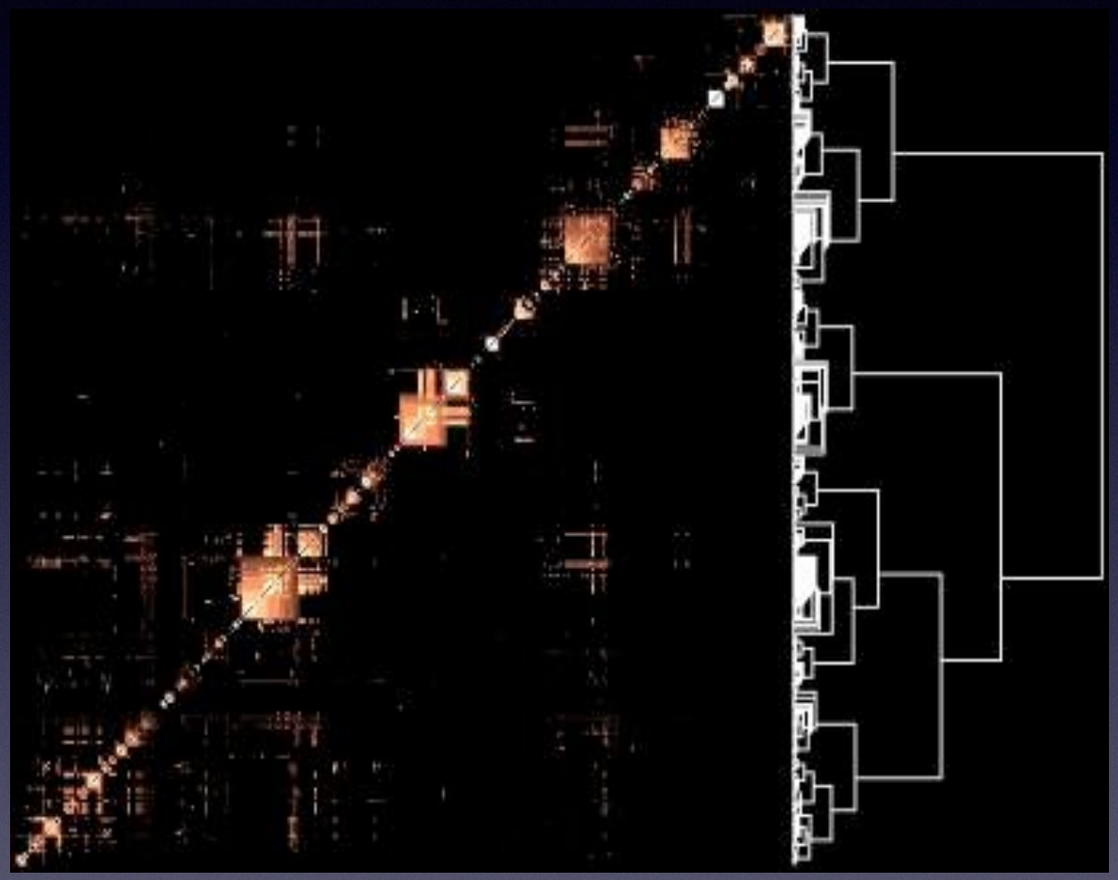
Patent Citation Networks Are Modular

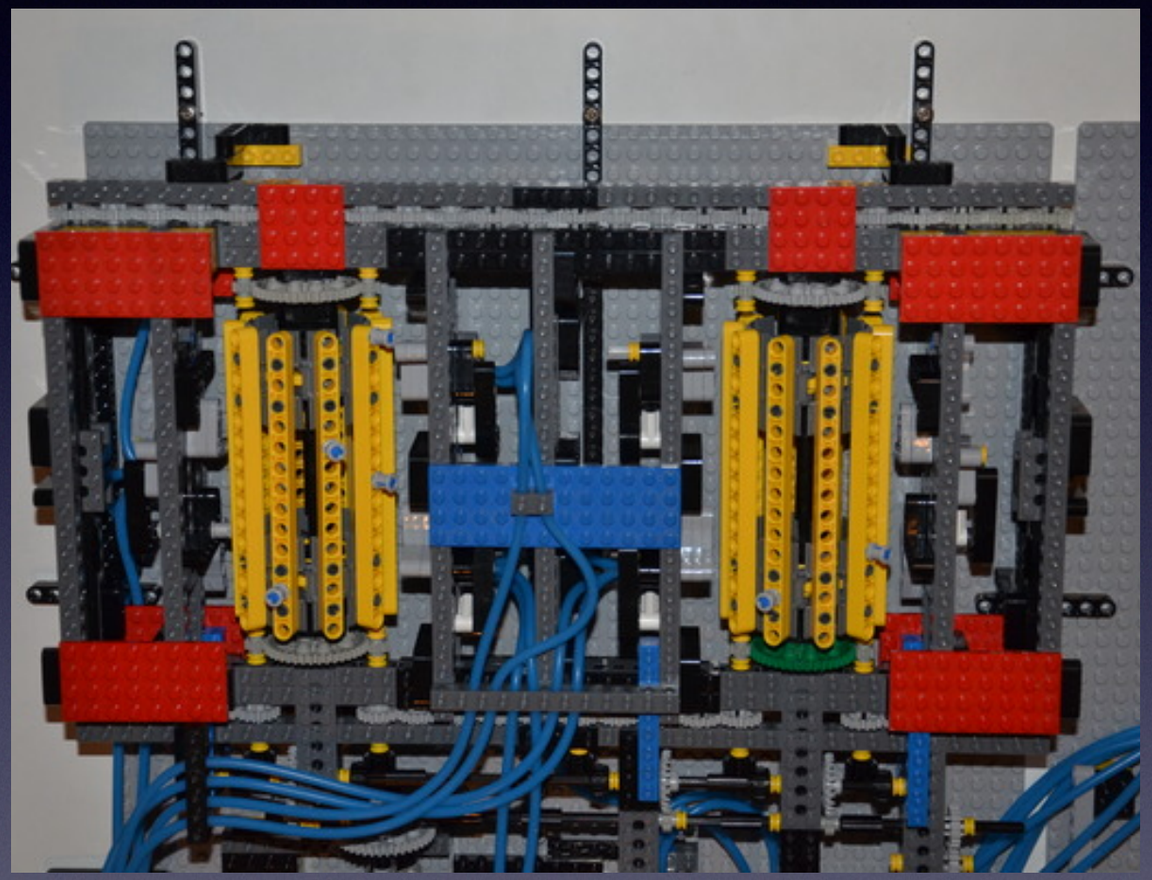


USPTO

Hierarchical Organisation of Networks

