

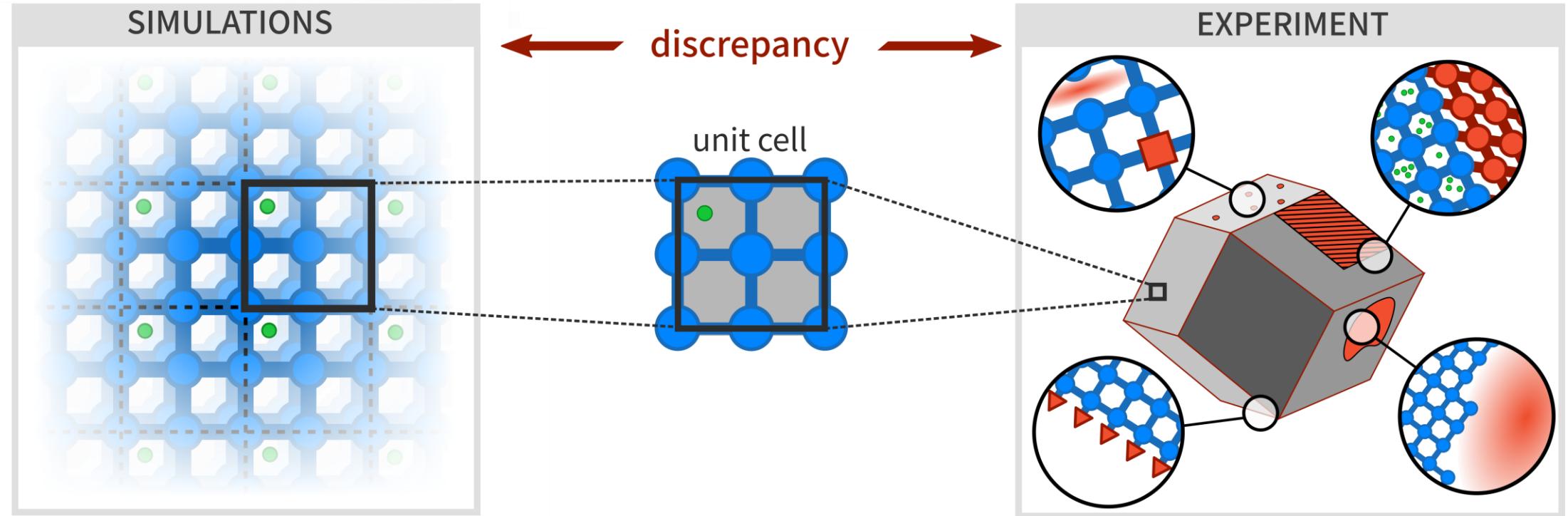
Systematic coarse-graining in metal-organic frameworks

Sander Vandenhaute

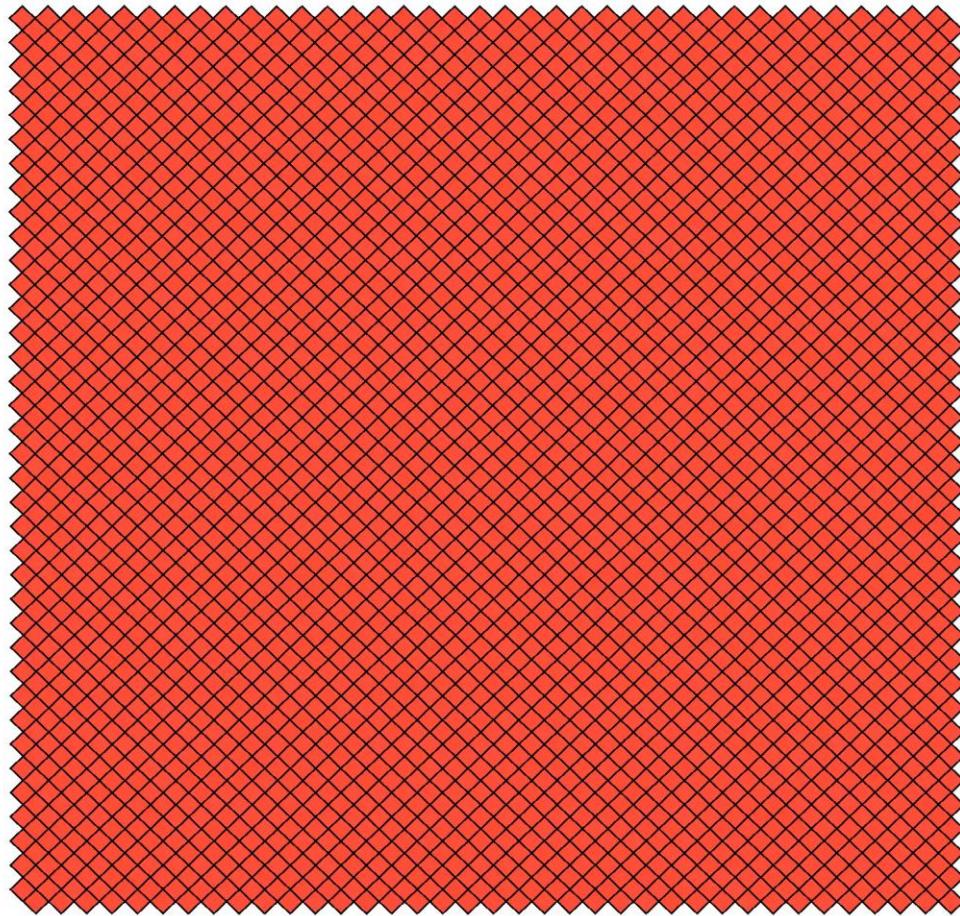
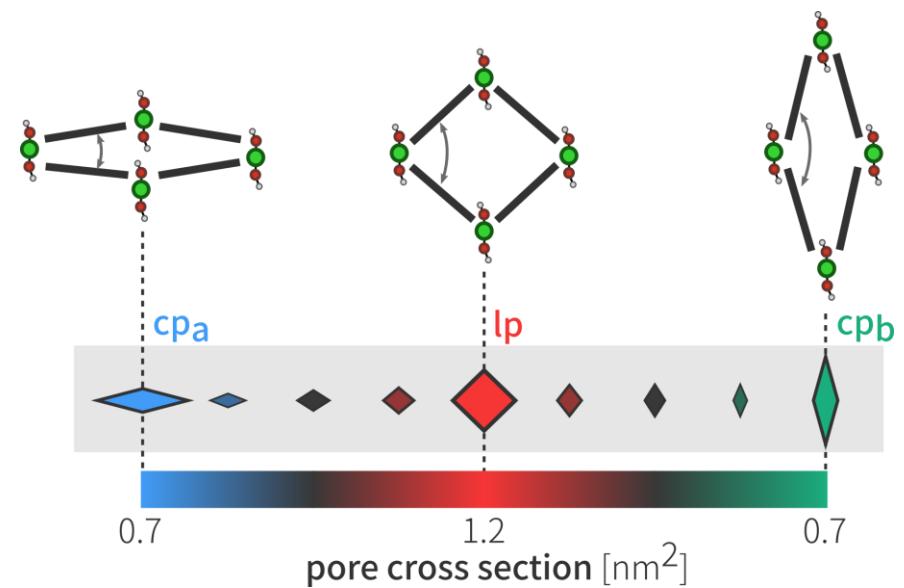
Sven M. J. Rogge

Veronique Van Speybroeck

Why?

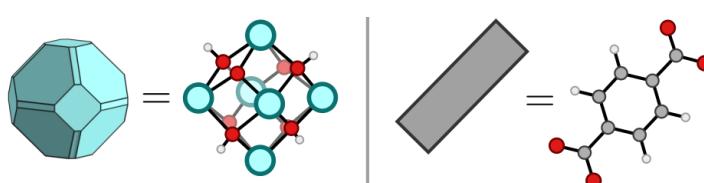
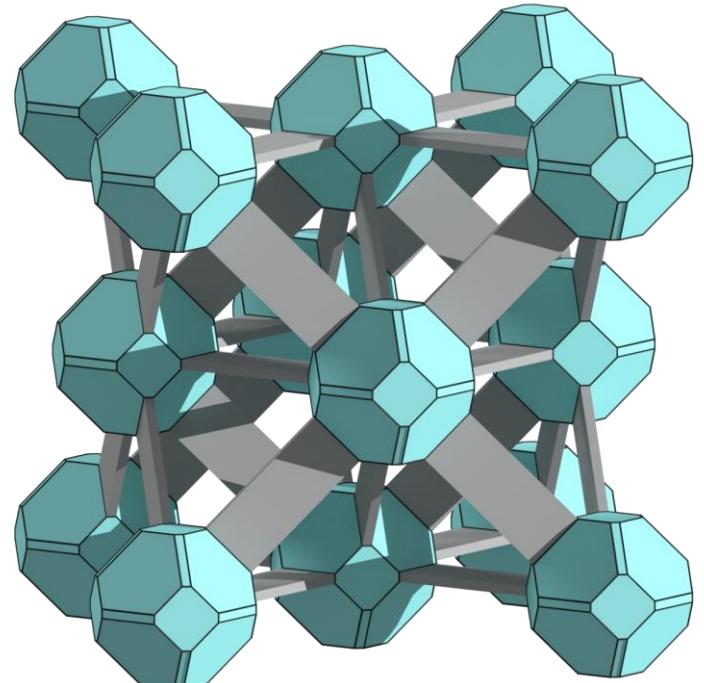


Beyond one million atoms



Coarse-graining in MOFs

an example: UiO-66(Zr)