To build complex things, both nature and engineers use modules





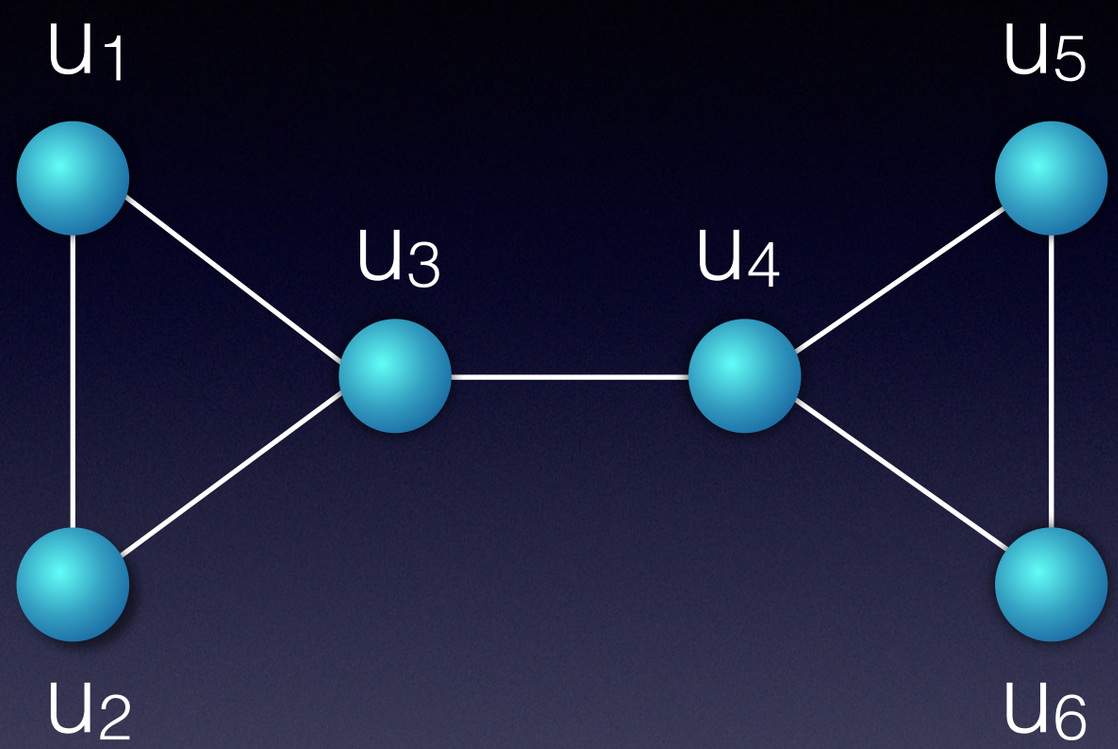
H. A. Simon, “The Architecture of Complexity” (1962)