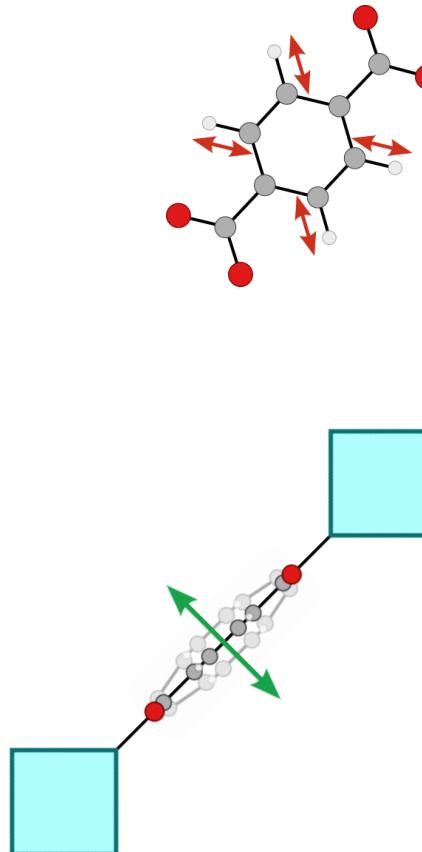


● Zr

● O

● C

○ H

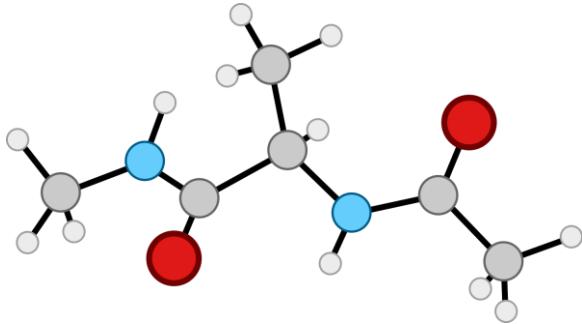


rigid, high-frequency modes
irrelevant

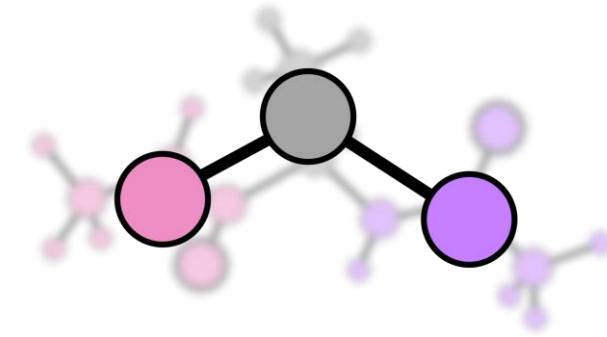
flexible, low-frequency modes;
relevant

1. partition atoms into CG beads
2. determine effective interaction potentials

The mapping entropy



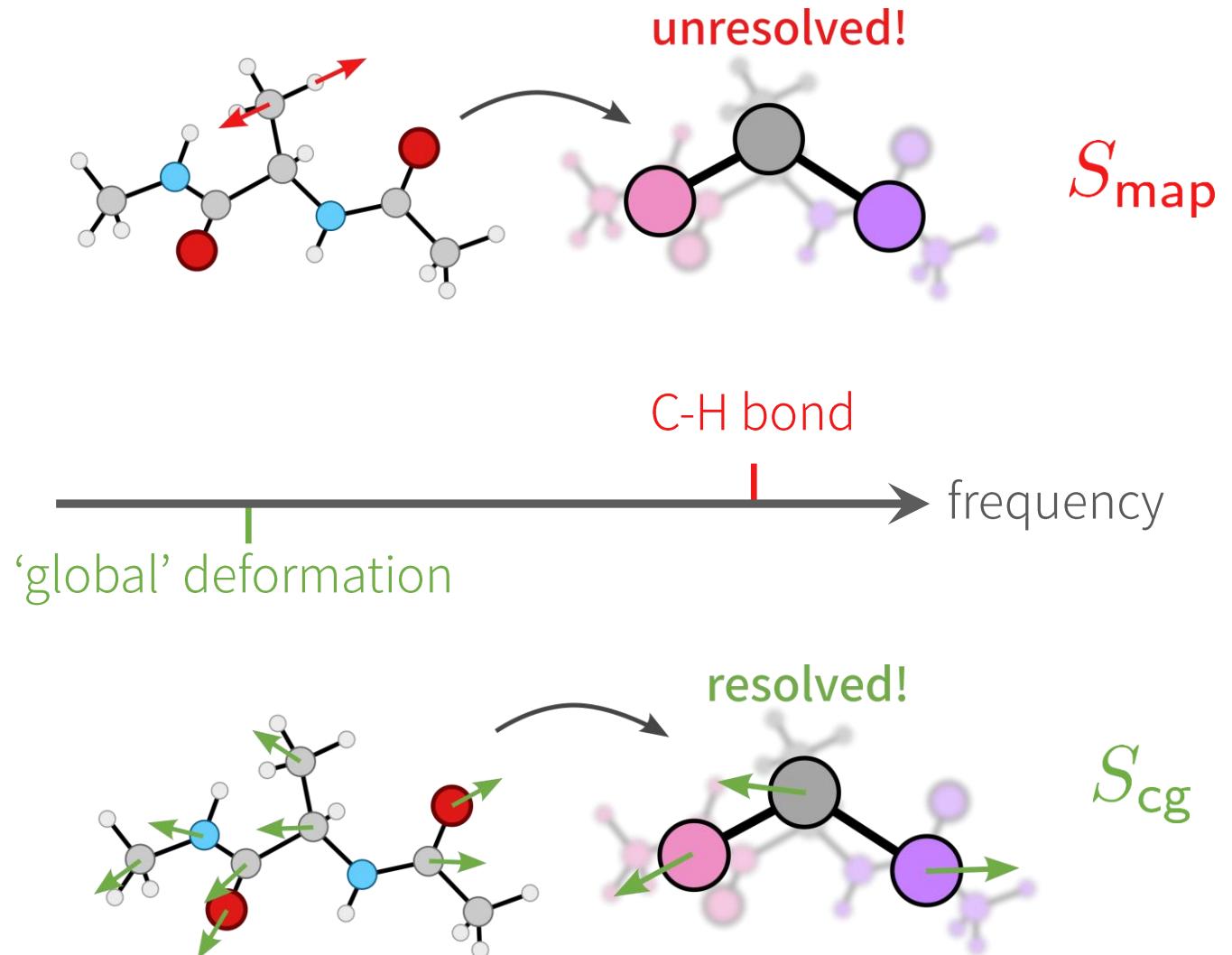
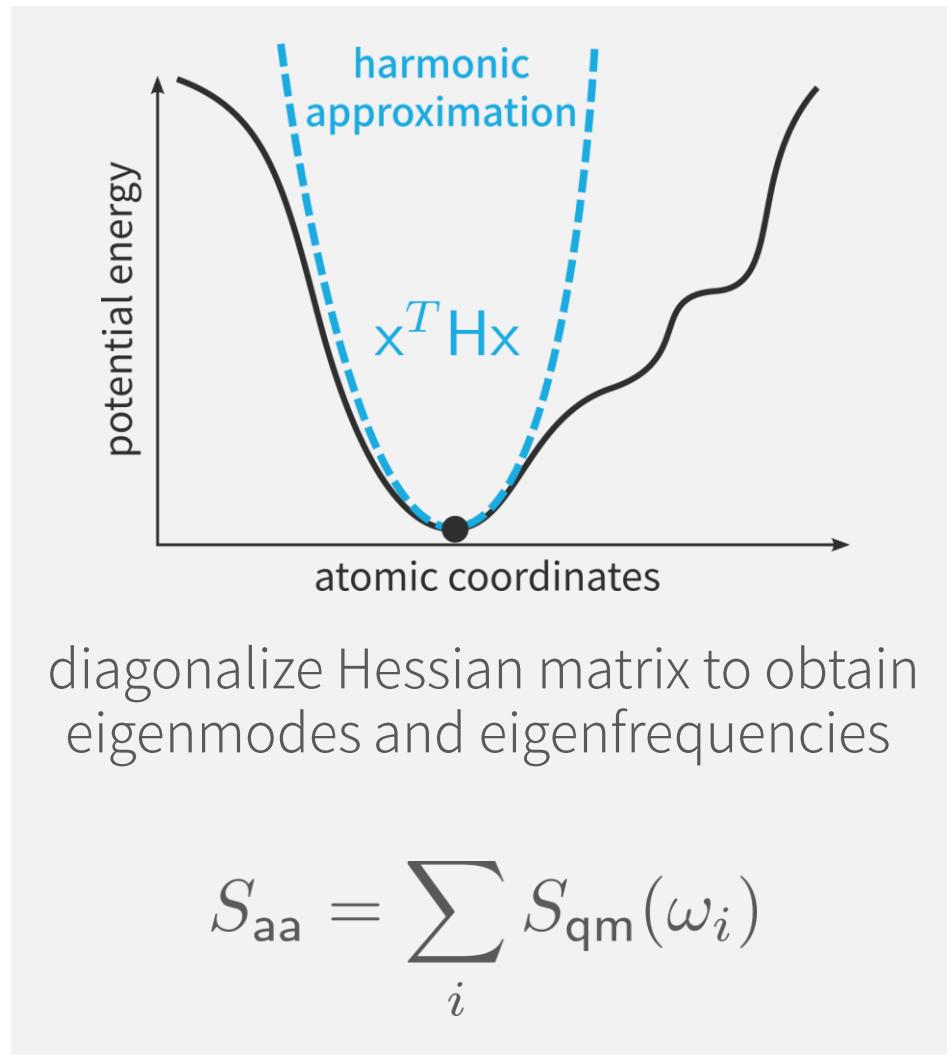
mapping M
→



$$\frac{S_{aa}}{\text{total atomic entropy}} \rightleftharpoons \frac{S_{cg}}{\text{total CG entropy}} + \frac{S_{\text{map}}}{\text{mapping entropy}}$$

ideally very small!