H. Jeong et al., Nature 407, 651-654 (2000)

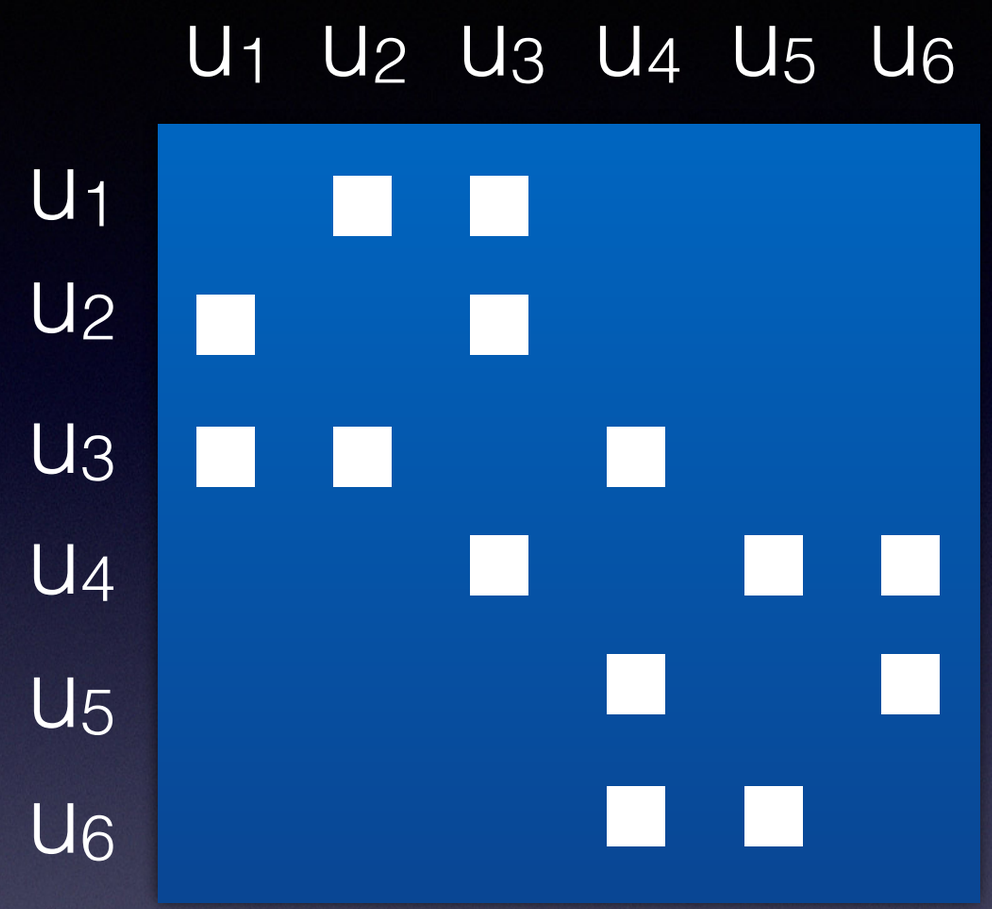
E. Ravasz and A.-L. Barabási (2003)