Physical intuition about the mapping entropy



Cool... how do we formalize this intuition?

the mapping is a **linear** operator!

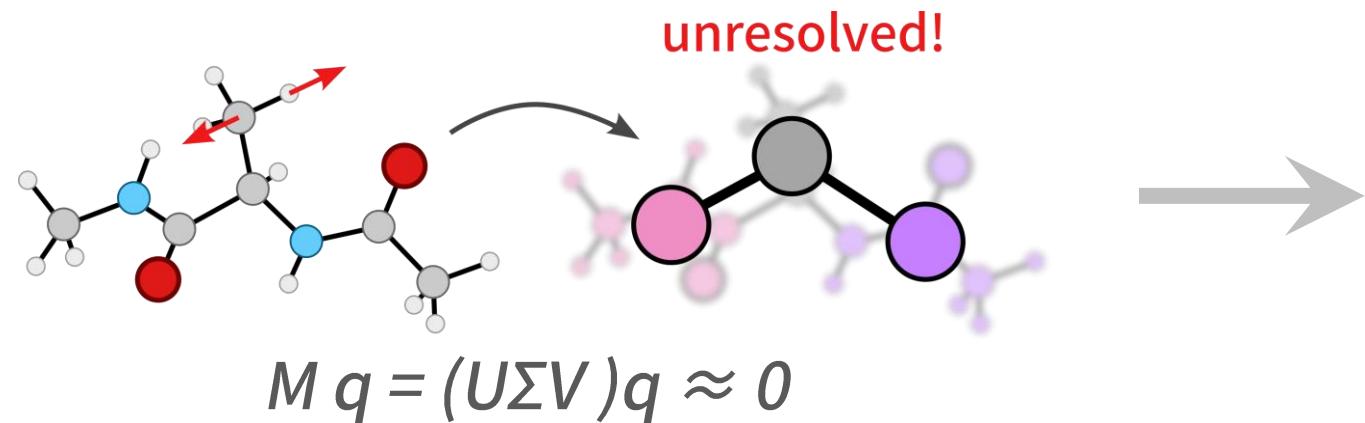
$$3N \begin{array}{c} | \\ Q \end{array} = \boxed{M} \begin{array}{c} | \\ q \end{array} \quad 3n$$

consider its **singular value decomposition** (SVD):

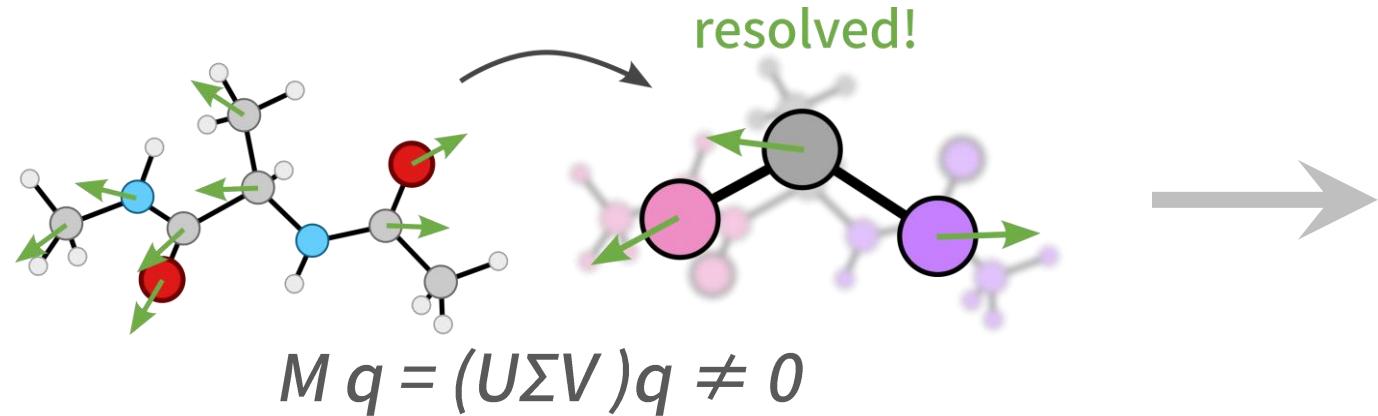
$$\boxed{M} = 3N \begin{array}{c} U \\ \vdots \\ \vdots \end{array} \Sigma \begin{array}{c} 0 \\ \vdots \\ 0 \end{array} \begin{array}{c} V \\ \vdots \\ \vdots \end{array} \quad 3n \quad 3n$$

— basis vectors for the **row space**
— basis vectors for the **null space**

Unresolved modes reside in the null space



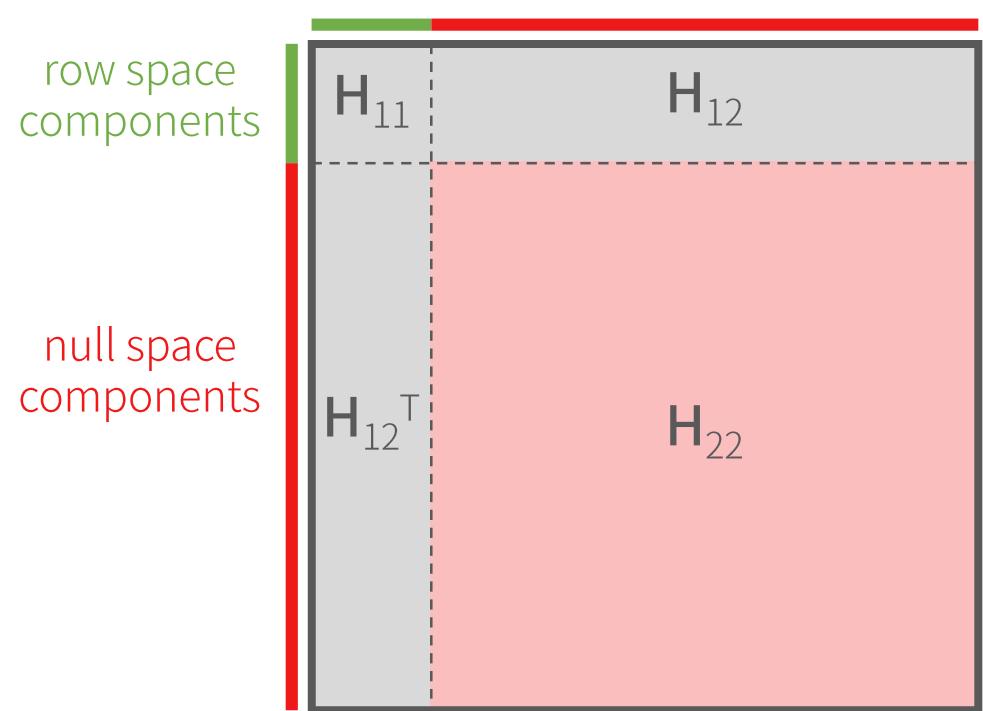
unresolved modes reside
in the null space of M



resolved modes reside
in the row space of M

Represent the Hessian in the SVD basis!

transform the Hessian using V :



$$S_{aa} = \sum_i^{3n} S(\omega_i)$$

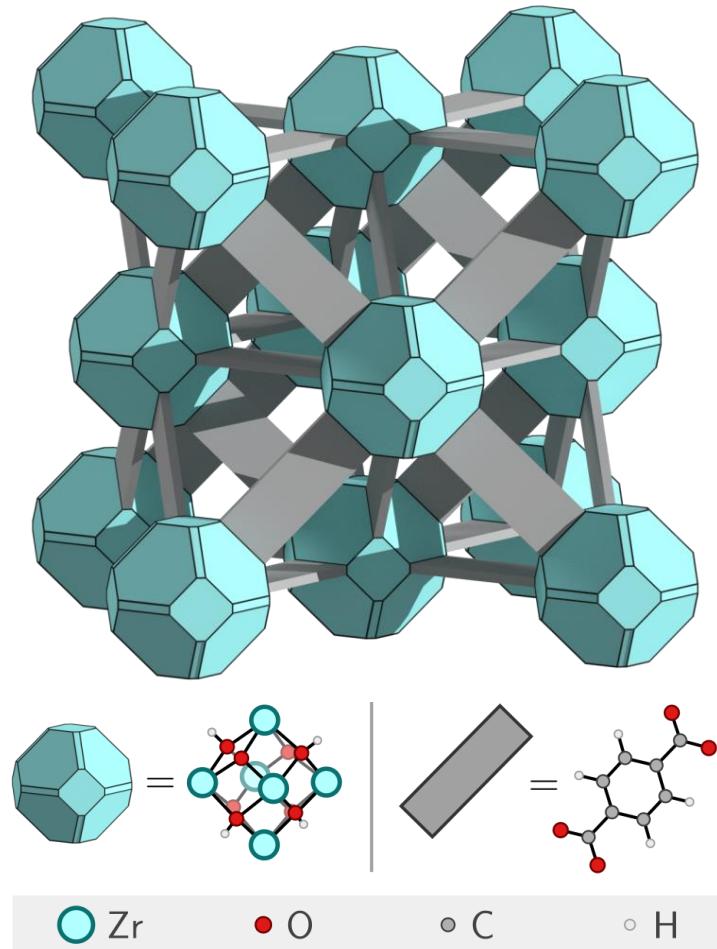
depends on:

- I. the Hessian
- II. the SVD of the mapping

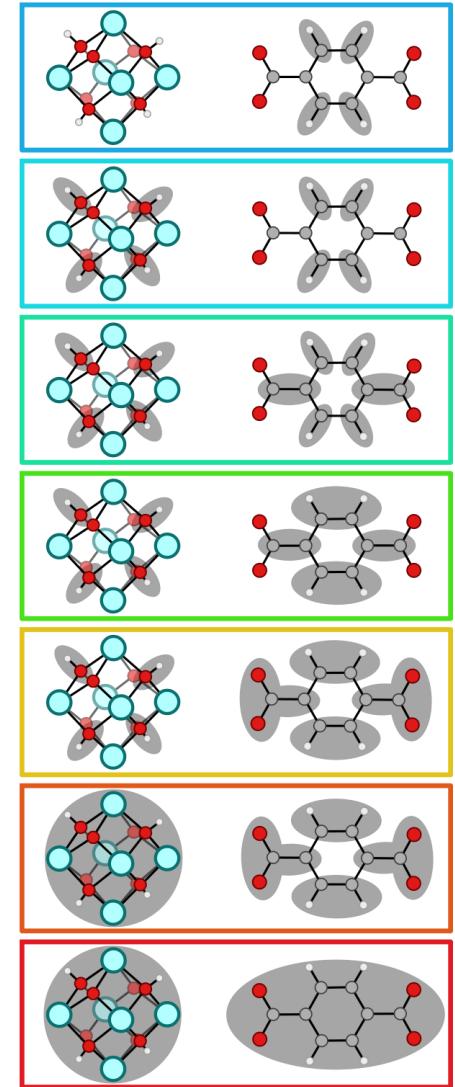
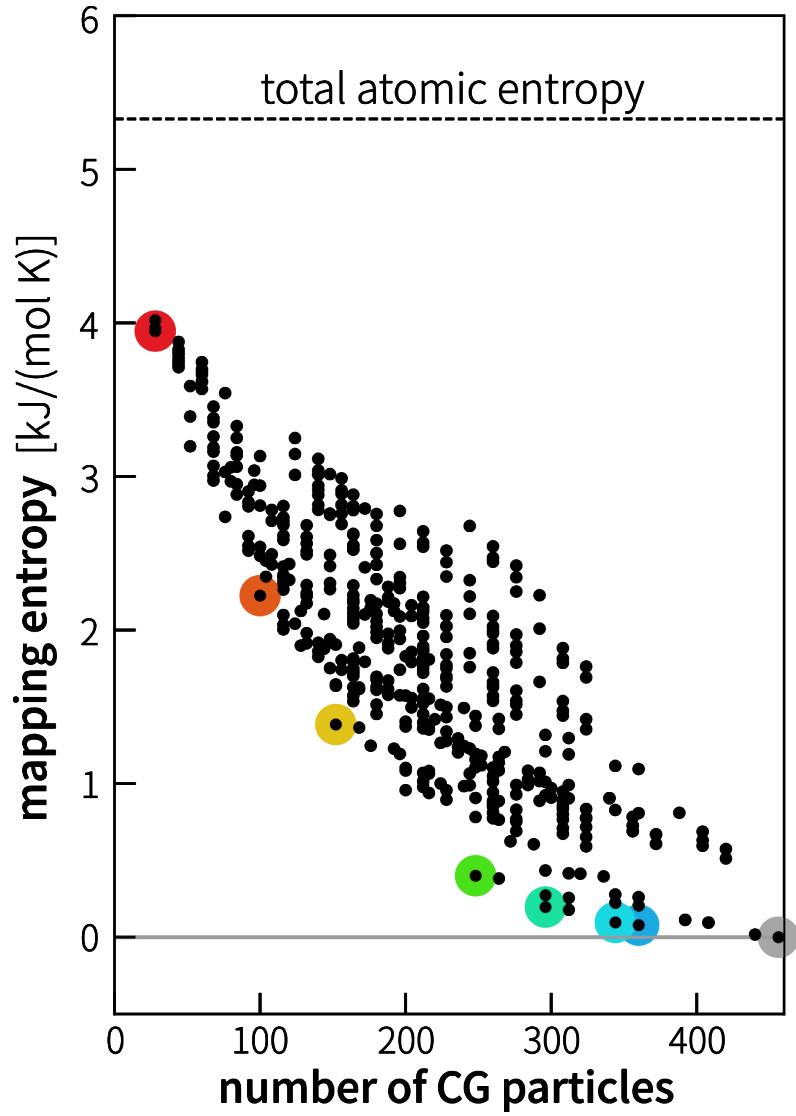
$$S_{\text{map}} = \sum_i^{3n-3N} S(\omega_i)$$