LEGO Turing Machine

Community Detection



Network



Matrix

Modularity measures the presence of dense clusters of interdependent nodes

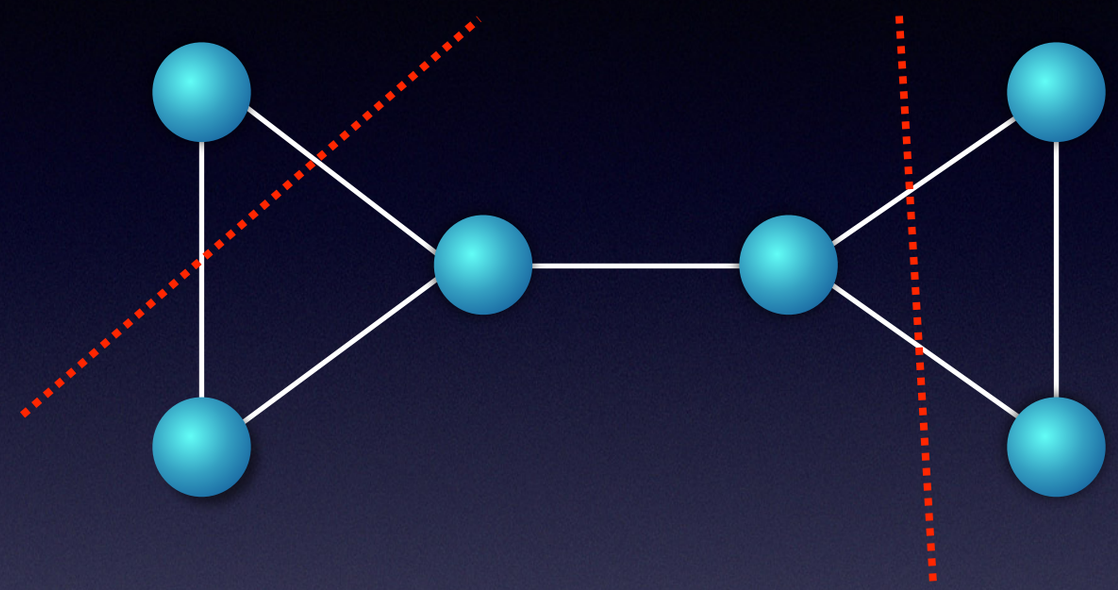
How to find modularity?

(1) Divide up the network

(2) Calculate the modularity value (Q)

(3) Repeat until a solution is optimised

(1) Divide up the network



(2) Calculate the modularity value (Q)

$$Q = \sum_{\text{for each of the modules}} \left[\begin{array}{c} \textbf{Observed} \text{ fraction} \\ \text{of links in group} \end{array} - \begin{array}{c} \textbf{Expected} \text{ fraction} \\ \text{of links in group} \end{array} \right]$$

(2) Calculate the modularity value (Q)

Number of Modules

Number of links between nodes in module 's'

Sum of degrees of nodes in module 's'

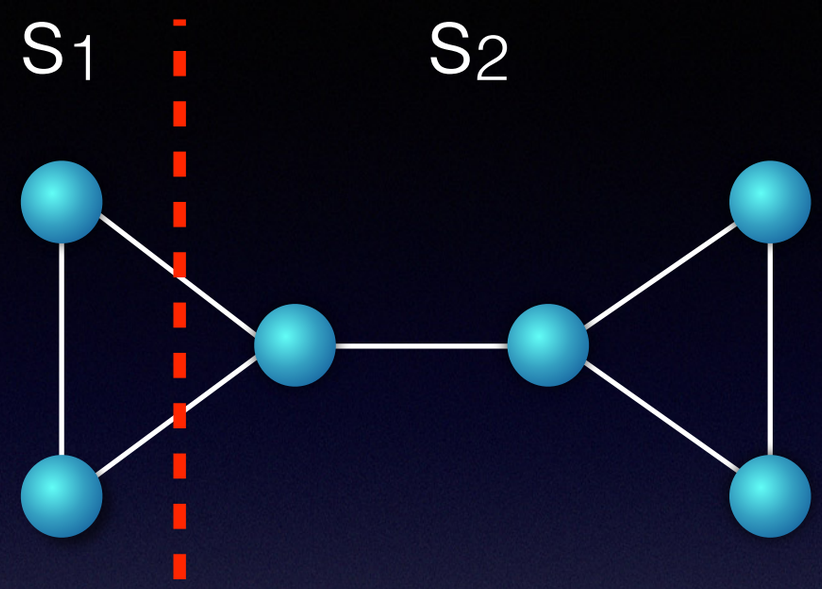
Taking square to obtain link probability

$$Q = \sum_{s=1}^{N_m} \left[\frac{l_s}{L} - \left(\frac{d_s}{2L} \right)^2 \right]$$