(proof based on linear algebra and stat. mech.)

UiO-66(Zr): evaluate all possible mappings



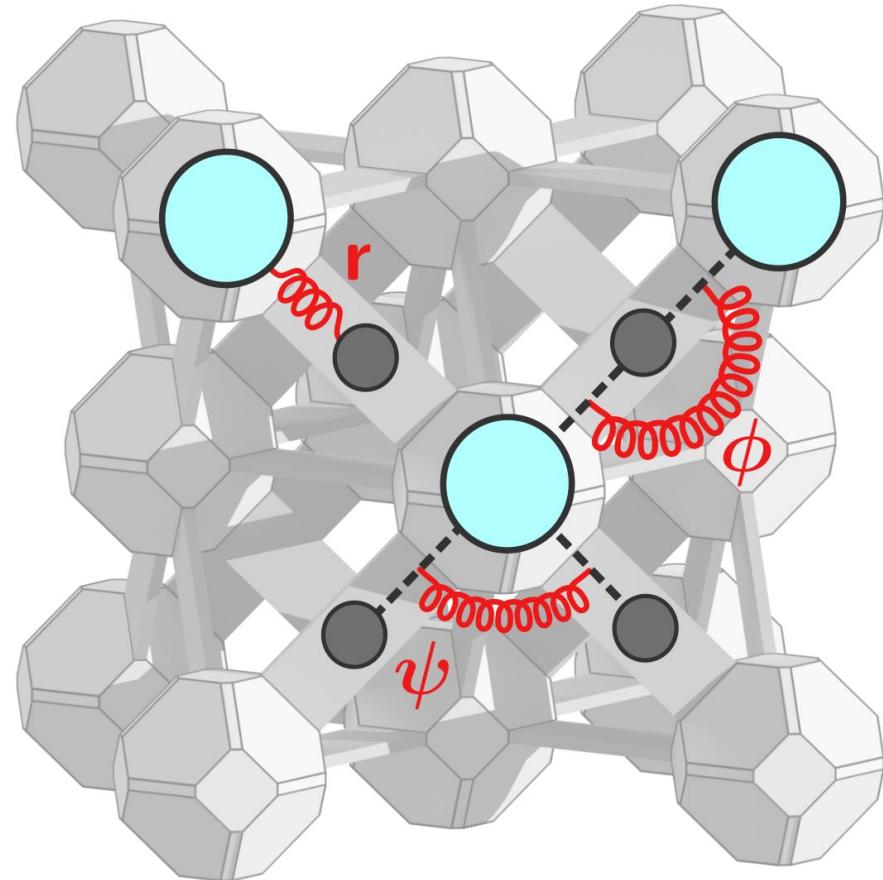
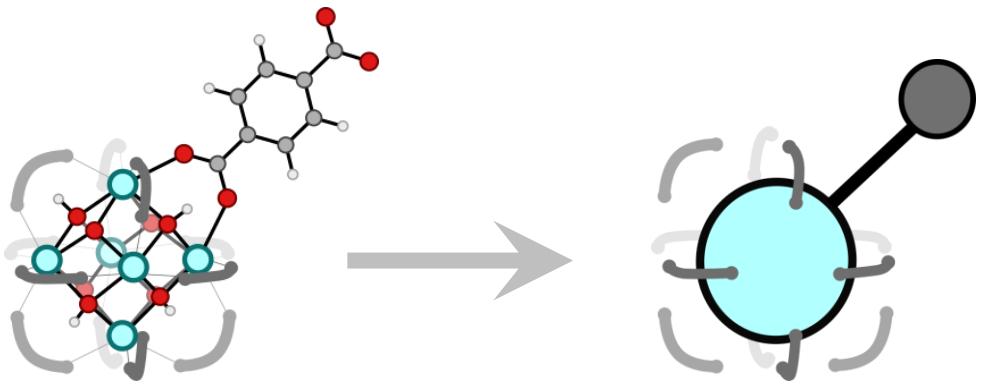
456 atoms in the unit cell;
~ 512 possible mappings!



1. partition atoms into CG beads
2. determine effective interaction potentials

A minimal model for UiO-66(Zr)

consider the minimal mapping for UiO-66(Zr):



how do we parameterize the interactions?

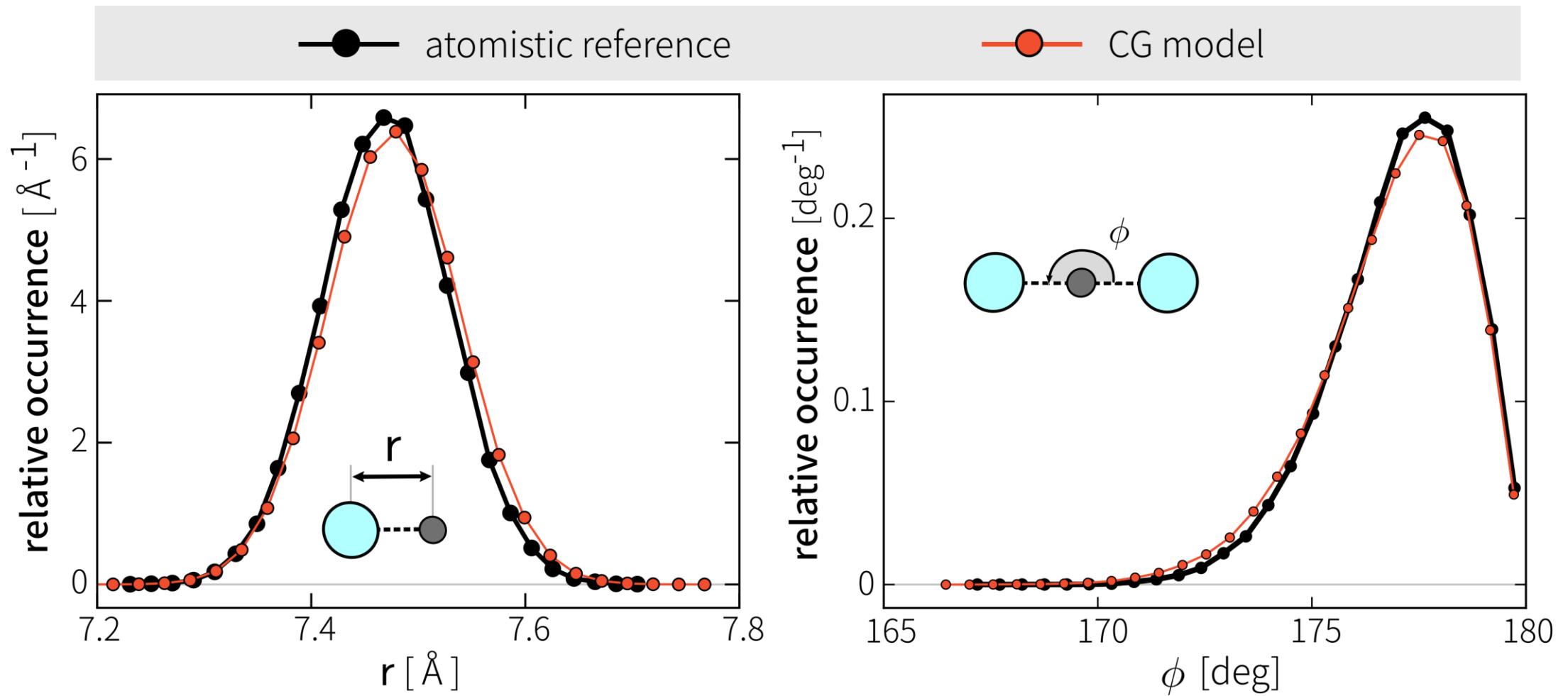
Interactions determined by the free energy surface

$$\begin{aligned} F(\mathbf{R}) &= -kT \log \int e^{-\beta U_{\text{aa}}(\mathbf{r})} \delta(\mathbf{M}(\mathbf{r}) - \mathbf{R}) d\mathbf{r} \\ &= \underline{U(\mathbf{R})} - \underline{T \cancel{S(\mathbf{R})}} \end{aligned}$$

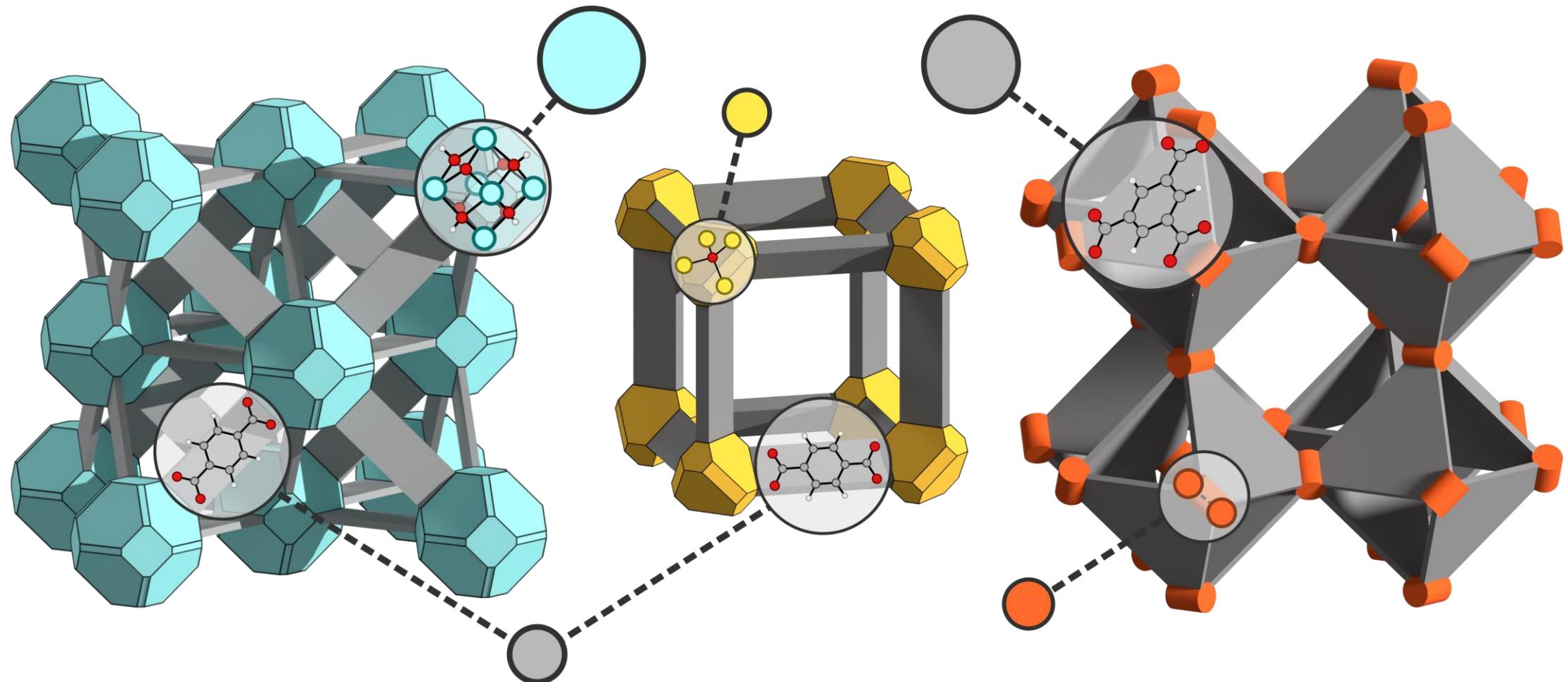
easy to approximate
using energy matching!