Number of links in the network

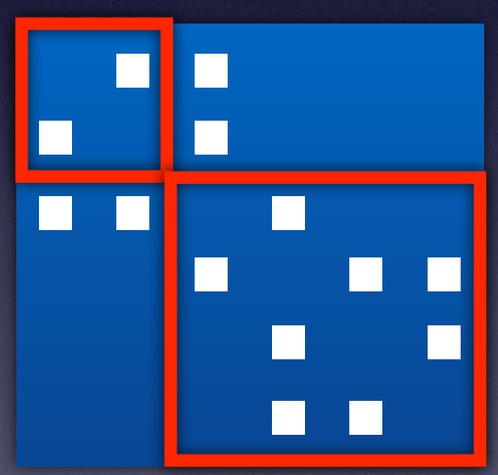
Girvan and Newman **PNAS** 99:7821 (2002)

Example (1/2)



$$Q_{s_1} = \frac{1}{7} - \left(\frac{4}{14}\right)^2 = 0.06$$

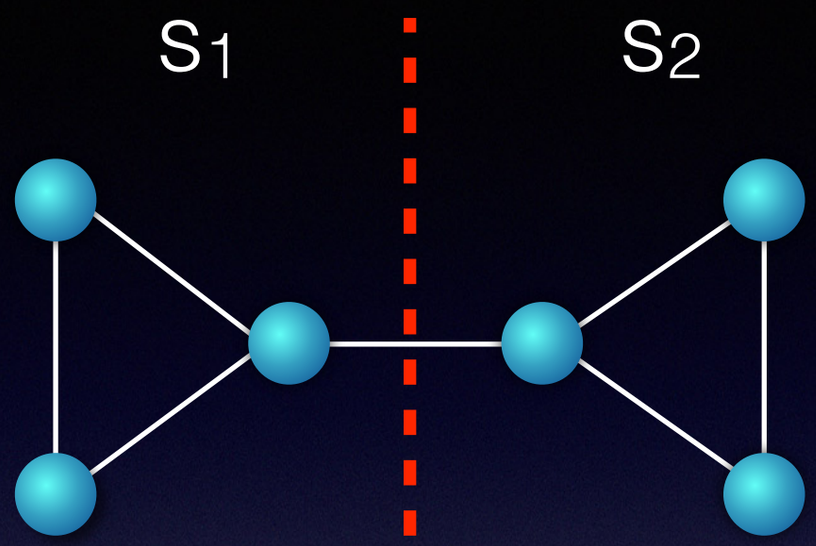
$$Q_{s_2} = \frac{4}{7} - \left(\frac{10}{14}\right)^2 = 0.06$$



$$Q = Q_{s_1} + Q_{s_2} = 0.12$$

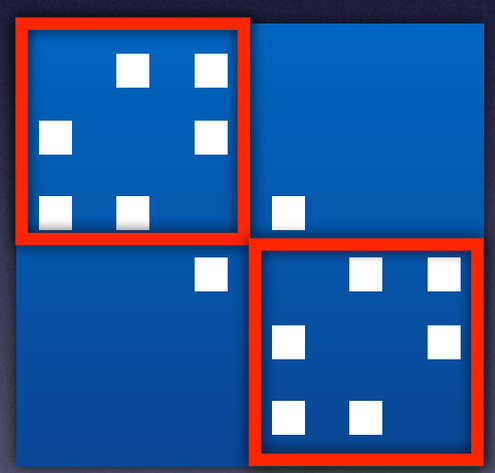
$$Q = \sum_{s=1}^{N_m} \left[\frac{l_s}{L} - \left(\frac{d_s}{2L}\right)^2 \right]$$