if the atomic interactions are
harmonic, then the entropic
contribution to $F(\mathbf{R})$ is constant!

This works quite well ...



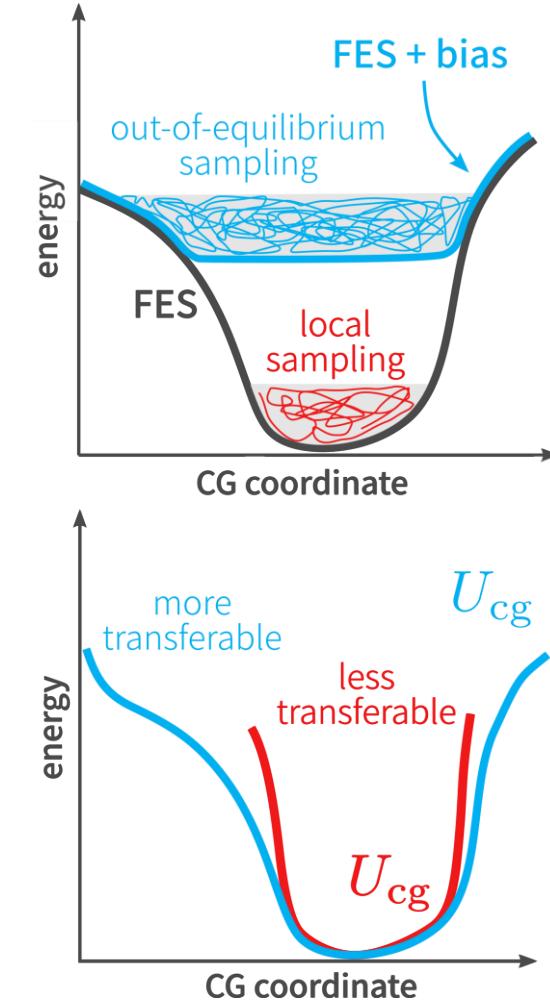
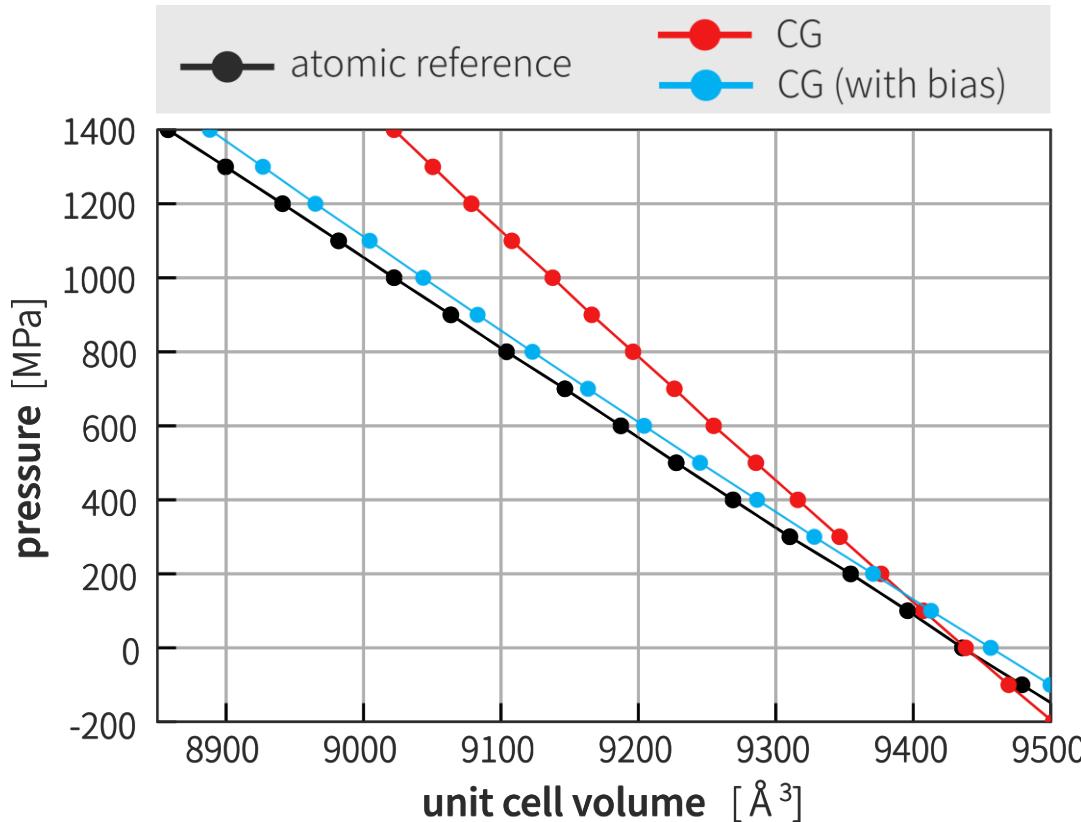
... and is generally applicable



Aye, there's the rub

are these CG models **transferable**?

yes, but we need **enhanced sampling!**



1. partition atoms into CG beads
transform the **Hessian** using the **eigendecomposition** of the mapping
2. determine effective interaction potentials
incorporate **enhanced sampling** to obtain **transferable** models

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