Example (2/2)



$$Q_{s_1} = \frac{3}{7} - \left(\frac{7}{14}\right)^2 = 0.18$$

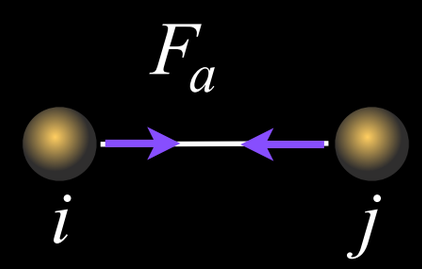
$$Q_{s_2} = Q_{s_1} = 0.18$$



$$Q = Q_{s_1} + Q_{s_2} = 0.36 > 0.12$$

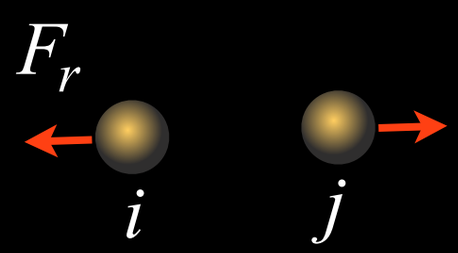
$$Q = \sum_{s=1}^{N_m} \left[\frac{l_s}{L} - \left(\frac{d_s}{2L}\right)^2 \right]$$

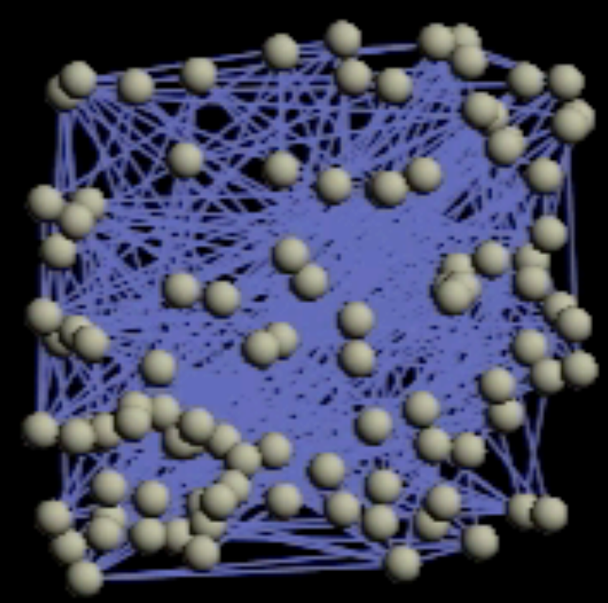
Energy models and modularity reveal communities



$Q=0$

$$Q = \frac{1}{2m} \sum_{i,j} \left[A_{ij} - \frac{k_i k_j}{2m} \right] \delta(c_i, c_j)$$

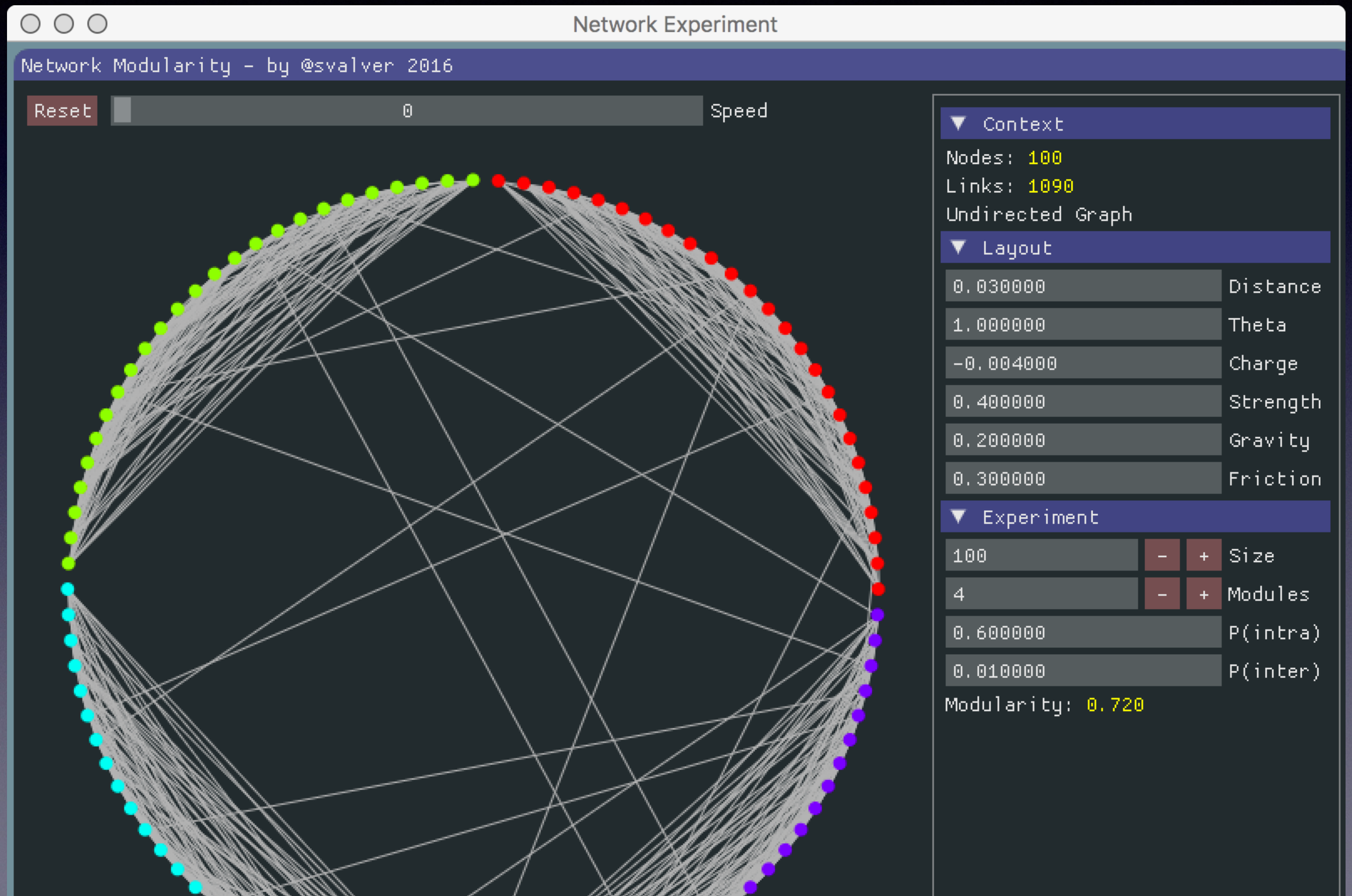
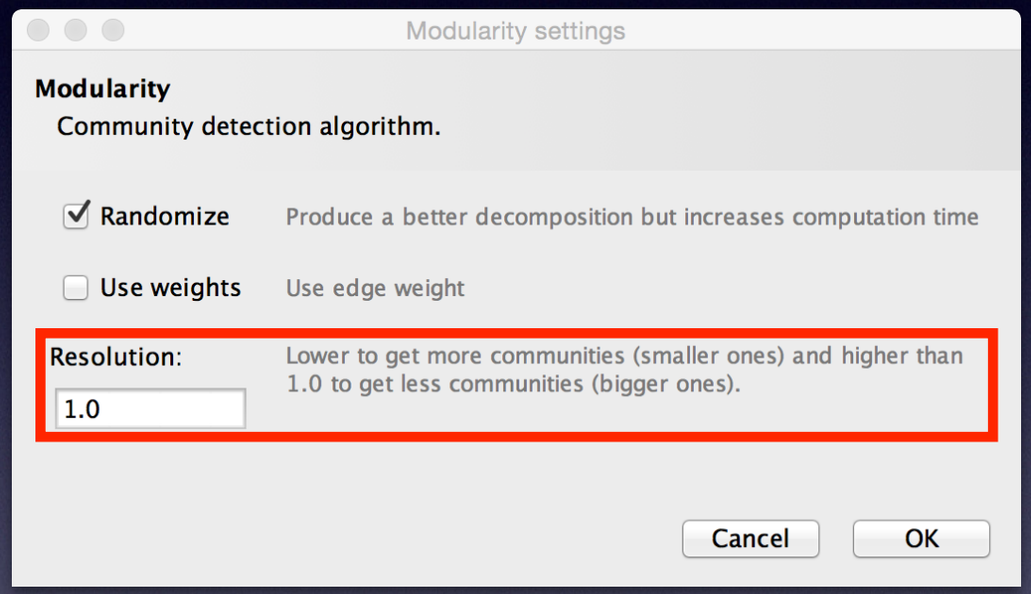




Complex Systems Lab (2010)

Network Modularity

Finding communities in random modular networks



Multiresolution Analysis of Networks